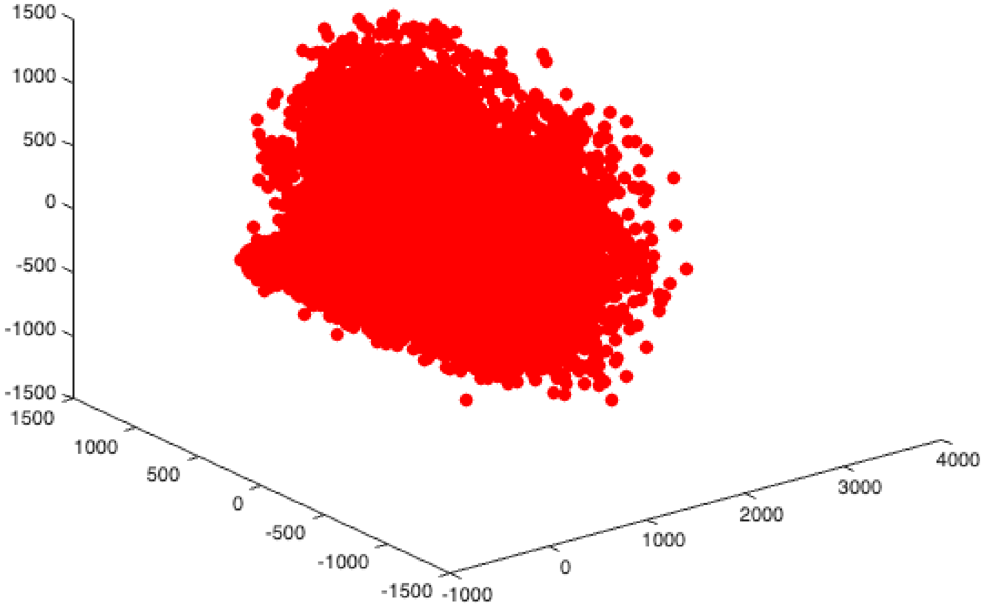
```
clear all
clc
warning("off")
load ('mnist.mat')
colormap(gray(256))
images = rescale(test.images(:,:,:),0,255);
data = reshape(images,784,10000);


inputs = data';

[coeff,~,~,~,explained,~] = pca(inputs);
Z=inputs*coeff(:,1:3);

figure;
%plot(Z(:,1),Z(:,2),'r.','MarkerSize',10)
view(3);
plot3(Z(:,1),Z(:,2),Z(:,3),'r.','MarkerSize',20)
```



```
% generate synaptic weights
SetRNG(111);
n_src = 784;
n_dst = 500;
n_per_src = 20;
synaptic_weights_mat = randn(n_src,n_dst);
```

```
[srcIdx,dstIdx] = ConnectHypergeometric(n_dst, n_src, n_per_src);
index = [srcIdx;dstIdx];
for i = 1:n_dst;
    nonzero_idx = index(2,find(index(1,:) == i));
    zero_idx = setdiff(1:n_src,nonzero_idx);
    synaptic_weights_mat(zero_idx,i) = 0;
end
cells = synaptic_weights_mat; %original
```

```
% cycle 1, find the cluster winners
ori_cycle1_cells = cells;
cycle1_cells = normc(cells);
cycle1_cells_iter = normc(cells);

% for k = 1:10;
%     cycle1_winners = [];
%     sampled_data = data(:, randperm(size(data, 2)));
%     winner_len_mat = [];
%     winner_average = zeros(784,1,8000);
%     for col = 1:size(sampled_data,2); % loop over all inputs
%         lr = 0.001;
%         input1 = sampled_data(:,col);% each input
%         len_input = norm(input1);
%         input1 = normc(input1);
%         winner_per_cycle = [];
%         product = input1'*cycle1_cells; % the dot products of the input
and all cells
%         winning_value = max(product); % max dot product value
%         winning_idx = find(product == winning_value); % index(indices) of
winning cell(s)
%         winning_cell = cycle1_cells(:,winning_idx); % the winning cell
set, which may contain more than one winning cell
%         % loop over each winning cell in the set, in case of tied winners,
%         % which may not happen
%         updated_winningset = []; % in case of tied winners
%         for cell_idx = winning_idx;
