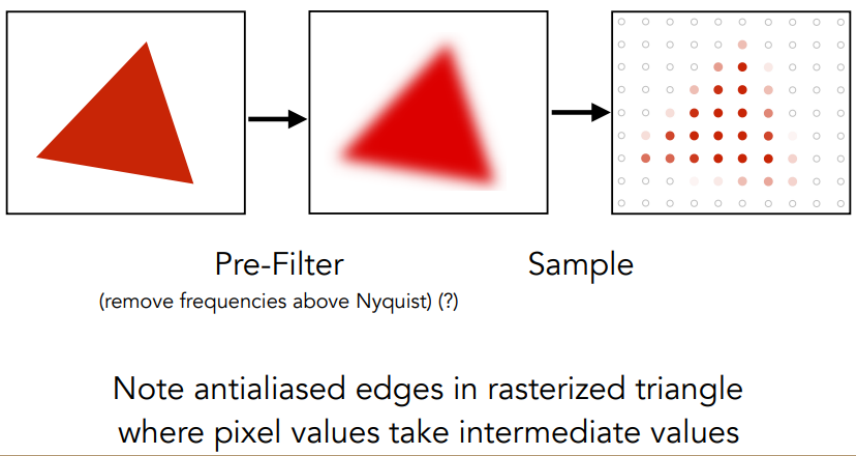
Aliasing Artifacts

Signals are changing too fast (high frequency),but sampled too slowly



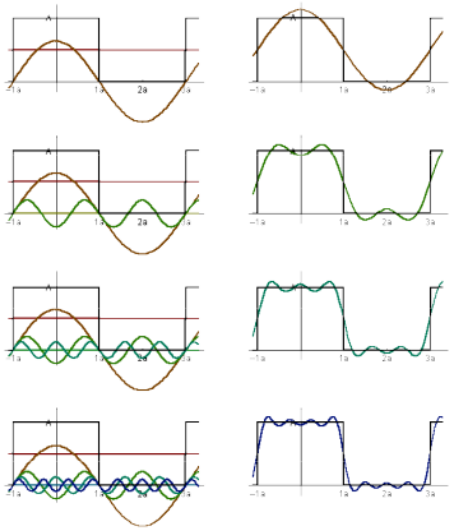
Antialiasing Idea: Blurring (Pre-Filtering) Before Sampling

Fourier Transform

Fourier Transform

Represent a function as a weighted sum of sines and cosines

Joseph Fourier 1768 - 1830

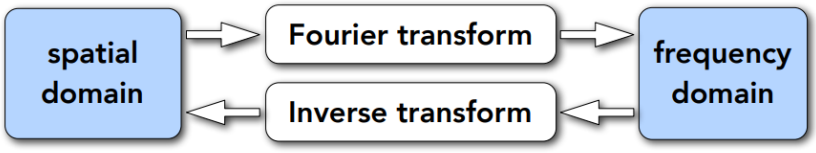


将周期函数展开成由三角函数组成的级数

$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

任何一个函数可以分解成不同的频率

$f(x)$ $\quad F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i\omega x}dx \quad$ $F(\omega)$

