Parcial 1: Conceptos Básicos y serie de Fourier SyS

① $x(t) = 20 \operatorname{sen}\left(7t - \frac{\pi}{2}\right) - 3\cos(5t) + 2\cos(10t)$

R=// identidad

$\operatorname{sen}(x) = \cos\left(x - \frac{\pi}{2}\right) \Rightarrow \cos(x) = \operatorname{sen}\left(x + \frac{\pi}{2}\right) \Rightarrow -\cos(x) = \operatorname{sen}\left(x - \frac{\pi}{2}\right)$

con esto

$\Rightarrow -20\cos(7t) - 3\cos(5t) + 2\cos(10t) = x(t)$

Rango de $x(t) = |x(t)| = 20 + 3 + 2 = 25 \to -25V$ a $25V$

Rango del microprocesador $\Rightarrow -3{,}3V$ a $5V$

- La señal acondicionada al microprocesador va a ser $X_{ac}(t)$

$$X_{ac}(t) = a \cdot x(t) + b$$

$$\begin{cases} -3{,}3 = a(-25) + b \\ 5 = a(25) + b \end{cases} \Rightarrow 5 - (-3{,}3) = 25a + b - (-25a + b)$$

$\to 8{,}3 = 50a \to a = \frac{8{,}3}{50} = 0{,}166$

- $5 = 0{,}166(25) + b \Rightarrow 5 = 4{,}15 + b \Rightarrow b = 5 - 4{,}15 = 0{,}85$

$X_{ac}(t) = 0{,}166 \cdot x(t) + 0{,}85$ (acondicionamiento)

- Para la Digitalización

5 bits $\Rightarrow (2)^5 = 32$ niveles de la señal (cuantización)

mínimo cambio detectable $\Rightarrow \frac{V_{max} - V_{min}}{2^n - 1}$ con n = bits

- $\frac{5 - (-3{,}3)}{2^5 - 1} = \frac{8{,}3}{32 - 1} \to \frac{8{,}3}{31} = 0{,}26774$

- Muestreo (discretización)

las frecuencias son 5, 7 y 10

$\omega_1 = 5 \rightarrow f_1 = \left(\frac{5}{2\pi}\right) = 0{,}7957\ Hz \rightarrow T_1 = 1{,}257\ s$

$\omega_2 = 7 \rightarrow f_2 = (7/2\pi) = 1{,}114\ Hz \rightarrow T_2 = 0{,}897\ s$

$\omega_3 = 10 \rightarrow f_3 = (10/2\pi) = 1{,}5915\ Hz \rightarrow T_3 = 0{,}628\ s$

- Teorema de Nyquist $\rightarrow f_s \geq 2 f_{max}$

$$f_s \geq 2(1{,}5915) \rightarrow f_s \geq 3{,}183\ Hz$$

la frecuencia de muestreo debe ser 3,183 Hz ó más.

$$T_s = \frac{1}{f_s} \rightarrow T_s = 0{,}31416\ s$$

2.

$$x(t) = 3\cos(1000\pi t) + 5\,sen(2000\pi t) + 10\cos(11000\pi t)$$

$f_s = 5\,kHz$ $\quad t = T_s \rightarrow T_s = \frac{1}{f_s} \rightarrow T_s = \frac{1}{5000}$

$$x(n) = x(nT_s) = x\left(\frac{n}{5000}\right)$$

$$x(n/f_s) = 3\cos\left(1000\pi \cdot (n/5000)\right) + 5\,sen\left(2000\pi \cdot (n/5000)\right) + 10\cos\left(11000\pi (n/5000)\right)$$

$$x(n/f_s) = 3\cos\left(\pi \cdot \frac{n}{5}\right) + 5\,sen\left(\frac{2\pi}{5} \cdot n\right) + 10\cos\left(\frac{11\pi}{5} \cdot n\right)$$