%             winner = cycle1_cells_iter(:,cell_idx);
%             update_winner_ori = winner+(input1-winner)*lr;
%             winner_average(:,:,cell_idx) = update_winner_ori +
winner_average(:,:,cell_idx);
%
%             len_update_winner = norm(update_winner_ori);
%             update_winner = update_winner_ori/len_update_winner;
%             updated_winningset = [updated_winningset;update_winner];
%             winner_len_mat = [winner_len_mat;
[cell_idx,len_input]];
%             cycle1_cells(:,cell_idx) = update_winner;
%             cycle1_cells_iter(:,cell_idx) = update_winner_ori;
%             %ori_cycle1_cells(:,cell_idx) = update_winner*len_input;
%         end
%         cycle1_winners = [cycle1_winners;winning_idx];
%     end
% end
%
% [~,~,ix] = unique(cycle1_winners,"stable");
% winner_stats = [unique(cycle1_winners,"stable"),accumarray(ix,1)];
%
% for i = 1:size(winner_stats,1);
% winner_average(:,:,winner_stats(i,1)) =
winner_average(:,:,winner_stats(i,1))./winner_stats(i,2);
% cycle1_cells(:,winner_stats(i,1)) =
winner_average(:,:,winner_stats(i,1));
% end
%
%
% cycle1_winner=cycle1_winners;
% cycle1_winners = unique(cycle1_winners,"stable");
% [~,idx] = max(normr(inputs)*cycle1_cells(:,cycle1_winners),[],2);
% cycle1_winners = cycle1_winners(unique(idx,"stable"));
% cycle1_winners
%
% for c = cycle1_winners';
%     each_input = winner_len_mat(winner_len_mat(:,1)== c,2);
%     ori_cycle1_cells(:,c) = cycle1_cells(:,c)*mean(each_input);
% end
% cycle1_winner_demask = ori_cycle1_cells(:,cycle1_winners);
%
%
% % after cycle 1 plots
% final_all_data = [inputs;cells(:,cycle1_winners)'];
% [coeff,~,~,~,explained,~] = pca(final_all_data);
% Za1=final_all_data*coeff(:,1:3);
% subplot(2,2,2)
% hold on
% view(3)
%
plot3(Za1(1:length(data),1),Za1(1:length(data),2),Za1(1:length(data),3),'r.'
,'MarkerSize',10)
%
plot3(Za1(end,1),Za1(end,2),Za1(end,3),'bp','MarkerFaceColor','blue','Marker
