Quantum Walk

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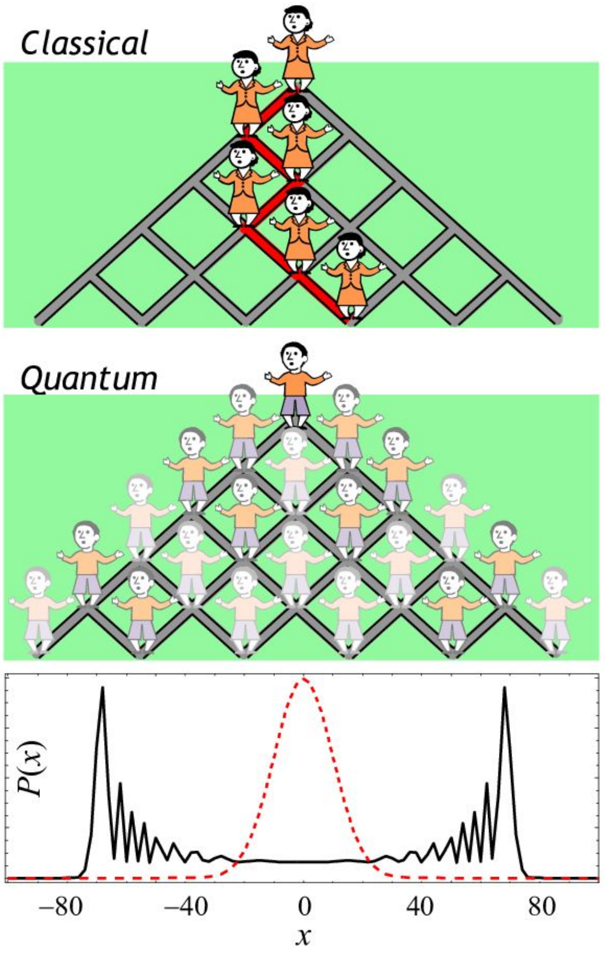
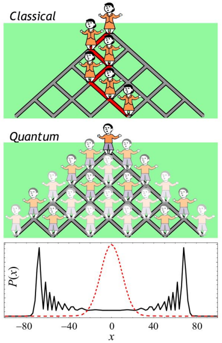
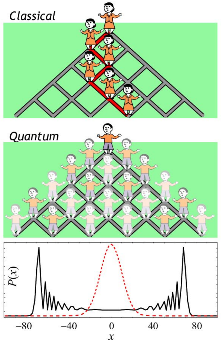


https://www.researchgate.net/figure/A-comparison-of-quantum-vs-classical-random-walk-A-classical-random-walk-leads-to-a_fig3_23418414

Motivation

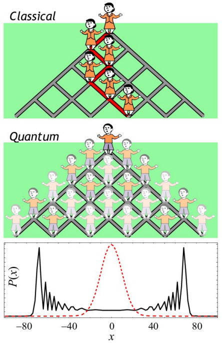
- Gaining a deep understanding and giving example usage.
- Demonstrate what was learned in this course.
- QW is a powerful tool for creating new algorithms and simulating complex physical systems.

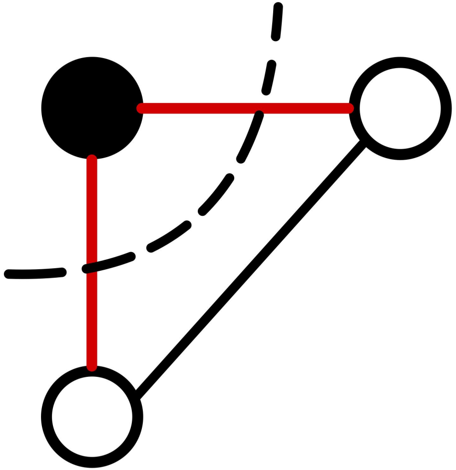


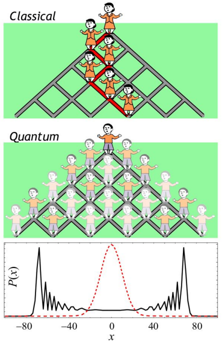


The Problem: Maxcut

- A Classically NP-Hard problem (O(2n)).
- Given graph G = (V, E), connected vertices with differing values add 1 to the cut.
- We want a solution with the most amount of cuts.

