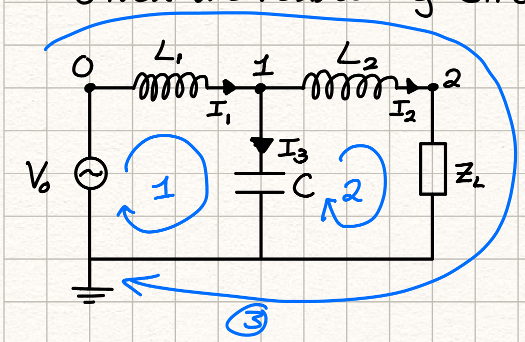
11.1.1:

Given the Following Circuit:



$* L_1 = L_2$

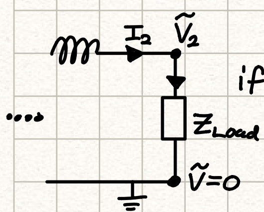
Applying KCL at Node 1:

$$\Sigma \tilde{I}_n(t) = \tilde{I}_1 - \tilde{I}_2 - \tilde{I}_3 = 0$$

$$\tilde{I}_1 = \tilde{I}_2 + \tilde{I}_3 \quad * \ \tilde{I}_3 = j\omega C \tilde{V}_1 \ , \ Z_C = 0 + \frac{1}{j\omega C} \text{ (only reactive component)}$$

$$\therefore \tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1$$

Looking at Node 2:



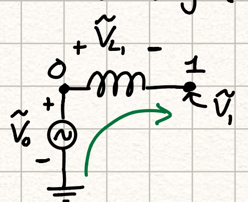
if considering Ground as bottom load:

$$\tilde{I}_2 = \frac{\tilde{V}_2 - 0V}{\tilde{Z}_{Load}} \quad * \text{ Ohms Law states } \tilde{V} = \tilde{I}\tilde{Z}$$

$$\boxed{\therefore \tilde{I}_1 = \tilde{V}_2 / \tilde{Z}_{Load}}$$

This should be I_2

Similarly this can be done in node 1 & node 0:



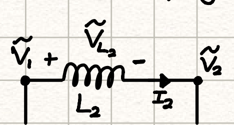
at 1 we have $\tilde{I}_1$ enter

$$\tilde{I}_1 = \frac{\tilde{V}_0 - \tilde{V}_1}{\tilde{Z}_{L_1}}$$

$$\tilde{V}_1 = \tilde{V}_0 - \tilde{Z}_{L_1}\tilde{I}_1 \quad * \ Z_{L_1} = R_{L_1} + jX_{L_1} = 0 + j\omega L_1$$

$$\boxed{\therefore \tilde{V}_1 = \tilde{V}_0 - j\omega L \tilde{I}_1} \quad * \ L_1 = L_2 = L$$

One Can apply the Same Method to find a relationship between $\tilde{V}_1$ & $\tilde{V}_2$:



Applying KCL at 2 we see I_2 enter

$$\tilde{I}_2 = \frac{\tilde{V}_1 - \tilde{V}_2}{Z_{L_2}} \quad * \ \tilde{V}_{L_2} = \tilde{Z}_2 \tilde{I}_2$$

$$\tilde{V}_2 = \tilde{V}_1 - \tilde{Z}_{L_2}\tilde{I}_2 \quad * \ Z_{L_2} = \cancel{R_L}^{0} + jX_{L_2} = j\omega L_2$$

$$\boxed{\therefore \tilde{V}_2 = \tilde{V}_1 - j\omega L_2 \tilde{I}_2}$$

11.1.2

Given $\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0}$, Z_0 is the equivalent impedence of the whole circuit

Z_1 will be the equivalent impedance of L_2, C, Z_{Load}

$$Z_{L_2} = j\omega L_1, \quad Z_C = \frac{1}{j\omega C}, \quad Z_L = R_L + X_L$$

$$\boxed{\tilde{Z}_1 = \left(\frac{1}{\tilde{Z}_{L_2} + \tilde{Z}_{Load}} + \frac{1}{\tilde{Z}_C}\right)^{-1}}$$

$$\tilde{Z}_1 = \frac{\tilde{Z}_C + \tilde{Z}_{L_2} + \tilde{Z}_{Load}}{\tilde{Z}_C(\tilde{Z}_{L_2} + \tilde{Z}_{Load})} \qquad * \ \tilde{Z}_{L_2} = j\omega L, \ \tilde{Z}_C = \frac{1}{j\omega C} = -jX_C$$

$$\tilde{Z}_1 = \frac{\tilde{Z}_{Load} + j(X_{L_2} - X_C)}{(L/C - jX_C \tilde{Z}_{Load})} \qquad * \ Z_{L_2} \cdot Z_C = L/C$$