$\omega_1 = 1000\pi \rightarrow f_1 = 500\ Hz$

$\omega_2 = 2000\pi \rightarrow f_2 = 1000\ Hz$

$\omega_3 = 11000\pi \rightarrow f_3 = 5500\ Hz$

del teorema de Nyquist tenemos que $f_s \geq f_{max}$, en este caso

$fs = 5000\ Hz$ y $f_{max} = 5500\ Hz$ por lo que no se cumple el teorema y hay aliasing. en el componente 3.

para resolver el aliasing usamos $f_{alias} = |f - N \cdot fs|$
con N como el entero más cercano a f/fs

$$N = \frac{5500}{5000} = 1 \rightarrow f_{alias} |5500 - 1 \cdot 5000| = \underline{500\ Hz}$$

entonces $10\cos(11000\pi t) \Rightarrow 10\cos(2\pi \cdot 500 n Ts)$

$$= 10\cos\left(2\pi \cdot \frac{500}{5000} n\right) = \underline{10\cos\left(\frac{\pi}{5} n\right)}$$

la señal quedaría

$$x(n/fs) = 3\cos\left(\frac{\pi}{5} n\right) + 5 sen\left(\frac{2\pi}{5} \cdot n\right) + 10\cos\left(\frac{\pi}{5} n\right)$$

$$\underline{x(n/fs) = 13\cos\left(\frac{\pi}{5} n\right) + 5 sen\left(\frac{2\pi}{5} \cdot n\right)} \Rightarrow$$ señal obtenida en tiempo discreto con un conversor análogo digital con $fs = 5kHz$

3.

$$d(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

$$x_1(t) = A\cos(w_0 t), \quad w_0 = \frac{2\pi}{T}$$

$$x_2(t) = \begin{cases} 1 & 0 \leq t < \frac{T}{4} \\ -1 & \frac{T}{4} \leq t < \frac{3T}{4} \\ 1 & \frac{3T}{4} \leq t < T \end{cases}$$

• Distancia media entre señales

R=// Dividimos la integral en 3 partes según $X_2(t)$:

$$d(x_1,x_2)=\frac{1}{T}\left[\int_0^{T/4}(A\cos(w_0t)-1)^2dt+\int_{T/4}^{3T/4}(A\cos(w_0t)+1)^2dt+\right.$$

$$\left.+\int_{3T/4}^{T}(A\cos(w_0t)-1)^2dt\right]$$

ya que en los intervalos $[0,T/4]$ y $[3T/4,T]$ son iguales por simetría.

$$d(x_1,x_2)=\frac{1}{T}\left[2\cdot\int_0^{T/4}(A\cos(w_0t)-1)^2dt+\int_{T/4}^{3T/4}(A\cos(w_0t)+1)^2dt\right]$$

• $(A\cos(w_0t)\pm1)^2=A^2\cos^2(w_0t)\pm2A\cos(w_0t)+1$

$$\rightarrow d(x_1,x_2)=\frac{1}{T}\left[2\underbrace{\int_0^{T/4}(A^2\cos^2(w_0t)-2A\cos(w_0t)+1)\,dt}_{I_1}+\underbrace{\int_{T/4}^{3T/4}(A^2\cos^2(w_0t)+2A\cos(w_0t)+1)\,dt}_{I_2}\right]$$

• sabemos que

$$\cos^2(w_0t)=\frac{1+\cos(2w_0t)}{2}$$

• $\int\cos(w_0t)\,dt=\frac{1}{w_0}\,sen(w_0t)$

• $I_1=\int_0^{T/4}(A^2\cos^2(w_0t)-2A\cos(w_0t)+1)\,dt$

- $\int_0^{T/4}A^2\cos^2(w_0t)=\int_0^{T/4}A^2\,\frac{1+\cos(2w_0t)}{2}\,dt=\frac{A^2}{2}\int1+\cos(2w_0t)\,dt$

$$\int_0^{T/4}\cos(2w_0t)\,dt=\left[\frac{sen(2w_0t)}{2w_0}\right]_0^{T/4}=\frac{1}{2w_0}\left(sen(2(T/4)(t)-sen(0)\right)$$

