College of Science and Engineering
School of Physics and Astronomy

Modelling and Visualisation in Physics

SCQF Level 10, U01414, PHY-4-ModVis

Friday 24th April, 2009

9.30 a.m. – 12.30 p.m.

Chairman of Examiners
Professor R D Kenway

External Examiner
Professor M Green

Answer the question overleaf.

Completed codes should be uploaded via WebCT as a single zip file after the examination has finished. If technical problems arise with the submission process, you should email the zip fie to r.a.blythe@ed.ac.uk instead. Electronic figures may also be submitted electronically provided they are described in the script book. You may use any resources available on the internet at the beginning of the examination, or present in your CPlab home directory but you may not communicate with any other person electronically or otherwise.

The bracketed numbers give an indication of the value assigned to each portion of a question.

Only the supplied Electronic Calculators may be used during this examination.

A model for traffic flow down a one-way street running from west to east is defined as follows. The road is represented as a one-dimensional lattice of L sites. Each of these sites may either be empty, or occupied by a car, so that at any given time there are $M \le L$ cars in total. In each timestep, every car attempts to move into the site immediately to its east with probability p. Each of these moves is a statistically independent event. We assume that drivers do not wish to collide with each other, and hence that no two cars may occupy the same site simultaneously. We further assume open boundary conditions: in timesteps where the west-most site is empty, a car appears in it with probability α; meanwhile, a car that occupies the east-most site has a probability β per timestep of exiting the system.

(a) Draw a sketch of these dynamical rules. Write down an algorithm for performing a single update of the lattice from one timestep to the next by means of a *fully parallel updating scheme.* **[5]**

(b) Describe the configuration of the lattice that will be reached as time $t \to \infty$ for the following parameter combinations: (i) $p > 0$, $\alpha = 0$ and $\beta > 0$; (ii) $p > 0$, $\alpha > 0$ and $\beta = 0$. **[2]**

(c) Consider the initial condition that has all even-numbered sites occupied by a car and all odd-numbered sites empty. What is the subsequent evolution of this system when $\alpha = \beta = p = 1$? **[3]**

(d) Write a Java code that simulates the process for general α, β and p and that presents a graphical display of the system's state as it evolves. Write a brief note in your script to explain how to run the simulation. **[25]**

Use your code to answer the remaining questions. For these questions, the graphs and relevant screenshots may be submitted electronically with your code, but you should also sketch and comment on them in the exam script, noting special features and parameter values.

(e) With $p = 0.8$, $\alpha = 0.3$, and a lattice comprising a few hundred sites, plot the steady state values of mean density $\rho = \langle \frac{M}{L} \rangle$, and the density fluctuations $(\Delta\rho)^2 = \langle (\frac{M}{L})^2 \rangle - \rho^2$ as a function of β, $0 < \beta < 1$. Use these measurements to argue for the existence of a phase transition at a critical value of $\beta = \beta_c$, describe its physical nature and provide an estimate of β_c (with appropriate errors). Note and explain the features of the graph that allow you to draw this conclusion, and how these are affected when the system size L is small. **[7]**

(f) Show that at $p = 0.8$ and the increased value of $\alpha = 0.7$ this phase transition appears to be absent. Estimate, with appropriate errors, the value of α at which the transition vanishes. **[5]**

(g) Describe the typical configurations seen at $p = 0.8$, $\alpha = \beta = 0.1$ and how these configurations evolve over time. **[3]**