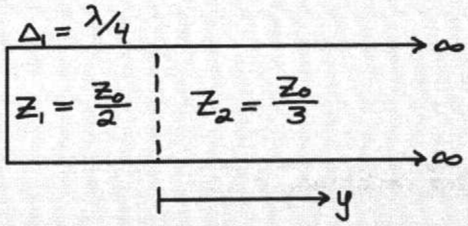
Darwin Quiroz

ECE 513
12/01/20

10.1 Impedence Transformation I

Given the Following Transmission line:



Compute $\tilde{\rho}_1(0)$ & $Z_1(-\lambda/4)$ by hand and using Smith Chart

10.1.1 Hand Calculation

Impedence Continuity states:

$$Z_1(0) = Z_2(0)$$

where Spatially dependence says:

$$Z_n(y) = Z_n\left(\frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)}\right)$$

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2\left(\frac{1+\tilde{\rho}_2(0)}{1-\tilde{\rho}_2(0)}\right)$$

Since Z_2 impedence line goes to infinity, then no reflected wave is produced

thus $\tilde{\rho}_2(y) = 0$ for all y

$$\therefore \frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)} = \frac{Z_2}{Z_1} \qquad * \ Z_1 = \frac{Z_0}{2},\ Z_2 = Z_0/3$$

$$\frac{1+\tilde{\rho}_1}{1-\tilde{\rho}_1} = \frac{2}{3} \qquad * \ \tilde{\rho}_n(y) = \tilde{\rho}_1 e^{2j\beta y} \to \tilde{\rho}_1(0) = \tilde{\rho} \ \text{(some complex const)}$$

$$1+\tilde{\rho}_1 = \frac{2}{3} - \frac{2}{3}\tilde{\rho}_1$$

$$\frac{5}{3}\tilde{\rho}_1 = -1/3$$

$$\boxed{\therefore \tilde{\rho}_1(0) = -1/5} \Rightarrow \tilde{\rho}_1 = -\frac{1}{5} = \frac{1}{5}e^{j\pi}$$

$$\tilde{\rho}_1(y) = -1/5\, e^{j\frac{4\pi}{\lambda_1}y} \qquad * \beta = \frac{2\pi}{\lambda}$$

$$Z_1(-\lambda/4) = Z_1\left(\frac{1+\tilde{\rho}_1(-\lambda/4)}{1-\tilde{\rho}_1(-\lambda/4)}\right) \qquad * \ \tilde{\rho}_1 = -1/5\, e^{-j\pi} = 1/5 \ ?$$

$$Z_1(-\lambda_1/4) = Z_1\left(\frac{1+1/5}{1-1/5}\right)$$ Signs wrong.

$$= Z_1\left(\frac{\cancel{6}^{3}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{4}_{2}}\right)$$

$$\boxed{Z_1(-\lambda_1/4) = (3/2)Z_1}$$

10.1.2 Smith Chart

1. Computing normalized impedance and associated normalized resistance r & x:

$$Z_1(0) = Z_2(0)$$

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2\left(\frac{1+\cancel{\tilde{\rho}_2(0)}}{1-\cancel{\tilde{\rho}_2(0)}}\right)^{1} \quad * \tilde{\rho}_2(y) = 0$$

$$\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = \frac{Z_2}{Z_1}$$

$$\therefore \tilde{\rho}_1(0) = \left(\frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}\right)$$

Normalized impedance is found as:

$$\frac{Z_2}{Z_1} = 2/3 = r + jx$$

$$\therefore r = 2/3, \; x = 0$$

2. from Smith Chart:

$$\tilde{\rho}_1(0) = -1/5$$

3. where:

$$\tilde{\rho}_1(-\lambda_1/4) = \tilde{\rho}_1(0)e^{j2\beta_1(-\Delta_1)} = \tilde{\rho}_1(0) \angle 2\beta_1(-\Delta_1) \qquad * \phi = -2\beta_1\Delta_1 = -\pi$$