$\rightarrow \omega_0 = 2\pi/T \rightarrow \frac{1}{2\omega_0}\left(sen\left(\frac{T}{2}\cdot\frac{2\pi}{T}\right) - sen(0)\right) = \underline{0}$

$\rightarrow \int_0^{T/4} A^2\cos^2(\omega_0 t) = \frac{A^2}{2}\cdot\frac{T}{4} = \underline{A^2\frac{T}{8}}$

$\rightarrow \int_0^{T/4} 2A\cos(\omega_0 t) = 2A\left|\frac{1}{\omega_0}sen(\omega_0 t)\right|_0^{T/4} = 2A\left(\frac{T}{2\pi}sen\left(\frac{2\pi}{T}\frac{T}{4}\right)\right)$

$= 2A\left(\frac{T}{2\pi}sen\left(\frac{\pi}{2}\right)\right) = \underline{A\frac{T}{\pi}}$

$\cdot\, I_1 = A^2\frac{T}{8} - A\frac{T}{\pi} + \frac{T}{4} \rightarrow 2I_1 = 2A^2\frac{T}{8} - 2A\frac{T}{\pi} + 2\frac{T}{4}$

$2I_1 = \underline{A^2\frac{T}{4} - 2A\frac{T}{\pi} + \frac{T}{2}}$

$\cdot\, I_2 = \int_{T/4}^{3T/4}\left(A^2\cos^2(\omega_0 t) + 2A\cos(\omega_0 t) + 1\right)dt$

$\cdot\, I_2 = \frac{1}{2\omega_0}\left(sen\left(\frac{2\pi}{T}\left(\frac{3T}{4}\right)\right) - sen\left(\frac{2\pi}{T}\left(\frac{T}{4}\right)\right)\right) + 2A\left(\frac{T}{2\pi}\left(sen\left(\frac{2\pi}{T}\left(\frac{3T}{4}\right)\right) - sen\left(\frac{2\pi}{T}\left(\frac{T}{4}\right)\right)\right)\right)$

$I_2 = \frac{T}{4\pi}\left(sen\left(\frac{3}{2}\pi\right) - sen\left(\frac{\pi}{2}\right)\right) + 2A\left(\frac{T}{2\pi}\left(sen\left(\frac{3}{2}\pi\right) - sen\left(\frac{\pi}{2}\right)\right)\right)$

$I_2 = \frac{-2T}{4\pi} + 2A\left(\frac{T}{2\pi}(-2)\right) \rightarrow + \frac{A^2T}{2\pi} + \left(-2A\frac{T}{\pi}\right) + \frac{T}{2}$

$\cdot\, I_2 = \underline{\frac{A^2}{2}\cdot\frac{T}{2} - 2A\frac{T}{\pi} + \frac{T}{2}}$

$$d(x_1, x_2) = \frac{1}{T}\left(2I_1 + I_2\right)$$ $\rightarrow$ como $2I_1 = I_2 \Rightarrow 2I_1 + I_2 = 2I_2$

$$d(x_1, x_2) = \frac{1}{T}\left(\left(A^2\frac{T}{4} - 2A\frac{T}{\pi} + \frac{T}{2}\right)2\right)$$

$$d(x_1, x_2) = \frac{1}{T}\left(2A^2\frac{T}{4} - 4A\frac{T}{\pi} + \frac{2T}{2}\right)$$

$$d(x_1, x_2) = \frac{1}{T}\left(T\left(A^2/2 - \frac{4A}{\pi} + 1\right)\right)$$

$$d(x_1, x_2) = \frac{A^2}{2} - \frac{4A}{\pi} + 1$$ = distancia media entre señales

