Quantum Walk: A Different Approach to Searching Solution Spaces

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Abstract

Search problems involve taking a computational task that allows all the solutions to be mapped to a graph and traversed. The search for an optimized way to traverse the solution space/graph is a big field of study for its vast applications in many other fields. The proposed project aims to demonstrate the results obtained by using the Quantum Walk (QW), which is a quantum approach to searching solution spaces. A classical random walk does not have the same ability to search a space as a quantum walk because it cannot leverage the properties of superposition, entanglement, and interference. As a result, we aim to find good solutions that can be benchmarked against the classical results in [3]. We map the max-cut problem as an instance of the Ising model and try to find the ground state of the Hamiltonian, i.e. the near optimal configuration for the max-cut. By benchmarking the results of the quantum and classical walks, we hope to provide evidence of the quantum walk's practicality. We will use our expertise in qiskit and Dirac notation learned throughout the course to derive the mathematical differences in the probability distribution of classical vs. quantum walk positions and to make an implementation of the QW.

1 Background

1.1 Unweighted Max-cut as Ising Model:

For the max-cut problem we define an undirected graph G = (V, E) where we seek to determine a bipartition of vertices V into two sets V_1, V_2 to maximize the number of edges between the two sets (the cut size). Figure 1 is an example maxcut [2].

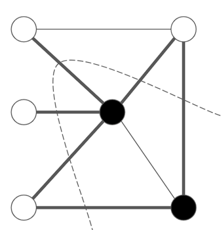

Figure 1: Partitions the black and white nodes into 2 sets [2].

To map the max-cut as an instance of the Ising model, each node contains $\sigma_i \in \{+1, -1\}$ (spin up or spin down) thereby determining the subset the vertices are in. The Ising model requires that a Hamiltonian be defined to evaluate the performance of the given max-cut configuration [2].

1.2 Classical Random Walks:

Before we go into quantum walks, it is good to understand how a classical system would traverse a graph, the intuition is that a particle transitions between vertices. A particle starts out in some initial position (vertex) on a graph G = (V, E) and transitions to neighboring vertices probabilistically, e.g. a pseudo-random number generator or a stochastic matrix for time evolution. The following equation portrays mathematically how a particle's position would be updated at each iteration according to a stochastic matrix [1, 4].

$$\mathbf{p}(t+1) = M\mathbf{p}(t) \tag{1}$$

Discrete-time classical walk: vector $\mathbf{p}$ is updated via stochastic matrix M.

The result after many iterations is a Gaussian distribution where the standard deviation increases the greater t is.

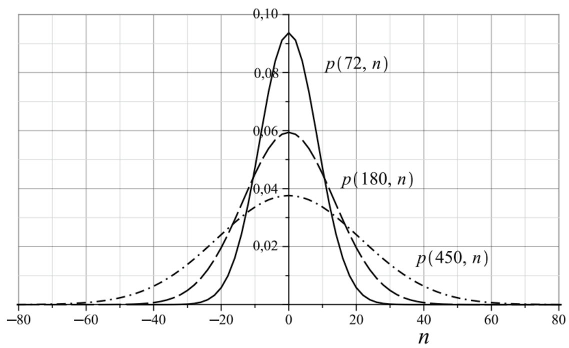

Figure 2: Probability distribution of classical random walk [9].

1.3 Continuous-Time Quantum Walk:

By turning the Eq. (1) into a differential equation we derive a variation of Schrödinger's equation, thereby making this continuous.

$$\frac{d\mathbf{p}(t)}{dt} = -H\mathbf{p}(t) \tag{2}$$

Excludes $\hbar$ and i and H is the Hamiltonian.

The solution to this differential equation is $\mathbf{p}(t) = e^{Ht}\mathbf{p}(0)$. From here we can get the following unitary evolution

$$U(t) = e^{iHt} \tag{3}$$

Time evolution operator: defining a spectrum of t's [1].

