Quantum Walk: A Different Approach to Searching Solution Spaces

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Abstract

Search problems involve taking a computational task for which all the solutions can be mapped to a graph and traversed. The search for an optimized way to traverse the solution space/graph is an important field of study for its vast applications in many other fields. This project demonstrates the results obtained by using a Quantum Walk to traverse a solution space for the max-cut problem (QW). QW's leverage the properties of superposition, entanglement, and interference to traverse solution spaces differently than classical walks. As a result, we obtain different probability distributions and higher standard deviation growth from the origin [8]. In practice (using small three and five-node graphs), we get the expected distribution of states but continue looking for methods to favor binary sequences of high cut values. Finally, we create an informative presentation, accompanied by an implementation with slight heuristic adjustments.

1 The Problem (Background)

1.1 Unweighted Max-cut

For the max-cut problem we define an undirected graph G = (V, E) where we seek to determine a bipartition of vertices V into two sets V_1, V_2 to maximize the number of edges between the two sets (the cut size). Figure 1 is an example maxcut [2].

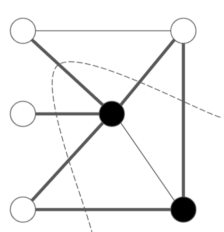

Figure 1: Partitions the black and white nodes into 2 sets [2].

Each solution is represented as a binary sequence of length N-nodes. The cut value is evaluated using the following equation:

$$\text{Cut}(G) = \frac{1}{2} \sum_{(i,j)\in E} (1 - \sigma_i \sigma_j)$$

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1.2 Classical Random Walks:

Before we go into QWs, it is good to understand how a classical system would traverse a graph. The intuition is that a particle transitions between vertices starting out in some initial position (vertex) on a graph G = (V, E) and transitions to neighboring vertices probabilistically, e.g. a pseudo-random number generator or a stochastic matrix for time evolution. The following equation portrays mathematically how a particle's position would be updated at each iteration according to a stochastic matrix [1, 4].

$$\mathbf{p}(t+1) = M\mathbf{p}(t) \tag{1}$$

Discrete-time classical walk: vector $\mathbf{p}$ is updated via stochastic matrix M.

This approach tends to get stuck at local maxima and only evaluates one solution at a time.

2 Motivation

Discrete-time QW's maintain a superposition of states starting at the origin and expanding with every unitary operator that is applied to the current state. Depending on the coin used, applying it can flip the phase of a qubit and cause another state to collapse via destructive interference. In contrast, the classical approach transitions between states, one by one, and cannot be efficiently used to see a bigger picture within the solution space. Thus, it is interesting to ask if adjustments can be made to a QW circuit such that high probabilities are attributed to max-cut solutions with higher cuts. Our team is motivated to leverage these properties of quantum states to create such a circuit.

3 Methodology: Discrete-Time Quantum Walk

Given an initial position $|n\rangle$, the Discrete-Time QW uses a coin followed by a shift operator to transition between nodes. A typical coin (not skewed) would be the Hadamard gate since it gives equal probability to the $|0\rangle$ and $|1\rangle$ states. Here is an example shift gate that moves the nth position n

$$\begin{aligned} S|0\rangle|n\rangle &= |0\rangle|n+1\rangle, \\ S|1\rangle|n\rangle &= |1\rangle|n-1\rangle. \end{aligned}$$

These operations are combined into the unitary operator U^t where t is the total amount of time iterations. The probability distribution in this case is not Gaussian like the classical approach, it is a more volatile superposition of all the previous states.

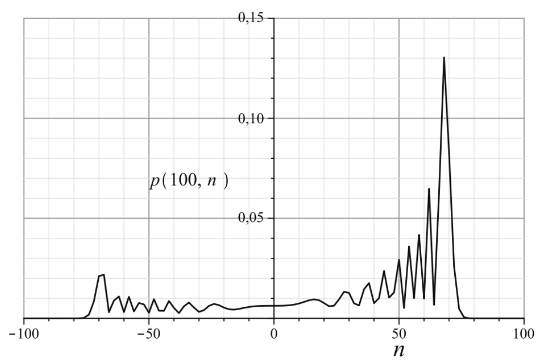


Figure 2: Probability distribution of Hadamard quantum walk [9].