The Equivalent impedence is then:

$$\boxed{Z_0 = Z_1 + j\omega L_1}$$

11.1.3

1. Reffering to previous equations found in 1:

$$\boxed{\therefore \tilde{I}_1 = \frac{\tilde{V}_0}{Z_0}}$$

$$\tilde{V}_1 = \tilde{V}_0 - j\omega L \tilde{I}_1$$

$$\tilde{V}_1 = \tilde{V}_0 - j\omega L \frac{\tilde{V}_0}{Z_0} = \tilde{V}_0\left(1 - \frac{j\omega L}{Z_0}\right)$$

$$\boxed{\therefore \tilde{V}_1 = \tilde{V}_0\left(1 - \frac{j\omega L}{Z_0}\right)}$$

$\tilde{I}_2$ can be found as:

$$\tilde{I}_2 = \frac{\tilde{V}_0}{Z_0} - j\omega C \tilde{V}_1 = \frac{\tilde{V}_0}{Z_0} - j\omega C \tilde{V}_0\left(1 - \frac{j\omega L}{Z_0}\right)$$

$$= \tilde{V}_0\left(\frac{1}{Z_0} - j\omega C - \frac{\omega^2 LC}{Z_0}\right)$$

$$\tilde{I}_2 = \tilde{V}_0\left(\frac{1}{Z_0}(1 - \omega^2 LC) - j\omega C\right) = \boxed{\tilde{V}_0\left(\frac{\alpha}{Z_0} - j\omega C\right)} \qquad * \text{ let } \alpha = (1 - \omega^2 LC)$$

Then $\tilde{V}_2$ can be found:

$$\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2$$

$$= \tilde{V}_0\left(1 - \frac{j\omega L}{Z_0}\right) - j\omega L \tilde{V}_0\left(\frac{\alpha}{Z_0} - j\omega C\right)$$

$$= \tilde{V}_0\left(1 - \frac{j\omega L}{Z_0} - \frac{j\omega L}{Z_0}\alpha - \omega^2 LC\right)$$

$$= \tilde{V}_0\left(1 - \omega^2 LC - \frac{j\omega L}{Z_0}(1 + \alpha)\right)$$

$$\tilde{V}_2 = \tilde{V}_0\left(\alpha - \frac{j\omega L}{Z_0}(1+\alpha)\right)$$

$$\tilde{V}_2 = \tilde{V}_0\left(\alpha\left(1-\frac{j\omega L}{Z_0}\right) - \frac{j\omega L}{Z_0}\right) = \alpha\tilde{V}_1 - j\omega L\frac{\tilde{V}_0}{Z_0}$$

Since I_2 goes across the load:

$$I_2 = \tilde{V}_2/Z_{Load}$$

$$\boxed{\tilde{I}_2 = \frac{\tilde{V}_0}{Z_{load}}\left(\alpha\left(1-\frac{j\omega L}{Z_0}\right) - \frac{j\omega L}{Z_0}\right)}$$

1. Listing equations:

$$\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0}$$

$$\tilde{I}_2 = \tilde{V}_0\left(\frac{\alpha}{Z_0} - j\omega C\right)$$

$$\tilde{V}_1 = \tilde{V}_0\left(1-\frac{j\omega L}{Z_0}\right)$$

$$\tilde{V}_2 = \tilde{V}_0\left(\alpha - \frac{j\omega L}{Z_0}(1+\alpha)\right)$$

2. Since the Source varies as $V_0\cos(\omega t)$

$$V_0(t) = \left(\tilde{V}_0 e^{j\omega t}\right) = V_0\cos(\omega t)$$

The time domain equations should not be complex.

