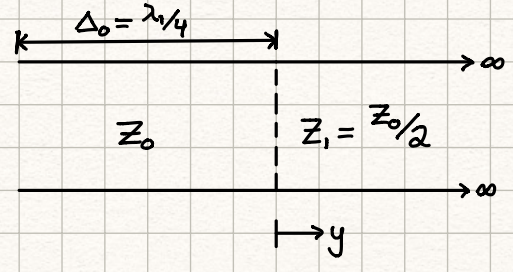
10.1 Impedence Transformation II

Given the following Transmission line:



Compute:

1. $\tilde{\rho}_0(0)$
2. $Z_0(-\Delta_1)$

10.2.1 Hand Calculation

1. Impedance Continuity states:

$$Z_0(0) = Z_1(0)$$

since Z_1 line exists and continues to ∞, then no reflected wave exists:

$$\therefore \tilde{\rho}_1(y) = 0$$

$$Z_0\left(\frac{1+\tilde{\rho}_0(0)}{1-\tilde{\rho}_0(0)}\right) = Z_1 \quad * Z_1 = Z_0/2$$

then $\tilde{\rho}_0(0)$ can be found as:

$$\tilde{\rho}_0(0) = \left(\frac{Z_1/Z_0 - 1}{Z_1/Z_0 + 1}\right)$$

$$= \left(\frac{1/2 - 1}{1/2 + 1}\right)$$

$$\therefore \tilde{\rho}_0(0) = -1/3$$

2. To find $Z_0(-\Delta_1)$, phase is added to $\tilde{\rho}_0(y)$:

$$\tilde{\rho}_0(-\Delta_0) = -1/3\, e^{j2\beta_0(-\Delta_1)} \quad * \beta_0 = \frac{2\pi}{\lambda_0} \;\&\; \Delta_1 = \lambda_1/4$$

$$\tilde{\rho}_0(-\Delta_0) = -1/3\, e^{-j\pi\, \lambda_1/\lambda_0} \quad * \text{if } \lambda_1 = \lambda_0$$

$$= 1/3 \quad * e^{-j\pi} = -1$$

$$Z_0(-\Delta_0) = Z_0\left(\frac{1+\tilde{\rho}_0(-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_0)}\right)$$

$$= Z_0\left(\frac{4/3}{2/3}\right) = 2Z_0$$

10.2.2 Using Smith Chart

1. Normalized impedance says:

$$\frac{Z_1(0)}{Z_0(0)} = \frac{1}{2}$$

then $r = 1/2$, $x = 0$

2. From Smith Chart

$$\tilde{\rho}_0(0) = -\frac{1}{3}$$

3. Finding $\rho_0(-\lambda_0/4)$:

$$\tilde{\rho}_0(-\lambda_0/4) = \tilde{\rho}_0(0)\, e^{-j\frac{4\pi}{\lambda_0}\frac{\lambda_0}{4}}$$

$$= \tilde{\rho}_0(0)\, e^{-j\pi}$$

$$\tilde{\rho}_0(-\tfrac{\lambda_0}{4}) = -\tilde{\rho}_0(0) = \frac{1}{3}$$

4. From a CW rotation of π, the Smith Chart reads:

$$r \approx 2.0, \quad x = 0$$

5. Then

$$Z_0(-\lambda/4) = Z_0\left(\frac{1+\tilde{\rho}_0(-\frac{\lambda_0}{4})}{1-\tilde{\rho}_0(-\frac{\lambda_0}{4})}\right)$$

$$= Z_0\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$$

$$= Z_0\left(\frac{4}{3}\cdot\frac{3}{2}\right)$$

$$\boxed{Z_0(-\lambda/4) = 2Z_0}$$

The Complete Smith Chart

Black Magic Design