College of Science and Engineering
School of Physics & Astronomy

Modelling and Visualisation in Physics

SCQF Level 10, PHYS10035

Monday 30th April, 2011

14:30 – 17:30

Chairman of Examiners
Professor S Playfer

External Examiner
Professor D McMorrow

Answer all four questions overleaf.

Completed codes should be uploaded via WebCT as a single zip file after the examination has finished. If technical problems arise with the submission process, you should email the zip file to r.a.blythe@ed.ac.uk instead. Figures may also be submitted electronically provided they are described in the script book. You may use any resources available on the internet at the beginning of the examination, or present in your CPlab home directory but you may not communicate with any other person electronically or otherwise.

The bracketed numbers give an indication of the value assigned to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

A simple model of voting preferences in the run-up to the US Presidential Election has eligible voters conveniently arranged on the sites of a two-dimensional square lattice. At any time t, each voter i may hold one of two preferences $s_i(t)$: Republican ($s_i = +1$) or Democrat ($s_i = -1$). As the campaign proceeds, voter preferences may change by repeatedly iterating the following update rule: (i) a voter i is chosen at random from the entire system; (ii) a second voter j is chosen at random from the set of i's nearest neighbours; (iii) voter i's preference at time $t + \delta t$ is set to agree with that of voter j at time t.

1. A macroscopic observable that encapsulates the overall level of agreement ('order') among voters is

$$m(t) = \frac{1}{N} \sum_i s_i(t)$$

where N is the total number of lattice sites. What values does m take in states total disorder (i.e., a totally random assignment of voter preferences) and total order (all preferences the same)? **[2]**

2. An *inactive state* is a microscopic configuration that, once entered, cannot be exited via the dynamical rules of a model. A property of the voting model is that any inactive state that can be accessed from the initial condition by some sequence of updates has a nonzero probability of eventually being entered.

a. Identify all inactive states in the voting model, and for each one write down the set of initial configurations under which it is accessible. Hence, given an initially disordered system, identify the possible values that $m(t)$ may take in the $t \to \infty$ limit. **[3]**

b. Suppose there is one stubborn voter who never changes his or her preference, but may nevertheless affect neighbouring voters in the usual way. How does this affect the accessibility of the inactive states, and the possible values of $m(t)$ as $t \to \infty$? **[2]**

c. Now suppose there is more than one stubborn voter in the system. Enumerate the possible $t \to \infty$ limiting behaviours of $m(t)$, and how these depend on the preferences adopted by the stubborn voters. **[3]**

3. Write a Java program to simulate the voting model on an $L \times L$ two-dimensional square lattice with periodic boundary conditions. Your program should provide some means to change the linear dimension L, the initial amount of order $m(0)$ in the system, and the number of stubborn voters n to include at least the cases $n = 0, 1$ and 2. It should also display the state of the system in real time as it is running. **[25]**

4. Use your code to answer the following questions:

a. On a small lattice (e.g., L around 10) and for $n = 0$, run your code multiple times for different values of $m(0)$ until an inactive state is reached. Use your data to plot the probability of ending up in each of the inactive states as a function of $m(0)$. **[5]**

b. For $n = 0$ and $m(0) = 0$, plot the average time T, measured in Monte Carlo sweeps, that is needed to reach an inactive state as a function of L up to about $L = 30$. Use your data to suggest a value of the exponent χ in the relationship

$$T \propto L^{\chi} ,$$

explaining how you obtained it, and how confident you are in your answer. **[7]**

c. Run your code for the cases $n = 1$ and $n = 2$ and, by using your visualisation, describe how the system evolves over time. Comment in particular on any dependence on the preferences assigned to the stubborn voter(s). **[3]**