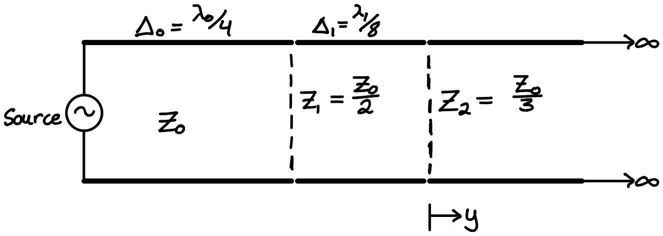
Darwin Quiroz



# 10.3 Impedence Transformation III

Given the Following Transmission line:




## 10.1.3 Hand Calculation

Impedence Continuity states:

$$Z_1(0) = Z_2(0)$$

Since $Z_2$ line continues to $\infty$, $\tilde{\rho}_2(y) = 0$ due to no reflected wave, then:

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2$$

then $\tilde{\rho}_1(0)$ can be found as:

$$\tilde{\rho}_1(0) = \left(\frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}\right) \qquad * \ \frac{Z_2}{Z_1} = \frac{Z_0}{3} \cdot \frac{2}{Z_0} = 2/3$$

$$\tilde{\rho}_1(0) = \left(\frac{2/3 - 1}{2/3 + 1}\right) = -1/5$$

Then:

$$\tilde{\rho}_1(-\lambda_1/8) = \tilde{\rho}_1(0)\, e^{-j2\frac{2\pi}{\cancel{\lambda_1}}\frac{\cancel{\lambda_1}}{8}}$$

$$= \tilde{\rho}_1(0)\, e^{-j\pi/2}$$

$$= +j\,1/5$$

From this the impedence $Z_1(-\lambda_1/8)$ can be found:

$$Z_1(-\lambda_1/8) = Z_1\left(\frac{1+\tilde{\rho}_1(-\lambda_1/8)}{1-\tilde{\rho}_1(-\lambda_1/8)}\right)$$

$$= Z_1\left(\frac{1+j\,1/5}{1-j\,1/5}\right)$$

Using MatLAB:

$$Z_1(-\lambda_1/8) = Z_1\left(12/13 - j\,5/13\right) = Z_1\left(1e^{-j0.1257\pi}\right) = \tfrac{1}{2} Z_0 \angle +22.62 \qquad * \tan^{-1}\left(\tfrac{5}{12}\right) \approx 0.1257\pi$$

then from $2^{nd}$ Boundary at $-\lambda_1/8$:

Impedence Continuity

$$Z_0(-\lambda_1/8) = Z_1(-\lambda_1/8)$$

$$Z_0\left(\frac{1+\rho_0(-\lambda_1/8)}{1-\rho_0(-\lambda_1/8)}\right) = Z_1\, e^{-j0.1257\pi} \qquad * \; Z_1 = \frac{Z_0}{2}$$

$$\frac{1+\tilde{\rho}_0(-\lambda_1/8)}{1-\tilde{\rho}_0(-\lambda_1/8)} = \frac{1}{2}\, e^{-j0.1257\pi}$$

$$\rho_0(-\lambda_1/8) = \left(\frac{\frac{1}{2}e^{j0.1257}-1}{\frac{1}{2}e^{j0.1257}+1}\right) \qquad * \; Z_1 = Z_0/2$$

$$\tilde{\rho}_0(-\lambda_1/8) = \left(\frac{\frac{1}{2}e^{j0.1257}-1}{\frac{1}{2}e^{j0.1257}+1}\right)$$

Computing w/ MATLAB

$$\therefore \; \tilde{\rho}_0(-\lambda_1/8) \approx 0.388 \measuredangle 152.85^\circ = 0.8492\pi$$

then:

$$\rho_0(-\Delta_1-\Delta_0) \approx 0.388e^{-j0.8492\pi} e^{-j\frac{\cancel{4}\pi}{\cancel{\lambda_0}}\frac{\cancel{\lambda_1}}{\cancel{4}}} \qquad * \text{ if } \lambda_0 = \lambda_1$$

$$\tilde{\rho}_0(-\Delta_1-\Delta_0) = 0.388e^{-j0.8492\pi} e^{-j\pi}$$

$$= 0.388 \measuredangle -27.15^\circ = 0.388e^{-j1.8492\pi}$$

impedence at source can be found as:

$$Z_0(-\Delta_1-\Delta_0) = Z_0\left(\frac{1+\tilde{\rho}_0(-\Delta_1-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_1-\Delta_0)}\right)$$

$$= Z_0\left(\frac{1-0.388e^{j1.85\pi}}{1+0.388e^{j1.8\pi}}\right)$$

Using MatLAB:

$$Z_0(-\Delta_1-\Delta_0) = 2Z_0 \measuredangle -22.62^\circ$$

Calculating using Smith chart

Normalized i,

$$\frac{Z_2}{Z_1} = \frac{1/3}{1/2} = 2/3 \Rightarrow r = 2/3,\; x = 0$$

Using Smith Chart :

Then the reflection is found as:

$\tilde{\rho}_1(0) = -\frac{1}{5}$

then $\tilde{\rho}_1(-\lambda/4) = \frac{1}{5}j$ $\ast$ CW rotation of $90°$ due to $-\lambda/8$ position

From this impedence of $Z_1$ can be found at $-\lambda/8$

$$Z_1(-\lambda/8) = Z_1 \left( \frac{1+\tilde{\rho}_1(-\lambda/8)}{1-\tilde{\rho}_1(-\lambda/8)} \right)$$

$$\frac{Z_1(-\lambda/8)}{Z_1} = \left( \frac{1+\frac{1}{5}j}{1-\frac{1}{5}j} \right) = 0.9231 + 0.3846j$$

$$Z_1(-\lambda/8) = \frac{Z_0}{2}(0.9231 + 0.3846j)$$

$$Z_1(-\lambda/8) = Z_0\,(0.4615 + 0.1923j)$$

so at $-\lambda/8$

$r = 0.4615$ , $x = -0.1923j$

then $\rho_0$ can be found as:

$\tilde{\rho}_0(-\lambda/8) = 0.3875 + 0.1875j$

then moving to Source :

$\tilde{\rho}_1(-\lambda/8 - \lambda/4) = 0.3875 - 0.1875j$

then finally $Z_0$ at source can be found as:

$$Z_0(-\lambda/8 - \lambda_0/4) = \left( \frac{1+(\ 3875-0.1875j)}{1-(0.3875-0.1875j)} \right)$$

$$\approx 2.186 \measuredangle -24.72°$$

# The Complete Smith Chart

Black Magic Design