Review

Modeling and Validation of Solar Hot Water System for Space Heating Application

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Abstract: Solar hot water systems for space heating are becoming increasingly popular thanks to its environmental and economic advantages. However, the economic feasibility depends significantly on the capacity to perform the accurate assessment of the system operation under different conditions. In this work, a comprehensive modular modeling approach of the solar hot water system used for space heating is presented. This modeling approach evaluates the dynamics defined using the first law of thermodynamics. These derived models are then simulated in Matlab-Simulink® environment. Simultaneously, with the help of a , similar system has been simulated and . The simulations exploit the hourly sampled weather data from Lisbon, Portugal for the year of 2019. To investigate the accuracy of these derived models, the simulation data is compared against the commercial software platform called Polysun®. This platform serves as benchmark for this study as it is widely used for sizing and design purpose by the engineers and the enterprises. The RSME index is used to measure the accuracy of the derived models. Finally, the results demonstrate that the derived models are successfully validated against the simulation data obtained by commercial platform. These accurate derived models have extensive usability from from development of state-of-the-art control strategies like Model Predictive Control to the generation of synthetic data to analyze the energy and cost consumption.

Keywords: Solar Collectors; Buildings; Thermal Energy Storage System;

1. Introduction

Governments have implemented numerous initiatives and strategic policies at international and national levels to guide efforts for climate change mitigation. To reduce the greenhouse gas emissions, it is absolutely necessary to eliminate the use of fossil fuels and integrate more and more clean energy sources. Thanks to decreasing investment costs and increased market demand, technologies related to energy generation using renewable resources, like wind and solar, have become favoured and affordable alternatives. Statistics show that building heating systems account for around 30% of energy consumption in Europe[?]. To reduce the use of fossil fuel and the energy costs, solar hot water systems for space heating are becoming as a novel substitute today. Hence, the betterment of their design and performance have become a popular topic among researchers.

An efficient system design can significantly affect its capability and energy saving benefits. Moreover, the design of optimal controllers is another vital contributor to maximize the energy gain for minimal cost. In a nutshell, the streamlined functional models of the solar energy systems are seen as the tools that help to analyze the system behaviour and controller design in the design and prototyping phase. There are numerous efforts seen in the literature pre senting various approaches of modeling and simulation[1][2]. These approaches are widely divided in data based and first principle based methods. Data based techniques are very popular these days but data based modelling approaches have their own challenges like availability of data, inaccuracies and noise in available data, insufficient understanding of system parameters and their effects on system behaviour etc.. Contrarily, first principle based methods provides a deep insight into the dynamic behaviour of the system, can incorporate numerous variables, and are easy to analyze as compared to data based methods.

The modeling of solar systems for space heating as been addressed in the literature, but to the best of our knowledge, there are very few references that describe the full stack of components as an integrated model and that can be used to develop advanced management strategies. For space heating,[1] proposed a model based on first-principle equations for several subsystems: a thermal flat-solar collector field, a hot water accumulation system, and a gas heater. The model also includes discrete dynamics to develop a hybrid model that is able to simulate the different configuration modes of the solar system for both simulation and control design purposes. More recently, [2] presented a study of a building heating and comfort based on solar thermal system consisting of a solar thermal collector, a water storage tank, a boiler and a low temperature radiator. The formalism of Bond Graph was used (a graphical representation of first-principle equations that allows to develop state-space representations of physical systems) for modeling of the solar water heater using a thermostat regulated hot water radiator for heating a building. More recently, solar systems for space heating have been coupled with heat pumps, as reviewed by [3]. This works suggests that the integrated modeling needs to be improved for better economic and environmental assessment.

Regarding the modeling tools that are used, there have been multiple approaches, like TRNSYS®as in [4] when the scope is to optimize the design of components, or MATLAB/SIMULINK®like in [2] and the objective is to develop further the system operation strategies. Other example of applications of MATLAB/SIMULINK®are [5], where a model has been developed to predict the storage water temperature, greenhouse indoor temperature and the amount of auxiliary fuel, as a function of various design parameters of the greenhouse such as location, dimensions, and meteorological data of the region; or [6] where a model of a solar water heater has been developed and simulated for an aviculture building in Tehran, Iran. Polysun®, a commercial tool for solar systems design, is rarely used in the literature. An example can be found in [7] to analysis of the influence of hot water consumption profiles and hot water tank size in solar thermal systems.

This paper presents a thorough modeling approach for the solar thermal system for space heating installed in LNEG Lab in Lisbon. The modular modeling approach has been adopted in this work which implies that the mathematical models for each unit of underlying solar system has been developed in Matlab-Simulink®. These models can be used in the development and implementation of advanced, state-of-the-art control algorithms, like Model Predictive Control[8][9]. The validation of these models is performed against a *Polysun*® commercial software platform for the design of the solar thermal systems. Comparative study between these models is presented considering different weather conditions.

