College of Science and Engineering
School of Physics & Astronomy

Modelling and Visualisation in Physics

SCQF Level 10, PHYS10035

Wednesday 20th April, 2011

9.30 a.m. – 12.30 p.m

Chairman of Examiners
Professor R D Kenway

External Examiner
Professor D McMorrow

Answer the question overleaf.

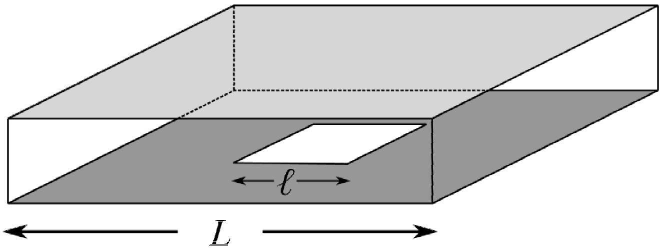
Completed codes should be uploaded via WebCT as a single zip file after the examination has finished. If technical problems arise with the submission process, you should email the zip file to r.a.blythe@ed.ac.uk instead. Figures may also be submitted electronically provided they are described in the script book.

You may use any resources available on the internet at the beginning of the examination, or present in your CPlab home directory but you may not communicate with any other person electronically or otherwise.

The bracketed numbers give an indication of the value assigned to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

1. A cuboid-shaped 'funnel' for a gas has been constructed by placing two square plates of side length L close together, one vertically above the other and both centred on the origin in the x–y plane. The gas is funnelled through the system by maintaining a constant density ρ_0 along all the vertical faces. Gas can escape through a square hole in the bottom plate. This hole has side length ℓ and is in the centre of the plate.



a. If gas particles escape at a rate α from any point vertically above the hole, explain why the density of the gas $\rho(x, y, t)$ is independent of the vertical coordinate z. If the gas further has a diffusion constant D, argue that $\rho(x, y, t)$ is governed by the partial differential equation

$$\frac{\partial \rho}{\partial t} = D\left[\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}\right] - \alpha H(x, y)\rho \qquad (1)$$

where the 'hole' function $H(x, y) = 1$ if $|x| < \ell/2$ and $|y| < \ell/2$, and zero otherwise. What are the boundary conditions on this equation? **[4]**

b. Explain why we may set $\rho_0 = 1$, $L = 1$ and $D = 1$ with no loss of generality. **[2]**

c. An approximate form of the solution $u(x, y)$ of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

can be found at points (i, j) on a square lattice with spacing δ by iterating the equations

$$u_{ij} = \frac{1}{4}\left[u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - f_{i,j}\delta^2\right] .$$

Use this fact to write down an iterative algorithm for finding the steady-state density profile of the gas in the funnel from Equation (1). Comment on any special cases that arise at the exterior faces of the funnel, or at the edge of the hole. **[4]**

d. Write a Java code that allows you to find the steady-state density profile of the gas in the funnel for arbitrary values of the hole size ℓ, escape rate α and lattice spacing δ. **[25]**

Use your code to answer the remaining questions. For these questions, the graphs and relevant screenshots may be submitted electronically with your code, but you should also sketch and comment on them in the exam script, noting special features and parameter values.

e. Plot the steady-state profile as a function of x along the line $y = 0$ for $\ell = \frac{1}{5}$, $\alpha = 1$ and of order 50 grid points per unit length, demonstrating that further iterations of the algorithm do not visibly change the plot you produce. Explain the general form of the profile obtained in physical terms. **[4]**

f. Determine any changes to the solution that occur as the spatial resolution δ is varied whilst $\ell = \frac{1}{5}$ and $\alpha = 1$ remain fixed. Estimate the number of grid points that is needed for the solution to become largely independent of the grid spacing. **[4]**

g. Write down an expression for the total steady-state flux of gas through the funnel in terms of the numerically-determined densities ρ_{ij}. Plot this quantity as a function of α for a fixed combination of ℓ and δ of your choice. **[4]**

h. Whilst in *The Magnet* cafe, you overhear a mathematical physicist remark that "when α is large, the gas flux should increase with α as $\ln(\alpha)$". Can you confirm or exclude this possibility using your numerical data? Why (not)? **[3]**