1.4 Discrete-Time Quantum Walk:

This approach uses a coin followed by a shift operator to transition between nodes given a position $|n\rangle$. An example of an unbiased coin (not skewed) would be the Hadamard gate since it gives equal probability to the $|0\rangle$ and $|1\rangle$ states. Here is an example shift gate that moves the initial position n

$$S|0\rangle|n\rangle = |0\rangle|n+1\rangle,$$
$$S|1\rangle|n\rangle = |1\rangle|n-1\rangle.$$

These operations are combined into the unitary operator U^t where t is the total amount of time iterations. The probability distribution in this case is not Gaussian rather it is a more volatile superposition of all the previous states.

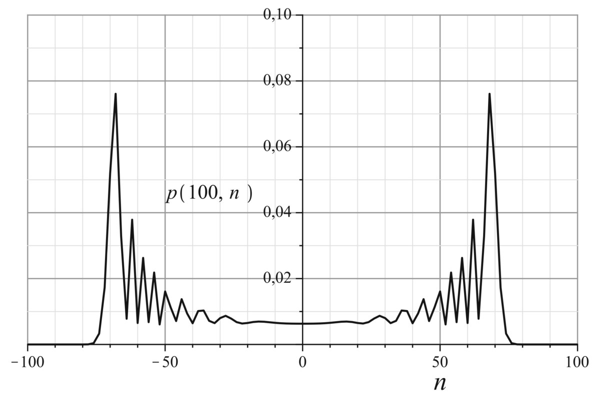

Figure 3: Probability distribution of Hadamard quantum walk [9].

2 Motivation

Our team is motivated by the prospect of demonstrating a deep understanding of the QW and giving an example of its usage. The QW is an important algorithm to understand as it can be used to simulate complex physical systems and create new quantum algorithms [9]. Our work could be developed into a workshop that fosters learning within the academic community. If the results are not as we expect we will investigate where mistakes were made and record them. We will present our findings in the last few days of this course and hope that students are able to learn about the viability of the QW for searching through solution spaces.

3 Tentative Plan

3.1 Research

To understand how to formulate our problem more clearly we will read Lucas. A [8], which contains details on Ising formulations specifically for NP complete problems (max-cut). We will continue reading Portugal, R. [9] and using it as a reference for the theoretical aspects of the quantum walk, such as probability distributions and formulas. We have found a few quantum walk implementations [1][6][7] and plan of find more to see what the similarities are in implementation. These research steps will help us formulate our problem and gain confidence in implementing our own quantum walk for a max-cut-specific use case.

3.2 Implementation

The implementation on [7] comes directly from the creators of Qiskit Community and performs a walk on a 1-D graph. This is a perfect introduction to the QW and will lay the foundation for the rest of our work. The implementation on [6] is especially useful because the algorithm performs a walk on a 3-dimensional cube, which will give us intuition on increasing the search space of our QW. [1] contains an older but still useful continuous-time implementation and the accompanying mathematics. We will get all the implementations working and compare them side by side to then create our own that traverses a graph of maxcut configurations. We will start by simply and iteratively adjusting our program based on the results.

3.3 QAOA Collaboration

We will share our findings and implementation with the group working on finding the ground state of the Ising model using the Quantum Approximate Optimization Algorithm (QAOA). We will benchmark our results against theirs as our problem is also equivalent to finding the ground state of the Ising model. Comparing results will give us useful intuitions about each algorithm.

3.4 Documentation

We will document the similarities we notice in the QW implementations from [1][6][7]. As we implement our own QW we will note down our mistakes and what we learn throughout the process. Finally, we will benchmark our results with the results from the classical implementation found in [3].

4 Expected Results & Take-Home Message

We expect to get similar if not better results than classical random walks. The properties of the QW will allow us to escape local minima that solution spaces tend to have, which can yield better solutions. If the results are as we expect then our work serves as evidence that the quantum walk is a viable approach to searching solution spaces.

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