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ECE 513
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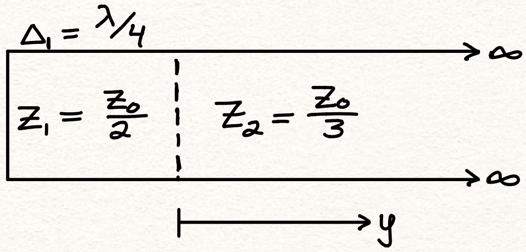
10.1 Impedence Transformation I

Given the Following Transmission line:



Compute $\tilde{\rho}_1(0)$ & $z_1(-\lambda/4)$ by hand and using Smith Chart

10.1.1 Hand Calculation

Impedence Continuity states:

$$Z_1(0) = Z_2(0)$$

where Spatially dependence says:

$$Z_n(y) = z_n\left(\frac{1+\tilde{\rho}_n(y)}{1-\tilde{\rho}_n(y)}\right)$$

$$z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = z_2\left(\frac{1+\tilde{\rho}_2(0)}{1-\tilde{\rho}_2(0)}\right)$$

Since z_2 impedence line goes to infinity, then no reflected wave is produced

thus $\tilde{\rho}_2(y) = 0$ for all y

$$\therefore \quad \frac{1+\tilde{\rho}_1(0)}{1-\rho_1(0)} = \frac{z_2}{z_1}$$

$* \; z_1 = \frac{z_0}{2}, \; z_2 = z_0/3$

$$\frac{1+\tilde{\rho}_1}{1-\tilde{\rho}_1} = \frac{2}{3}$$

$* \; \tilde{\rho}_n(y) = \tilde{\rho}_1 e^{2j\beta y} \rightarrow \tilde{\rho}_1(0) = \tilde{\rho}$ (some complex const)

$$1+\tilde{\rho}_1 = \tfrac{2}{3} - \tfrac{2}{3}\tilde{\rho}_1$$

$$\tfrac{5}{3}\tilde{\rho}_1 = -\tfrac{1}{3}$$

$$\boxed{\therefore \tilde{\rho}_1(0) = -1/5}$$

$$\tilde{\rho}_1(y) = -\tfrac{1}{5} e^{j\frac{4\pi}{\lambda_1}y}$$

$* \; \beta = \frac{2\pi}{\lambda}$

$$z_1(-\lambda/4) = z_1\left(\frac{1+\tilde{\rho}_1(-\lambda/4)}{1-\tilde{\rho}_1(-\lambda/4)}\right)$$

$* \; \tilde{\rho}_1 = -\tfrac{1}{5}e^{-j\pi} = 1/5$

$$Z_1(-\lambda/4) = Z_1\left(\frac{1+1/5}{1-1/5}\right)$$

$$= Z_1\left(\frac{\cancel{6}^{3}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{4}_{2}}\right)$$

$$\boxed{Z_1(-\lambda/4) = (3/2)Z_1}$$

10.1.2 Smith Chart

1. Computing normalized impedance and associated normalized resistance r & x:

$$Z_1(0) = Z_2(0)$$

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2\overset{1}{\cancel{\left(\frac{1+\tilde{\rho}_2(0)}{1-\tilde{\rho}_2(0)}\right)}} \qquad * \tilde{\rho}_2(y) = 0$$

$$\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = \frac{Z_2}{Z_1}$$

$$\therefore \tilde{\rho}_1(0) = \left(\frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}\right)$$

Normalized impedance is found as:

$$\frac{Z_2}{Z_1} = 2/3 = r + jx$$

$$\therefore r = 2/3,\ x = 0$$

2. from smith Chart:

$$\tilde{\rho}_1(0) = -1/5$$

3. where:

$$\tilde{\rho}_1(-\lambda_1/4) = \tilde{\rho}_1(0)e^{j2\beta_1(-\Delta_1)} = \tilde{\rho}_1(0)\measuredangle 2\beta_1(-\Delta_1) \qquad * \phi = -2\beta_1\Delta_1 = -\pi$$

$= 1/5$ from Smith Chart using CW rotation of $-\pi$ from $\tilde{\rho}_1(0)$

4. from the smith Chart this yields:

$$\boxed{r = 3/2,\ x = 0}$$

5. Z_1 can be determined at $y = -\Delta_1$ as:

$$Z_1(-\Delta_1) = Z_1\left(\frac{1+\tilde{\rho}_1(-\Delta_1)}{1-\tilde{\rho}_1(-\Delta_1)}\right) \qquad * \tilde{\rho}_1(-\Delta_1) = 1/5$$

$$= Z_1\left(\frac{1+1/5}{1-1/5}\right)$$

$$\boxed{Z_1(-\Delta_1) = \frac{3}{2}Z_1}$$

The Complete Smith Chart

Black Magic Design