Then Equations Become:

$$I_1(t) = \frac{V_0}{Z_0}\cos(\omega t)$$

$$I_2(t) = V_0\left(\frac{\alpha}{Z_0} - j\omega C\right)\cos(\omega t)$$

$$V_1(t) = V_0\left(1-\frac{j\omega L}{Z_0}\right)\cos(\omega t)$$

$$V_2(t) = V_0\left(\alpha - \frac{j\omega L}{Z_0}(1+\alpha)\right)\cos(\omega t)$$

To Verify Derived Relationships:

* KCL at 1 says:

$$\tilde{I}_1 = \tilde{I}_2 + j\omega C\tilde{V}_1$$

* Using relationships, in terms of Z_0 & $\tilde{V}_0$

$$\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0},\quad \tilde{I}_2 = \tilde{V}_0\left(\frac{\alpha}{Z_0} - j\omega C\right),\quad \tilde{V}_1 = \tilde{V}_0\left(1-\frac{j\omega L}{Z_0}\right) \qquad * \alpha = (1-\omega^2 LC)$$

$$\tilde{V}_0/Z_0 = \tilde{V}_0\left(\alpha/Z_0 - j\omega C\right) + j\omega C\tilde{V}_0\left(1-\frac{j\omega L}{Z_0}\right)$$

$$\frac{\tilde V_0}{Z_0} = \tilde V_0 (\alpha/Z_0) - \cancel{\tilde V_0 j\omega C} + \cancel{j\omega C \tilde V_0} - j\omega C \tilde V_0 \left(\frac{j\omega L}{Z_0}\right)$$

$$\frac{\tilde V_0}{Z_0} = \frac{\tilde V_0}{Z_0}(\alpha) + \omega^2 LC \frac{\tilde V_0}{Z_0}$$

$$\frac{\tilde V_0}{Z_0} = \frac{\tilde V_0}{Z_0}(1-\omega^2LC+\omega^2LC)$$

$\frac{\tilde V_0}{Z_0} = \frac{\tilde V_0}{Z_0}$ * KCL Proves consistent using Relationships derived

* Analytically Using KVL across outer loop ③ from Circuit:

$$+\tilde V_0 - j\omega L \tilde I_1 - j\omega L\tilde I_2 - Z_{Load}\tilde I_2 = 0$$

$$\tilde V_0 = j\omega L(\tilde I_1+\tilde I_2) + Z_{Load}\tilde I_2$$

Substituting Expressions:

$\tilde I_1 = \frac{\tilde V_0}{Z_0}$, $\tilde I_2 = V_0\left(\frac{\alpha}{Z_0} - j\omega C\right) = \frac{\tilde V_2}{Z_{load}}$, $\tilde V_2 = \tilde V_0\left(\alpha\left(1-\frac{j\omega L}{Z_0}\right) - \frac{j\omega L}{Z_0}\right)$

$$\tilde V_0 = j\omega L\left(\frac{\tilde V_0}{Z_0} + \tilde V_0\left(\frac{\alpha}{Z_0} - j\omega C\right) + \cancel{Z_{Load}}\left(\frac{\tilde V_2}{\cancel{Z_{Load}}}\right)\right.$$

$$\tilde V_0 = j\omega L\frac{\tilde V_0}{Z_0} + j\omega L\alpha\frac{\tilde V_0}{Z_0} - j\omega L(j\omega C\tilde V_0) + \tilde V_0\left(\alpha\left(1-\frac{j\omega L}{Z_0}\right) - \frac{j\omega L}{Z_0}\right)$$

$$\tilde V_0 = \cancel{j\omega L}\frac{\tilde V_0}{Z_0} + \cancel{j\omega L}\frac{\alpha \tilde V_0}{Z_0} + \omega^2LC\tilde V_0 + \alpha\tilde V_0 - \cancel{j\omega L\frac{\alpha\tilde V_0}{Z_0}} - \cancel{j\omega L}\frac{\tilde V_0}{Z_0}$$