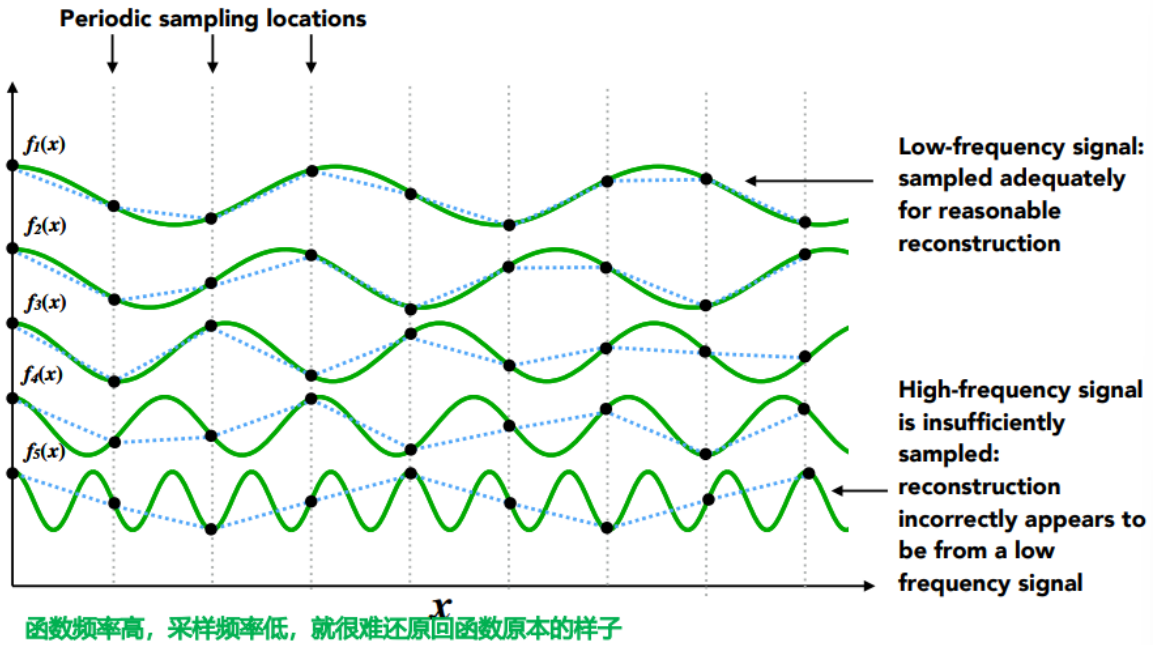


$$f(x) = \int_{-\infty}^{\infty} F(\omega)e^{2\pi i\omega x}d\omega$$

Recall $e^{ix} = \cos x + i\sin x$

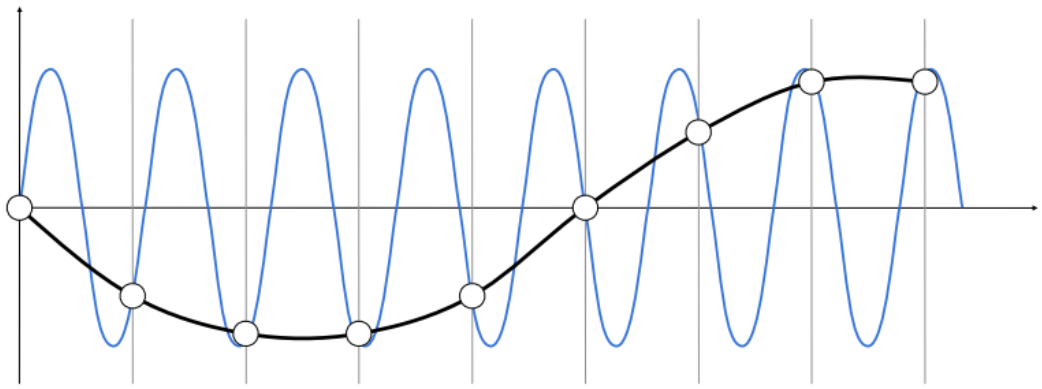
可以利用傅里叶变换与逆变换在时域与频域之间进行转换

Higher Frequencies Need Faster Sampling



函数频率高，采样频率低，就很难还原回函数原本的样子

Undersampling Creates Frequency Aliases

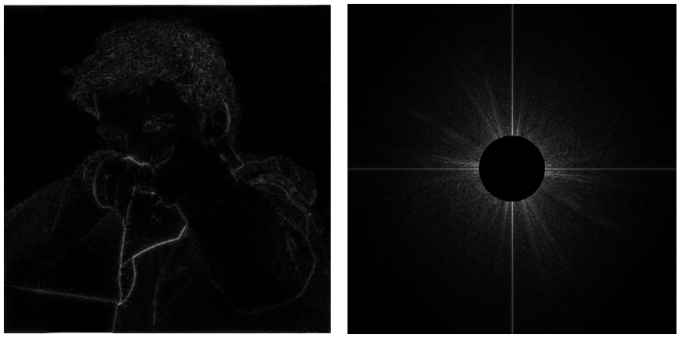


High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

Two frequencies that are indistinguishable at a given sampling rate are called “aliases”

使用同一种采样方法，采样两个完全不同的函数，得到的结果完全一样，被称为走样

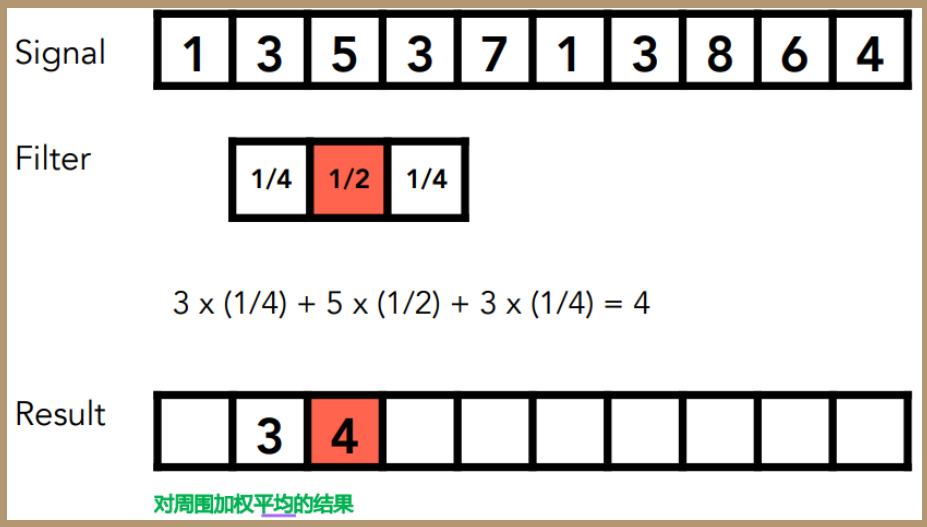
Filtering = Getting rid of certain frequency contents



