Contents

0.1 Matrix Definitions and Operations

Definition 1.1 (Matrix). A real-valued (m, n) matrix A with $m, n \in \mathbb{N}$ is an $m \cdot n$-tuple of elements a_{ij}, where $i = 1, \ldots, m$ and $j = 1, \ldots, n$, arranged in a rectangular pattern with m rows and n columns:

$$
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
$$

Matrix addition is an operation performed on two matrices of the same size. Given matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{m \times n}$, adding the matrices together involves adding their corresponding elements (Element-Wise):

$$
A + B = \begin{bmatrix}
a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\
a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn}
\end{bmatrix}
$$

Matrix multiplication is performed between two matrices, $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times k}$, and results in $C = AB \in \mathbb{R}^{m \times k}$. The product C is calculated by taking the dot product of each row of matrix A with each column of matrix B. We sum the products of the multiplication to obtain an entry in the corresponding row i and column j of C.

$$
c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}
$$

- a_{ik} is the element at row i and column k of matrix A,

- b_{kj} is the element at row k and column j of matrix B,

- n is the number of columns in matrix A (which is also the number of rows in matrix B).

such that

$$
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
a_{31} & a_{32}
\end{bmatrix} \in \mathbb{R}^{3 \times 2}, \quad
B = \begin{bmatrix}
b_{11} & b_{12} & b_{13} \\
b_{21} & b_{22} & b_{23}
\end{bmatrix} \in \mathbb{R}^{2 \times 3}
$$

$$
C = \begin{bmatrix}
a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\
a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\
a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23}
\end{bmatrix}
$$

Definition 1.2 (Identity Matrix). A matrix in $\mathbb{R}^{n \times n}$ that contains only 1's on its diagonal entries.

$$
I_n := \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}
$$

Definition 1.3 (Linear Independence). In a linearly independent matrix, each column holds the property that it cannot be expressed by performing linear combinations of the other columns. i.e. if a matrix has 3 columns, you cannot multiply columns 2 or 3 by any value $c \in \mathbb{R}$ such that the columns would be the same as column 1 and vice versa. It also holds that adding two columns together with the ability to scale them also cannot equal another column in the matrix.

The formal definition states: A set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ in a vector space V over a field F is said to be linearly independent if the only scalars $c_1, c_2, \ldots, c_n$ satisfying the equation

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \ldots + c_n \mathbf{v}_n = \mathbf{0}$$

the only solution to this should be $c_1 = c_2 = \ldots = c_n = 0$. In other words, the only way to obtain the zero vector $\mathbf{0}$ as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ is by setting all the scalar coefficients to zero.
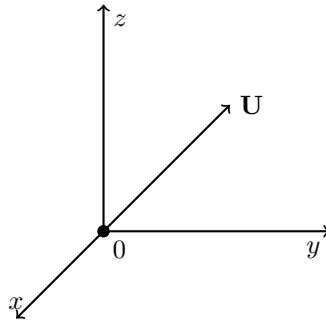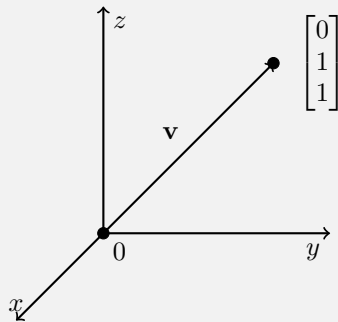
0.2   3D Vector Space Representation

Figure 1: A vector in three-dimensional space. Starting at fixed point 0, the vector v represents the point **U**.

EXAMPLE

Notice that the 1st value in the vector (representing x), $v \in \mathbb{R}$ is 0 so the vector stays on the z and y axis.

Definition 1.3 (Commutative Property). In quantum computing, operations on qubits are represented

by matrices called quantum gates.

$$T_1 \otimes T_2 = \begin{bmatrix} t_{11} \cdot T_2 & t_{12} \cdot T_2 & \cdots & t_{1n} \cdot T_2 \\ t_{21} \cdot T_2 & t_{22} \cdot T_2 & \cdots & t_{2n} \cdot T_2 \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} \cdot T_2 & t_{m2} \cdot T_2 & \cdots & t_{mn} \cdot T_2 \end{bmatrix} \tag{1}$$

$T_1 \otimes T_2 = T_2 \otimes T_1$

0.3   Tensors

0.4