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CS 161 HW #7

1\. (a) Generalized product rule: $Pr(A, B \mid K) = Pr(A \mid B, K)\, Pr(B \mid K)$

☆ $Pr(\alpha \mid \beta) = \frac{Pr(\alpha \wedge \beta)}{Pr(\beta)}$ ☆ $\wedge \equiv ,$

LHS: $Pr(A, B \mid K) = \frac{Pr(A, B \wedge K)}{Pr(K)} = \frac{Pr(A, B, K)}{Pr(K)}$ ✓

RHS: $Pr(A \mid B, K)\, Pr(B \mid K) = \frac{Pr(A \wedge B, K)}{Pr(B, K)} \cdot \frac{Pr(B \wedge K)}{Pr(K)}$

$= \frac{Pr(A, B, K)}{\cancel{Pr(B, K)}} \cdot \frac{\cancel{Pr(B, K)}}{Pr(K)} = \frac{Pr(A, B, K)}{Pr(K)}$ ✓

$\frac{Pr(A, B, K)}{Pr(K)} = \frac{Pr(A, B, K)}{Pr(K)} \Rightarrow LHS = RHS$ ✓

(b) Generalized Bayes' rule: $Pr(A \mid B, K) = Pr(B \mid A, K)\, Pr(A \mid K) / Pr(B \mid K)$

LHS: $Pr(A \mid B, K) = \frac{Pr(A \wedge B \wedge K)}{Pr(B \wedge K)} = \frac{Pr(A, B, K)}{Pr(B, K)}$ ✓

RHS: $\frac{Pr(B \mid A, K)\, Pr(A \mid K)}{Pr(B \mid K)} = \frac{\frac{Pr(B \wedge A \wedge K)}{\cancel{Pr(A \wedge K)}} \cdot \frac{\cancel{Pr(A \wedge K)}}{\cancel{Pr(K)}}}{\frac{Pr(B \wedge K)}{\cancel{Pr(K)}}}$

order doesn't matter for multiplying

$= \frac{Pr(B \wedge A \wedge K)}{Pr(B \wedge K)} = \frac{Pr(A, B, K)}{Pr(B, K)}$ ✓

$\frac{Pr(A, B, K)}{Pr(B, K)} = \frac{Pr(A, B, K)}{Pr(B, K)} \Rightarrow LHS = RHS$ ✓

2\. $Pr(oil) = 0.5$ $Pr(\neg oil) = 0.5$
$Pr(\text{only natural gas}) = 0.2$ $Pr(\neg \text{only natural gas}) = 0.8$
$Pr(\text{neither}) = 0.3$ $Pr(\neg \text{neither}) = 0.7$

if oil is present, $Pr(pos) = 0.9$
if only gas is present, $Pr(pos) = 0.3$
if neither are present, $Pr(pos) = 0.1$

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If test is positive, Pr(oil) = ?

Pr (pos | oil) = ?

$$\frac{Pr(pos \wedge oil)}{Pr(oil)}$$

oil	gas	
T	T	$\left.\right\}$ 0.5
T	F	
F	T	} 0.2
F	F	} 0.3

★ Probability of either oil or gas or both:

$Pr(E) = 0.5 + 0.2 = 0.7$

★ Probability of neither oil nor gas

$Pr(\neg E) = 1 - 0.7 = 0.3$

$Pr(pos \mid oil) = 0.9$

$Pr(pos \mid neither) = 0.1$

$\Downarrow$

$Pr(neg \mid oil) = 1 - 0.9 = 0.1$

$Pr(neg \mid both) = 1 - 0.1 \quad 0.9$

$Pr(oil \mid pos) = \boxed{\frac{Pr(oil, pos)}{Pr(pos)}}$ A B

★ Bayes rule $= \frac{Pr(pos \mid oil)\, Pr(oil)}{Pr(pos)} = \frac{0.45}{0.54} = 0.83$

A $Pr(pos \mid oil)\, Pr(oil) = 0.9 \times 0.5$

B $Pr(pos) = Pr(pos, oil)\, Pr(oil) + Pr(pos, \text{only natural gas})\, Pr(\text{only natural gas})$
$+ Pr(pos, neither)\, Pr(neither)$