4. $$c_n = \frac{1}{T}\int_{t_i}^{t_f} f(t)e^{-jn\omega_0 t}dt \rightarrow x(t) = \sum_{n} c_n e^{jn\omega_0 t}$$

$$x'(t) = \frac{d}{dt}x(t) = \frac{d}{dt}\left\{\sum_n c_n e^{jn\omega_0 t}\right\} = \sum_n c_n e^{jn\omega_0 t} jn\omega_0$$

$$x''(t) = \frac{d}{dt}\left\{\sum_n c_n e^{jn\omega_0 t}(jn\omega_0)\right\} = \sum_n c_n e^{jn\omega_0 t}(jn\omega_0)^2$$

$$\tilde{c}_n = \frac{(x''(t), e^{jn\omega_0 t})}{\|e^{jn\omega_0 t}\|^2} = \int_{t_i}^{t_f} \frac{x''(t)e^{-jn\omega_0 t}}{T}dt \; ; \; T = t_f - t_i$$

$$\tilde{c}_n = c_n(jn\omega_0)^2 = \int_{t_i}^{t_f}\frac{x''(t)e^{-jn\omega_0 t}}{T}dt$$

$$c_n = \frac{1}{(t_f - t_i)(jn\omega_0)^2}\int_{t_i}^{t_f} x''(t)e^{-jn\omega_0 t}dt = \frac{1}{(t_i - t_f)n^2\omega_0^2} \times \int_{t_i}^{t_f} x''(t)e^{-jn\omega_0 t}dt$$

$$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(n w_0 t) + b_n \operatorname{sen}(n w_0 t)$$

$$x'(t) = \sum_{n=1}^{N} a_n (-n w_0) [\operatorname{sen}(n w_0 t)] + b_n (n w_0)(\cos(n w_0 t))$$

$$x''(t) = \sum_{n=1}^{N} a_n (-n w_0)(n w_0) \cos(n w_0 t) + b_n (n w_0)(-n w_0) \operatorname{sen}(n w_0 t)$$

$$\tilde{a}_n = \frac{2}{T} \int_{ti}^{tf} x''(t) \cos(n w_0 t)\, dt \; ; \; \tilde{b}_n = \frac{2}{T} \int_{ti}^{tf} x''(t) \operatorname{sen}(n w_0 t)\, dt$$

$$a_n (-n^2 w_0^2) = \frac{2}{T} \int_{ti}^{tf} x''(t) \cos(n w_0 t)\, dt$$

$$a_n = \frac{2}{-T n^2 w_0^2} \int_{ti}^{tf} x''(t) \cos(n w_0 t)\, dt$$

$$b_n = (-n^2 w_0^2) = \frac{2}{T} \int_{ti}^{tf} x''(t) \operatorname{sen}(n w_0 t)\, dt$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt = \frac{1}{t} \int_{-d2}^{-d1} \frac{A}{d2-d1}(t+d2)\, dt + \frac{1}{T} \int_{-d_1}^{d1} 4\, dt \; +$$

$$+ \frac{1}{T} \int_{d1}^{d2} -\frac{A}{d2-d1} (t-d2)\, dt$$

$$= \frac{1}{T} \left[\frac{A}{d2-d1} \left(\left(\frac{t^2}{2} \right) + d2\, t \right) \Big|_{-d2}^{-d1} + At \Big|_{-d_1}^{d1} - \frac{A}{d2-d1} \right.$$

$$\left. \left(\frac{t^2}{2} - d2\, t \right) \Big|_{d1}^{d2} \right] = \frac{1}{T} \left[\frac{A}{d2-d1} \left(\frac{d1^2}{2} - d1 d2 - \frac{d2^2}{2} + d2 d2 \right) + A(d1+d1) - \right.$$

$$\left. - \frac{A}{d2-d1} \left(\frac{d2^2}{2} - d2 d2 - \frac{d1^2}{2} + d1 d2 \right) \right]$$

