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ECE 513
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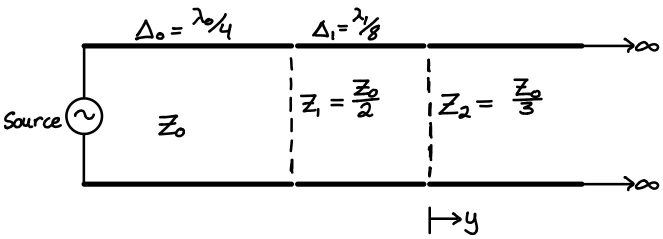
10.3 Impedence Transformation II

Given the Following Transmission line:



10.1.3 Hand Calculation

Impedence Continuity states:

$$Z_1(0) = Z_2(0)$$

Since Z_2 line continues to ∞, $\tilde{\rho}_2(y) = 0$ due to no reflected wave, then:

$$Z_1\left(\frac{1+\tilde{\rho}_1(0)}{1-\tilde{\rho}_1(0)}\right) = Z_2$$

then $\tilde{\rho}_1(0)$ can be found as:

$$\tilde{\rho}_1(0) = \left(\frac{Z_2/Z_1 - 1}{Z_2/Z_1 + 1}\right) \quad * \ \frac{Z_2}{Z_1} = \frac{Z_0}{3} \cdot \frac{2}{Z_0} = \frac{2}{3}$$

$$\tilde{\rho}_1(0) = \left(\frac{2/3 - 1}{2/3 + 1}\right) = -1/5$$

Then:

$$\tilde{\rho}_1(-\lambda_1/8) = \tilde{\rho}_1(0)\, e^{j2 \frac{2\pi}{\cancel{\lambda_1}} \frac{\cancel{\lambda_1}}{8}}$$

$$= \tilde{\rho}_1(0)\, e^{j\pi/2}$$

$$= -j\, 1/5$$

From this the impedence $Z_1(-\lambda_1/8)$ can be found:

$$Z_1(-\lambda_1/8) = Z_1\left(\frac{1+\tilde{\rho}_1(-\lambda_1/8)}{1-\tilde{\rho}_1(-\lambda_1/8)}\right)$$

$$= Z_1\left(\frac{1-j\,1/5}{1+j\,1/5}\right)\left(\frac{1-j\,1/5}{1-j\,1/5}\right)$$

$$= Z_1\left(\frac{(1-j\,1/5)^2}{1+1/25}\right)$$

$$= Z_1\left[\left(\frac{1-\frac{1}{25}}{1+1/25}\right) - j\left(\frac{2/5}{1+1/25}\right)\right]$$

$$Z_1(-\lambda_1/8) = Z_1\left(12/13 - j\,5/13\right) = Z_1\left(1e^{-j0.1257\pi}\right) \qquad * \tan^{-1}\left(\tfrac{5}{12}\right) \approx 0.1257\pi$$

then from 2^{nd} Boundary at $-\lambda_1/8$:

$$Z_0(-\lambda_1/8) = Z_1(-\lambda_1/8)$$

$$Z_0\left(\frac{1+\rho_0(-\lambda_1/8)}{1-\rho_0(-\lambda_1/8)}\right) = Z_1\, e^{-j0.1257\pi}$$

$$\frac{1+\tilde{\rho}_0(-\lambda_1/8)}{1-\tilde{\rho}_0(-\lambda_1/8)} = \frac{Z_1}{Z_0}\, e^{-j0.1257\pi}$$

$$\rho_0(-\lambda_1/8) = \left(\frac{Z_1/Z_0\, e^{-j0.1257}-1}{Z_1/Z_0\, e^{-j0.1257}+1}\right) \quad * Z_1 = Z_0/2$$

$$\tilde{\rho}_0(-\lambda_1/8) = \left(\frac{1/2\, e^{-j0.1257}-1}{1/2\, e^{-j0.1257}+1}\right)$$

Computing w/ MATLAB

$$\therefore\ \tilde{\rho}_0(-\lambda_1/8) \approx 0.388 \measuredangle -152.85^\circ = -0.8492\pi$$

then:

$$\rho_0(-\Delta_1-\Delta_0) \approx 0.388 e^{-j0.8492\pi} e^{j\frac{4\pi}{\cancel{\lambda_0}}\frac{\cancel{\lambda_1}}{\cancel{4}}} \qquad * \text{if } \lambda_0 = \lambda_1$$

$$\tilde{\rho}_0(-\Delta_1-\Delta_0) = 0.388 e^{-j0.8492\pi} e^{j\pi}$$

$$= 0.388 e^{j0.1508\pi}$$

impedence at source can be found as:

$$Z_0(-\Delta_1-\Delta_0) = Z_0\left(\frac{1+\tilde{\rho}_0(-\Delta_1-\Delta_0)}{1-\tilde{\rho}_0(-\Delta_1-\Delta_0)}\right)$$

$$= Z_0\left(\frac{1-0.388e^{j0.1508\pi}}{1+0.388\,e^{j0.1508\pi}}\right)$$

$$Z(-[\Delta_1+\Delta_0]) = Z_0\left(2e^{-j0.8492\pi}\right)$$

$$= 2Z_0 \measuredangle -152.85^\circ$$