4 Implementation

We used the 1-D QW implementation in [7] from the qiskit community. The code was out of date and required adapting to the most recent version of qiskit. We used this new version to implement the quantum walk on a simple five-node graph. This involved applying Hadamard gates followed by the shift operator. The shift operation was separated into an increment and decrement set of gates to increment n in the system. These gates generalize to U^t, which we applied between five to twelve steps adjusting heuristically to optimize the outcomes. Some other heuristic improvement involved applying gates in between the shift operations, specifically controlled Z-gates.

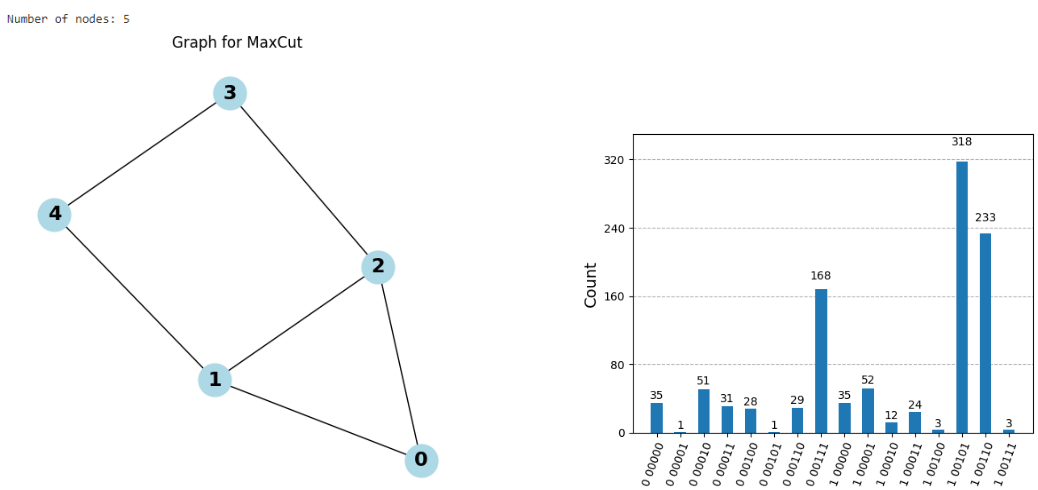
link: https://colab.research.google.com/drive/1nkfcaAJ7gNTaxkeanalJKvniF7-lcKJ3?usp=sharing

5 Milestone

Our target was to achieve specific bit sequences that would result in the highest cut values. The target bit sequences we looked for were $\{00101, 11010, 01010, 10101\}$. The QW initially does not favor any solution over another, making heuristic adjustments necessary. We were only able to find one of the four optimal solutions with a cut of five on the five-node graph. More experimentation is required to find gate placements and additions that favor better cut solutions.

6 Deliverables

The final deliverable of this project includes the Qiskit code formulation of the Discrete-time QW and heuristic adjustments. Additionally, we created an introductory presentation detailing the max-cut problem, classical limitations for searching solution spaces, and example coin and shift applications using Dirac notation. We discovered that between five and twelve iterations, the unitary operation (U^t) produced good results that ran within a few seconds. The probability distribution we obtained aligns with the expected distribution shown in Figure 2 except for the center value. This is likely due to the heuristic adjustments made to the circuit.



(a) five-node graph instance

(b) Results on a five-node graph

Figure 3: Comparison of results on a five-node graph.

7 Expected Results & Take-Home Message

More experimentation is required to create a circuit that collapses predominantly to solutions of high cut values. These results defy our initial expectations for the QW. Precisely because of the misconception that we already know something about the optimal solutions or that classical adjustments can be made so that the circuit gives preference to better cut solutions. The challenge is that the QW is in many states at a time, and one cannot simply measure midway through the circuit to see how the circuit is performing because the Qubits would collapse upon measurement. The differences between the QW and the classical random walk, present a unique opportunity for experimentation and discovery, considering that much more work can be done to improve how solution spaces are searched.

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