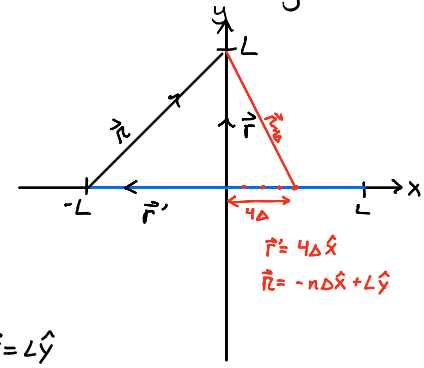
Continuos Charge Distributions



$\vec{r} = L\hat{y}$
$\vec{r}' = x\hat{x}$
$\vec{\mathscr{r}} = -x\hat{x} + L\hat{y}$
$|\vec{\mathscr{r}}| = \sqrt{x^2+L^2}$

$$E = \int \frac{K\,dq}{|\vec{\mathscr{r}}|^2}\hat{\mathscr{r}} \qquad dq \approx \lambda\, dl \rightarrow Q = \int \lambda\, dl \approx \sum_{i=1}^{n} \lambda(i\Delta) \approx \lambda \sum_{i=1}^{n}(i\Delta)$$

$$E = \frac{K\lambda_0}{1}\int_{-L}^{L} \frac{dl\,(-x\hat{x}+L\hat{y})}{(x^2+L^2)^{3/2}}$$

$$E_x = 2K\lambda_0 \int_0^L \frac{-x\, dl}{(x^2+L^2)^{3/2}}\hat{x}$$

$$E_y = 2K\lambda_0 \int_0^L \frac{L\, dl}{(x^2+L^2)^{3/2}}\hat{y}$$

$$E = \frac{KQ}{\mathscr{r}^2}\hat{\mathscr{r}}$$

$$E_x = \int_{-L}^{L} \frac{K\lambda_0(-x\, dl)}{(x^2+L^2)^{3/2}}$$

$$= \lim_{n\to\infty} \sum_{i=-n}^{n} \frac{K\lambda_0(x_i)(\Delta)}{(x_i)^2+L^2} \qquad * \; x_i = i\Delta$$

$$= \lim_{n\to\infty} \sum_{i=-n}^{n} \frac{K\lambda_0(i\Delta^2)}{((i\Delta)^2+L^2)^{3/2}}$$

$$E_x(0,L) \approx K\lambda_0 \sum_{i=-n}^{n} \frac{i\Delta^2}{((i\Delta)^2+L^2)^{3/2}}$$

$$E_y = \int_{-L}^{L} \frac{L\,dx\, K\lambda_0}{(x^2+L^2)^{3/2}}$$

$$= 2KL\lambda_0 \int_0^L \frac{dx}{(x^2+L^2)^{3/2}}$$

$$E_{y_a} \approx 2KL\lambda_0 \sum_{i=1}^{n} \frac{\Delta}{((i\Delta)+L^2)^{3/2}}$$

$$E_{y_e} = \frac{2K\cancel{L}\lambda_0}{\cancel{L}\sqrt{L^2+L^2}} = \frac{2K\lambda_0}{\sqrt{2}\,L}$$

$$\sum_{i=1}^{n} \frac{\Delta\, \cancel{2K\lambda_0 L}}{((i\Delta)^2+L^2)^{3/2}} = \frac{\cancel{2K\lambda_0 L}}{\sqrt{2}\,L^2}$$

$$\sum_{i=1}^{n} \frac{\Delta}{((i\Delta)^2+L^2)^{3/2}} = \frac{1}{\sqrt{2L^4}}$$

$$\frac{\Delta}{\quad} = \frac{(2L^4)^{1/2}}{\quad}$$

$$1 - \frac{E_{y_a}}{E_{y_e}}$$

$n\Delta = L$

$$\Delta = \frac{L}{n} \rightarrow E_{y_a} = \sum_{i=1}^{n} \frac{\frac{L}{n}}{\left(\left(\frac{iL}{n}\right)^2+L^2\right)^{3/2}}$$

$$dx = \lim_{n\to\infty} \frac{L}{n} \qquad \sum_{i=1}^{n} \frac{L}{n\left(\frac{i^2L^2}{n^2}+L^2\right)^{3/2}}$$

$$E_{y_a} \left(\frac{n}{?}\ \frac{\cancel{2K\lambda_0}}{\quad}\right) = \sum_{i=1}^{n} \frac{2K\lambda_0}{nL\left(\left[\left(\frac{i}{n}\right)^2+1\right]\right)^{3/2}}$$

