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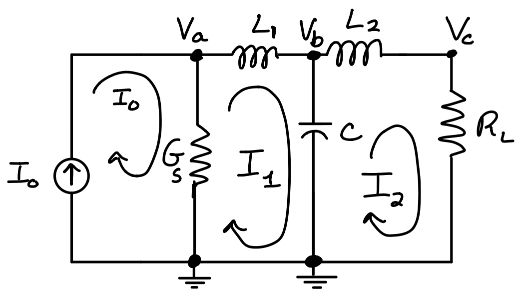
ECE 513

HW 7.2

10/20/20

4.3. b) Replacing Voltage soure and internal resistance to a current source w/ a paralled resistance:



Starting w/ the mesh equations:

Path of I_o:

Not included because it is conneeteel to Current source

Path of I_1:

$$\boxed{-L_1\frac{dI_1}{dt} - \frac{1}{C}\int(I_1 - I_2)\,dt - \frac{1}{G_S}(I_1 - I_o) = 0}$$

Path of I_2:

$$\boxed{-R_L I_2 - L_2\frac{dI_2}{dt} - \frac{1}{C}\int(I_2 - I_1)\,dt = 0}$$

∴ Loop/mesh equations derived from KVL still hold true

Node equations:

Current in = '+', Current out = '-'

at Node a:

$$I_o - \frac{1}{L_1}\int(V_b - V_a)\,dt + (V_a - 0)G_s = 0 \qquad * \; G_s = \frac{1}{R_s}$$

$$\boxed{I_o + \frac{1}{L_1}\int(V_a - V_b)\,dt + \frac{V_a}{R_s} = 0}$$

at node b:

$$\frac{1}{L_1}\int(V_b - V_a)\,dt + C\frac{d}{dt}(V_b - 0) - \frac{1}{L_2}\int(V_b - V_c)\,dt = 0$$

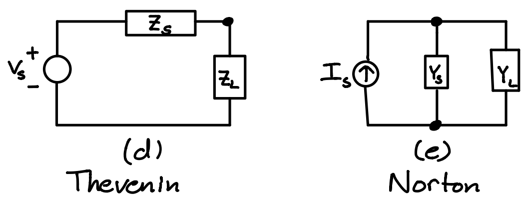
$$\boxed{\frac{1}{L_1}\int(V_b-V_a)dt + C\frac{dV_b}{dt} + \frac{1}{L_2}\int(V_c-V_b)dt = 0}$$

at node c:

$$+\frac{(V_c-0)}{R_L} - \frac{1}{L_2}\int(V_b-V_c)\,dt = 0$$

$$\boxed{\frac{V_c}{R_L} + \frac{1}{L_2}\int(V_c-V_b)\,dt = 0}$$

4.3c) Figure 4.3d and e are:



$* \; Y_s \equiv Z_s^{-1} \rightarrow I_s = V_s Y_s$

From Figure (d)

The current across load Z_L:

$$\boxed{I_L = I_s = \frac{V_s}{Z_s + Z_L}}$$

Voltage across load is given by voltage divider formula:

$$\boxed{V_L = V_s\left(\frac{Z_L}{Z_L+Z_s}\right)}$$

From Figure e:

Voltage delivered is given from:

$$V_s = I_s Z_{tot} = I_s(Z_s \parallel Z_L)$$

$$V_L = I_s\left(\frac{Z_L Z_s}{Z_L+Z_s}\right) \quad * \text{ due to both in parallel}$$

$$V_L = V_s \cancel{Y_s}\left(\frac{Z_L \cancel{Z_s}}{Z_L+Z_s}\right) \quad * \text{ using relationship: } I_s = V_s Y_s \;\; \& \;\; Z_s = \frac{1}{Y_s}$$

$$\boxed{V_L = V_s\left(\frac{Z_L}{Z_L+Z_s}\right)}$$

Current delivered to load is:

$$I_L = \frac{V_L}{Z_L} = \frac{V_s}{\cancel{Z_L}}\left(\frac{\cancel{Z_L}}{Z_L+Z_s}\right) = \boxed{\frac{V_s}{Z_L+Z_s}}$$

∴ Current & Voltage formulas are equivalent

4.3.d) For fixed V_s & Z_s, where Z_s is a conjugate match: $Z_L = Z_s^*$

$$P = \frac{1}{2} R |I|^2 \quad \text{for complex circuits}$$

Power delivered to load depends on current delivered to load:

$$P_L = R_L |I_L|^2$$

Using figure (d) Thevenin equivalent:

$$I_L = I_s = \frac{V_s}{Z_L + Z_s} \qquad * \; z = R + j\chi$$

$$I_L = \frac{V_s}{(R_L+R_s) + j(\chi_s + \chi_L)}$$

$$I_L^* = \frac{V_s}{(R_L+R_s) - j(\chi_s + \chi_L)}$$

$$P_L = \frac{R_L}{2} |I_L|^2 = \frac{R_L}{2}\left(\frac{V_s}{(R_L+R_s)^2 + (\chi_s+\chi_L)^2}\right)$$