Paper Organization: Section 2 presents the general configuration of the overall system under consideration that comprises solar collector, thermal storage tank, building and fan coil unit (FCU) systems. The modular mathematical modeling for all the units is then presented in detail in Section 3. The model mapping to the pilot site and its validation using the Polysun platform is then demonstrated in Section 4. Additionally, Section 4 provides the model behaviour for varied use cases considering different weather profiles. Finally, the work is concluded in the Section 5 with the remarks on the future use for the development of advance control strategies.

2. Solar Thermal System for Space Heating

The modeling solution developed under IMPROVEMENT Project are validated on the solar thermal pilot site situated at LNEG, Lisbon Portugal. The layout of this pilot plant is shown in Figure 2. A flat-plate type solar collector field absorbs the solar energy and stores it in a thermal storage tank. This stored heat energy is then used to supply a fan-coil unit that maintains the thermal comfort of the building. The schematic representation of the above plant is presented in Figure 2.

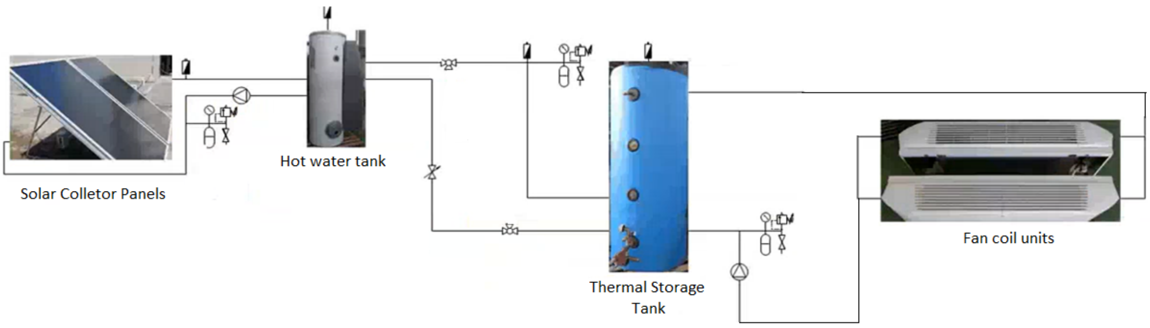

Figure 1. Solar Hot Water System : Pilot Plant, Lisbon Portugal

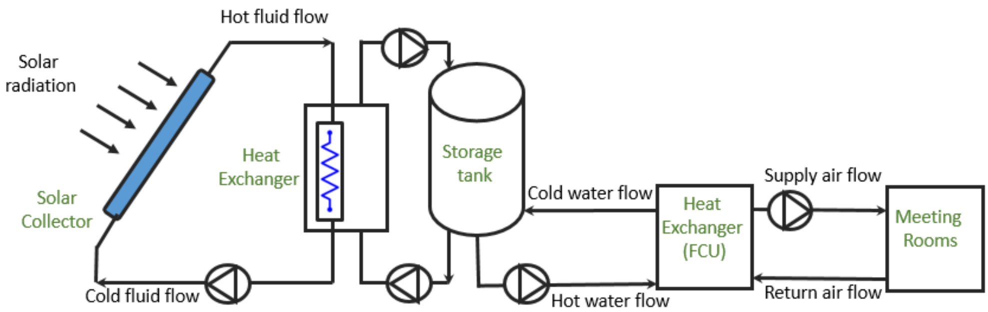

Figure 2. Solar Hot Water System : Schematic

3. Mathematical modeling and simulation

Inspired from the pilot plant at Lisbon, the benchmark model for solar hot water system is designed and simulations are performed that provided realistic imitation. It is essential to understand the dynamic thermal behavior of the units comprised in the solar hot water system. That allows to derive the mathematical interpretation of the complete solar hot water system. These thermal behaviors are well explained using first law of thermodynamics i.e. mass and energy balance equations [10] [11]. The overall notion of mass and energy balance equation can be expressed as below:

$$\begin{array}{l} \text{accumulation} \\ \text{of energy} \\ \text{(mass)} \end{array} = \begin{array}{l} \text{energy} \\ \text{(mass) in} \end{array} + \begin{array}{l} \text{energy} \\ \text{(mass) out} \end{array} - \begin{array}{l} \text{energy} \\ \text{(mass) loss} \end{array}$$

Mass and energy balance equations are written for each unit of the benchmark defined for the solar hot water system. The pilot plant data available describing the system parameters and characteristics is used for the simulation of these mathematical models. Further, these models are verified with the reference to the commercial platform of Polysun. In the next subsections, the mathematical models are discussed in details for each unit.

3.1. Solar Collector

Solar collectors capture the radiant solar energy and convert it into thermal energy that is transported using heat transfer fluid. The pilot plant under consideration has installed a flat-plate non-tracking type solar collector as shown in Figure 3.