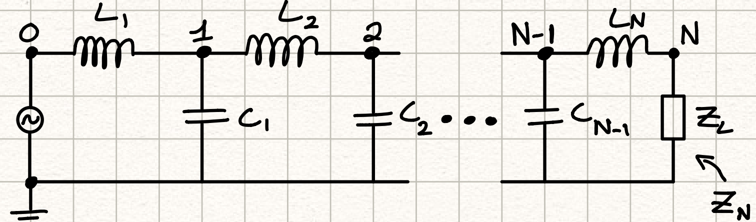
$$\tilde V_0 = \tilde V_0(\alpha+\omega^2LC)$$

$$\tilde V_0 = \tilde V_0(1-\omega^2LC+\omega^2LC)$$

$$\boxed{\therefore\ \tilde V_0 = \tilde V_0}$$

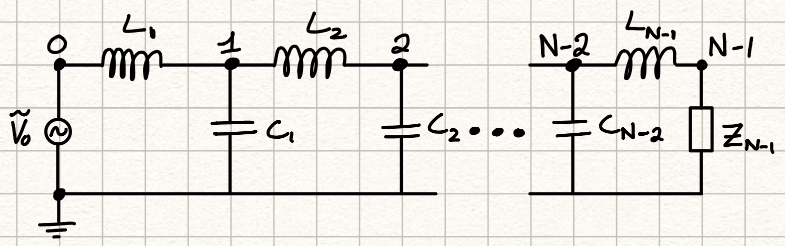
11.1.4:

For a Ladder Network of N nodes

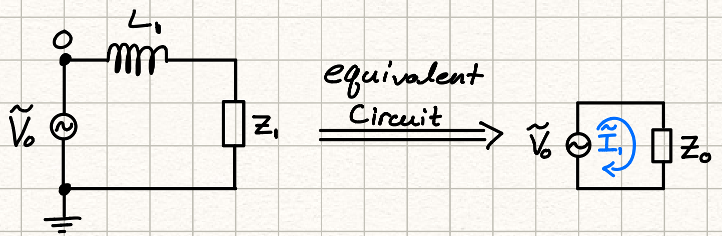


1. To account for all

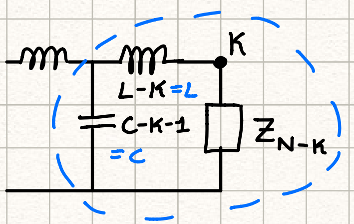
From Circuit Theory, one can find the equivalent impedance for node N to:



and can Continue Until:



during the K^{th} process of equiv. impedance, where $K \in (0,N)$



$$Z_{N-(K+1)} = \left(\frac{1}{Z_{N-K} + j\omega L} + \frac{1}{(1/j\omega C)} \right)^{-1}$$

$$Z_{N-(K+1)} = \left(\frac{1}{Z_{N-K} + j\omega L} + j\omega C \right)^{-1}$$

* Let $n = N-K$:

$$Z_{n-1} = \left(\frac{1}{Z_n + j\omega L} + j\omega C \right)^{-1}$$

Special Cases:

for $K=0$ process

$$Z_n = Z_{N-K} = Z_N = Z_L$$

for $K=N$ process

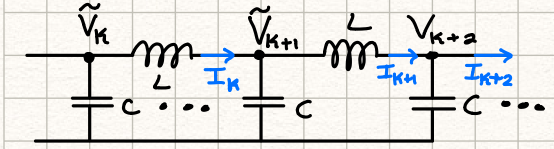
$$Z_n = Z_{N-K} = Z_0 = Z_1 + j\omega L$$

Otherwise:

$$Z_{n-1} = \left(\frac{1}{Z_n + j\omega L} + j\omega C \right)^{-1}$$

then All Z_n array is accounted for through recursion using Z_n, ω, L, & C.

2. Starting at the K^{th} node, where $K \in (0, N)$



$$\tilde{V}_{K+1} = \tilde{V}_K - j\omega L \tilde{I}_K$$

Special Case:

for $K=0 \rightarrow \tilde{V}_K = \tilde{V}_0$ is given

To check:

$$\tilde{V}_N = V_{N-1} - j\omega L \tilde{I}_{N-1}$$

$$\tilde{V}_N = \tilde{I}_N Z_L \quad * \; Z_N = Z_L$$

Applying KCL at the (K+1) node:

$$I_K - I_{K+1} - j\omega C V_{K+1} = 0$$

$$\therefore \; I_{K+1} = I_K - j\omega C V_{K+1}$$

Where finally:

$$I_N = I_{N-1} - j\omega C V_N$$

& $I_1 = \dfrac{\tilde{V}_0}{Z_0}$ * can be found