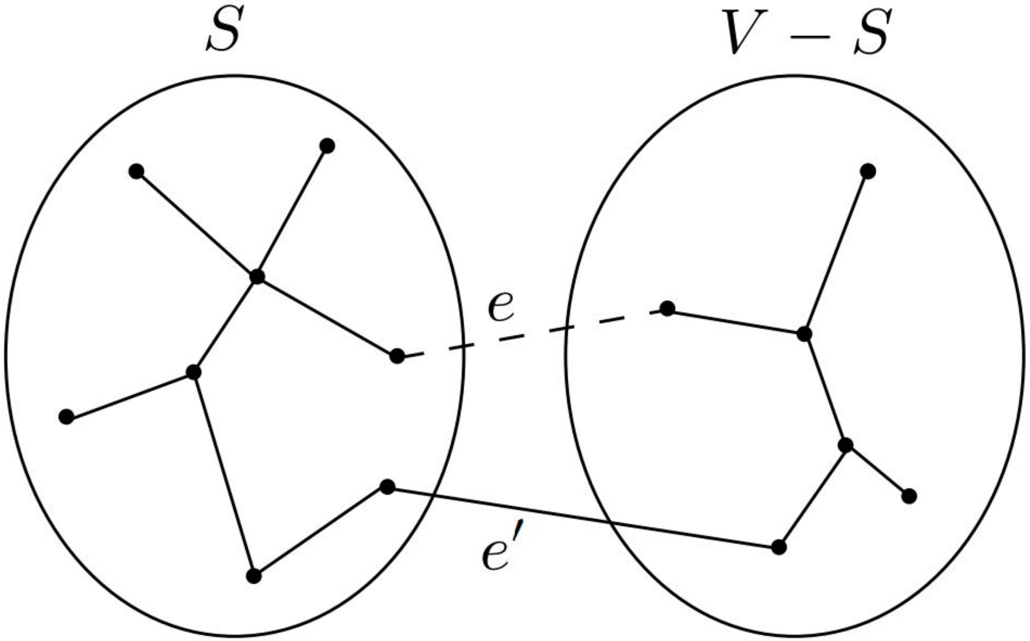






[2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem"

Another Perspective



We separate the vertices of a given graph into 2 sets, S and V and we want to maximize the # of edges between them.

Algorithms, algorithmics.lsi.upc.edu/docs/Dasgupta-Papadimitriou-Vazirani.pdf. Accessed 16 Aug. 2024.

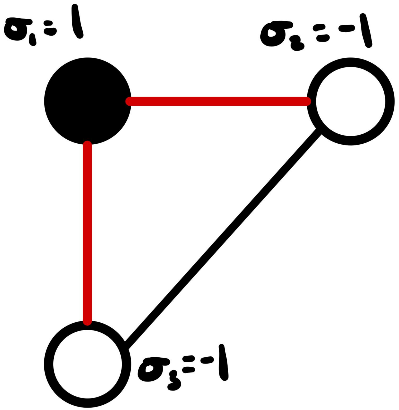
Evaluating the Configuration

- **Define a formula for the cut value**

$$\mathrm{Cut}(G) \quad = \quad \frac{1}{2} \sum_{(i,j)\in E} (1 - \sigma_i \sigma_j)$$

[2] A. Jin and X. -Y. Liu, "A Fast Machine Learning Algorithm for the MaxCut Problem"

Example: Using Ising



$$H(\boldsymbol{\sigma}) = -\sum_{(i,j)\in E} \sigma_i \sigma_j$$

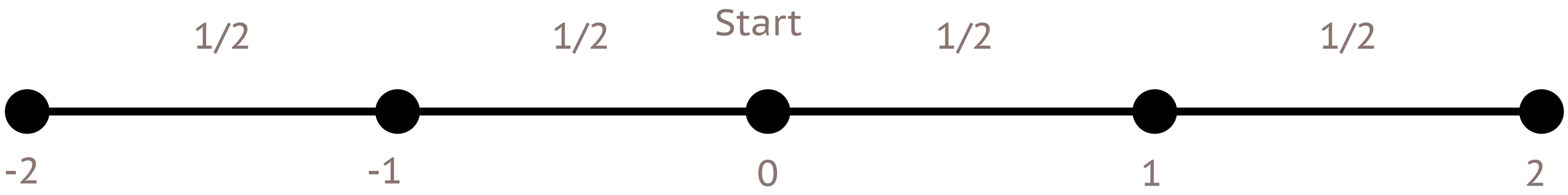
$$\mathrm{Cut}(G) = \frac{1}{2}|E| - \frac{1}{2}H(\boldsymbol{\sigma})$$