Figure 3. Solar Collector Panels

As there is no accumulation of mass in solar collectors, only the energy balance equation is considered in this case. The solar energy absorbed by the flat-plates is represented by $(Q_{sc,solar})$ while the carried heat transfer fluid is $Q_{sc,fluid}$ and $Q_{sc,loss}$ is the loss of energy to the atmosphere in the panel. The energy balance equation for the solar collector can be written as:

$$Q_{sc,acc} = Q_{sc,solar} - Q_{sc,fluid} + Q_{sc,loss} \tag{1}$$

Further, the energy absorbed by the solar plate collector $(Q_{sc,solar})$ is a function of solar radiation and a linear relation can be given as,

$$Q_{sc,solar} = A_{sc}\eta I \tag{2}$$

where A_{sc} is solar plate collector surface area (m^2), I is the solar radiance (W/m^2) and η is optical efficiency (dimensionless) [10].

The energy transferred to the fluid $Q_{sc,fluid}$ is formulated as following,

$$Q_{sc,fluid} = \dot{m}_{sc}c_{sc}(T_{sc,out} - T_{sc,in}) \tag{3}$$

where $\dot{m}_{sc}$ is the mass flow rate for the fluid (kg/s) and c_{sc} is the specific heat of the fluid $(J/kg/K)$, $T_{sc,in}$ and $T_{sc,out}$ are the inlet and outlet temperatures respectively.

Finally the heat loss in the solar collector can be given as,

$$Q_{sc,loss} = U_{sc}A_{sc}T_{sc,abs} \tag{4}$$

where U_{sc} is the heat loss coefficient (W/m^2K) and $T_{sc,abs}$ is the absolute temperature of solar collector surface. Although, it is quite challenging to measure surface temperature, hence for simplicity, the absolute temperature $T_{sc,abs}$ is replaced by the second order approximation:

$$Q_{sc,loss} = U_{sc}A_{sc}\left[\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right) + \left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right)^2\right] \tag{5}$$

Now, the complete energy balance can be written using equations (2), (3) and (5),

$$\rho_c c V_c \frac{dT_{sc,out}}{dt} = A_{sc}\eta I - \dot{m}_{sc}c_{cs}(T_{sc,out} - T_{sc,out}) + U_{sc}A_{sc}\left[\left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right) + \left(\frac{T_{sc,in} + T_{sc,out}}{2} - T_{oa}\right)^2\right] \tag{6}$$

Note that mass flow rate $\dot{m}_{sc}$ is expressed in terms of volumetric flow rate F_{sc} as $\dot{m}_{sc} = F_{sc}\rho_{sc}$, where ρ_{sc} is the density of the solar heat transfer fluid. Hence, after further rearranging the equation (6), we obtain the final formulation

$$\frac{dT_{sc,out}}{dt} = \frac{A_{sc}\eta}{\rho_{sc}c_{sc}V_{sc}}I - \frac{F_{sc}}{V_{sc}}(T_{sc,out} - T_{sc,in}) + \frac{U_{sc}A_{sc}}{\rho_{sc}c_{sc}V_{sc}}\left[\left(\frac{T_{sc,in}+T_{sc,out}}{2} - T_{oa}\right) + \left(\frac{T_{sc,in}+T_{sc,out}}{2} - T_{oa}\right)^2\right] \tag{7}$$

Note that from the system control perspective, the solar radiance I and the outside temperature T_{oa} are disturbances, while the solar heat fluid inlet $T_{sc,in}$ and outlet temperatures $T_{sc,out}$ are controllable variables and the solar heat fluid flow rate F_{sc} is a manipulating variable.

3.2. Hot water tank

In hot water tank, the heat energy is transferred from solar heat transfer fluid to the water. This heat exchange in the hot water tank is considered to be of the counter-current type where the inflows evolve in the opposite direction. These type of heat exchanger allows more efficient heat transfer [12]. We assume the temperature in the tank is uniform and heat losses are negligible. The energy balance equation can be written as follows :

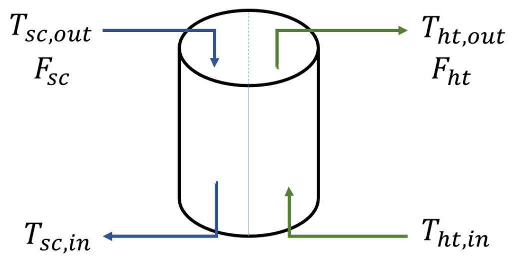

Figure 4. Hot water tank: schematic

$$\frac{dT_{sc,in}}{dt} = \frac{F_{sc}}{V_{ht}}(T_{sc,out} - T_{sc,in}) - \frac{U_{ht}A_{ht}}{\rho_{sc}c_{sc}V_{ht}}(T_{sc,in} - T_{ht,out}) \tag{8}$$

$$\frac{dT_{ht,out}}{dt} = \frac{F_{ht}}{V_{ht}}(T_{ht,in} - T_{ht,out}) - \frac{U_{ht}A_{ht}}{\rho_{w}c_{w}V_{ht}}(T_{sc,in} - T_{ht,out}) \tag{9}$$

3.3. Thermal Storage Tank

When solar energy is unavailable at nights and in cloudy weather, the continuous space heating is ensured using the Thermal Energy Storage (TES) tank. TES also allows to shift the peak hours energy consumption to off peak hours, significantly improving the system performance and saving energy costs. The pilot site in LNEG Lisbon is installed with a stratified TES that contains a dual heat exchanger. Hence, we present a detailed mathematical model of the stratified TES based on the energy balance equations.