Size',50)
% title("b) after cycle one")
% legend('input','cycle one winners','Location','NW')
% grid on
% hold off
```

```
%cycle 2, find the sub-cluster winners
% determine the clutser
```

```
ori_cycle2_cells = ori_cycle1_cells;
cycle2_cells = cycle1_cells;
cycle2_cells_iter = cycle1_cells;
%ori_cycle2_cells(:,cycle1_winners) = 0;
%cycle2_cells(:,cycle1_winners) = 0;
%cycle2_cells_iter(:,cycle1_winners) = 0;

cycle2_cells_copy = cycle2_cells;
cycle2_winner_mat = [];
this_winner_mat = [];


lr = 0.001;
%this_idx = c;
sampled_data_fix = data; %select each cluster data
%this_winner = ori_cycle1_cells(:,cycle1_winners(1)); % the cell that
"this" cluster wins in cycle 1
this_winner = mean(inputs)';
this_winner_mat = [this_winner_mat;this_winner';this_winner'];

%pull all inputs of this clutser to its center
sampled_data_fix = sampled_data_fix-this_winner;
assign_id = test.labels;

for k = 1:60; % each cluster learns 60 rounds
    winner_len_mat = [];
    cycle2_winners = [];
    sampled_data = sampled_data_fix(:, randperm(size(sampled_data_fix, 2)));
    winner_average = zeros(784,1,n_dst);
    for col = 1:size(sampled_data,2); % loop over all inputs
        input1 = sampled_data(:,col);
        len_input = norm(input1);
        input1 = normc(input1);
        product = input1'*cycle2_cells; % the  dot products of the input
and all cells
        winning_value = max(product); % max dot product value
        winning_idx = find(product == winning_value);
        winning_cell = cycle2_cells_iter(:,winning_idx);
        cycle2_winners = [cycle2_winners;winning_idx];
        winner_len_mat = [winner_len_mat;[winning_idx,len_input]];
        update_winner = winning_cell+(input1-winning_cell)*lr;
        winner_average(:,:,winning_idx) = update_winner +
winner_average(:,:,winning_idx);
        cycle2_cells_iter(:,winning_idx) = update_winner;
        update_winner = normc(update_winner);