σ1σ2 = (1)(-1) = -1
σ2σ1 = (-1)(1) = -1
σ2σ3 = (-1)(-1) = 1
σ3σ2 = (-1)(-1) = 1
σ3σ1= (-1)(1) = -1
σ1σ3= (1)(-1) = -1
⇒ H(σ) = -(-4 + 2)
⇒ H(σ) = -(-2) = 2

|E| = 6
H(σ) = 2
⇒ Cut(G) = ½ (6) - ½ (2)
⇒ Cut(G) = 3 - 1 = 2

Classical Random Walks

- **Probabilistic node transitions**



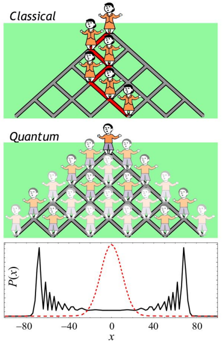
- **Higher dimensionality**

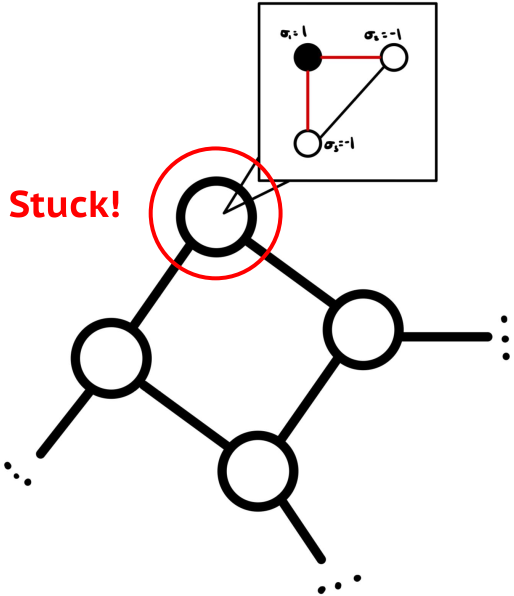
$$\mathbf{p}(t+1) = M\mathbf{p}(t)$$

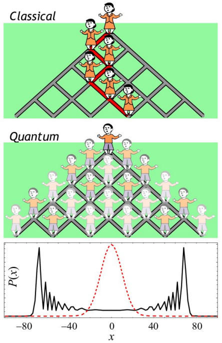
[1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.

Drawbacks

- No guarantee of finding the best solution.
- Some approaches get stuck at a local maximum.







Methodology: Discrete-Time Quantum Walk

- **Two operations are applied to a given position**

$$|\psi(0)\rangle = |0\rangle|n=0\rangle$$

- **Coin**

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- **Shift**

$$S|0\rangle|n\rangle = |0\rangle|n+1\rangle,$$
$$S|1\rangle|n\rangle = |1\rangle|n-1\rangle.$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Example

U^t applied once

$$|0\rangle \otimes |0\rangle \xrightarrow{H \otimes I} \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle$$

$$\xrightarrow{S} \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |1\rangle + |1\rangle \otimes |-1\rangle\big)$$

Generalized

$$|\psi(t)\rangle = U^t |\psi(0)\rangle$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Example

Up to the 3rd case

$$|\psi(1)\rangle = \frac{1}{\sqrt{2}}\big(|1\rangle|-1\rangle + |0\rangle|1\rangle\big),$$
$$|\psi(2)\rangle = \frac{1}{2}\Big(- |1\rangle|-2\rangle + (|0\rangle + |1\rangle)|0\rangle + |0\rangle|2\rangle\Big),$$
$$|\psi(3)\rangle = \frac{1}{2\sqrt{2}}\Big(|1\rangle|-3\rangle - |0\rangle|-1\rangle + (2|0\rangle + |1\rangle)|1\rangle + |0\rangle|3\rangle\Big)$$

[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Probability Distributions

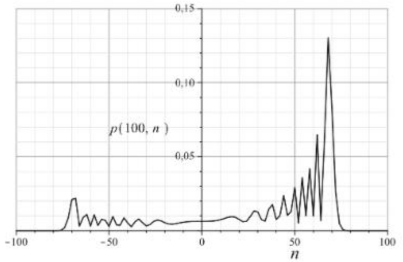


Fig. 3.4 Probability distribution after 100 steps of a quantum walk with the Hadamard coin starting from the initial condition $|\psi(0)\rangle = |0\rangle|n = 0\rangle$. The points where the probability is zero were excluded (n odd)