The principle of stratified thermal storage tank is based on thermal stratification process where hot water sits on the cold water as depicted in Figure 5. In this work, we use the node based energy balance method, i.e. the energy balance equation for each node (12 in this case) are evaluated [13]. Please note that the terms, node and layer are used interchangeably in the literature, hence for convenience we use the term layer. For the simplification of model, following assumptions are made [14]:

1. The fluid used in the thermal storage tank is incompressible;
2. Pressure in the tank is assumed to be appropriate to avoid any fluid phase changes.
3. The mixing of layers due to buoyancy force has been considered negligible;
4. There is no mass flowing in or our of the storage tank, hence the mass balance equation is not considered.

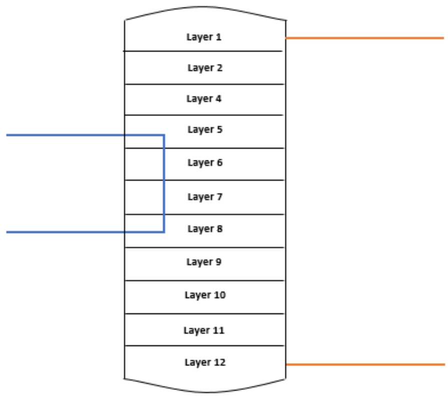

Figure 5. Stratified Thermal Storage Tank with 12 nodes

The energy balance equation will be different based on the position of the layer i.e. 1) boundary layers (layer 1, 12), 2) layers that are in direct contact with the heat transfer fluid of the solar collector (layer 5,6,7,8) or 3) the generic layer that only exchanges heat with the neighbouring layers (layer 2,3,4,9,10,11).

The temperature of generic layer is a function of the temperature of that layer itself and the temperatures of surrounding layers. The energy balance equation for i^{th} generic layer is given as follows (see Figure 5):

$$\frac{dT_{stl,i}}{dt} = \frac{\dot{V}_{stl}}{V_{stl}}(T_{stl,i+1} - T_{stl,i}) - \frac{U_{st}A_{stl}}{\rho_w c_w V_{stl}}(T_{stl,i} - T_{oa}) + \frac{k_{j,1}A_{stl}}{\rho_w c_w V_{stl}\Delta z}(T_{stl,i+1} - T_{stl,i}) + \frac{k_{j,2}A_{stl}}{\rho c V_{stl}\Delta x}(T_{stl,i-1} - T_{stl,i}) \tag{10}$$

where V_{stl} is the volume of the layer (m^3); $\dot{V}_{stl}$ is the volumetric flow rate (m^3/s); $T_{stl,i}$, $T_{stl,i-1}$ and $T_{stl,i+1}$ represent the temperature (^oC) of i^{th}, $i+1^{th}$ and $i-1^{th}$ layer; U_{st} is the heat transfer coefficient between the fluid in the storage tank and the atmospheric air considered constant (W/m^2K) and A_{stl} is the lateral area of the tank in contact with the layer (m^2); finally k_i is thermal conductivity of the water in the i^{th} layer (W/mK) and Δx is the height of the layer (m). It is clear from equation (10) that the thermal dynamics represent the energy gain due to the heat transfer fluid flow rate, energy transfer between lower and upper layers and finally the energy loss to the ambient.

Moreover, the layers in contact with the heat carrying fluid from solar collectors will have an additional term as shown in following equation representing the heat gain due to the solar collector fluid flow rate.

$$\frac{dT_{stl,i}}{dt} = \frac{\dot{V}_{stl}}{V_{stl}}(T_{stl,i+1} - T_{stl,i}) - \frac{U_{st}A_{stl}}{\rho_w c_w V_{stl}}(T_{stl,i} - T_{oa}) + \frac{k_{j,1}A_{stl}}{\rho_w c_w V_t\Delta z}(T_{stl,i+1} - T_{stl,i}) \tag{11}$$

$$+\frac{U_{st,coil}A_{sc,coil}}{\rho_{sc} c_{sc} V_{st,coil}}(T_{stl,j} - T_{sc,out}) \tag{12}$$

Note that the term $\frac{U_{st,coil}A_{sc,coil}}{\rho_{sc} c_{sc} V_{st,coil}}(T_{stl,j} - T_{sc,out})$ represents the heat flux due to the solar collector fluid.

