Nonlinear Model Predictive Control of the Solar Hot Water System

Tejaswinee Darure
University of Perpignan Via Dometia
PROMES-CNRS
Perpignan, France
tejaswinee.darure@univ-per.fr

Julien Eynard
University of Perpignan Via Dometia
PROMES-CNRS
Perpignan, France
julien.eynard@univ-perp.fr

Stéphane Grieu
University of Perpignan Via Dometia
PROMES-CNRS
Perpignan, France
grieu@univ-perp.fr

***Abstract*—Optimal control have paved the way into efficient operation of the systems of renewable energy generation. In recent years, model predictive controller has due to its inherent feature of constraint and disturbance handling. Moreover, the researchers are exploring the deployment of the model predictive control on renewable energy systems, specially solar thermal systems. Following the trend, in this paper, a novel approach of predictive controller implementation in solar thermal system has been explore. Primarily, to fulfil the prerequisite of the predictive control, the mathematical models based on first law of thermodynamics have been derived. Further, the design of predictive controller based on these models is provided where the nonlinearity of these models have been conserved, contrary to conventional approach of model linearization. The consideration of nonlinearity in the controller equips the control engineer with the in-depth system analysis as it imitates the real time dynamic behaviour. Finally, the performance of the proposed nonlinear model predictive controller has been verified through exhaustive simulations and the result are presented.**

***Index Terms*—Solar Hot water system, Nonlinear Model Predictive Control**

I. Introduction

Recently academic and industrial researchers have been focusing on the operational betterment of renewable energy systems. Especially, solar thermal systems have been popular due to its growing market in domestic and commercial sectors. The technological trend to use solar radiation to heat the supply water that can eventually be used for domestic purposes or space heating have been the center of the recent developments. Numerous publications in the literature discuss the dynamic modeling and simulation approaches of the solar thermal systems [1] [2]. Some researchers have investigated the performance of advanced control strategies and operation of solar thermal systems. References [3] [4] provide a brief review of possible advanced control methodologies that can be applied to solar collector field. Moreover due to intrinsic feature of model predictive control (MPC) and its trait for the handling constraints and disturbance rejection , its application to solar thermal plants becomes even more compelling. Publications like [5], [6] and [7] are some noted contributions of MPC implementation on solar thermal systems. Interestingly, [7] derives the linearized mathematical models of the solar collector and connected heat exchanger which is further used to demonstrate the energy efficient performance due to the deployment of predictive control strategy. It is worth to note that the linearization of mathematical model is generally carried out around the neighbourhood of the nominal operating point, Hence, evidently, the unaccounted change in the operating point of the system may perturb the performance of the MPC and the system.With the same motive, [8] demonstrates an approach of adaptive set-based approach for the systems with LPV model where parameters are optimized online to minimize model-process mismatches and then tube based MPC has been employed. Although, this approach provides the robustness , the computational complexity increases proportionally. Henceforth, to ensure the system performance at all times and all operating ranges, it is vital to avoid the linearization and consider the nonlinearities in the system while designing the model predictive control.

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In this work, we present an alternative approach of nonlinear predictive control where system dynamics is not compromised and the consideration of nonlinearities in the predictive control strengthens the robustness of the system performance in case of reference tracking. Section II presents the mathematical models of the solar thermal system under consideration. The controller design approach and problem statement has described in Section III and IV respectively. The Section V demonstrates the simulation result on the benchmark solar thermal system. Finally, Section VI concludes the discussion with future path forward.

II. Mathematical modeling and simulation

Inspired from the pilot plant at Lisbon, the benchmark model for solar hot water system is designed and simulations are performed that provided realistic imitation. It is essential to understand the dynamic thermal behavior of the units comprised in the solar hot water system That allows to derive the mathematical interpretation of the complete solar hot water system. These thermal behaviors are well explained using first law of thermodynamics i.e. mass and energy balance equations [**?**] [**?**]. The overall notion of mass and energy balance equation can be expressed as below:

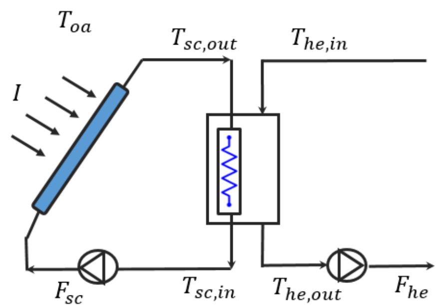

Fig. 1. Solar Collector Panels

accumulation of = energy (mass)	energy + (mass) in	energy - (mass) out	energy (mass) loss

Mass and energy balance equations are written for each unit of the benchmark defined for the solar hot water system. The pilot plant data available describing the system parameters and characteristics is used for the simulation of these mathematical models. Further, these models are verified with the reference to the commercial platform of Polysun. In the next subsections, the mathematical models are discussed in details for each unit.