High-pass filter

Convolution

Filtering = Convolution(卷积) (= Averaging)



卷积相当于对一个像素周围进行加权平均

Convolution Theorem

Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa

时域上对两个信号进行卷积反映在频域上是两个信号的频域的乘积

Option 1:

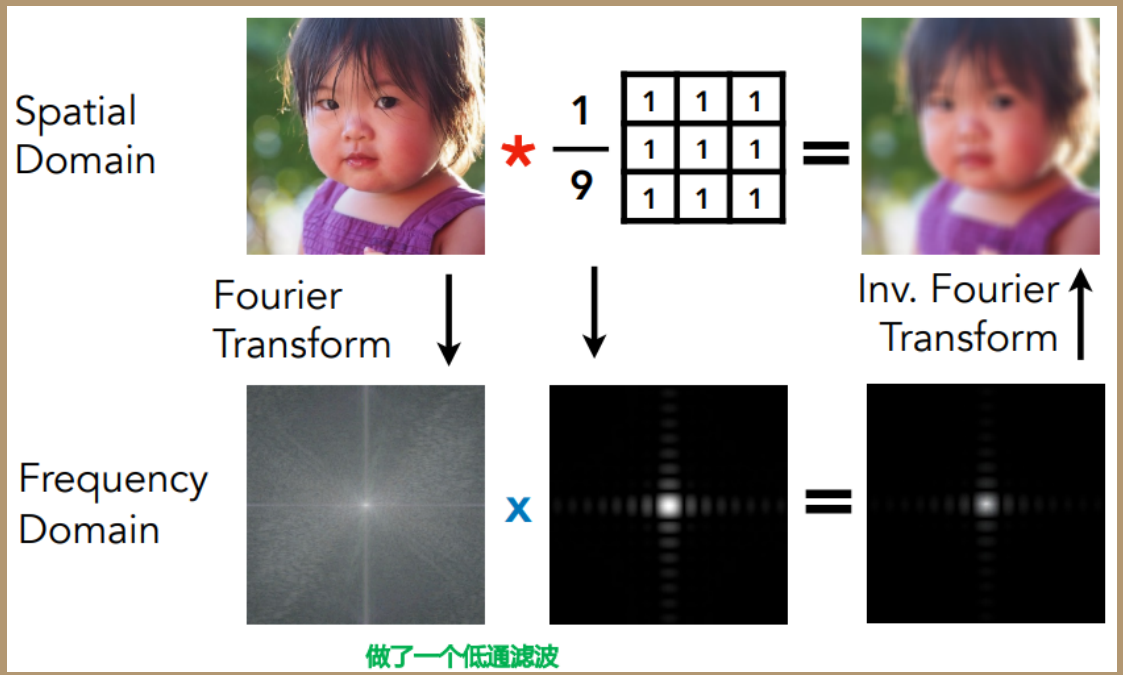
如何做一个卷积?
·可以拿到一幅图用一个卷积的滤波器做一个卷积操作
·也可以将图傅里叶变换到频域上，再将卷积的滤波器变换到频域上，将两者进行相乘得到频域的结果再逆傅里叶变换到时域上

- Filter by convolution in the spatial domain

Option 2:

- Transform to frequency domain (Fourier transform)
- Multiply by Fourier transform of convolution kernel
- Transform back to spatial domain (inverse Fourier)

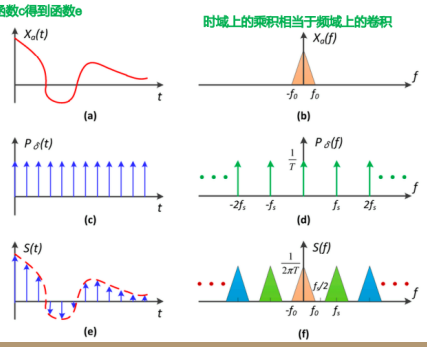
在时域上的乘积相当于频域上的卷积



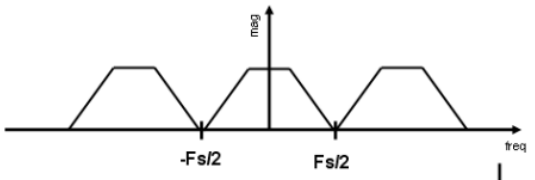
Sampling = Repeating Frequency Contents

用函数a乘以函数c得到函数e

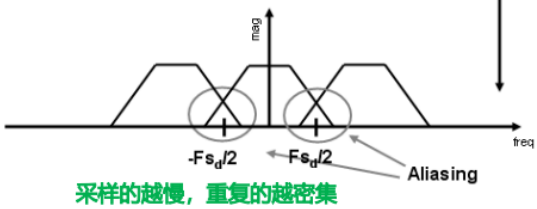
时域上的乘积相当于频域上的卷积



Dense sampling:



Sparse sampling:



采样的越慢，重复的越密集