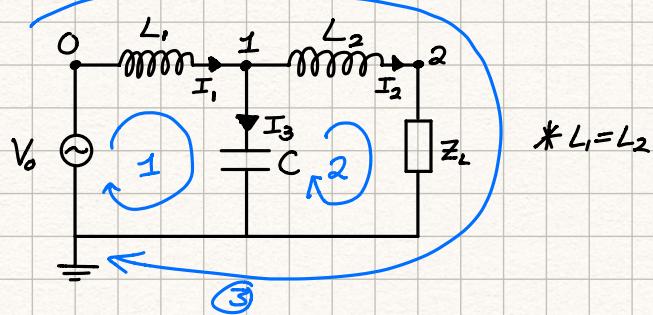


11.1.1:

Given the following Circuit:



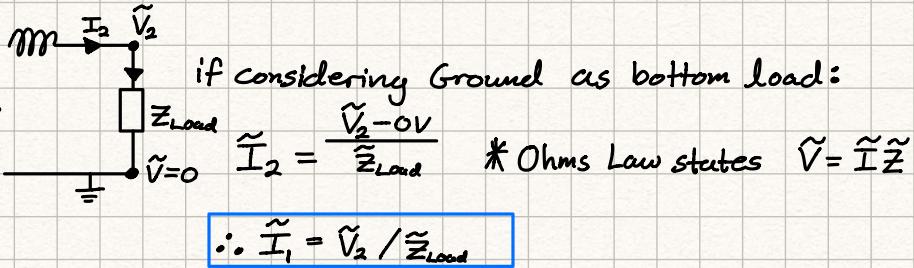
Applying KCL at Node 1:

$$\sum \tilde{I}_n(t) = \tilde{I}_1 - \tilde{I}_2 - \tilde{I}_3 = 0$$

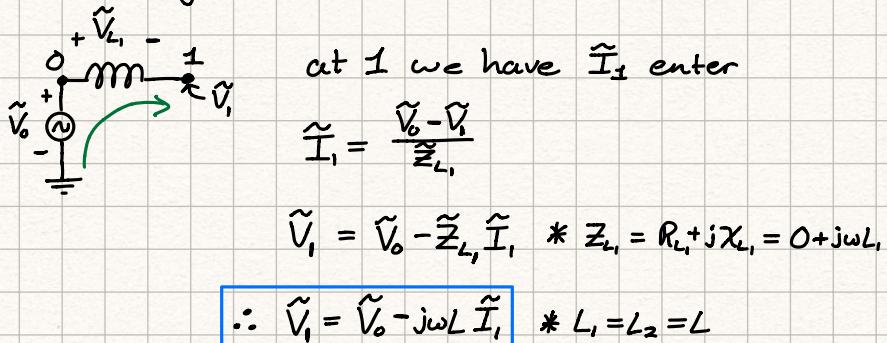
$$\tilde{I}_1 = \tilde{I}_2 + \tilde{I}_3 \quad * \quad \tilde{I}_3 = j\omega C \tilde{V}_1, \quad Z_C = 0 + j\omega C \quad (\text{only reactive component})$$

$$\therefore \tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1$$

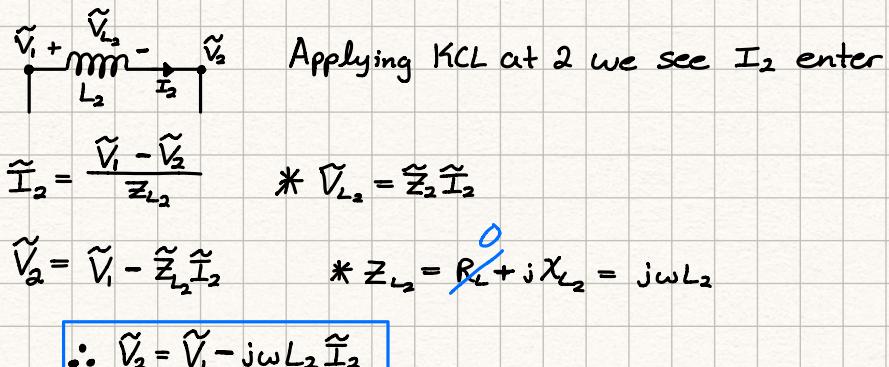
Looking at Node 2:



Similarly this can be done in node 1 & node 0:



One can apply the same method to find a relationship between \tilde{V}_1 & \tilde{V}_2 :



11.1.2

Given $\tilde{I}_1 = \frac{\tilde{V}_o}{Z_0}$, Z_0 is the equivalent impedance of the whole circuit