$((i\Delta)^2+L^2)^{-1/2}$ $(2L^4)^{-1/2}$ E_{y_e} $\left(\sum_{i=1} n\cancel{L}\left(\left(\frac{i}{n}\right)^2+1\right)^{3/2}\right) \Big/ \frac{\cancel{2K\lambda_0}}{\cancel{L}\sqrt{2}}$

$\hookrightarrow \sum_{i=1}^{n} \frac{\sqrt{2}}{\left(\left(\frac{i}{n}\right)^2+1\right)^{3/2}}$

where n represents the # of charges

* Exact solution by Griffiths for the electric field a distance y above the midpoint of a straight line segment

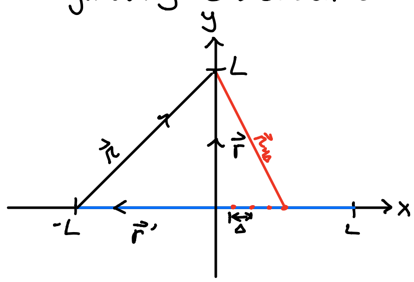
$$\vec{E}(0,y) = \frac{1}{4\pi\epsilon_0} \frac{2\lambda_0 L}{y\sqrt{y^2+L^2}}\,\hat{y} \qquad * K = 4\pi\epsilon_0$$

if $y = L$ then:

$$E(0,L) = \frac{2\lambda_0 K\cancel{L}}{\cancel{L}\sqrt{L^2+L^2}}$$

$$\vec{E}_{y_e}(0,L) = \frac{2K\lambda_0}{L\sqrt{2}}\,\hat{y}$$

$\vec{E}$ is originally calculated using Coulombs Law:



$\vec{r} = L\hat{y}$
$\vec{r}' = x\hat{x}$
$\vec{\mathscr{r}} = -x\hat{x} + L\hat{y}$
$|\vec{\mathscr{r}}| = \sqrt{x^2+L^2}$

$$\vec{E} = \frac{KQ}{|\vec{\mathscr{r}}|^2}\,\hat{\mathscr{r}}$$

$$Q = \int dq = \int \lambda_0\, dl \qquad * dl = dx$$

$$\therefore Q = \int_{-L}^{L} \lambda\, dx$$

$$\vec{E} = K\lambda_0 \int_{-L}^{L} \frac{dx\,(-x\hat{x}+L\hat{y})}{(x^2+L^2)^{3/2}}$$

$$E_y = K\lambda_0 \int_{-L}^{L} \frac{dx\, L}{(x^2+L^2)^{3/2}} \qquad * \text{ an even function}$$

$$E_y = 2K\lambda_0 L \int_0^L \frac{dx}{(x^2+L^2)^{3/2}}$$

$$E_y = 2K\lambda_0 L \lim_{N\to\infty} \sum_{n=0}^{N} \frac{\Delta}{((n\Delta)^2+L^2)^{3/2}} \qquad * X_i = n\Delta$$

* where N represents total amount of charges of one side
Δ represents distance between charges

$$X_N = N\Delta = L$$
$$\Delta = \frac{L}{N}$$ * as the # of charges increases, distance between charges decreases

$$E_{y_a} = 2K\lambda_0 L \sum_{n=1}^{N} \frac{\frac{L}{N}}{\left(n^2(\frac{L}{N})^2 + L^2\right)^{3/2}} \quad * \ \Delta = \frac{L}{N}$$
$$= 2K\lambda_0 L \sum_{n=1}^{N} \frac{L}{N\left(L^2\left((\frac{n}{N})^2+1\right)\right)^{3/2}}$$
$$= 2K\lambda_0 \sum_{n=1}^{N} \frac{\cancel{L^2}}{NL^3\left((\frac{n}{N})^2+1\right)^{3/2}}$$
$$= \frac{2K\lambda_0}{L} \sum_{n=1}^{N} \frac{1}{N\left((\frac{n}{N})^2+1\right)^{3/2}}$$