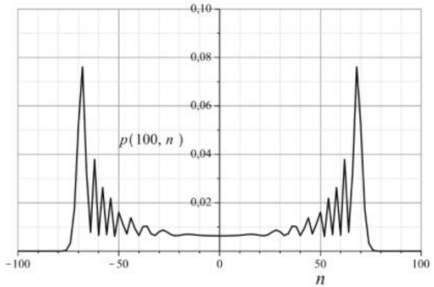


Fig. 3.5 Probability distribution after 100 steps of a Hadamard quantum walk starting from the initial condition (3.23)

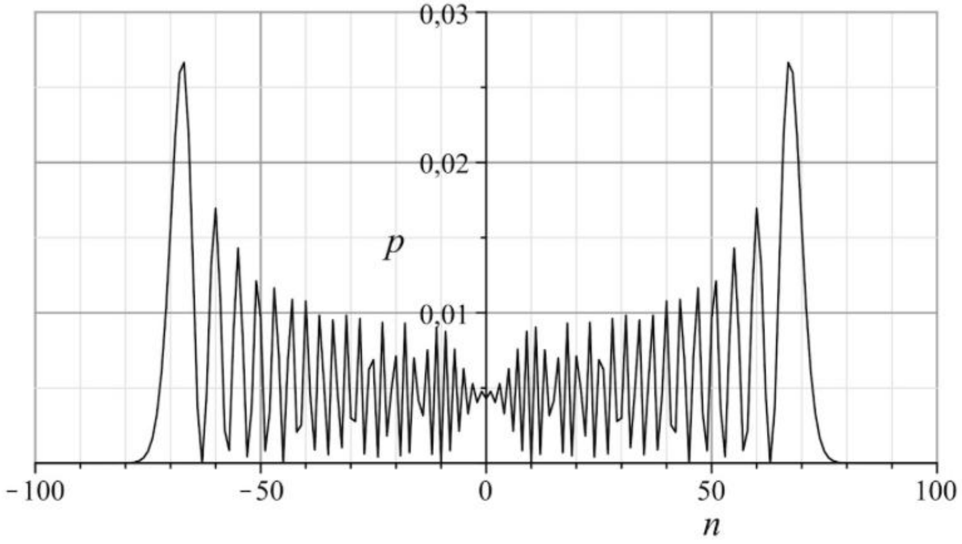
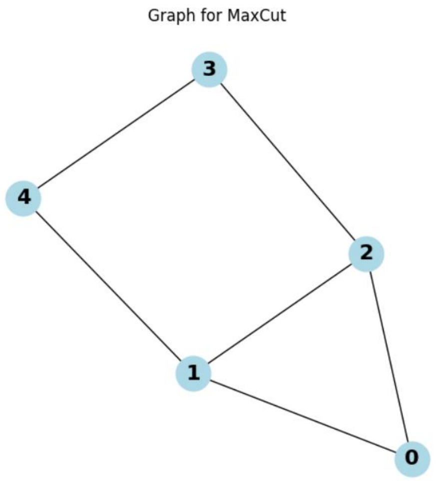


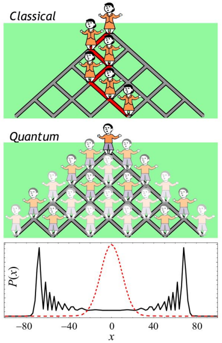
Fig. 3.7 Probability distribution at $t = 100$ with $\gamma = \left(2\sqrt{2}\right)^{-1}$ of a continuous-time quantum walk with initial condition $|\psi(0)\rangle = |0\rangle$

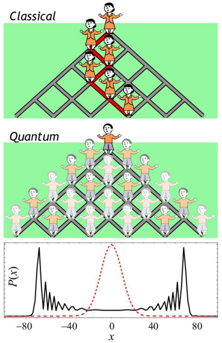
[9] Portugal, R. (2018). Introduction to Quantum Walks. In: Quantum Walks and Search Algorithms. Quantum Science and Technology. Springer, Cham. https://doi.org/10.1007/978-3-319-97813-0_3

Solution: Code Implementation

Google Colab:
https://colab.research.google.com/drive/1nkfcaAJ7gNTaxkeanalJKvniF7-lcKJ3?usp=sharing