A. Solar Collector

Solar collectors capture the radiant solar energy and convert it into thermal energy that is transported using heat transfer fluid. The pilot plant under consideration has installed a flat-plate non-tracking type solar collector as shown in Figure 1.

As there is no accumulation of mass in solar collectors, only the energy balance equation is considered in this case. The solar energy absorbed by the flat-plates is represented by $(Q_{sc,solar})$ while the carried heat transfer fluid is $Q_{sc,fluid}$ and $Q_{sc,loss}$ is the loss of energy to the atmosphere in the panel. The energy balance equation for the solar collector can be written as:

$$Q_{sc,acc} = Q_{sc,solar} - Q_{sc,fluid} + Q_{sc,loss} \tag{1}$$

Further, the energy absorbed by the solar plate collector $(Q_{sc,solar})$ is a function of solar radiation and a linear relation can be given as,

$$Q_{sc,solar} = A_{sc}\eta I \tag{2}$$

where A_{sc} is solar plate collector surface area (m^2), I is the solar radiance (W/m^2) and η is optical efficiency (dimensionless) [**?**].

The energy transferred to the fluid $Q_{sc,fluid}$ is formulated as following,

$$Q_{sc,fluid} = \dot{m}_{sc}c_{sc}(T_{sc,out} - T_{sc,in}) \tag{3}$$

where $\dot{m}_{sc}$ is the mass flow rate for the fluid (kg/s) and c_{sc} is the specific heat of the fluid $(J/kg/K)$, $T_{sc,in}$ and $T_{sc,out}$ are the inlet and outlet temperatures respectively.

Finally the heat loss in the solar collector can be given as,

$$Q_{sc,loss} = U_{sc}A_{sc}T_{sc,abs} \tag{4}$$

where U_{sc} is the heat loss coefficient (W/m^2K) and $T_{sc,abs}$ is the absolute temperature of solar collector surface. Although, it is quite challenging to measure surface temperature, hence for simplicity, the absolute temperature $T_{sc,abs}$ is replaced by the second order approximation:

$$Q_{sc,loss} = U_{sc}A_{sc}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right) + U_{sc}A_{sc}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right)^2 \tag{5}$$

Now, the complete energy balance can be written using equations (2), (3) and (6),

$$\rho_c c V_c \frac{dT_{sc,out}}{dt} = A_{sc}\eta I - \dot{m}_{sc}c_{cs}(T_{sc,out} - T_{sc,out}) + U_{sc}A_{sc}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right) + + U_{sc}A_{sc}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right)^2 \tag{6}$$

Note that mass flow rate $\dot{m}_{sc}$ is expressed in terms of volumetric flow rate F_{sc} as $\dot{m}_{sc} = F_{sc}\rho_{sc}$, where ρ_{sc} is the density of the solar heat transfer fluid. Hence, after further rearranging the equation (7), we obtain the final formulation

$$\frac{dT_{sc,out}}{dt} = \frac{A_{sc}\eta}{\rho_{sc}c_{sc}V_{sc}}I - \frac{F_{sc}}{V_{sc}}(T_{sc,out} - T_{sc,in}) + \frac{U_{sc}A_{sc}}{\rho_{sc}c_{sc}V_{sc}}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right) + + \frac{U_{sc}A_{sc}}{\rho_{sc}c_{sc}V_{sc}}\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right)^2 \tag{7}$$

Note that from the system control perspective, the solar radiance I and the outside temperature T_{oa} are disturbances, while the solar heat fluid inlet $T_{sc,in}$ and outlet temperatures $T_{sc,out}$ are controllable variables and the solar heat fluid flow rate F_{sc} is a manipulating variable.