$= 1/5$ from Smith Chart using CW rotation of $-\pi$ from $\tilde{\rho}_1(0)$

4. from the Smith Chart this yields:

$$\boxed{r = 3/2, \; x = 0}$$

5. Z_1 can be determined at $y = -\Delta_1$ as:

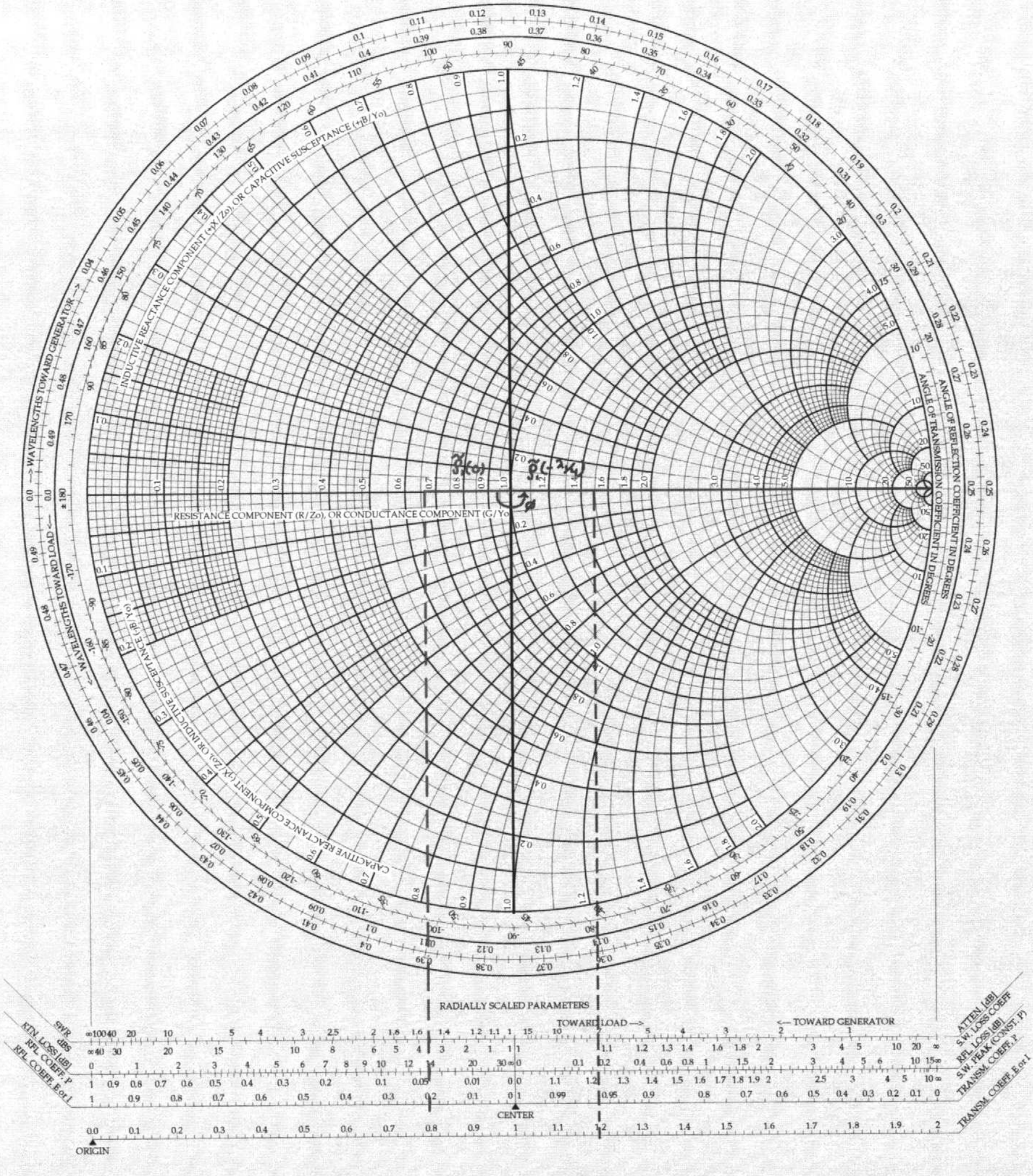
$$Z_1(-\Delta_1) = Z_1\left(\frac{1+\tilde{\rho}_1(-\Delta_1)}{1-\tilde{\rho}_1(-\Delta_1)}\right) \quad * \tilde{\rho}_1(-\Delta_1) = 1/5$$

$$= Z_1\left(\frac{1+1/5}{1-1/5}\right)$$

$$\boxed{Z_1(-\Delta_1) = \frac{3}{2} Z_1}$$ write in terms of Z_0! (which is given.)