Requirement:

$$\left| \frac{E_{y_e}(0,L) - E_{y_a}(0,L)}{E_{y_e}(0,L)} \right| \le 0.01$$

$$= \left| 1 - \frac{E_{y_a}(0,L)}{E_{y_e}(0,L)} \right| \le 0.01$$

* $E_{y_a}(0,L)$ will have a finite N such that:

$$E_{y_e}(0,L) = \frac{2K\lambda_0}{L\sqrt{2}}$$
$$E_{y_a}(0,L) = \frac{2K\lambda_0}{L} \sum_{n=1}^{N} \frac{1}{N\left((\frac{n}{N})^2+1\right)^{3/2}}$$

$$\frac{E_{y_a}(0,L)}{E_{y_e}(0,L)} = \sum_{n=1}^{N} \frac{\sqrt{2}}{N\left((\frac{n}{N})^2+1\right)^{3/2}}$$

$$\therefore \left| \frac{E_{y_e}(0,L) - E_{y_a}(0,L)}{E_{y_e}(0,L)} \right| \le 0.01$$

$$\Rightarrow \left| 1 - \sum_{n=1}^{N} \frac{\sqrt{2}}{N\left((\frac{n}{N})^2+1\right)^{3/2}} \right| \le 0.01$$

to find value, must test different N values

where $\Delta = \frac{L}{N}$ as Δ decreases approximation becomes closer to exact value

Such that a table is created.

N	$\Delta = \frac{L}{N}$	$\left\lvert \frac{E_{y_e} - E_{y_a}}{E_{y_e}} \right\rvert$
1	L	0.99
2	$L/2$	0.98
3	$L/3$	⋮
⋮	⋮	0.01

$$Q = \lambda_0 L = N \Delta \lambda_0$$

$$\sum_{n=1}^{N} N\left(\left(\frac{n}{N}\right)^2 + 1\right)^{3/2}$$

$$E_y = \frac{2K\lambda_0}{\sqrt{2}L}$$

$$E_{y_a} = 2K\lambda_0 L \lim_{N\to\infty} \sum_{i=0}^{n} \frac{\Delta}{((x_i)^2 + L^2)}$$

$$E_{y_a} = 2K\lambda_0 L \sum_{i=0}^{n} \frac{\Delta}{((\Delta i)^2 + L^2)^{3/2}}$$

$$\frac{E_{y_a}}{E_{y_e}} = \frac{\cancel{2K\lambda_0} L \sum \frac{\Delta}{((\Delta i)^2 + L^2)^{3/2}}}{\frac{\cancel{2K\lambda_0}}{\sqrt{2}L}}$$

$$= \sqrt{2} L^2 \sum_{i=0}^{n} \frac{\Delta}{((\Delta i)^2 + L^2)^{3/2}}$$

$$= \sqrt{2} L^2 \sum_{i=1}^{n} \frac{\Delta}{\left(L^2\left(\left(\frac{\Delta i}{L}\right)^2 + 1\right)\right)^{3/2}}$$

$$= \sqrt{2} L^2 \sum \frac{\Delta}{L^3\left(\left(\frac{\Delta i}{L}\right)^2 + 1\right)^{3/2}} \qquad \Delta = \frac{L}{N}$$

$$= \frac{\sqrt{2}}{\cancel{L}} \sum_{i=0}^{N} \frac{\cancel{L}}{N\left(\left(\frac{\cancel{L} i}{N \cancel{L}}\right)^2 + 1\right)^{3/2}}$$

$$= \sqrt{2} \sum_{i=1}^{N} \frac{1}{N\left(\left(\frac{i}{N}\right)^2 + 1\right)^{3/2}}$$

$$\frac{E_{y_a}}{E_{y_e}} = \sum_{i=1}^{N} \frac{\sqrt{2}}{N\left(\left(\frac{i}{N}\right)^2 + 1\right)^{3/2}}$$