However, the boundary layers 1 and 12 have increased surface contact with the outside temperature, resulting in extra heat loss. The energy balance equations for these boundary layers are represented as follows:

$$\frac{dT_{stl,1}}{dt} = \frac{\dot{V}_{stl}}{V_{stl}}(T_{stl,2} - T_{stl,1}) - \frac{U_{st}A_{st,lateral}}{\rho c V_{stl}}(T_{stl,1} - T_{oa}) + \frac{k_{1,1}A_{stl}}{\rho c V_t\Delta z}(T_{stl,2} - T_{stl,1}) \tag{13}$$

$$\frac{dT_{stl,12}}{dt} = \frac{\dot{V}_{stl}}{V_{stl}}(T_{st,in} - T_{st,12}) - \frac{U_{st}A_{st,lateral}}{\rho c V_{stl}}(T_{stl,12} - T_{oa}) + \frac{k_{1,1}A_c}{\rho c V_t \Delta x}(T_{stl,11} - T_{stl,12}) \tag{14}$$

Note that $A_{st,lateral}$ represent the area of the lateral tank in contact with the layer plus area of the top section of the tank (m^2). Thus, the agglomeration of equations (10), (13) and (14) represents the dynamic behaviour of the stratified TES. It is worth to note that all the system parameters are used derived from the available pilot plant data.

3.4. Fan Coil Unit (Heating coil)

Fan Coil Units (FCU) are one of the favoured HVAC systems widely used in the buildings to its ease of installation, lower noise levels and versatility in type of mounting (floor or ceiling). A typical FCU unit contains a supply fan, a heat exchanger coil, filters and noise alternators. However, the FCU installed on the pilot site of LNEG has a simplified configuration, where the supply fan of the FCU is running on constant speed and creates a supply air flow that passes through the heating coil. In the heating coil, the temperature of the supply air is increased to the predefined temperature. This conditioned air is then used to heat the space in occupied thermal zones. Note that the air filters do not directly affect the thermal behavior of the air, hence the dynamics of the filter is neglected.

In this work, we consider the shell and tube counter-current type heating coil.The mathematical model of the air-water heating coil is presented in the equations (15) (16) that is essentially the energy balance equation is applied for both shell and tube side fluid flow rates. The hot water in the shell from the thermal storage tanks heats the supply air in the tube side to required temperature. [15] presents the detailed model of the heat exchanger, which is is extended for this pilot plant case study. The required air flow rate entering the thermal zones is manipulated through the thermal zone controller. Hence, for this heat coil model, the demand air flow rate is considered as the demand. Please note that the heat energy loss to the outside environment is considered negligible.

The following equations represent the shell side and tube side energy balance equations respectively:

$$\rho_{sa}c_{sa}V_{fcu}\frac{dT_{fcu,out}}{dt} = \rho_{sa}c_{sa}F_{fcu}T_{fcu,in} - \rho_{sa}c_{sa}F_{fcu}T_{fcu,out} - \frac{k_{fcu}A_{fcu,a}}{\Delta z}(T_{fcu,out} - T_{st,in}) \tag{15}$$

$$\rho_w c_w V_{fcu}\frac{dT_{st,in}}{dt} = \rho_w c_w F_{st}T_{st,in} - \rho_w c_w F_w T_{st,out} - \frac{k_{fcu}A_{fcu,w}}{\Delta z_{fcu}}(T_{st,in} - T_{fcu,out}) \tag{16}$$

Note that $A_{fcu,a}$ and $A_{fcu,w}$ represent the ares of the tube/air side and shell/water side (m^2) respectively and k_{fcu} is conductive heat transfer coefficient of FCU (dimensionless). Finally Δz is the length of the tube (m). ρ_{sa} denotes the density of supply air (kg/m^3), $T_{fcu,in}$ $T_{fcu,out}$ are the inlet and outlet air temperature for the the heating coil. The inlet and outlet water temperature () from stratified TES is denoted by $T_{st,in}$ and $T_{st,out}$ respectively. V_{fcu} is the volume of the heating coil (m^3) where F_{fcu} and F_{st} is the air and water flow rate (m^2/S) respectively.

4. Simulation and Validation

The derived mathematical models have been simulated using numerical data representing the pilot plant. All the relevant numerical data of the system parameters and characteristics are presented in Table 1. Further, the dynamics of these models have been verified against *Polysun*®, a professional software provided by VelaSolaris.This Swiss platform is very popular among researchers and professionals as it provides the complex and thorough models for real time operating conditions. The use of this software is particularly known for the application of design and sizing of solar thermal plants for different applications, including space heating. Hence considering its credibility, the derived mathematical models in this work have been tested and verified against the identical system parameters and weather data in Polysun. Note that Polysun software packages also allows the use of an exhaustive weather database to demonstrate the dynamic model behaviour for different weather conditions and locations. Further, the derived mathematical models are simulated in the MATLAB Simulink platform (Version 2017a). The example screenshot is shown in Figure 6. As shown in the figure, the equations representing dynamics of each unit is implemented and are interconnected as depicted in Figure 2.