The Complete Smith Chart

Black Magic Design



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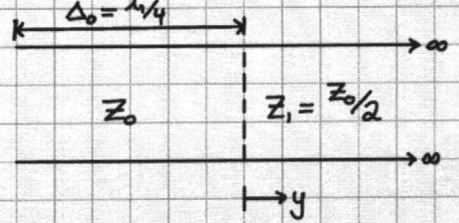
10.1 Impedence Transformation II

Given the following Transmission line:



Compute:

1. $\tilde{\rho}_0(0)$
2. $Z_0(-\Delta_1)$

10.2.1 Hand Calculation

1. Impedance Continuity states:

$$Z_0(0) = Z_1(0)$$

Since Z_1 line exists and continues to ∞, then no reflected wave exists:

$$\therefore \tilde{\rho}_1(y) = 0$$

$$Z_0\left(\frac{1+\tilde{\rho}_0(0)}{1-\tilde{\rho}_0(0)}\right) = Z_1 \quad * Z_1 = Z_0/2$$

then $\tilde{\rho}_0(0)$ can be found as:

$$\tilde{\rho}_0(0) = \left(\frac{Z_1/Z_0 - 1}{Z_1/Z_0 + 1}\right)$$

$$= \left(\frac{1/2 - 1}{1/2 + 1}\right)$$

$$\boxed{\therefore \tilde{\rho}_0(0) = -1/3}$$

2. To find $Z_0(-\Delta_1)$, phase is added to $\tilde{\rho}_0(y)$:

$$\tilde{\rho}_0(-\Delta_0) = -1/3\, e^{j2\beta(-\Delta_1)} \quad * \beta = \frac{2\pi}{\lambda_0} \ \& \ \Delta_1 = \lambda_1/4$$

$$\tilde{\rho}_0(-\Delta_0) = -1/3\, e^{-j\pi \lambda_1/\lambda_0} \quad * \text{ if } \lambda_1 = \lambda_0$$

$$= 1/3 \quad * e^{-j\pi} = -1$$

$$Z_0(-\Delta_0) = Z_0\left(\frac{1+\tilde{\rho}_0(-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_0)}\right)$$

$$= Z_0\left(\frac{4/3}{2/3}\right) = 2Z_0$$

10.2.2 Using Smith Chart

1. Normalized impedance says:

$$\frac{Z_1(0)}{Z_0(0)} = \frac{1}{2}$$

then $r = \frac{1}{2}$, $x = 0$

2. From Smith Chart

$$\tilde{\rho}_0(0) = -\frac{1}{3}$$

3. Finding $\rho_0(-\lambda_0/4)$:

$$\tilde{\rho}_0(-\lambda/4) = \tilde{\rho}_0(0)e^{-j\frac{4\pi}{\lambda_0}\frac{\lambda_0}{4}}$$

$$= \tilde{\rho}_0(0)e^{-j\pi}$$

$$\tilde{\rho}_0(-\tfrac{\lambda}{4}) = -\tilde{\rho}_0(0) = \tfrac{1}{3}$$

4. From a CW rotation of π, the Smith Chart reads:

$r \approx 2.0$, $x = 0$

5. Then

$$Z_0(-\lambda/4) = Z_0\left(\frac{1+\tilde{\rho}_0(-\frac{\lambda}{4})}{1-\tilde{\rho}_0(-\frac{\lambda}{4})}\right)$$