$$\lambda_0 = \frac{Nq}{2L}$$

$$2K\lambda_0 L \sum_{i=0}^{n} \frac{\Delta}{((\Delta i)^2 + L^2)^{3/2}}$$

$$E_{y_i} = 2K\lambda L \sum \frac{\Delta}{((\Delta i)^2+L^2)^{3/2}}$$

$$= \cancel{2}K\cancel{L}\left(\frac{Nq}{\cancel{2L}}\right)$$

$$E_{y_a} = Kq\sum \frac{\Delta N}{((\Delta i)^2+L^2)^{3/2}}$$

$$E_{y_e} = \frac{2K\lambda}{\sqrt{2}L} = \frac{\cancel{2}K Nq}{\cancel{2}L^2\sqrt{2}} = \frac{KNq}{\sqrt{2}L^2}$$

$$\frac{E_{y_a}}{E_{y_e}} = \sum_{i=1}^{N} \frac{\Delta\sqrt{2}L^2}{((\Delta i)^2+L^2)^{3/2}} = \frac{\frac{2L}{N}\sqrt{2}L^2}{\left(\left(\frac{2L}{N}\right)^2 i^2 + L^2\right)^{3/2}}$$

$$E_{y_a} = \sum_{i=1}^{N}\frac{2Kq}{d_i^2} = \sum_{i=0}^{N} \frac{2Kq}{(x_i^2+L^2)^{3/2}} = \sum_{i=0}^{N}\frac{2Kq}{((x_i)^2+L^2)^{3/2}}$$

$$\lambda = \frac{Nq}{2L}$$

$$\alpha\Delta = \frac{N}{2L}$$

$$\lambda = \frac{\text{charge}}{\text{Length}} = \frac{Nq}{2L} \Big| \; E_{y_a} \Rightarrow \sum_{i=0}^{N} \frac{2Kq}{((\Delta i)^2+L^2)^{3/2}}$$

$$E_{y_e} = \frac{\sqrt{2}K\lambda_0}{L} = \frac{\sqrt{2}KNq}{2L^2}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2KNq}{2\sqrt{2}L^2} = \frac{KNq}{\sqrt{2}L^2}$$

$$\frac{E_{y_a}}{E_{y_e}} = \sum_{i=0}^{N} \frac{2\cancel{Kq}}{\left(\left(\frac{N}{L}i\right)^2+L^2\right)^{3/2}} \cdot \frac{\sqrt{2}L^2}{\cancel{K}N\cancel{q}}$$

$$= \sum_{i=0}^{N} \frac{2\sqrt{2}L^2}{N\left(\left(\frac{N}{L}i\right)^2+L^2\right)^{3/2}}$$

$$= \sum_{i=0}^{N} \frac{2\sqrt{2}L^2}{N\left(L^2\left(\frac{\Delta i}{L}\right)^2+1\right)^{3/2}}$$

$$= \sum_{i=0}^{N} \frac{2\sqrt{2}}{NL\left(\left(\frac{\Delta i}{L}\right)^2+1\right)^{3/2}}$$

$x_i = \Delta i$

$\Delta = \frac{N}{2L}$

$\lambda = \frac{Nq}{2L} \to dq = \lambda dl$

$\Rightarrow q = \lambda\Delta$

$\Delta = \frac{q}{\lambda}$

$$= \sum_{i=0}^{N} \frac{2\sqrt{2}}{NL\left(\left(\frac{2\cancel{L}}{N}\cdot\frac{i}{\cancel{L}}\right)^2+1\right)^{3/2}}$$

$N = \Delta 2L$

$X_N = L = \alpha N \quad \therefore \alpha = \frac{L}{N}$

$X_i = \alpha i \to X_1 = \alpha 1$

$$\sum_{i=0}^{N} \frac{4}{\sqrt{2}NL\left(\left(\frac{Ni}{2L^2}\right)^2+1\right)^{3/2}}$$

$\Delta = \frac{N}{2L}$

$$\frac{4}{\sqrt{2N^2L^2}\left(\left(\frac{Ni}{2L^2}\right)^2+1\right)^{3/2}}$$

$$E_{y_a} = \sum_{i=0}^{N} \frac{2Kq}{((x_i)^2+L^2)^{3/2}}$$

$$= \sum_{i=0}^{N} \frac{2Kq}{\left(\left(\frac{L}{N}i\right)^2+L^2\right)^{3/2}}$$