Z_1 will be the equivalent impedance of L_2, C, Z_{load}

$$Z_{L_2} = j\omega L_1, \quad Z_c = \frac{1}{j\omega C}, \quad Z_L = R_L + jX_L$$

$$\tilde{Z}_1 = \left(\frac{1}{\tilde{Z}_{L_2} + \tilde{Z}_{\text{load}}} + \frac{1}{\tilde{Z}_c} \right)^{-1}$$

$$\tilde{Z}_1 = \frac{\tilde{Z}_c + \tilde{Z}_{L_2} + \tilde{Z}_{\text{load}}}{\tilde{Z}_c(\tilde{Z}_{L_2} + \tilde{Z}_{\text{load}})} * \tilde{Z}_{L_2} = j\omega L, \quad \tilde{Z}_c = \frac{1}{j\omega C} = -jX_C$$

$$\tilde{Z}_1 = \frac{\tilde{Z}_{\text{load}} + j(X_{L_2} - X_C)}{(L/C - jX_C \cdot \tilde{Z}_{\text{load}})} * Z_{L_2} \cdot Z_c = L/C$$

The Equivalent impedance is then:

$$Z_0 = Z_1 + j\omega L_1$$

11.1.3

1. Referring to previous equations found in 1:

$$\therefore \tilde{I}_1 = \frac{\tilde{V}_o}{Z_0}$$

$$\tilde{V}_1 = \tilde{V}_o - j\omega L \tilde{I}_1$$

$$\tilde{V}_1 = \tilde{V}_o - j\omega L \frac{\tilde{V}_o}{Z_0} = \tilde{V}_o \left(1 - \frac{j\omega L}{Z_0} \right)$$

$$\therefore \tilde{V}_1 = \tilde{V}_o \left(1 - \frac{j\omega L}{Z_0} \right)$$

\tilde{I}_2 can be found as :

$$\tilde{I}_2 = \frac{\tilde{V}_o}{Z_0} - j\omega C \tilde{V}_1 = \frac{\tilde{V}_o}{Z_0} - j\omega C \tilde{V}_o \left(1 - \frac{j\omega L}{Z_0} \right)$$

$$= \tilde{V}_o \left(\frac{1}{Z_0} - j\omega C - \frac{\omega^2 LC}{Z_0} \right)$$

$$\tilde{I}_2 = \tilde{V}_o \left(\frac{1}{Z_0} (1 - \omega^2 LC) - j\omega C \right) = \boxed{\tilde{V}_o \left(\frac{\alpha}{Z_0} - j\omega C \right)} * \text{let } \alpha = (1 - \omega^2 LC)$$

Then \tilde{V}_2 can be found:

$$\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2$$

$$= \tilde{V}_o \left(1 - \frac{j\omega L}{Z_0} \right) - j\omega L \tilde{V}_o \left(\frac{\alpha}{Z_0} - j\omega C \right)$$

$$= \tilde{V}_o \left(1 - \frac{j\omega L}{Z_0} - \frac{j\omega L}{Z_0} \alpha - \omega^2 LC \right)$$

$$= \tilde{V}_o \left(1 - \omega^2 LC - \frac{j\omega L}{Z_0} (1 + \alpha) \right)$$

$$\tilde{V}_2 = \tilde{V}_o \left(\alpha - \frac{j\omega L}{Z_o} (1+\alpha) \right)$$

$$\tilde{V}_2 = \tilde{V}_o \left(\alpha \left(1 - \frac{j\omega L}{Z_o} \right) - \frac{j\omega L}{Z_o} \right) = \alpha \tilde{V}_1 - j\omega L \frac{\tilde{V}_o}{Z_o}$$

Since I_2 goes across the load:

$$I_2 = \frac{\tilde{V}_2}{Z_{\text{load}}}$$

$$\tilde{I}_2 = \frac{\tilde{V}_o}{Z_{\text{load}}} \left(\alpha \left(1 - \frac{j\omega L}{Z_o} \right) - \frac{j\omega L}{Z_o} \right)$$

1. Listing equations :

$$\tilde{I}_1 = \frac{\tilde{V}_o}{Z_o}$$

$$\tilde{I}_2 = \tilde{V}_o \left(\frac{\alpha}{Z_o} - j\omega C \right)$$

$$\tilde{V}_1 = \tilde{V}_o \left(1 - \frac{j\omega L}{Z_o} \right)$$

$$\tilde{V}_2 = \tilde{V}_o \left(\alpha - \frac{j\omega L}{Z_o} (1+\alpha) \right)$$