B. Heat Exchanger

In hot water tank, the heat energy is transferred from solar heat transfer fluid to the water. This heat exchange in the hot water tank is considered to be of the counter-current type where the inflows evolve in the opposite direction. These type of heat exchanger allows more efficient heat transfer [9]. We assume the temperature in the tank is uniform and heat losses are negligible. The energy balance equation can be written as follows :

$$\frac{dT_{sc,in}}{dt} = \frac{F_{sc}}{V_{ct}}(T_{sc,out} - T_{sc,in}) - \frac{U_{ht}A_{ht}}{\rho_{sc}c_{sc}V_{ct}}(T_{sc,in} - T_{ht,out}) \tag{8}$$

$$\frac{dT_{he,out}}{dt} = \frac{F_{he}}{V_{ce}}(T_{he,in} - T_{he,out}) - \frac{U_{he}A_{he}}{\rho_w c_w V_{ce}}(T_{sc,in} - T_{he,out}) \tag{9}$$

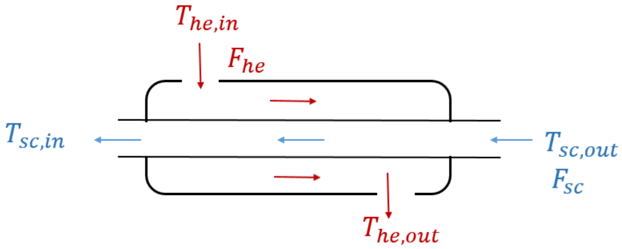

Fig. 2. Heat Exchanger

The system of equations is implemented in the MATLAB simulink platform (Version 2017a) and tested against the test inputs. The example screenshot is shown in Figure 3.

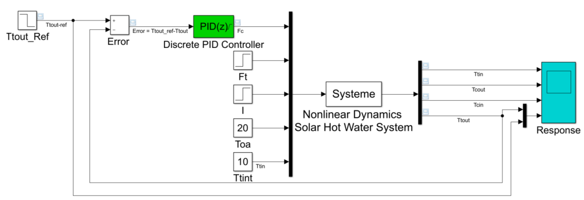

Fig. 3. Screenshot- MATLAB Simulink implementation of the SHWS

III. Problem Formulation

IV. Control Design

The economic and efficient operation of the solar thermal system has become the popular topic among the researchers. Although, the commercial products [10] available still offer the conventional control solutions like proportional-integral-derivative (PID) schemes. As, this conventional solution proposes simplicity in the implementation and can be easily manageable by the operators without having in-depth understanding of the dynamics. However, on the other hand, PID control scheme is reactive control strategy where the error between reference trajectories and actual outputs will be minimized that implies the control scheme is not robust to large changes in reference or the behavior of disturbance like solar radiation and outside temperature. Hence, the performance of the PID control is limited. Optimal control like MPC offers the solution where the performance of the system is not compromised due to variations in the references or the behaviour of the disturbances. As discussed in the introduction section, to avoid the challenges due to linearization of the system model, in this work we propose a novel approach of NMPC. The comparative analysis between PID and NMPC control performance have been presented in the forthcoming sections with the help of simulation results on the benchmark solar thermal system.

A. PID control scheme

Figure 3 shows an implementation of PID control scheme equivalent to commercial implementations [10]. Figure 4 depicts the the heat exchanger output temperature is controller by manipulating the solar heat fluid flow rate though the solar collector. Clearly, the water flow rate through the heat exchanger is considered constant and may act as disturbance.

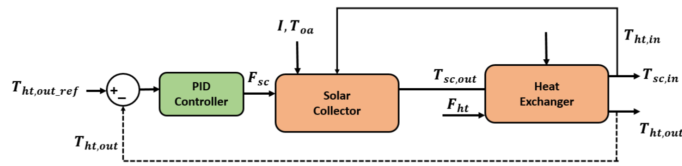

Fig. 4. PID Control Scheme

B. NMPC Control scheme

As described in the section II, the dynamics of the considered solar hot water system is given as,

$$\frac{dT_{sc,out}}{dt} = \frac{A_{sc}\eta}{\rho_{sc}c_{sc}V_{sc}}I - \frac{F_{sc}}{V_{sc}}(T_{sc,out} - T_{sc,in}) + \frac{U_{sc}A_{sc}}{\rho_{sc}c_{sc}V_{sc}}\left(\frac{T_{sc,in}+T_{sc,out}}{2} - T_{oa}\right) \tag{10}$$

$$\frac{dT_{sc,in}}{dt} = \frac{F_{sc}}{V_{ct}}(T_{sc,out} - T_{sc,in}) - \frac{U_{ht}A_{ht}}{\rho_{sc}c_{sc}V_{ct}}(T_{sc,in} - T_{ht,out}) \tag{11}$$

$$\frac{dT_{he,out}}{dt} = \frac{F_{he}}{V_{ce}}(T_{he,in} - T_{he,out}) - \frac{U_{he}A_{he}}{\rho_{w}c_{w}V_{ce}}(T_{sc,in} - T_{he,out}) \tag{12}$$