$\sqrt{2}K\lambda \quad \sqrt{2}K\,Nq \quad \sqrt{2}KNq$

$$E_{y_c} = \frac{\cdots}{L} = \frac{\cdots}{L}\,\frac{\cdots}{2L} = \frac{\cdots}{2L^2}\,\frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}KNq}{\cancel{2}\sqrt{2}L^2} = \frac{KNq}{\sqrt{2}L^2}$$

$$\frac{E_{y_a}}{E_{y_c}} = \sum_{i=0}^{N} \frac{2\cancel{Kq}}{L^{\cancel{3}}\left(\left(\frac{i}{N}\right)^2+1\right)^{3/2}}\;\frac{\sqrt{2}\cancel{L^2}}{\cancel{K}N\cancel{q}} \qquad E_{y_c} = \frac{\sqrt{2}K}{\Delta L}$$

$$= \sum_{i=0}^{N} \frac{2\sqrt{2}}{NL\left(\left(\frac{i}{N}\right)^2+1\right)^{3/2}} \qquad \lambda = \frac{NQ}{2L} = \frac{Q}{\Delta}$$

$$\Delta = \frac{q}{\lambda} = \frac{2L}{N} \qquad \Delta = \frac{q}{\lambda}$$

$$N = \frac{2L}{\Delta} \qquad \sum_{i=0}^{N} \frac{2^{3/2}}{LN\left(\left(\frac{i}{N}\right)^2+1\right)^{3/2}}$$

$$\sum_{i=0}^{N} \frac{1}{LN}\left(\frac{2}{\left(\frac{i}{N}\right)^2+1}\right)^{3/2}$$

$$L = \frac{\Delta N}{2} \qquad = \sum_{i=0}^{N} \frac{2}{N^2\Delta}\left(\frac{2}{\left(\frac{i}{N}\right)^2+1}\right)^{3/2}$$

$$N = \frac{2L}{\Delta}$$

$$\Delta = \frac{2L}{N}$$

$$L = \frac{N\Delta}{2}$$

Coulomb's Law:

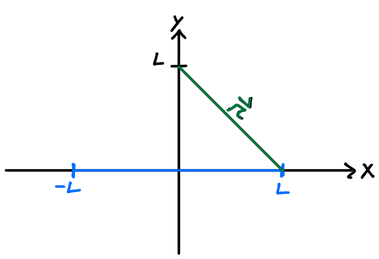
$$\vec{E} = \frac{Kq}{r^2}\hat{r}$$

When dealing with N charges

$$\vec{E}_i = \frac{Kq_i}{r_i^3}\vec{r}_i$$

Now let $q_i = q$ for all i and distributed evenly across a line of $2L$

Where the $\vec{E}$ field is to be calculated L above the midpoint



Using Griffiths pg. 64

$$\vec{E}(0,y) = \frac{1}{4\pi\epsilon_0}\,\frac{2\lambda_0 L}{y\sqrt{y^2+L^2}}\,\hat{y} \qquad * \; K = \frac{1}{4\pi\epsilon_0}$$

When a distance L from the midpoint:

$$\vec{E}(0,L) = \frac{2K\lambda_0 \cancel{L}}{\cancel{L}\sqrt{L^2+L^2}}\,\hat{y}$$

$$= \frac{2K\lambda_0}{L\sqrt{2}}\,\hat{y}$$

$$E_{y_e}(0,L) = \frac{\sqrt{2}K\lambda_0}{L}$$ * exact solution

For aproximate solution N charges exist

* $\lambda = \frac{\text{Total charge}}{\text{Total Length}}$

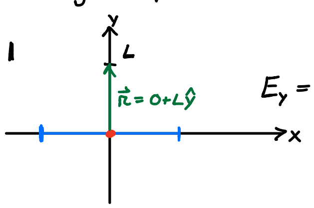
$$\lambda = \frac{Nq}{2L}$$

$$E_{y_e}(0,L) = \frac{\sqrt{2}K}{L}\frac{Nq}{2L}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$E_{y_e}(0,L) = \frac{KNq}{\sqrt{2}L^2}$$

For N charges spread evenly over 2L:

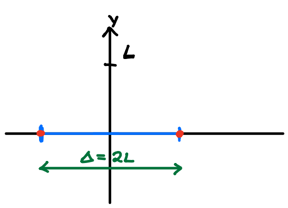
for N = 1



$$E_y = \frac{KqL}{(0+L^2)^{3/2}} = \frac{Kq}{L^2}$$

* where $\vec{r} = L\hat{y}$

for N = 2



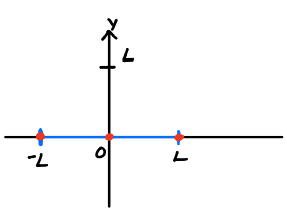
$$E_y = E_{y_1} + E_{y_2}$$

$$= \frac{KqL}{(L^2+L^2)^{3/2}} + \frac{KqL}{((-L)^2+L^2)^{3/2}}$$

$$= \frac{KqL}{L^3\,2\sqrt{2}} + \frac{KqL}{L^3\,2\sqrt{2}}$$

$$E_y = \frac{Kq}{L^2\sqrt{2}}$$

for N = 3

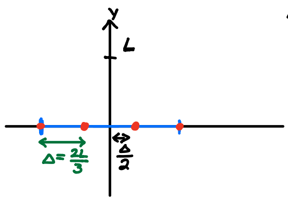


$$E_y = \sum_{i=1}^{3} E_{y_i}$$

$$= \frac{KqL}{(L^2+L^2)^{3/2}} + \frac{Kq}{L^2} + \frac{KqL}{(L^2+L^2)^{3/2}}$$

$$= \frac{Kq}{L^2} + \frac{Kq}{\sqrt{2}L^2} = \frac{Kq}{L^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

for N=4



$$E_Y = \sum_{i=1}^{4} E_{Y_i} = \frac{2KqL}{(L^2+(\frac{L}{3})^2)^{3/2}} + \frac{2KqL}{(L^2+L^2)^{3/2}}$$

$$= \frac{2Kq}{L^2(1+(\frac{1}{3})^2)^{3/2}} + \frac{Kq}{L^2\sqrt{2}}$$

$$= \frac{Kq}{L^2\sqrt{2}} + \frac{Kq}{L^2}\frac{2}{(\frac{10}{9})^{3/2}}$$

$$= \frac{Kq}{L^2}\left(\frac{1}{\sqrt{2}} + \frac{2}{(\frac{10}{9})^{3/2}}\right)$$

$$E_{Y_i} = \frac{KqL}{(x_i^2+L^2)^{3/2}}$$

$x_i = x_0 + i\Delta$ where $i = 1, 2, \dots, N-1$

$$x_0 = \begin{cases} \frac{\Delta}{2} & \text{for } N=\text{even} \\ 0 & \text{for } N=\text{odd} \end{cases}$$

$$\therefore E_{Y_a} = \sum_{i=0}^{\frac{N-1}{2}} \frac{2KqL}{((x_0+i\Delta)^2+L^2)^{3/2}} \qquad \Delta = \frac{2L}{(N-1)}$$

$$= \sum_{i=0}^{N-1} \frac{2KqL}{((\frac{x_0+i\Delta}{L})^2+1)^{3/2}}$$

for N=odd and N>1

$$E_Y = E_0 + \sum_{i=1}^{\frac{N-1}{2}} \frac{2KqL}{\left(L^2\left[(\frac{x_0+i\Delta}{L})^2+1\right]\right)^{3/2}}$$

$$E_{Y_a} = \frac{Kq}{L^2} + \sum_{i=1}^{\frac{N-1}{2}} \frac{2Kq}{L^2\left(\left[\frac{x_0+i\Delta}{L}\right]^2+1\right)^{3/2}} \qquad * \; x_0 = 0, \; \Delta = \frac{2L}{(N-1)}$$

$$E_{Y_a} = \frac{Kq}{L^2}\left(1 + \sum_{i=1}^{\frac{N-1}{2}} \frac{2}{\left[(\frac{i}{L}\frac{2L}{(N-1)})^2+1\right]^{3/2}}\right)$$

$$E_{Y_a} = \frac{Kq}{L^2}\left(1 + \sum_{i=1}^{\frac{N-1}{2}} \frac{2}{\left[(\frac{2i}{(N-1)})^2+1\right]^{3/2}}\right) \qquad E_{Y_e}(0,L) = \frac{KNq}{\sqrt{2}L^2}$$