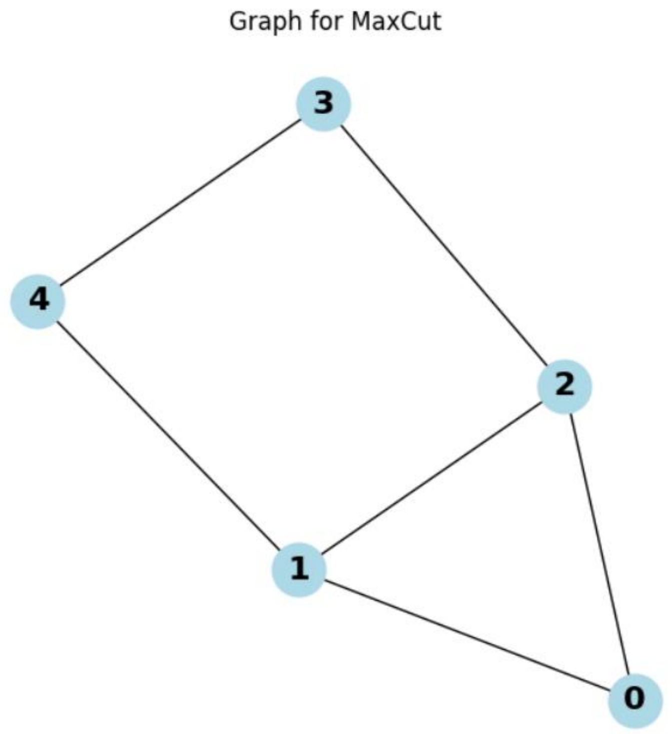






MileStone

- Performed a quantum walk on a 5 node graph
- Target Bit_squences: b1 b2 b3 b4 b5
  - 00101
  - 11010
  - 01010
  - 10101
- We were only able to find one of these sequences, more testing is required...



Deliverables

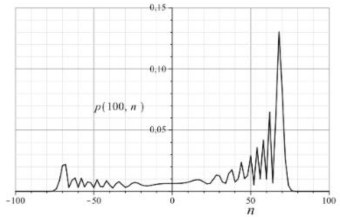
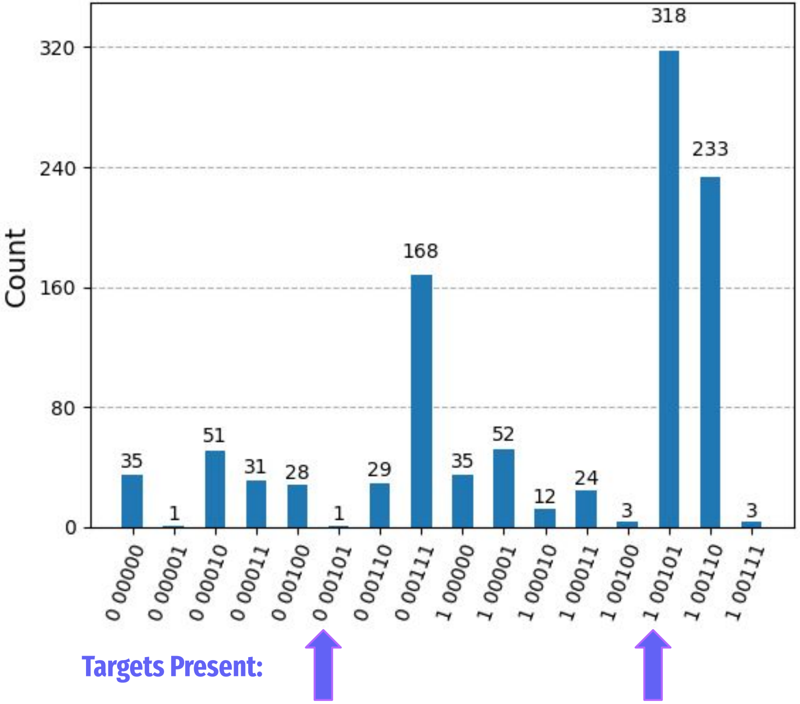


Fig. 3.4 Probability distribution after 100 steps of a quantum walk with the Hadamard coin starting from the initial condition $|\psi(0)\rangle = |0\rangle|n = 0\rangle$. The points where the probability is zero were excluded (n odd)

- 1 out of 4 optimal solutions found
- Quantum Walk Steps: 8
- 1 Qubit per node
- We get the expected probability distribution except for center. (We added heuristic improvements)



Targets Present:

More testing needed to find missing ones

Expected Results & Take Home Message

- Quantum random walk is completely different from classical approaches.
- More “Tricks” and Heuristic improvements can be made.

References

[1] J. D. Hidary, "Quantum Walks" in Quantum Computing: An Applied Approach, Cham, Switzerland:Springer, 2019.
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others:

- https://vixra.org/pdf/1909.0131v1.pdf
- https://lucaman99.github.io/blog/2019/08/03/Quantum-Random-Walks.html