$*$ $Z_L = R_L + j\chi_L$, minimizing each individually:

$$\frac{\partial P_L}{\partial \chi_L} = \frac{V_s R_L}{2} \frac{\partial}{\partial \chi_L}\left(\frac{1}{(R_L+R_s)^2 + (\chi_s+\chi_L)^2}\right) = 0$$

$$= \frac{V_s R_L}{\cancel{2}}\left(\frac{-\cancel{2}(\chi_s+\chi_L)}{[(R_L+R_s)^2 + (\chi_s+\chi_L)^2]^2}\right) = 0$$

$$\chi_L = -\chi_s$$

Minimizing R_L

$$\frac{\partial P_L}{\partial R_L} = \left(\frac{V_s}{2}\right)\frac{\partial}{\partial R_L}\left(\frac{R_L}{(R_L+R_s)^2 + (\chi_s+\chi_L)^2}\right) = 0$$

$$\frac{V_s}{2}\left(\frac{1}{\cancel{(R_L+R_s)^2 + (\chi_s+\chi_L)^2}} + \frac{R_L(-1)(2(R_L+R_s))}{[(R_L+R_s)^2 + (\chi_s+\chi_L)^2]^{\cancel{2}}}\right) = 0$$

$$1 - \frac{2R_L(R_L+R_s)}{(R_L+R_s)^2 + (\chi_s+\chi_L)^2} = 0$$

$$R_L(R_L+R_s) = [(R_L+R_s)^2 + (\cancel{\chi_s+\chi_L})^2]\,\frac{1}{2} \qquad * \text{ min } \chi_L \text{ is when } \chi_L = -\chi_s$$

$$R_L(\cancel{R_L+R_s}) = (R_L+R_s)^{2}\,\frac{1}{2}$$

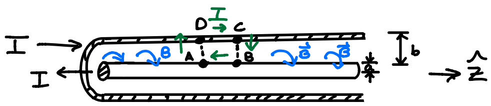
$$2R_L = R_L + R_s$$

$$\therefore R_L = R_s$$

$\therefore$ for P_L to be at max value:

$$Z_L = R_s - jX_s = Z_s^*$$

4.6e) Figure 3.17



From Faradays Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$$

$$\int_A^B \vec{E}\cdot d\vec{l} + \int_B^C \vec{E}\cdot d\vec{l} + \int_C^D \vec{E}\cdot d\vec{l} + \int_D^A \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$$

in region between coaxial cable:

if considering external inductance independent of time:

$$\vec{B} = \vec{B}_0 e^{j\omega t} \rightarrow -\frac{\partial \vec{B}}{\partial t} = -j\omega\vec{B}$$

$$-\frac{\partial \Phi_B}{\partial t} = -j\omega\,\Phi_B = j\omega L_e \cdot I$$ * external inductance per unit length due to B-field

Left hand integrals can be related to potential difference across points:

$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = j\omega L_e I \Delta z$$

Finding V_{CD}:

$$E_z = I(Z_{int})_{outer}$$ * $(Z_{int})_{outer}$ = internal impedence of outer conductor

where interal impedence of a conductor is given as

$$\frac{Z_{in}}{\Delta z} = \left(\frac{R_s}{2\pi r_0} + \frac{j\omega L_s}{2\pi r_0}\right)\frac{\Omega}{m}$$

at path $C \rightarrow D$, $r_0 = b$

$$(Z_{int})_{outer} = \left(\frac{R_{s_b}}{2\pi b} + \frac{j\omega L_{s_b}}{2\pi b}\right)\frac{\Omega}{m}$$

$\therefore V_{CD} = -I(Z_{in})_{outer}\Delta z$ * KVL says going w/ path of current, gives neg. Voltage

Similarly one can obtain an expression of impedence for path between $A \rightarrow B$

$$V_{AB} = I(Z_{int})_{inner}\Delta z = I\left(\frac{R_{s_a}}{2\pi a} + \frac{j\omega L_{s_a}}{2\pi a}\right)\Delta z$$

Substituting into equation:

$$V_{DA} - I\left(\frac{R_{sb} + j\omega L_{ib}}{2\pi b}\right)\Delta z + V_{BC} - I\left(\frac{R_{sa} + j\omega L_{ia}}{2\pi a}\right)\Delta z = (-j\omega L_e I)\Delta z$$

$$I\left(\frac{R_{sb}}{2\pi b} + \frac{R_{sa}}{2\pi a}\right)\Delta z + I\left(\frac{j\omega L_{ib}}{2\pi b} + \frac{j\omega L_{ia}}{2\pi a} + j\omega L_e\right)\Delta z = V_{DA} + V_{BC}$$

$V_{DA} + V_{BC} = (I R_{eq} + I L_{eq})\Delta z$ * L_{eq} & R_{eq} hold same formula as circuit diagram
* Inductors & Resistors add in series

where Δz is dependent on length of path, as $\Delta z \to 0$

$V_{DA} + V_{BC} = 0$, *circuit diagram matches voltage gains/drops

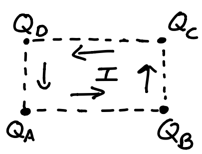
Considering paths $B \to C$ & $D \to A$

- Coaxial cylinder conductors act as capacitors and as such:

$$|V_{BC}| = \frac{Q_{BC}}{C}$$

$$|V_{DA}| = \frac{Q_{DA}}{C}$$

due to conservation of charge:



$Q_{DA} = Q_{BC}$ *current along path is continuous

$\therefore \ V_{DA} = -V_{BC} \Rightarrow V_{DA} = V_{CB}$

$\therefore$ Circuit equivalent of $\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t}\int B \cdot dA$