

Reflection Matrix

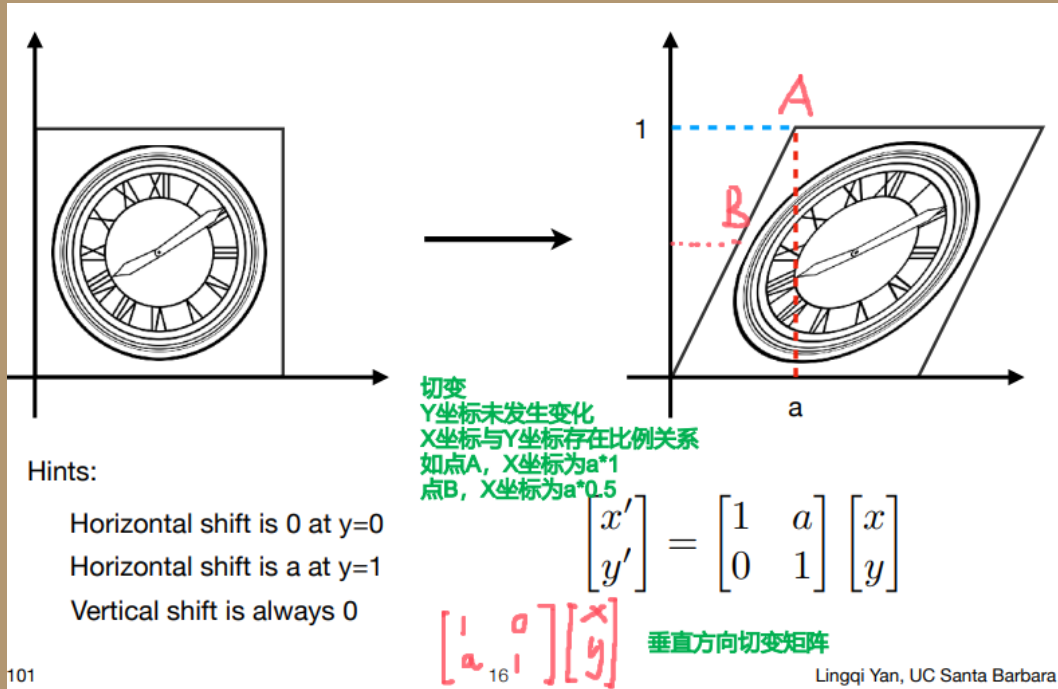
Horizontal reflection:

$$x' = -x$$

$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

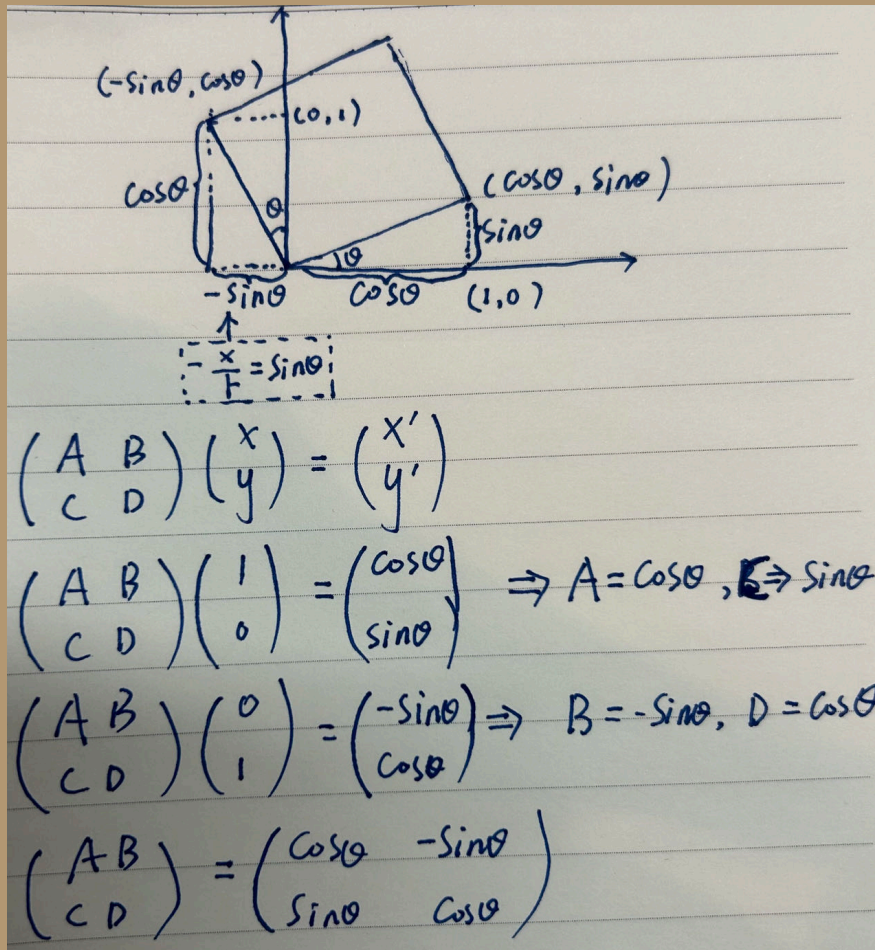
Shear Matrix



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Rotation Matrix



旋转-θ角度

正交矩阵

$$R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

cosθ对称函数

$$R_{-\theta} = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = (R_\theta)^T$$

$$R_{-\theta} = (R_\theta)^{-1} \text{ (by definition)}$$

旋转-θ

齐次坐标

$$\text{2D point} = (x, y, 1)^T$$

$$\text{2D vector} = (x, y, 0)^T$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

对点会进行平移改变，对向量则不会改变其位置，向量只有方向和长度没有位置

Valid operation if w-coordinate of result is 1 or 0

• vector + vector = vector

• point - point = vector

• point + vector = point

• point + point = ??

在齐次坐标中，两个点相加相当于这两个点的中心点

In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \text{ is the 2D point } \begin{pmatrix} x/w \\ y/w \\ 1 \end{pmatrix}, w \neq 0$$

仿射变换

Affine map = linear map + translation 仿射变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

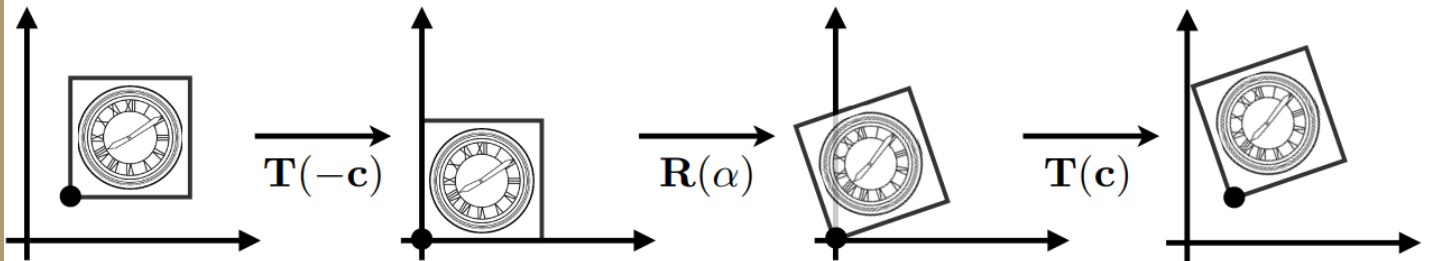
Using homogenous coordinates: 齐次坐标

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

先做线性变换后做平移变换

How to rotate around a given point c ?

1. Translate center to origin
2. Rotate
3. Translate back



Matrix representation?

以任意点为旋转中心

移动至原点后旋转后平移

$$T(c) \cdot R(\alpha) \cdot T(-c)$$