2. Since the Source varies as $V_o \cos(\omega t)$

$$V_o(t) = (\tilde{V}_o e^{j\omega t}) = V_o \cos(\omega t)$$

Then Equations Become :

$$I_1(t) = \frac{V_o}{Z_o} \cos(\omega t)$$

$$I_2(t) = V_o \left(\frac{\alpha}{Z_o} - j\omega C \right) \cos(\omega t)$$

$$V_1(t) = V_o \left(1 - \frac{j\omega L}{Z_o} \right) \cos(\omega t)$$

$$V_2(t) = V_o \left(\alpha - \frac{j\omega L}{Z_o} (1+\alpha) \right) \cos(\omega t)$$

To Verify Derived Relationships :

* KCL at 1 says :

$$\tilde{I}_1 = \tilde{I}_2 + j\omega C \tilde{V}_1$$

* Using relationships, in terms of Z_o & \tilde{V}_o

$$\tilde{I}_1 = \frac{\tilde{V}_o}{Z_o}, \quad \tilde{I}_2 = \tilde{V}_o \left(\frac{\alpha}{Z_o} - j\omega C \right), \quad \tilde{V}_1 = \tilde{V}_o \left(1 - \frac{j\omega L}{Z_o} \right) \quad * \alpha = (1 - \omega^2 L C)$$

$$\frac{\tilde{V}_o}{Z_o} = \tilde{V}_o \left(\frac{\alpha}{Z_o} - j\omega C \right) + j\omega C \tilde{V}_o \left(1 - \frac{j\omega L}{Z_o} \right)$$

$$\frac{\tilde{V}_o}{Z_o} = \tilde{V}_o \left(\frac{\alpha}{Z_o} \right) - \tilde{V}_o j\omega C + j\omega C \tilde{V}_o - j\omega C \tilde{V}_o \left(\frac{j\omega L}{Z_o} \right)$$

$$\frac{\tilde{V}_o}{Z_o} = \frac{\tilde{V}_o}{Z_o} (\alpha) + \omega^2 LC \frac{\tilde{V}_o}{Z_o}$$

$$\frac{\tilde{V}_o}{Z_o} = \frac{\tilde{V}_o}{Z_o} (1 - \omega^2 LC + \omega^2 LC)$$

$$\frac{\tilde{V}_o}{Z_o} = \frac{\tilde{V}_o}{Z_o} * KCL \text{ Proves consistent using Relationships derived}$$

* Analytically Using KVL across outer loop ③ from Circuit:

$$+\tilde{V}_o - j\omega L \tilde{I}_1 - j\omega L \tilde{I}_2 - Z_{load} \tilde{I}_2 = 0$$

$$\tilde{V}_o = j\omega L (\tilde{I}_1 + \tilde{I}_2) + Z_{load} \tilde{I}_2$$

Substituting Expressions:

$$\tilde{I}_1 = \frac{\tilde{V}_o}{Z_o}, \quad \tilde{I}_2 = V_o \left(\frac{\alpha}{Z_o} - j\omega C \right) = \frac{\tilde{V}_2}{Z_{load}}, \quad \tilde{V}_2 = \tilde{V}_o \left(\alpha \left(1 - \frac{j\omega L}{Z_o} \right) - \frac{j\omega L}{Z_o} \right)$$

$$\tilde{V}_o = j\omega L \left(\frac{\tilde{V}_o}{Z_o} + \tilde{V}_o \left(\frac{\alpha}{Z_o} - j\omega C \right) + Z_{load} \left(\frac{\tilde{V}_2}{Z_{load}} \right) \right)$$

$$\tilde{V}_o = j\omega L \frac{\tilde{V}_o}{Z_o} + j\omega L \alpha \frac{\tilde{V}_o}{Z_o} - j\omega L (j\omega C \tilde{V}_o) + \tilde{V}_o \left(\alpha \left(1 - \frac{j\omega L}{Z_o} \right) - \frac{j\omega L}{Z_o} \right)$$

$$\tilde{V}_o = j\omega L \frac{\tilde{V}_o}{Z_o} + j\omega L \frac{\alpha \tilde{V}_o}{Z_o} + \omega^2 LC \tilde{V}_o + \alpha \tilde{V}_o - j\omega L \frac{\alpha \tilde{V}_o}{Z_o} - j\omega L \frac{\tilde{V}_o}{Z_o}$$