        cycle2_cells(:,winning_idx) = update_winner;
    end

end
```

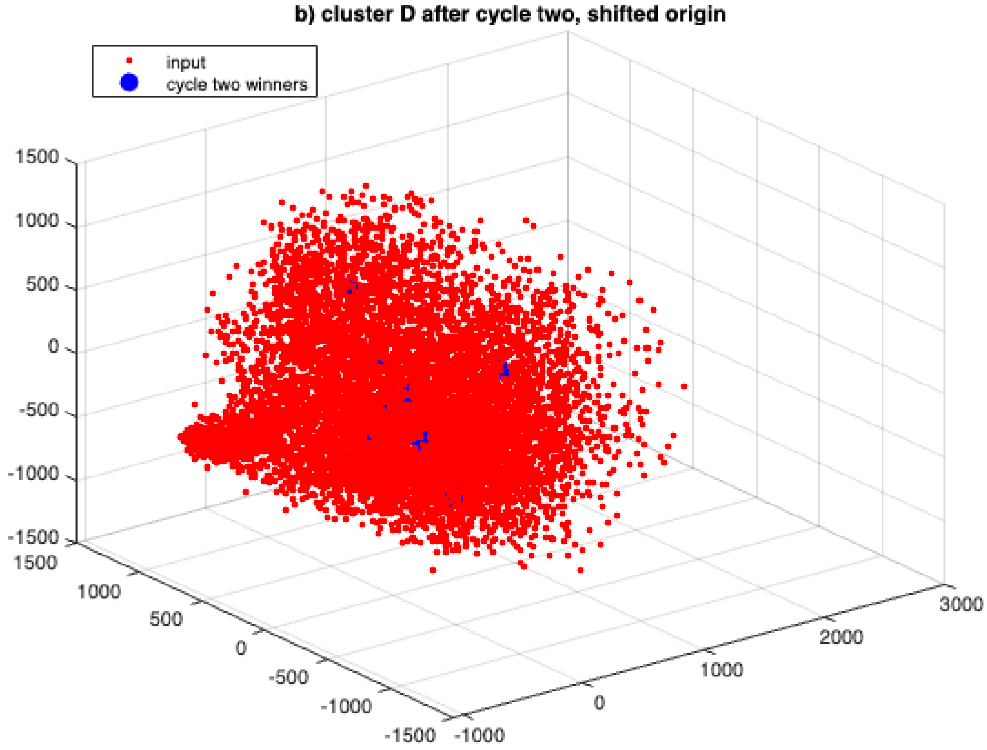
```
[~,~,ix] = unique(cycle2_winners,"stable");
winner_stats = [unique(cycle2_winners,"stable"),accumarray(ix,1)];
for i = 1:size(winner_stats,1);
winner_average(:,:,winner_stats(i,1)) =
winner_average(:,:,winner_stats(i,1))./winner_stats(i,2);
cycle2_cells_iter(:,winner_stats(i,1)) =
winner_average(:,:,winner_stats(i,1));
end

cycle2_winners = unique(cycle2_winners);
for c = cycle2_winners';
    each_input = winner_len_mat(winner_len_mat(:,1)== c,2);
    ori_cycle2_cells(:,c) = cycle2_cells_iter(:,c)*mean(each_input);
end



final_all_data = [sampled_data';ori_cycle2_cells(:,cycle2_winners)'];
[coeff,~,~,~,explained,~] = pca(final_all_data);
Za2=final_all_data*coeff(:,1:3);
explained = round(explained);

figure;
hold on
view(3)
plot3(Za2(1:10000,1),Za2(1:10000,2),Za2(1:10000,3),'r.','MarkerSize',10)
plot3(Za2(10001:end,1),Za2(10001:end,2),Za2(10001:end,3),'b.','MarkerSize',3
0)
title("b) cluster D after cycle two, shifted origin")
legend('input','cycle two winners','Location','NW')
grid on
hold off
```



```
[~,idx] = max(normc(sampled_data_fix)'*cycle2_cells(:,cycle2_winners),[],2);

[GC,GR] = groupcounts(idx); % stats of all categories
cluster_distribution = [GR,GC];
T = array2table(cluster_distribution)
```

T = 24×2 table

	cluster_distribution1	cluster_distribution2
1	1	1007
2	2	1
3	3	1
4	4	626
5	5	1202
6	6	1
7	7	1
8	8	769
9	9	1
10	10	2

	cluster_distribution1	cluster_distribution2
11	11	1274
12	12	824
13	13	629
14	14	791

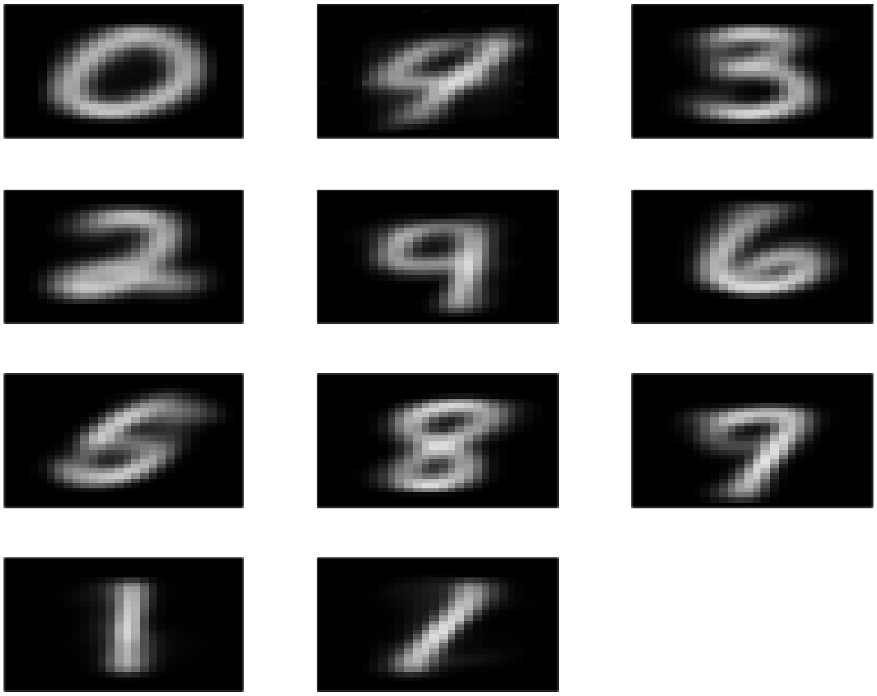
⋮

```
cluster_distribution =
cluster_distribution(find(cluster_distribution(:,2)>=10),:);
cluster_distribution =
[cluster_distribution(:,1),cluster_distribution(:,2)];
T2 = array2table(cluster_distribution)
```

T2 = 11×2 table

	cluster_distribution1	cluster_distribution2
1	1	1007
2	4	626
3	5	1202
4	8	769
5	11	1274
6	12	824
7	13	629
8	14	791