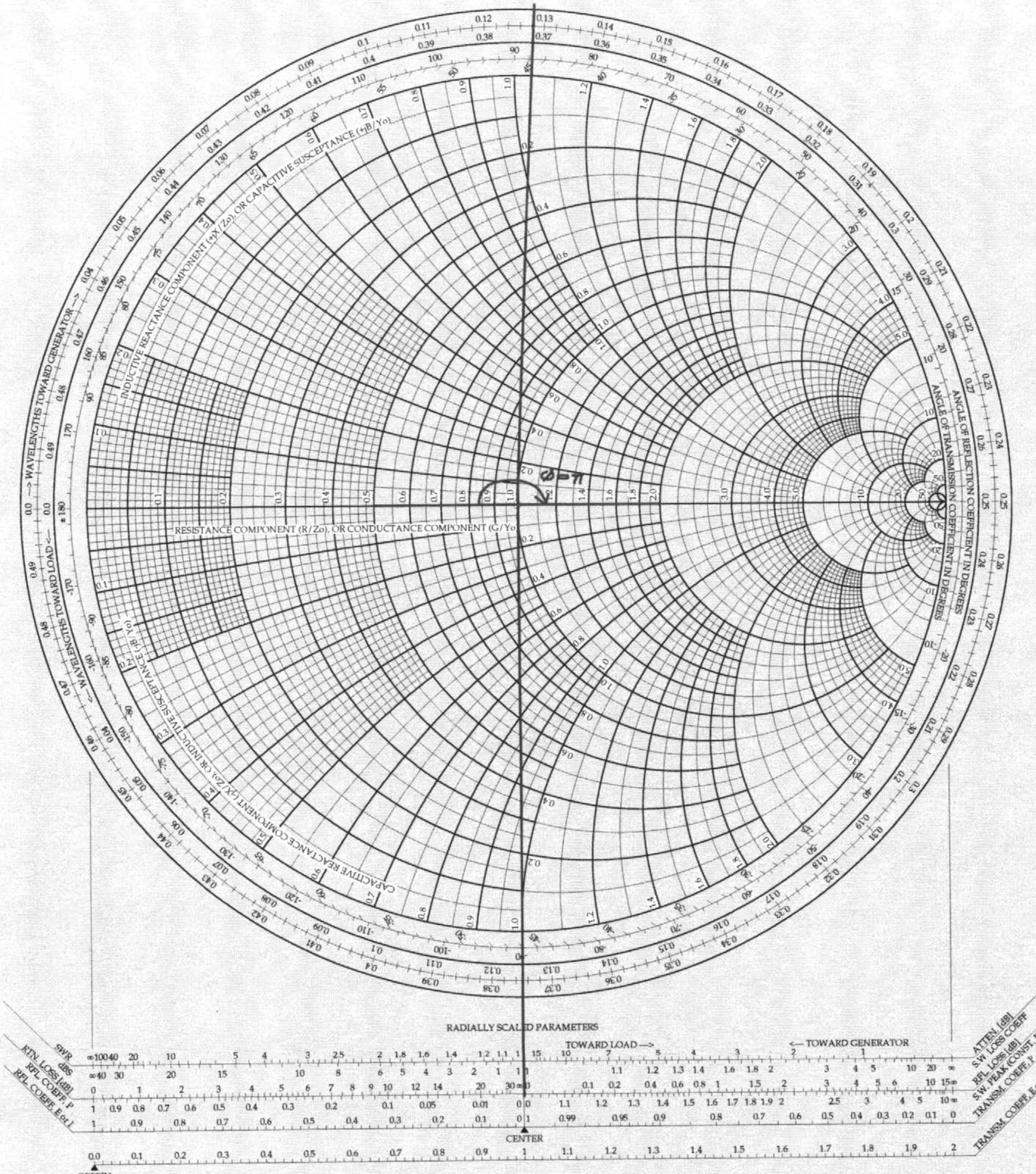
$$= Z_0\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$$

$$= Z_0\left(\frac{4}{3}\cdot\frac{3}{2}\right)$$

$$\boxed{Z_0(-\lambda/4) = 2Z_0}$$

The Complete Smith Chart

Black Magic Design



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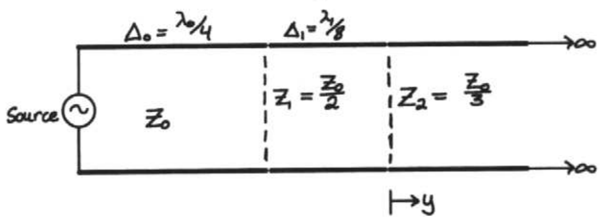
10.3 Impedence Transformation III

Given the Following Transmission line:



10.1.3 Hand Calculation

Impedence Continuity states:

$$Z_1(0) = Z_2(0)$$

Since Z_2 line continues to ∞, $\tilde{\rho}_2(y) = 0$ due to no reflected wave, then:

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2$$

then $\tilde{\rho}_1(0)$ can be found as:

$$\tilde{\rho}_1(0) = \left(\frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}\right) \qquad * \frac{Z_2}{Z_1} = \frac{Z_0}{3} \cdot \frac{2}{Z_0} = \frac{2}{3}$$

$$\tilde{\rho}_1(0) = \left(\frac{2/3 - 1}{2/3 + 1}\right) = -1/5$$

Then:

$$\tilde{\rho}_1(-\lambda_1/8) = \tilde{\rho}_1(0)\, e^{-j2\frac{2\pi}{\lambda_1}\cdot\frac{\lambda_1}{8}}$$

$$= \tilde{\rho}_1(0)\, e^{-j\pi/2}$$

$$= +j\,1/5$$

From this the Impedence $Z_1(-\lambda_1/8)$ can be found:

$$Z_1(-\lambda_1/8) = Z_1\left(\frac{1+\tilde{\rho}_1(-\lambda_1/8)}{1-\tilde{\rho}_1(-\lambda_1/8)}\right)$$

$$= Z_1\left(\frac{1+j\,1/5}{1-j\,1/5}\right)$$

Using MatLAB:

$$Z_1(-\lambda_1/8) = Z_1\left(12/13 - j\,5/13\right) = Z_1\left(1e^{-j0.1257\pi}\right) = \frac{1}{2} Z_0 \angle +22.62 \qquad * \tan^{-1}\left(\frac{5}{12}\right) \approx 0.1257\pi$$

why is this (+)?
given this (−)

then from 2^{nd} Boundary at $-\lambda_1/8$:

