8.3.1

5.8.f from Wiley states to rewrite eqn 5.8(1):

$$V = V_+ e^{-j\beta z} + V_+ |\rho| e^{j(\theta_p + \beta z)}$$

in phasor notation as a standing wave + travelling wave. Rewrite as a real function of time

ρ is the reflection coefficient

$$\rho \triangleq \frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

& β is the phase constant:

$$\beta = \frac{\omega}{V_p} = \omega\sqrt{LC} = \frac{2\pi}{\lambda} \quad * \ \beta\lambda \text{ must repeat every } 2\pi$$

& θ_p is the phase angle between reflected & incident wave at $z=0$

$$V = V_+ (e^{-j\beta z} + |\rho| e^{j\theta_p} e^{j\beta z})$$

* Want to represent as the sum of:

	Real Form	Complex form
Standing wave =>	$2A_1 \cos(Kz)\cos(\omega t)$	$\rightarrow 2A_1\left(\frac{e^{jKz} + e^{-jKz}}{2}\right)$
+ travelling wave =	$A_2 \cos(\omega t - Kz)$	$= Re\{A_2 e^{j\omega t - Kz}\}$

* not including phases of δ

Looking to convert eqn 5.8(1) into the general form:

$$V(z,t) = 2A_1 \cos(Kz)\cos(\omega t) + A_2 \cos(Kz - \omega t)$$

$$V = V_+ (|\rho| e^{j(\theta_p + \beta z)} + e^{-j\beta z})$$

$$= V_+ e^{j\frac{\theta_p}{2}} (|\rho| e^{j(\frac{\theta_p}{2} + \beta z)} + e^{-j(\frac{\theta_p}{2} + \beta z)}) \quad * \text{ let } \phi \equiv \frac{\theta_p}{2} + \beta z$$

$$= V_+ e^{j\frac{\theta_p}{2}} \left[(|\rho| e^{j\phi} + (|\rho| - |\rho| + 1) e^{-j\phi} \right]$$

$$= V_+ e^{j\frac{\theta_p}{2}} \left[|\rho| (e^{j\phi} + e^{-j\phi}) + (1 - |\rho|) e^{-j\phi} \right] \quad * \text{ using inverse euler's identity}$$

$$= V_+ e^{j\frac{\theta_p}{2}} \left[2|\rho| \cos(\phi) + (1 - |\rho|) e^{-j\phi} \right] \quad * \text{expanding } \phi$$

$$= 2 V_+ e^{j\frac{\theta_p}{2}} |\rho| \cos(\tfrac{\theta_p}{2} + \beta z) + (1 - |\rho|) V_+ e^{j(-\frac{\theta_p}{2} - \beta z + \frac{\theta_p}{2})}$$

$$V = 2 V_+ e^{j\frac{\theta_p}{2}} |\rho| \cos(\tfrac{\theta_p}{2} + \beta z) + (1 - |\rho|) V_+ e^{-j\beta z}$$

Comparing first term to General solution form of Standing wave

$$2V_+ e^{j\frac{\theta_p}{2}} |g| \cos\left(\frac{\theta_p}{2} + \beta z\right) = 2A_1 \cos(\omega t + \delta_1)\cos(kz + \delta_2)$$

then $A_1 = V_+ |g| e^{j\frac{\theta_p}{2}}$

$\delta_2 = \frac{\theta_p}{2}$, $k = \beta$

$\Bigg\}$ * standing wave w/ phase shift of $\frac{\theta_p}{2}$ at $t=0$

Doing Similarly w/ General Solution to travelling wave & 2^{nd} term

$$(1-|g|)V_+ e^{-j\beta z} = A_2 e^{j(\omega t - kz)}$$

* at $\omega t = 0$, then

$A_2 = (1-|g|)V_+$

$K = \beta$

To rewrite as a function of time:

$$Re\{V e^{j\omega t}\} = Re\left\{\left[2V_+ e^{j\frac{\theta_p}{2}} |g| \cos\left(\frac{\theta_p}{2} + \beta z\right) + (1-|g|)V_+ e^{-j\beta z}\right] e^{j\omega t}\right\}$$

* will convert to general solution form as:

$$= Re\left\{2V_+ |g| \cos\left(\frac{\theta_p}{2} + \beta z\right) e^{j\left(\frac{\theta_p}{2} + \omega t\right)} + (1-|g|)V_+ e^{-j(\beta z + \omega t)}\right]\Big\}$$ * $e^{j\theta} = \cos\theta + j\sin(\theta)$

$$= 2V_+ |g| \cos\left(\beta z + \frac{\theta_p}{2}\right)\cos\left(\omega t + \frac{\theta_p}{2}\right) + (1-|g|)V_+ \cos(\omega t - \beta z)$$

