SOC Week-1 Operators

Lakshya Gadhwal

July 2023

1 Solutions

1.1 Part(a)

To Prove: $V \otimes F \cong V$
Proof: Consider an arbitrary element of the vector space $V \otimes F$: $v \otimes \alpha$ where $v \in V$ and $\alpha \in F$. Now define a bijection $T : V \otimes F \to V$ such that $T(v \otimes \alpha) = \alpha v$. Now since αv is always an element of V, we are done.

1.2 Part(b)

To Prove: Show that $tr \in \mathcal{L}(V,V)^\dagger$ has the Riesz representation $\langle I|$
Proof: By definition, $tr(M) = tr(\sum_i \sum_j a_{ij} A_{ij}) = \sum_k a_{kk} = \sum \langle A_{kk}|M\rangle = \langle I|M\rangle$.

To Prove: Riesz form of the trace operator when it is described as a function $tr : \mathrm{vec}(\mathcal{L}(V,V)) \to F$ is $\langle \mathrm{vec}(I)|$
Proof: This is trivial because of the isomorphism $A \otimes B \cong \mathcal{L}(V,V)$. Otherwise follow a similar procedure to the upper part.

To Prove: Reisz representation of the partial trace defined by
$tr_B(A_i \otimes B_j) \equiv tr(B_j)A_i$ is $|I\rangle\langle I|$.
Proof: From the last part we have $tr(B_j) = \langle I|B_j$. We know that $A_i = |I\rangle\langle I|A_i$. And finally from the property, $\langle v_1 \otimes v_2|w_1 \otimes w_2\rangle = \langle v_1|w_1\rangle\langle v_2|w_2\rangle$ we get, $tr_B(A_i \otimes B_j) = |I\rangle\langle I|(A \otimes B)$

To Prove: $\mathcal{L}(A \otimes B, A) \equiv \mathcal{L}(A \otimes B, A \otimes F) \equiv \mathcal{L}(A, A) \otimes \mathcal{L}(B, F)$ **Proof:** The first relation holds by Part (a) and the first and third quantities are equivalent because of the upper proof. And the final proof of mapping is trivial by the definition of the partial trace.