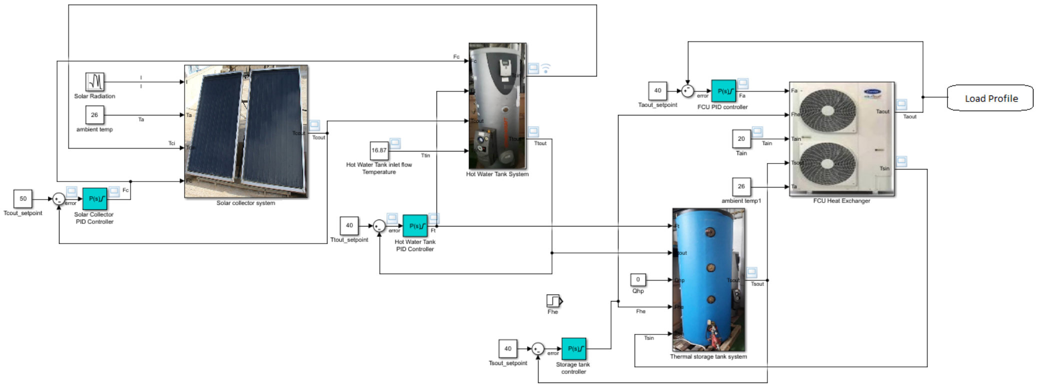

Figure 6. Screenshot- MATLAB Simulink Implementation of the SHWS

Weather data: The simulation and validation exercises for both platforms have been carried out for two different seasonal conditions, i.e. month of February and November. The selected weather data is collected at the location of Lisbon where the pilot plant is situated. Figure 7 and 8 show the weather pattern for outside temperature (^oC) and solar radiation (W/m^2). This data is collected for the sampling time of 60 minutes.

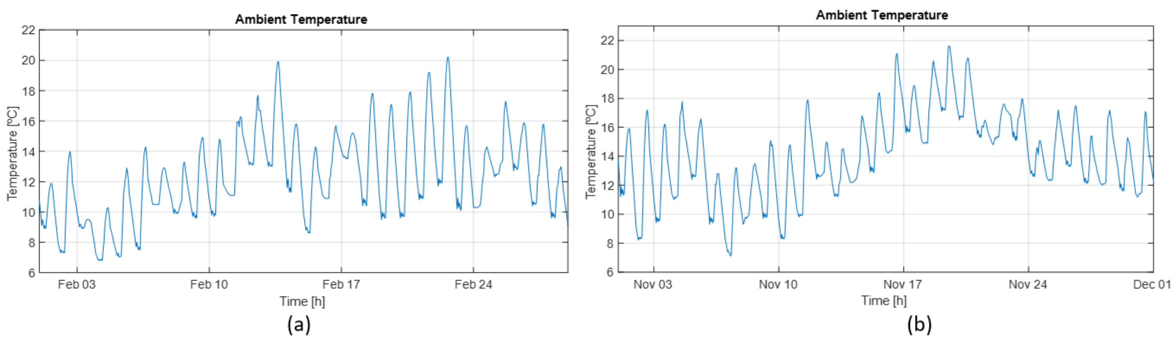

Figure 7. Outside Temperature Data: (a) 1st Feb 2019 to 28th Feb 2019 (b) 1st Nov 2019 to 1st Dec 2019

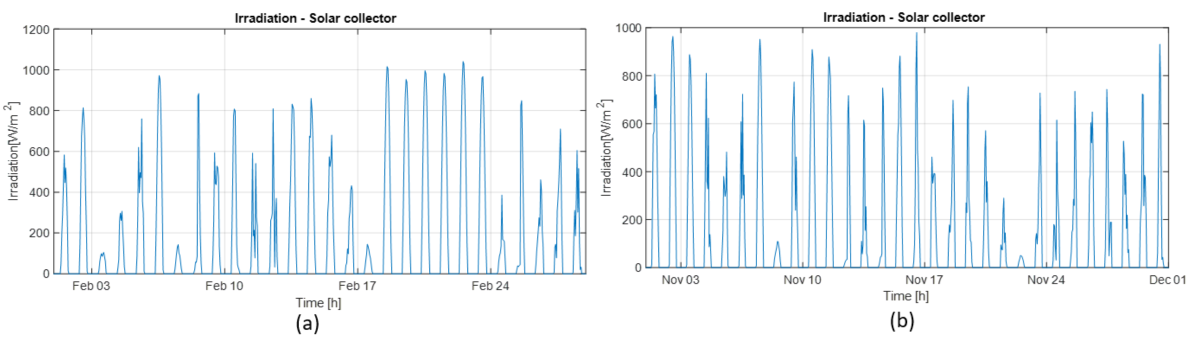

Figure 8. Solar Radiance Data: (a) 1st Feb 2019 to 28th Feb 2019 (b) 1st Nov 2019 to 1st Dec 2019

Using the same weather conditions, thermal behaviour of the each unit is then simulated in MATLAB Simulink and in Polysun respectively. Note that, the accuracy of the mathematical models have been compared with Polysun models using Root Mean Square Error (RSME) index. RSME is square root of the average of squared errors and its formula is given as:

$$RSME = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)} \tag{17}$$

where n represents the data point length, $\hat{y}$ represents the data simulated from MATLAB simulink where y represents the data simulated by Polysun software platform. RSME is For more information on the definition of this index,the reader is advised to refer [16].