Or rewritten in general form:

$$V(z,t) = 2A_1 \cos(\omega t + \delta_1)\cos(\beta z + \delta_2) + A_2 \cos(\omega t - \beta z)$$

$\delta_1 = \delta_2 = \frac{\theta_p}{2}$, $A_1 = V_+ |g|$, $A_2 = (1-|g|)V_+$

Now calculate $V(z)$ from Example in Fig. 5.8 at ωt shifted by $\pi/4$ from ωt_1

From Fig 5.8. it is given:

$\omega t_1 = -\frac{\theta_p}{2}$

then $\omega t = \frac{\pi}{4} - \frac{\theta_p}{2}$

$$\therefore V(z,t) = 2V_+ |g| \cos\left(\beta z + \frac{\theta_p}{2}\right)\cos\left(\frac{\pi}{4}\right) + (1-|g|)V_+ \cos\left(\frac{\pi}{4} - \frac{\theta_p}{2} - \beta z\right)$$

$$= \sqrt{2} V_+ |g| \cos\left(\beta z + \frac{\theta_p}{2}\right) + (1-|g|)V_+ \cos\left(\frac{\pi}{4} - \frac{\theta_p}{2} - \beta z\right)$$

$* \cos(\alpha+\beta) = \cos(A)\cos(B) - \sin(A)\sin(B)$, let $A = \frac{\pi}{4}$, $B = -\frac{\theta_\rho}{2} - \beta z$

$= \sqrt{2} V_+ |g| \cos(\beta z + \frac{\theta_\rho}{2}) + (1-|g|) V_+ [\cancel{\cos(\frac{\pi}{4})}^{\frac{\sqrt{2}}{2}} \cos(\frac{\theta_\rho}{2} + \beta z) + \cancel{\sin(\frac{\pi}{4})}^{\frac{\sqrt{2}}{2}} \sin(\frac{\theta_\rho}{2} + \beta z)]$

$= 2\frac{\sqrt{2}}{2} V_+ |g| \cos(\beta z + \frac{\theta_\rho}{2}) + (1-|g|) V_+ (\frac{\sqrt{2}}{2}) [\cos(\frac{\theta_\rho}{2} + \beta z) + \sin(\frac{\theta_\rho}{2} + \beta z)]$

$*$ combining cosine terms:

$= [(1+|g|) V_+ \cos(\beta z + \frac{\theta_\rho}{2}) + (1-|g|) V_+ \sin(\frac{\theta_\rho}{2} + \beta z)] \frac{\sqrt{2}}{2}$

Wiley states that at pg. 234, Fig 5.8 was plotted for $S=3$

$* \; |g| = \frac{S-1}{S+1} = \frac{2}{4} = 1/2$

Substituting into $V(z)$

$\Rightarrow \frac{3\sqrt{2}}{4} V_+ \cos(\beta z + \frac{\theta_\rho}{2}) + \frac{\sqrt{2}}{4} V_+ \sin(\beta z + \frac{\theta_\rho}{2})) \frac{\sqrt{2}}{2}$

$\therefore V(z) = \frac{3\sqrt{2}}{4} V_+ [\cos(\beta z + \frac{\theta_\rho}{2}) + \frac{1}{3}\sin(\beta z + \frac{\theta_\rho}{2})]$ at $\omega t = \omega t_1 + \pi/4$

8.3.2

$*$ Plot the standing wave in region 1 on the image shown for 8.2.

Recall the standing wave is:

$2A_1 \cos(Kz + \delta_1)\cos(\omega t + \delta_2)$

must define $A_1, \omega t, K, \delta_1, \delta_2$

From given plot, $E_1 = E_y(1:250)$

$A_1 = |E_y|$

E' exists for indexes 1-250

$X \in (0, 2.5)$

$i_t = 500$, then $t_1 = dt \cdot i_t = \frac{5}{3} \times 10^{-8}$ s

$\omega t = (2\pi \cdot f_s)(\frac{5}{3} \times 10^{-8})$ $* \; f_s \cdot t_1 = 5$

$\therefore \; \omega t = 10\pi$

periodic w/ 100 time steps

E_y starts w/ negative slope and asumed to be shifted in phase by 180°