For uniformity and simplicity of the nomenclature, the states of the system are $x = [T_{sc,out}, T_{sc,in}, T_{he,out}]$, the inputs $u = [F_{sc}, F_{he}]$ where disturbances $d = [I, T_{oa}, T_{he,in}]$. Hence, the nonlinear system dynamics can be represented as below:

$$x(k+1) \quad = \quad f(x(k), u(k), d(k)) \tag{13}$$

where f represents the equations 11, 12 and 13. It is worth to note than these set of differential equations are solved for the time span of 60 minutes, coherent to the disturbance data sampling. Further, we present the optimization problem for the standard NMPC as below,

$$\begin{aligned}
&\underset{U_k}{\text{minimize}} \qquad \mathcal{J}\left(U_k, x(k)\right)\\
&\text{subject to}\\
&x\left(k+j+1|\, k\right) = f(x\left(k+j|\, k\right), u\left(k+j|k\right), d\left(k+j|k\right)\\
&\qquad\qquad\qquad\qquad\qquad\qquad j = 0, \ldots, N-1\\
&x^{min} \leq x\left(k+j|\, k\right) \leq x^{max} \qquad j = 0, \ldots, N-1\\
&u^{min} \leq u\left(k+j|k\right) \leq u^{max} \qquad j = 0, \ldots, N-1\\
&x\left(k|\, k\right) = x(k)
\end{aligned}$$

where $\mathcal{J}$ is the functional representing the overall cost function, N is the prediction horizon, $U_k = \{u(k|k), \ldots, u(k+N-1|k)\}$ is the sequence of predicted control inputs at time k. The bounds u^{min}, u^{max} on the input vector u, i.e. on the solar heat fluid flow rate and water flow rate that represent the fluid pump limits. The bounds on states x^{min}, x^{max} represent the soft bounds on the temperatures to compel the values to remain the predefined range. Optimization problem is solved repetitively at each time k using the receding horizon principle for the current measured state $x(k)$ along with the predicted state variables $\{x(k+j|\, k)\}_{j=1}^{N}$. Assume that the forecast for the disturbances $\{d(k|k), \ldots,\ d(k+N-1|k)\}$ is available a prior. then the corresponding optimal sequence $U_k^{\star} = \{u^{\star}(k), ..., \ u^{\star}(k+N-1)\}$ is obtained and the first element $u^{\star}(k)$ of this sequence is applied to the system. The procedure is repeated at time $k+1$, based on the new measured state $x(k+1)$.

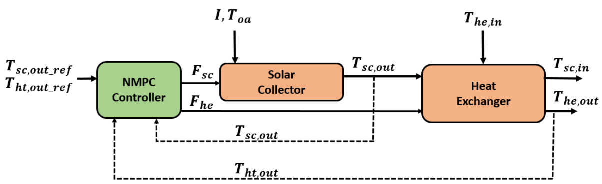

Fig. 5. NMPC Control Scheme

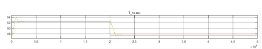

Fig. 6. NMPC Control Scheme

V. Simulation Results

Solar collector	U_{sc}	7
	η	0.8
	A_{sc}	2
	ρ_{sc}	1043
	c_{sc}	4180
	V_{sc}	0.0075
Heat Exchanger	A_{ht}	0.5
	U_{ht}	0.5
	U_{ht}	0.5