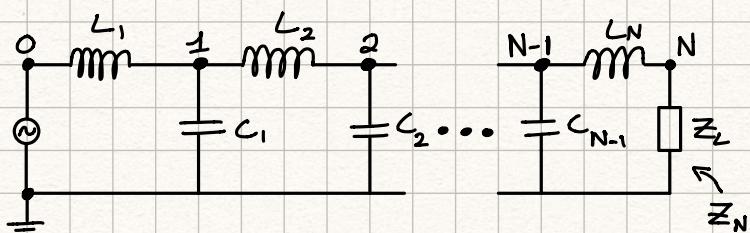
$$\tilde{V}_o = \tilde{V}_o (\alpha + \omega^2 LC)$$

$$\tilde{V}_o = \tilde{V}_o (1 - \omega^2 LC + \omega^2 LC)$$

$$\therefore \tilde{V}_o = \tilde{V}_o$$

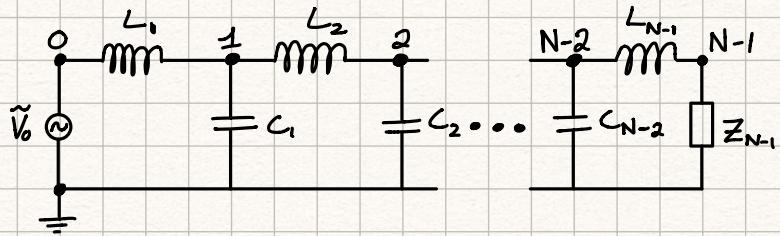
11.1.4:

For a Ladder Network of N nodes

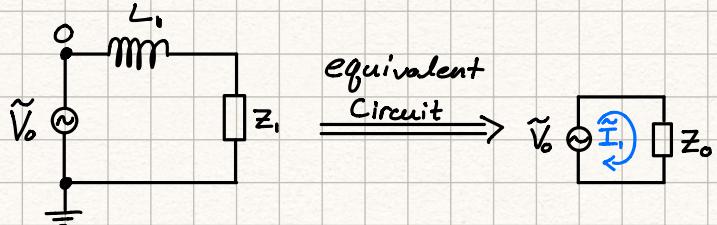


1. To account for all

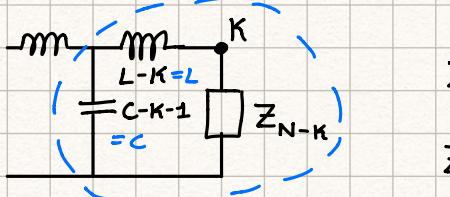
From Circuit Theory, one can find the equivalent impedance for node N to:



and can Continue Until:



during the k^{th} process of equiv. impedance, where $K \in (0, N)$



$$Z_{N-(K+1)} = \left(\frac{1}{Z_{N-K} + j\omega L} + \frac{1}{j\omega C} \right)^{-1}$$

$$Z_{N-(K+1)} = \left(\frac{1}{Z_{N-K} + j\omega L} + j\omega C \right)^{-1}$$

* Let $n = N - k$:

$$Z_n = \left(\frac{1}{Z_n + j\omega L} + j\omega C \right)^{-1}$$

Special Cases:

for $K=0$ process

$$Z_n = Z_{N-K} = Z_N = Z_L$$

for $K=N$ process

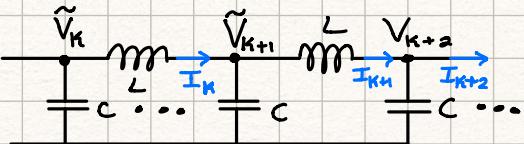
$$Z_n = Z_{N-K} = Z_0 = Z_1 + j\omega L$$

Otherwise:

$$Z_{n-1} = \left(\frac{1}{Z_n + j\omega L} + j\omega C \right)^{-1}$$

then All Z_n array is accounted for through recursion using $Z_n, \omega, L, \& C$.

2. Starting at the k^{th} node, where $k \in (0, N)$



$$\tilde{V}_{k+1} = \tilde{V}_k - j\omega L \tilde{I}_k$$

Special Case:

for $k=0 \rightarrow \tilde{V}_k = \tilde{V}_0$ is given

To check:

$$\tilde{V}_N = V_{N-1} - j\omega L \tilde{I}_{N-1}$$

$$\tilde{V}_N = \tilde{I}_N Z_L * Z_N = Z_L$$

Applying KCL at the $(k+1)$ node:

$$I_k - I_{k+1} - j\omega C V_{k+1} = 0$$

$$\therefore I_{k+1} = I_k - j\omega C V_{k+1}$$

Where finally:

$$I_N = I_{N-1} - j\omega C V_N$$

$$\& I_1 = \frac{\tilde{V}_0}{Z_0} * \text{can be found}$$