Unit	Variable	Value	Description
Solar collector	U_{sc}	7	solar heat loss coefficient $(Wm-2K-1)$
	η	0.8	optical efficiency (dimensionless)
	A_{sc}	2	solar collector plate surface area (m^2)
	ρ_{sc}	1043	solar collector fluid density (kgm^-3)
	c_{sc}	4180	solar collector plate specific heat $(JkgC)$
	V_{sc}	0.0075	solar collector fluid volume (m^3)
Hot water tank	A_{ht}	0.5	area of hot water tank wall (m)
	U_{ht}	0.5	heat transfer coefficient of the losses in the tank (W/m^2K)
	V_{ht}	0.5	hot water tank volume (m^3)
TES	Δz_{fcu}	10^5	constant for the stratification
	V_{st}	0.004	volume in the coil (m^3)
	U_{st}	250	heat transfer coefficient in the coil (W/m^2K)
	A_{st}	0.9	area of the coil (m^2)
FCU	A_{fcu}	0.4	area of the tank (m^2)
	V_{fcu}	0.4	volume of heating coil (m^3)
	$A_{fcu,a}$	0.4	shell outside area (m^2)

Table 1. System Parameters and Their Values

Figure 9 shows the comparison between the response of dynamic model of the solar collector and its presentation in Polysun model. This comparison is done for solar collector outlet temperature i.e.$T_{sc,out}$. It is evident from the given plot, that the temperature $T_{sc,out}$ is higher for higher solar radiance can reach up to 70^oC. The estimated RSME index is about 0.7484 that confirms the effective accuracy of the mathematical model with respect to the Polysun model.

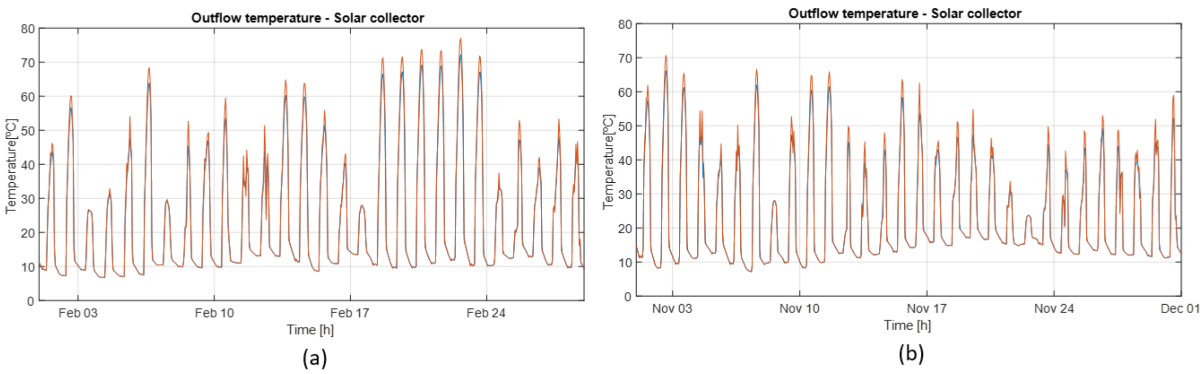

Figure 9. Solar Collector Outlet Temperature ($T_{sc,out}$) for (a) 1st Feb 2019 to 28th Feb 2019 (b) 1st Nov 2019 to 1st Dec 2019

Similarly, Figure 10 presents the model comparison for the top layer of the stratified TES. Referring to the dynamics of TES, the model comparison is done for the temperature of all the layers

205 $T_{st,1}, T_{st,2}, \ldots, T_{st,12}$ but due to space constraints and to avoid duplication, only top layer temperature
206 $(T_{st,12})$ dynamics have been presented. The estimated RSME index is about 1.842 that validates the
207 accuracy of the stratified TES model simulated in the MATLAB Simulink.