TABLE I
System Parameters and Their Values

VI. Conclusion and Path Forward

Solar thermal systems have become pioneer energy saving systems due to its wide applicability in domestic and industrial areas. Evidently, the efficient and economic operation of these systems has become dominant concern for the research community. With similar intention, in this work we propose the novel approach of nonlinear model predictive control for the solar collector and connected heat exchanger system. Primarily, the mathematical model of the solar hot water system have been derived using first law of thermodynamics. It is apparent that the thermal behaviour of the solar collector and heat exchanger have nonlinearities. Unlike the conventional approach of linearization around the operating point, we propose the predictive control strategy that takes into account these system nonlinearities. The performance of the deployed nonlinear model predictive control on the solar hot water system is then demonstrated on the benchmark solar hot water system and simulation results are presented. Moreover, the comparative analysis of the performance of nonlinear model predictive control against traditional proportional-integral-derivative control is presented. This clearly manifest the robust behaviour of nonlinear model predictive control in case of variations of reference trajectory and uncertainty imposed due to weather conditions. The extension of the

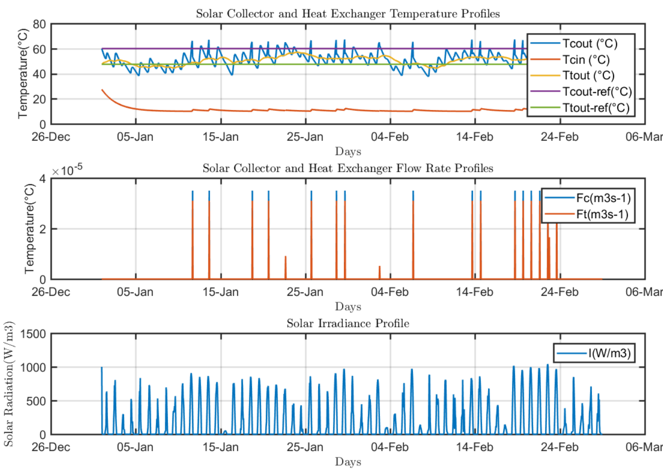

Fig. 7. NMPC Control Scheme

proposed control strategy can be extended to further units of the solar thermal system like thermal storage tanks, heat pumps and heating ventilation systems. Moreover, the adaptive nature of the proposed control strategy can be studied further in case of occurrences of faults or malfunctions in the units, their effects on the system and eventually the fault tolerant control schemes.

Nomenclature

T	temperature (^{o}C)
F	volumetric Flow rate (m^3/S)
A	area (m^2)
ρ	density (kg/m^3)
c	specific heat capacity $(J/kg/K)$
U	heat loss coefficient (W/m^2K)
V	volume (m^3)

subscripts

in	inlet
out	outlet
sc	solar collector
he	heat exchanger
w	water
oa	outside/weather
I	solar radiation

References

[1] Buzás, J., et al. "Modelling and Simulation of a Solar Thermal System." IFAC Proceedings Volumes 30.5 (1997): 143-147.

[2] Bojić, M., S. Kalogirou, and K. Petronijević. "Simulation of a solar domestic water heating system using a time marching model." Renewable energy 27.3 (2002): 441-452.

[3] Camacho, E. F., et al. ”A survey on control schemes for distributed solar collector fields. Part II: Advanced control approaches.” Solar Energy 81.10 (2007): 1252-1272.

[4] Camacho, E. F., et al. ”A survey on control schemes for distributed solar collector fields. Part I: Modeling and basic control approaches.” Solar Energy 81.10 (2007): 1240-1251.

[5] Finck, Christian, Rongling Li, and Wim Zeiler. ”Identification of a dynamic system model for a building and heating system including heat pump and thermal energy storage.” MethodsX 7 (2020): 100866.

[6] Ferhatbegović, Tarik, et al. ”Modelling and design of a linear predictive controller for a solar powered HVAC system.” 2012 IEEE International Symposium on Industrial Electronics. IEEE, 2012.

[7] Ferhatbegović, Tarik, Gerhard Zucker, and Peter Palensky. ”Model based predictive control for a solar-thermal system.” IEEE Africon’11. IEEE, 2011.

[8] Pipino, Hugo A., et al. ”Nonlinear temperature regulation of solar collectors with a fast adaptive polytopic LPV MPC formulation.” Solar Energy 209 (2020): 214-225.

[9] Woolf, Peter. ”CHE 466-Process Dynamics and Controls, Fall 2008.” 2008.

[10] Manuel of Solar controller DeltaSol CS Plus https://www.resol.de/Produktdokumente/11202674_DeltaSol_CS_Plus.monfr.pdf, 2020 ,