This is a good discussion of the background, but you have the ability to test if the expectation you have are consistent with an experiment based on the results from Part I. In particular, in Part II.3, you should compare the exact solution with the discrete solution. Need to discuss how the plots that you presented support the claims.

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ECE 513

Justifications:

1. When $Z_L = \sqrt{\frac{L}{C}}$, the ladder Network Solution is a discrete Approximation to an infinitely long transmission line w/ characteristic Impedance of $\sqrt{\frac{L}{C}}$ When calculating Z_n array. The reflection coefficient was also calculated for different values of L & C.

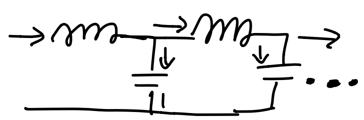
$\rho \approx 0$ for $Z_L = \sqrt{\frac{L}{C}}$

Then the reflected wave is expected to be minimal. This behavior was seen in wave mechanics and transmission line conditions where the new impedance or boundary goes on forever.

This can be conifirmed by the plots obtained when looking at the voltage Standing Wave Ratio for $Z_L = \sqrt{\frac{L}{C}}$ Should include this plot

The voltage is seen to be constant throughout position of N evolving over positions. magnitude of the complex V_k values

From an infinite ladder Network, what occurs is that the Voltage wave & current wave continue to propogate and energy is absorbed and transfered to the reactive components.



Although the waves never reach the load itself for an infinite ladder network (as mentioned in attached article that references analogy to radiation from an antenna which absorbs from its driving source).

Considering ideal capacitors & inductors, energy is not lost, but is continually transffered forever. This is because the load is infinite and can continue absorbing energy from the source.

The method derived from the article follows the derivation for the equivalent impedance of an infinite ladder network of ideal L & C components as:

$$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \left(\frac{\omega L}{2}\right)^2}$$

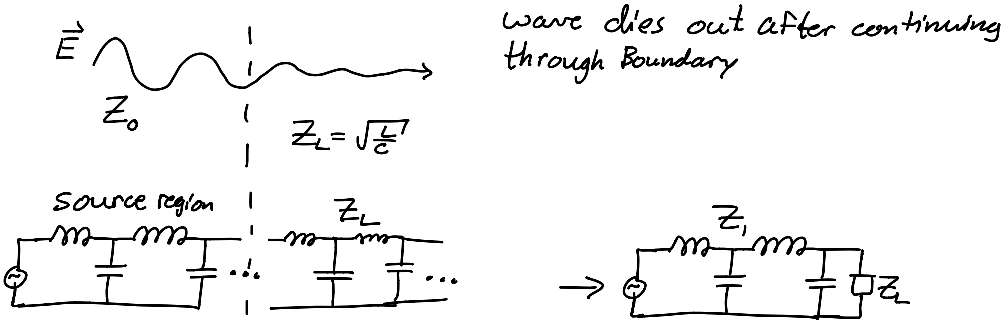
When approximation used looks at small frequencies:

$$Z_0 \approx \sqrt{\frac{L}{C}}$$

If rho is not exactly zero, you should see evidence of this in your plots.

* This is why the ρ is not exactly 0 because there is the extra term of: $\left(\frac{\omega L}{2}\right)^2$ that should be accounted for

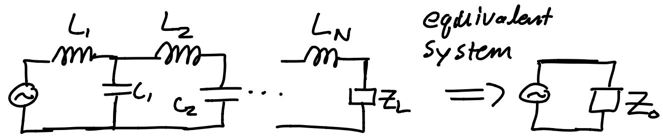
Overall the equivalent E&M wave system is for when encountering a conductor:



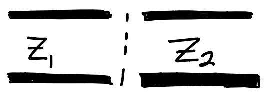
this is similar to a source that connects a finite transmission line to an infinite one since the infinite network can be directly solved and approximated to an equivalent impedance

2) $Z_L \neq \sqrt{\frac{L}{C}}$, ladder network is an approximate solution of the continuous transmission line that has a change in characteristic impedance

Following derivations from part 1. One can always find the equivalent impedance of the circuit configuration using parallel & series equivalences individually from Right to left of load.

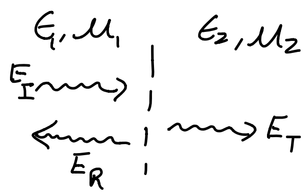


Similarly Z_L could also be the equivalent impedance of a seperate system, and instead of simplifying, one can expand Z_L as a connected transmission line as such:



From this, it is simple to see that as the voltage from the source propagates through Z_1, it will encounter Z_2 and cause a reflected and transmitted wave once encountered.

This is supported by the voltage plots showing large changes in amplitude over time and in position, when $Z_L \neq \sqrt{\frac{L}{C}}$. The equivalent reflection is also found to be non-zero, further providing an argument of this claim. The equivalent wave-mechanics situation is encountered for an E&M wave propogating and encountering a change in ϵ & μ such that the wave will produce reflection & transmitted waves based on the incident wave.



where the reflected & transmitted waves are dependent on Refl & Trans. Coefficients & Amplitude of Incidence.

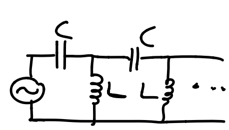
Similar analysis is seen when looking for ρ & τ using Z_L & Z_0. Or analyzing Standing Wave Ratio (VSWR). From plots obtained using a $Z_L \neq \sqrt{\frac{L}{C}}$ we see large changes in amplitude over time and different values of voltage amplitude delivered to load when comparing to source (change in position). This is largely due to reflected wave also being accounted for that is minimizing the total wave amplitud that is delivered to the load.

3 The discrete approximation to the continuous solution is better for small ω.

This is true due to several factors. The impedance of L & C components are largely dependent on a factor of ω. The equivalent impedance of a system is shown as:

$Z_0 = \sqrt{\left(\frac{L}{C}\right) - \frac{\omega^2 L^2}{4}}$, then for $\lim_{\omega \to 0}$, we obtain the discrete approximation of $Z_0 = \sqrt{\frac{L}{C}}$.

For large frequencies, the load becomes imaginary and causes current & voltage propagation to not occur. This low pass filter transmission line then shows that the ladder network can only function for small frequencies. This is largely due to a L-C configuration making this low-pass filter, since the inductors have an imaginary impedance of $j\omega L$. In reality many transmission lines work at high frequencies (GHz range) and use a high-pass filter configuration of a C-L Network.



This is because of the impedance of C working as $\frac{1}{j\omega C}$, and thus very high at low frequencies.