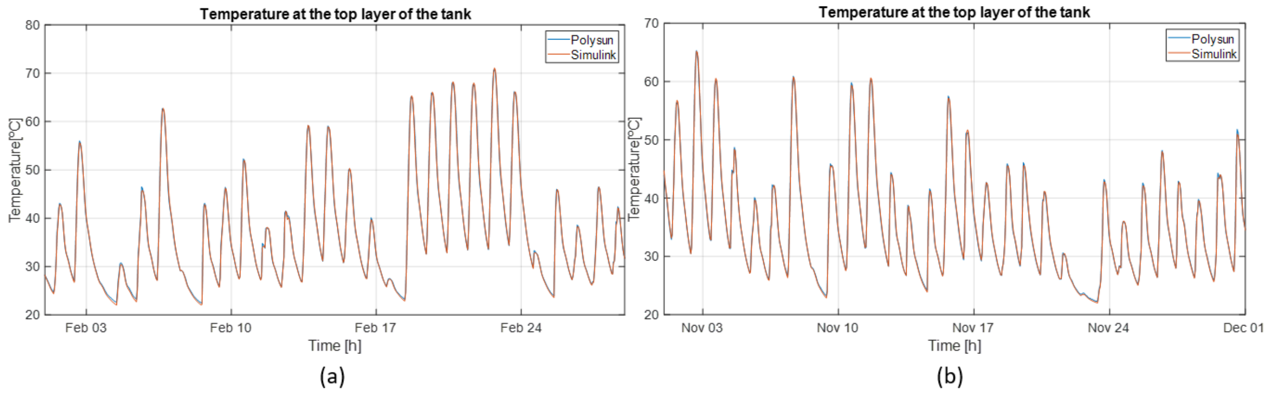

Figure 10. Top Layer storage tank temperature $(T_{st,12})$ for (a) 1st Feb 2019 to 28th Feb 2019 (b) 1st Nov 2019 to 1st Dec 2019

208 Finally, the figure 11 presents the model comparison for outlet temperature of FCU $T_{fcu,out}$. For
209 the simulation purpose, the supply air flow F_{fcu} and supply air inlet temperature $T_{fcu,in}$ are considered
210 constant that represent the thermal load of the building. The RSME index depicting the accuracy
211 of mathematical models is about 1.055 and it is worth to note that the dynamic behaviour of these
212 models are constant over varied seasonal conditions. Thus, this work show that these modular
213 validation exercises ascertain the authenticity of the derived mathematical models, Moreover, the
214 thermal behaviour of complete system where the all units are interconnected, as shown in Figure 6,
215 have been simulated and preliminary PID control strategies have been successfully tested. However,
216 the study of control strategies and their performance for these models is outside the scope of this paper
217 but paves the way towards the future perspective of this article.

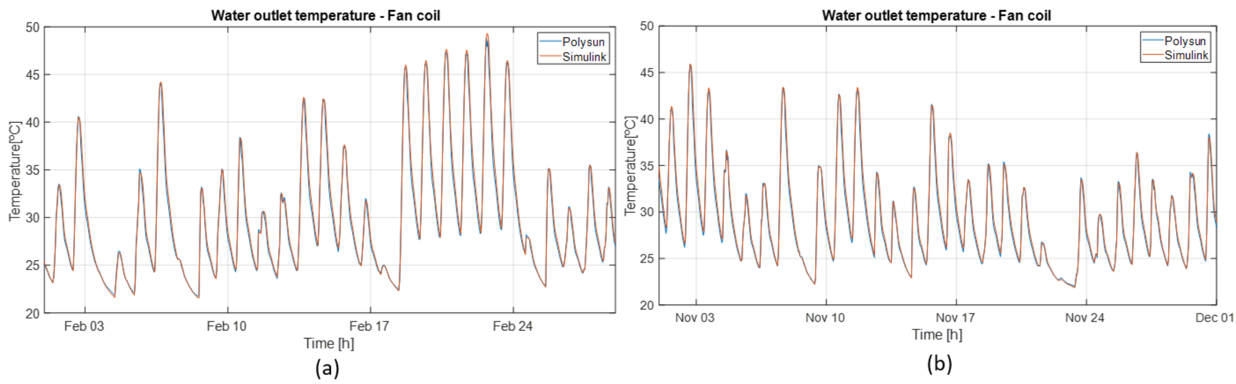

Figure 11. Outlet temperature fan coil $(T_{fcu,out})$ for (a) 1st Feb 2019 to 28th Feb 2019 (b) 1st Nov 2019 to 1st Dec 2019

218 ## 5. Conclusions

219 This paper proposes an integrated mathematical modeling approach for solar hot water systems
220 for space heating based on the first law of thermodynamics. The configuration of the solar hot water
221 system comprises a solar collectors, a hot water tank, a thermal storage tank and a fan coil unit.
222 These mathematical model are simulated in MATLAB Simulink that mimics the dynamics of the
223 pilot plant situated in LNEG,Lisbon Portugal. Then, these models have been validated against the

commercial modeling software Polysun. The comparisons for each unit of solar hot water system have been presented and discussed. It is worth to note that unlike commercial software platforms, these mathematical models allow the user deeper insight to perform sensitivity and behavioural analysis. Having the extensive knowledge about the dynamics and their parameters will allow the engineers to reason and test thoroughly the design and sizing phase of the system. Moreover, the these models will help in great deal for the accurate available selection for the control methods. These models can also support the generation of synthetic data for given weather and location, which can be further analyzed for the investigation of energy consumption and related cost effectiveness.

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Nomenclature

T	temperature (^oC)
F	volumetric Flow rate (m^3/S)
A	area (m^2)
ρ	density (kg/m^3)
c	specific heat capacity ($J/kg/K$)
U	heat loss coefficient (W/m^2K)
V	volume (m^3)

subscripts

in	inlet
out	outlet
sc	solar collector
ht	hot water tank
he	heat exchanger
st	thermal storage tank
fcu	fan coil unit
z	thermal zone
w	water
sa	supply air
oa	outside/weather
I	solar radiation

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