Let N=3

$$E_{Y_a} = \frac{Kq}{L^2} + \frac{2Kq}{L^2}\left(\frac{1}{\left[(\frac{2}{2}(1))^2+1\right]^{3/2}}\right)$$

$$= \frac{Kq}{L^2} + \frac{\cancel{2}Kq}{L^2}\frac{1}{\cancel{2}\sqrt{2}}$$

$$= \frac{Kq}{L^2} + \frac{Kq}{L^2\sqrt{2}} = \frac{Kq}{L^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

For N = even

$$E_{Y_a} = \sum_{i=0}^{N/2} \frac{2Kq}{L^2((\frac{x_0+i\Delta}{L})^2+1)^{3/2}} = \frac{2Kq}{L^2}\sum_{i=0}^{(N/2-1)} \frac{1}{\left((\frac{\frac{\Delta}{2}+i\Delta}{L})^2+1\right)^{3/2}} \qquad \Delta = \frac{2L}{N-1} \rightarrow \frac{\Delta}{L} = \frac{2}{N-1}$$

$$= \frac{2Kq}{L^2}\sum_{i=0}^{(\frac{N}{2}-1)} \frac{1}{\left((\frac{\Delta}{L}(\frac{1}{2}+i))^2+1\right)^{3/2}}$$

Let $N=4$

$$= \frac{2Kq}{L^2} \sum_{i=0}^{1} \frac{1}{\left(\left(\frac{2}{3}\right)^2\left(\frac{1}{2}+i\right)^2+1\right)^{3/2}}$$

$$= \frac{2Kq}{L^2}\left(\frac{1}{\left(\frac{4}{9}\left(\frac{1}{4}\right)+1\right)^{3/2}}\right) + \frac{2Kq}{L^2}\left(\frac{1}{\left(\left(\frac{2}{3}\right)^2\left(\frac{3}{2}\right)^2+1\right)^{3/2}}\right)$$

$$= \frac{2Kq}{L^2}\left[\left(\frac{1}{9}+1\right)^{-3/2} + \left(\left(\frac{4}{9}\right)\left(\frac{9}{4}\right)+1\right)^{-3/2}\right]$$

$$= \frac{2Kq}{L^2}\left[\frac{1}{\left(10/3\right)^{3/2}} + \frac{1}{(2)^{3/2}}\right]$$

$$= \frac{Kq}{L^2}\left[\frac{2}{\left(10/3\right)^{3/2}} + \frac{1}{\sqrt{2}}\right]$$ * works

$$\therefore E_{Y_a} = \begin{cases} \dfrac{Kq}{L^2}\left[\displaystyle\sum_{i=0}^{\left(\frac{N}{2}-1\right)} \frac{2}{\left(\left(\frac{1}{N-1}(1+2i)\right)^2+1\right)^{3/2}}\right] & \text{for } N = \text{even} \\[2ex] \dfrac{Kq}{L^2}\left(1+\displaystyle\sum_{i=1}^{\frac{N-1}{2}} \frac{2}{\left[\left(\frac{2i}{(N-1)}\right)^2+1\right]^{3/2}}\right) & \text{for } N = \text{odd} \end{cases}$$

* $E_{Y_e}(0,L) = \frac{KNq}{\sqrt{2}\,L^2}$

$$\frac{E_{Y_a}}{E_{Y_e}} = \frac{\sqrt{2}}{N}\sum_i (\text{odd or even})$$

$$0.01 \geq \left|1-\frac{E_{Y_a}}{E_{Y_e}}\right| \,?$$

* determine N such that the above ratio holds true

$$\Delta = \frac{2}{(N-1)}L$$

$$\Delta = \frac{2L}{(N-1)}$$

* found $N=51$

$$\therefore \Delta = \frac{1}{50}\cdot 2L$$

$$\Delta = 0.04L$$

For N=1:

$$E_{y_a} = \frac{Kq}{L^2}$$

$$\left|1 - \frac{E_{y_a}}{E_{y_e}}\right| = |1-\sqrt{2}| = 0.4142$$

For N=2:

$$E_{y_a} = \frac{Kq}{\sqrt{2}L^2}$$

$$E_{y_e} = \frac{KNq}{\sqrt{2}L^2} = \frac{2Kq}{\sqrt{2}L^2} = \frac{\sqrt{2}Kq}{L^2}$$

$$\left|1 - \frac{E_{y_a}}{E_{y_e}}\right| = 0.5$$