School of Physics & Astronomy

Modelling and Visualisation in Physics

PHYS10035 (SCQF Level 10)

Wednesday 3rd May, 2017 09:30 – 12:30
(May Diet)

Please read full instructions before commencing writing.

Examination Paper Information

Answer all questions overleaf.
Completed codes should be uploaded via Learn as a single zip file immediately after the examination has finished. If technical problems arise with the submission process, you can email the zip file to dmarendu@ph.ed.ac.uk.
Figures should be submitted electronically, they should also be described in the script book.
You may use any resources available on the internet at the beginning of the examination, or present in your CPlab home directory but you may not communicate with any other person electronically or otherwise.

Special Instructions

- Only authorised Electronic Calculators may be used during this examination.
- A sheet of physical constants is supplied for use in this examination.
- Attach supplied anonymous bar codes to *each* script book.

Special Items

- School supplied Constant Sheets
- School supplied barcodes

Chairman of Examiners: Prof S Playfer
External Examiner: Prof S Clark

A simple model for consensus spreading can be formulated in terms of a cellular automaton, defined on a two-dimensional square $L \times L$ lattice with periodic boundary conditions, as follows. Each cell in the lattice represents an individual (or agent), which can be in one of three states: "green" (G), "red" (R) or "blue" (B), corresponding for instance to three different opinions.

The update rule defining the cellular automaton works as follows. First, a cell (lattice site) is chosen randomly: we denote this cell as i and its corresponding state as S_i (i.e., S_i =R, G or B). Then, we randomly select one of the four lattice nearest neighbours of i: we denote this cell as j, and the corresponding state as S_j. Finally, cell i "converts" cell j to its state, S_i, with probability $p(S_i, S_j)$, to model spreading of opinions through directional interactions. Assume the conversion matrix $p(S_i, S_j)$ is determined by only two parameters, p_1 and p_2, and that

$$\begin{aligned} p(R,G) &= p(G,B) = P(B,R) = p_1, \\ p(G,R) &= p(B,G) = P(R,B) = p_2. \end{aligned}$$

Note that the values of $p(R,R)$, $p(G,G)$ and $p(B,B)$ are irrelevant as the state of cell j will remain invariant whatever the choice of these probabilities (for simplicity you can set these to zero in the code).

We here wish to simulate the dynamics and steady state of this model. Some useful quantities which we refer to below are the fraction of "red", "green" and "blue" sites, which we call f_R, f_G and f_B respectively. The quantity f_R, for a given configuration, is simply defined as the number of red cells divided by the total number of cells in the lattice (i.e., L^2); f_G and f_B are defined in a similar way. We will also consider the "majority" fraction, f_M, which, for a given configuration, is the largest of f_R, f_G and f_B.

a. Write a Java program to simulate the dynamics of this opinion spreading model. You should take $L = 50$ (i.e., a square 50×50 lattice). Your code should allow you to set the value of p_1 and p_2 as arguments, and it should also display the state of the system in real time as it is running. To initialise the system you can start with a random state with equal probability of having a state equal to R, G or B at each of the lattice points. **[25]**

b. Let us denote a state where R, G, and B states are all present in an equal or similar fractions as a "mixed state". For concreteness, we shall define a configuration to be in a mixed state if all three of f_R, f_G and f_B are above 0.2. Your initial state is therefore a "mixed state". On the other hand, let us denote a configuration where cells are either all R, all G or all B as a "coherent state". Set $p_1 = p_2 = 1$, and describe what happens in the model. Show a typical plot of f_R, f_B, and f_G as a function of time. Characterise each of the coherent and mixed states as either metastable or absorbing states, explaining why this is the case. **[5]**

c. Let us now set $p_1 = 1$, and vary p_2. Use your code to compute the probability that a simulation *reaches* an absorbing state within 5000 sweeps as a function of p_2. [Recall that a sweep equals L^2 attempted conversion moves.] To compute this probability, you should run at least 10 simulations for each p_2, and simply count the number of times your simulation leads to absorption. You should vary p_2 from 0.5 to 1, in steps of 0.05. Plot the graph corresponding to the absorption probability as a function of p_2. Explain the physical reason for the qualitative behaviour of this plot. **[8]**

d. Modify your code so that, in addition to the nearest-neighbour directed conversion, there is another way to change a cell's state, via random conversion. We model this as follows. At each update, after selecting a cell randomly, we perform either a directed conversion, with probability $1-p_r$, or a random conversion, with probability p_r. We model random conversion of a cell i by changing its state to either R, G or B with equal probability (i.e., the state may remain the same with probability 1/3). By fixing $p_r = 10^{-5}$, and by varying p_2 again from 0.5 to 1 (again in steps of 0.05), you should measure the fraction of time the system stays in a mixed state. As in b., we shall say that a given configuration is in a mixed state if the corresponding values of f_R, f_G and f_B are all above 0.2. Plot the graph corresponding to the fraction of time spent in the mixed state as a function of p_2. [To measure the fraction of time in the mixed state, you can perform a long simulation, of more than 10000 sweeps, and ideally 100000 sweeps, for each value of p_2.] **[7]**

e. Run your code for $p_2 = 0.7$ for between 100000 and 1000000 sweeps (keeping $p_1 = 1$), and calculate the probability $P(f_M)$ that a configuration has a certain majority fraction f_M. [Normalisation of the probability is not required.] To do this, you will need to bin the values of f_M, and to calculate the number of configurations associated with each bin. Show a plot of $P(f_M)$, either as a continuous curve or as histograms. **[5]**