9	17	1058
10	19	936
11	24	865

```
figure;
for i = 1:length(cluster_distribution);%length(id);
subplot(4,3,i)
k = cluster_distribution(i,1);
colormap(gray(256))
images = ori_cycle2_cells(:,cycle2_winners(k))+this_winner;
%images(images<0) = 0;
image(reshape(images,28,28))
set(gca,'XTick',[], 'YTick', [])
%pause(0.2);
end
```

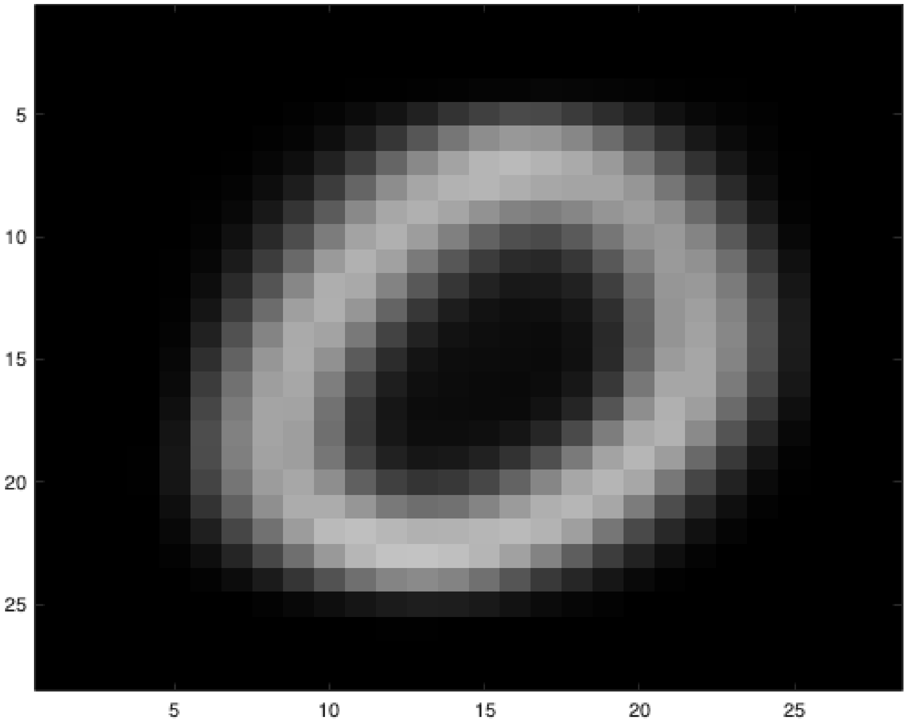


```
figure;
k = 1;
id = find(idx == k);
[GC,GR] = groupcounts(test.labels(id)); %stats of this category
T3 = array2table([GC,GR])
```

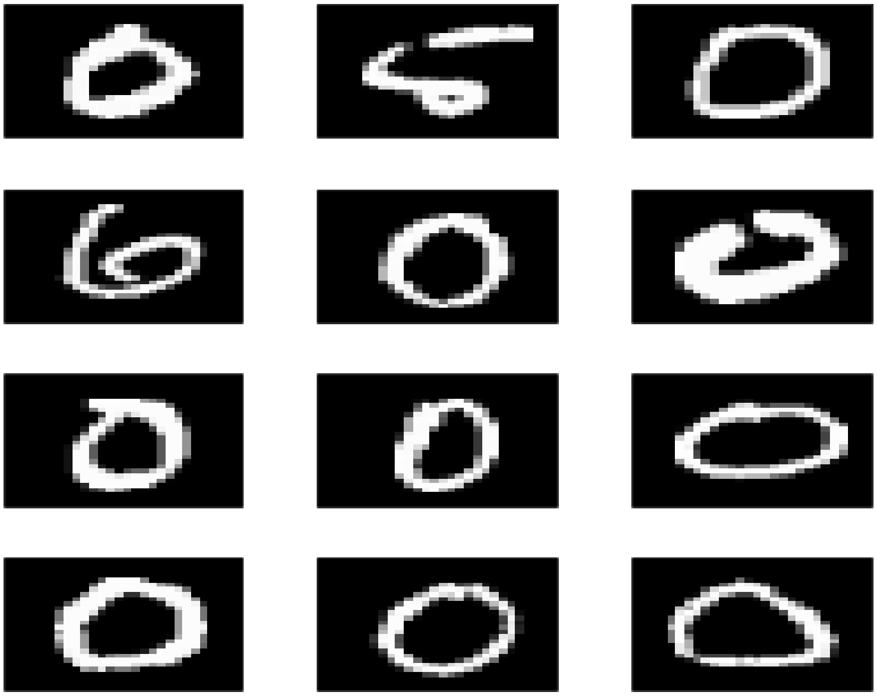
T3 = 9×2 table

	Var1	Var2
1	836	0
2	34	2
3	14	3
4	5	4
5	34	5
6	42	6
7	8	7
8	16	8
9	18	9

```
colormap(gray(256))
image(reshape(ori_cycle2_cells(:,cycle2_winners(k))+this_winner,28,28))
%center of this category
```



```
figure;
for i = 1:12;%length(id);
subplot(4,3,i)
colormap(gray(256))
image(reshape(data(:,id(i)),28,28))
set(gca,'XTick',[], 'YTick', [])
%pause(0.2);
end
```



```
% for index
[~,idx] = max(normc(sampled_data_fix)'*cycle2_cells(:,cycle2_winners),[],2);
[GC,GR] = groupcounts(idx);
delete_idx = find(ismember(idx,find(GC<10)));
X = data;
X(:,delete_idx) = [];
assign_id = test.labels;
assign_id(delete_idx) = [];
clustAssignments = idx;
clustAssignments(delete_idx) = [];
cluster_label = [clustAssignments,assign_id];

unique_cluster = unique(clustAssignments);
subdivi_winner_idx = [];
for i = 1:length(unique_cluster)
    assigned = unique_cluster(i);
    labels = cluster_label(find(cluster_label(:,1) == assigned),2);
    num_max_class = numel(find(labels==mode(labels)));

    if length(unique(labels)) == 1 | num_max_class/length(labels) >= 0.7;
        disp('cluster with winner index '+string(assigned)+' is not
subdivisible')
    else
```

```
        disp('cluster with winner index '+string(assigned)+' is
subdivisible')
        subdivi_winner_idx = [subdivi_winner_idx;assigned];
    end
end
```

```
cluster with winner index 1 is not subdivisible