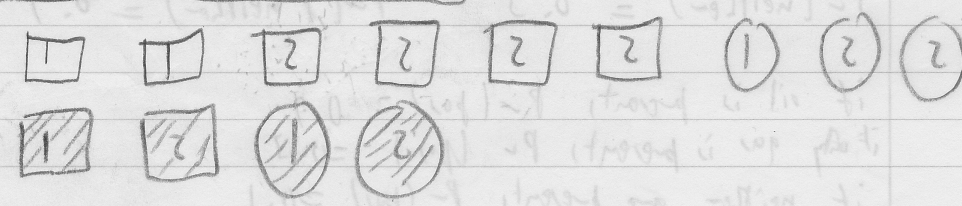
$Pr(pos) = 0.9 \times 0.5 + 0.3 \times 0.2 + 0.1 \times 0.3$
$= 0.45 + 0.06 + 0.03 = 0.54$

$$\boxed{Pr(oil \mid pos) = 0.83}$$

3.

yellow →

blue →



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α_1: the object is yellow;

α_2: the object is square;

α_3: if the object is one or yellow, then it is also square.

(1) Construct the joint probability distribution of problem

(2) Compute $\alpha_1, \alpha_2, \alpha_3$ by explicitly identifying the worlds at which each α_i holds

(1)

color	shape	Number	Pr
Y	S	1	2/13
Y	S	2	4/13
Y	C	1	1/13
Y	C	2	2/13
B	S	1	1/13
B	S	2	1/13
B	C	1	1/13
B	C	2	1/13

(2) α_1 $Pr(\text{yellow}) = 9/13$

α_2 $Pr(\text{square}) = 8/13$

α_3 $Pr(\text{square} \mid \text{one} \vee \text{yellow}) = \dfrac{Pr(\text{square}, \text{one} \vee \text{yellow})}{Pr(\text{one} \vee \text{yellow})}$

$$\alpha_3 = \frac{\frac{2}{13} + \frac{4}{13} + \frac{1}{13}}{\frac{9}{13} + \frac{2}{13}} = \frac{\frac{7}{13}}{\frac{11}{13}} = \frac{7}{\cancel{13}} \cdot \frac{\cancel{13}}{11} = \frac{7}{11}$$

☆ $Pr(\alpha \mid \gamma) = Pr(\alpha \mid \gamma, \beta) \equiv \alpha$ is independent of β given γ

1. $Pr(\alpha = \text{square} \mid Y = \text{not blue}) = Pr(\alpha = \text{square} \mid Y = \text{not blue}, B = 1)$

$\dfrac{Pr(\text{square}, \text{not blue})}{Pr(\text{not blue})} = \dfrac{6/\cancel{13}}{9/\cancel{13}} = \dfrac{2}{3}$ ✓

$\dfrac{Pr(\text{square}, \text{not blue}, 1)}{Pr(\text{not blue}, 1)} = \dfrac{2/\cancel{13}}{3/\cancel{13}} = \dfrac{2}{3}$ ✓

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If Y = not blue, α = square, $\beta = 1$, then α is independent of β given Y ✓

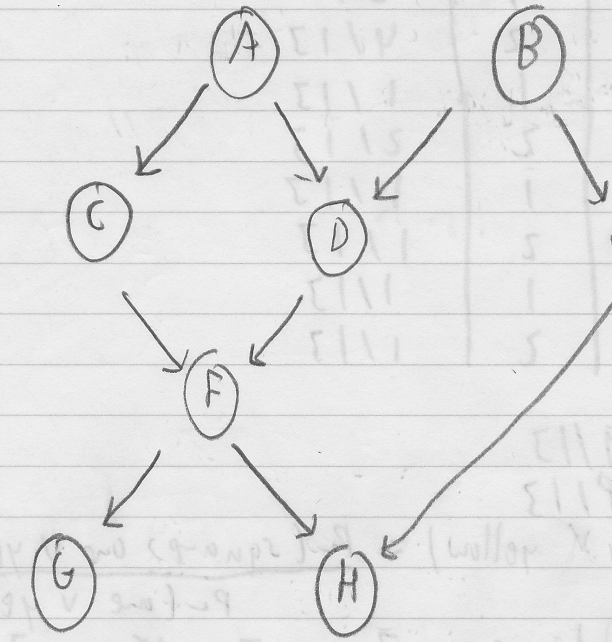
2. $Pr(\alpha = 1 \mid Y = \text{blue}) = Pr(\alpha = 1 \mid Y = \text{blue}, \beta = \text{circle})$