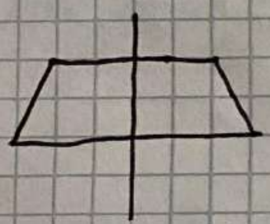
$$= \frac{1}{T}\left[\frac{2A}{d_2-d_1}\left(\frac{d_1^2}{2} - d_1 d_2 - \frac{d_2^2}{2} + d_2^2\right) + 2Ad_1\right]$$

si $A=1$, $d_1=1$, $d_2=2$, $T=2$, $d_2=4$

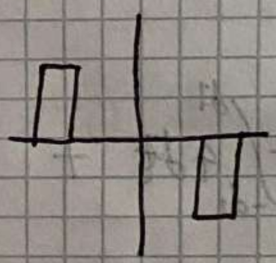
$$c_n = \frac{1}{\frac{2n^2\pi^2}{2}}\left(\cos cn\frac{2\pi}{4}\cdot 2\right) - \left(\cos cn\frac{2\pi}{4}\cdot 1\right)$$

$$b_n = \frac{2}{-Tn^2\omega_0^2}\int_{ti}^{tf} x''(t)\, sen(n\omega_0 t)\, dt$$

encontramos el espectro de fourier

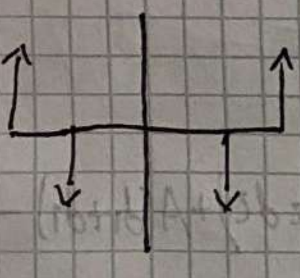


$$x''(t) = A\delta(t+d_2) - A\delta(t+d_1) - A\delta(t-d_1) + A\delta(t-d_2)$$



$$c_n = \frac{1}{-Tn^2\omega_0^2}\int_{-T/2}^{T/2} x''(t)e^{-jn\omega_0 t}\, dt$$

$$c_n = -\frac{1}{-Tn^2\omega_0^2}\int_{-T/2}^{T/2} A[\delta(t+d_2) - \delta(t+d_1) - \delta(t-d_1) + \delta(t-d_1)]\, e^{-jn\omega_0 t}\, dt$$



$$c_n = \frac{-A}{Tn^2\omega_0^2}\left(e^{-jn\omega_0(-d_2)} - e^{-jn\omega_0(-d_1)} - e^{-jn\omega_0 d_1} + e^{-jn\omega_0 d_2}\right)$$

$$c_n = \frac{-A}{Tn^2\omega_0^2}\left(e^{jn\omega_0 d_2} + e^{-jn\omega_0 d_2} - \left(e^{jn\omega_0 d_1} + e^{-jn\omega_0 d_1}\right)\right)$$

$$c_n = -\frac{A}{Tn^2\omega_0^2}\left(2\cos(n\omega_0 d_2) - 2\cos(n\omega_0 d_1)\right) \Rightarrow \frac{-2A}{Tn^2\frac{4\pi^2}{T_2}}$$

$$\left(\cos\left(n\frac{2\pi}{T}d_1\right) - \cos\left(n\frac{2\pi}{T}d_1\right)\right)$$

$$c_0 = \frac{1}{4}\left[\frac{2\cdot 1}{1}\left(\frac{1}{2}-2-2+4\right)+2\cdot 1\cdot 1\right] = \frac{1}{4}\left[2\left(\frac{1}{2}\right)+2\right] = \frac{3}{4}$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2}(f(t))^2\,dt = \frac{2}{T}\int_{-T/2}^{0}(x(t))^2\,dt = \frac{2}{t}\int_{-d_2}^{d_1}\left(\frac{A}{d_2\cdot d_1}\right)^2$$

$$(t+d_2)^2\,dt + \frac{2}{t}\int_{-d_1}^{0}A^2\,dt \rightarrow P_x = \frac{2}{t}\left(\frac{A}{d_2 d_1}\right)^2$$

$$[t^2+2td_2+d_2^2]\Big|_{-d_2}^{-d_1} + \frac{2}{T}A^2 t\Big|_{-d_1}^{0}$$

$$P_x = \frac{2}{T}\left(\frac{A}{d_2-d_1}\right)^2(d_1^2-2d_2d_1+d_2^2-d_2^2+2d_2^2-d_2^2)+\frac{2}{T}A^2(0+d_1)$$

$$P_x = \frac{1}{6}+\frac{1}{2} = \frac{2}{3}$$