cluster with winner index 4 is subdivisible
cluster with winner index 5 is subdivisible
cluster with winner index 8 is not subdivisible
cluster with winner index 11 is subdivisible
cluster with winner index 12 is not subdivisible
cluster with winner index 13 is subdivisible
cluster with winner index 14 is not subdivisible
cluster with winner index 17 is subdivisible
cluster with winner index 19 is subdivisible
cluster with winner index 24 is subdivisible
```

```

% k = 16;
% id = find(idx == k);
% [GC,GR] = groupcounts(test.labels(id)); %stats of this category
% T3 = array2table([GC,GR], 'VariableNames', {'counts', 'digits'})
% colormap(gray(256))
% image(reshape(ori_cycle2_cells(:,cycle2_winners(k))+this_winner,28,28))
%
% k = 0;
% for i = 101:136;%length(id);
% k = k+1;
% subplot(6,6,k)
% colormap(gray(256))
% image(reshape(data(:,id(i)),28,28))
% %pause(0.2);
% end
```

```
winner_id = cycle2_winners(clustAssignments);
PTY = purity(clustAssignments,assign_id)
```

```
PTY = 0.6409
```

```
NMI = nmi(assign_id, clustAssignments)
```

```
NMI = 0.5228
```

```
[RI, ARI] = randindex(assign_id, clustAssignments)
```

```
RI = 0.8972
ARI = 0.4175
```

```
[s] = ClusterEvalSilhouette (X', clustAssignments, 'cosine')
```

```
s = 0.1416
```

```
ch = ClusterEvalCalinskiHarabasz(X', clustAssignments)
ch = 369.7483
```

```
[db] = ClusterEvalDaviesBouldin (X', clustAssignments)
```

```
db = 2.7565
```

```



K_means_assign = kmeans(X',11);
PTY = purity(K_means_assign,assign_id)
```

```
PTY = 0.6448
```

```
NMI = nmi(assign_id, K_means_assign)
```

```
NMI = 0.5249
```

```
[RI, ARI] = randindex(assign_id, K_means_assign)
```

```
RI = 0.8955
ARI = 0.4230
```

```
[s] = ClusterEvalSilhouette (X', K_means_assign, 'cosine')
```

```
s = 0.1243
```

```
ch = ClusterEvalCalinskiHarabasz(X', K_means_assign)
```

```
ch = 368.4537
```

```
[db] = ClusterEvalDaviesBouldin (X', K_means_assign)
```

```
db = 2.8937
```