$\dfrac{Pr(1, \text{blue})}{Pr(\text{blue})} = \dfrac{1}{2} \dfrac{\cancel{2/13}}{\cancel{4/13}} = \dfrac{1}{2}$ ✓

$\dfrac{Pr(1, \text{blue}, \text{circle})}{Pr(\text{blue}, \text{circle})} = \dfrac{1/\cancel{13}}{2/\cancel{13}} = \dfrac{1}{2}$ ✓

If Y = blue, $\alpha = 1$, β = circle, then α is independent of β given Y ✓

4.



(a) ☆ Markovian assumption: $V \perp\!\!\!\perp$ non-descendant of V | all the parents of V

☆ Markovian assumption: $I(V, \text{Parents}(V), \text{Non-descendants}(V))$

$I(A, \{\}, \{B, E\})$

$I(B, \{\}, \{A, C\})$

$I(C, A, \{B, D, E\})$

$I(D, \{A, B\}, \{C, E\})$

$I(E, B, \{A, C, D, F, G\})$

$I(F, \{C, D\}, \{A, B, E\})$

$I(G, F, \{A, B, C, D, E, H\})$

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$I(H, \{E, F\}, \{A, B, C, D, G\})$

(b) · d-separated (A, BH, E)

no, F is a descendant of A and F & E converge on H, so A & E are not independent.

· d-separated (G, D, E)

yes, G & E are separated & blocked by D because there is a common divergent path through B

· d-separated (AB, F, GH)

no, F blocks G from A since F is divergent. However, B can still connect to H through the path $B \to E \to H$, so they are dependent.

(c) $Pr(a, b, c, d, e, f, g, h) = Pr(a \mid b, c, d, e, f, g, h)$
$\cdot Pr(b \mid c, d, e, f, g, h)$
$\cdot Pr(c \mid d, e, f, g, h)$
$\cdot Pr(d \mid e, f, g, h)$
$\cdot Pr(e \mid f, g, h)$
$\cdot Pr(f \mid g, h)$
$\cdot Pr(g \mid h)$
$\cdot Pr(h)$

☆ Chain rule

$$Pr(\alpha_1 \wedge \underbrace{\alpha_2 \wedge \ldots \wedge \alpha_n}_{B}) = Pr(\alpha_1 \mid \alpha_2 \ldots \alpha_n) \; Pr(\alpha_2 \mid \alpha_3 \ldots \alpha_n) \; \vdots \; Pr(\alpha_n)$$

(d) $Pr(A = 0, B = 0)$ and $Pr(E = 1 \mid A = 1)$

$Pr(A=0)$	$Pr(A=1)$
.8	.2

$Pr(B=0)$	$Pr(B=1)$
.3	.7

	$Pr(E=0 \mid B)$	$Pr(E=1 \mid B)$
$B=0$	.1	.9
$B=1$	.9	.1

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	$Pr(D=0 \mid A,B)$	$Pr(D=1 \mid A,B)$
$A=0, B=0$	.2	.8
$A=0, B=1$	.9	.1
$A=1, B=0$	.4	.6
$A=1, B=1$	.5	.5

$$Pr(A=0, B=0) = 0.8 \times 0.3 = \boxed{0.24}$$

Justification: A and B are independent

$$Pr(E=1 \mid A=1) = \frac{Pr(E=1 \wedge A=1)}{Pr(A=1)}$$

$$= \frac{\cancel{Pr(A=1)} \cdot Pr(E=1)}{\cancel{Pr(A=1)}} = Pr(E=1)$$

$$Pr(E=1, B=0) + Pr(E=1, B=1)$$
$$= Pr(E=1 \mid B=0) \cdot Pr(B=0) + Pr(E=1 \mid B=1) \cdot Pr(B=1)$$
$$= 0.9 \times 0.3 + 0.1 \times 0.7$$
$$= 0.27 + 0.07 = \boxed{0.34}$$

Justification: A and E are independent, so $Pr(E=1 \mid A=1) = Pr(E)$