Impedence Continuity

$$Z_0(-\lambda_1/8) = Z_1(-\lambda_1/8)$$

$$Z_0\left(\frac{1+\rho_0(-\lambda_1/8)}{1-\rho_0(-\lambda_1/8)}\right) = Z_1 e^{-j0.1257\pi} \quad * \; Z_1 = \frac{Z_0}{2}$$

$$\frac{1+\tilde{\rho}_0(-\lambda_1/8)}{1-\tilde{\rho}_0(-\lambda_1/8)} = \frac{1}{2} e^{-j0.1257\pi}$$

$$\rho_0(-\lambda_1/8) = \left(\frac{1/2\, e^{j0.1257} - 1}{1/2\, e^{j0.1257} + 1}\right) \quad * \; Z_1 = Z_0/2$$

$$\tilde{\rho}_0(-\lambda_1/8) = \left(\frac{1/2\, e^{j0.1257} - 1}{1/2\, e^{j0.1257} + 1}\right)$$

Computing w/ MATLAB

$$\therefore \; \tilde{\rho}_0(-\lambda_1/8) \approx 0.388 \angle 152.85^\circ = 0.8492\pi$$

then:

$$\rho_0(-\Delta_1-\Delta_0) \approx 0.388 e^{-j0.8492\pi} e^{j\frac{4\pi}{\lambda_0}\frac{\lambda_1}{4}} \qquad * \text{ if } \lambda_0 = \lambda_1$$

$$\tilde{\rho}_0(-\Delta_1-\Delta_0) = 0.388 e^{-j0.8492\pi} e^{-j\pi}$$

$$= 0.388 \angle -27.15^\circ = 0.388 e^{-j1.8492\pi}$$

impedence at source can be found as:

$$Z_0(-\Delta_1-\Delta_0) = Z_0\left(\frac{1+\tilde{\rho}_0(-\Delta_1-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_1-\Delta_0)}\right)$$

$$= Z_0\left(\frac{1-0.388e^{j1.85\pi}}{1+0.388e^{j1.8\pi}}\right)$$

Using MatLAB:

$$Z_0(-\Delta_1-\Delta_0) = 2Z_0 \angle -22.62^\circ$$

Calculating using Smith chart

Normalized i,

$$\frac{Z_2}{Z_1} = \frac{1/3}{1/2} = 2/3 \Rightarrow r = 2/3, \; x = 0$$

Using Smith Chart:

Then the reflection is found as:

$\tilde{\rho}_1(0) = -1/5$

then $\tilde{\rho}_1(-\lambda/4) = 1/5 j$ * CW rotation of 90° due to $-\lambda/8$ position

From this impedence of Z_1 can be found at $-\lambda/8$

$$Z_1(-\lambda/8) = Z_1 \left(\frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)} \right)$$

$$\frac{Z_1(-\lambda/8)}{Z_1} = \left(\frac{1+1/5 j}{1-1/5 j} \right) = 0.9231 + 0.3846j$$

$$Z_1(-\lambda/8) = \frac{Z_0}{2}(0.9231 + 0.3846j)$$

$$Z_1(-\lambda/8) = Z_0\,(0.4615 + 0.1923j)$$

so at $-\lambda/8$

$$r = 0.4615\,,\ x = -0.1923j$$

then ρ_0 can be found as:

$$\tilde{\rho}_0(-\lambda/8) = 0.3875 + 0.1875j$$

then moving to Source:

$$\tilde{\rho}_1(-\lambda/8 - \lambda/4) = 0.3875 - 0.1875j$$

then finally Z_0 at source can be found as:

$$Z_0(-\lambda/8 - \lambda_0/4) = \left(\frac{1 + (\ 3875 - 0.1875j)}{1 - (0.3875 - 0.1875j)} \right)$$

$$\approx 2.186 \angle -24.72^\circ$$

The Complete Smith Chart

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