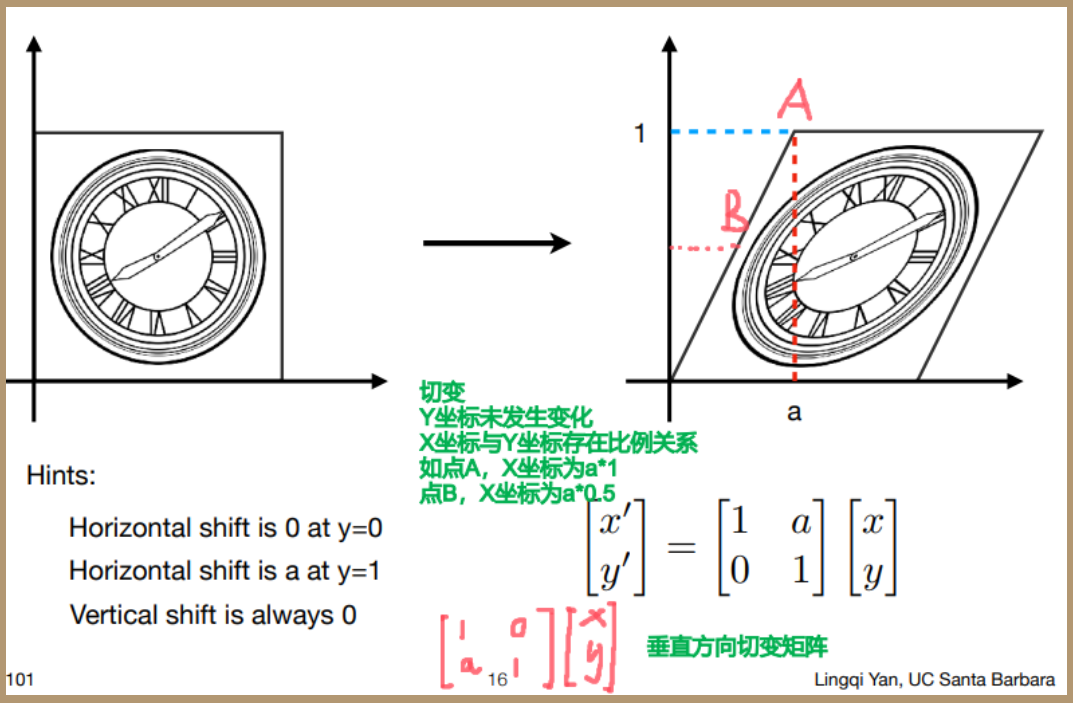
Reflection Matrix

Horizontal reflection:

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear Matrix



切变
Y坐标未发生变化
X坐标与Y坐标存在比例关系
如点A，X坐标为a*1
点B，X坐标为a*0.5

Hints:

Horizontal shift is 0 at y=0

Horizontal shift is a at y=1

Vertical shift is always 0

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ 垂直方向切变矩阵

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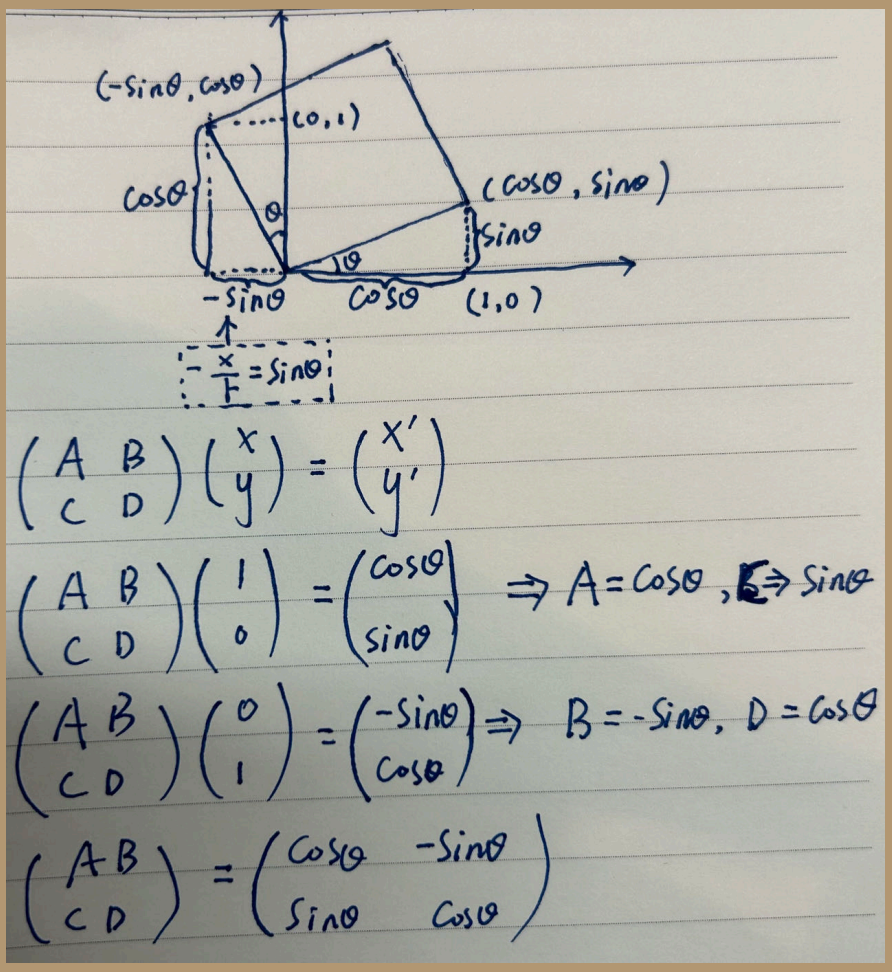
Rotation Matrix



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \Rightarrow A = \cos\theta, C \Rightarrow \sin\theta$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \Rightarrow B = -\sin\theta, D = \cos\theta$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

齐次坐标

2D point = (x, y, 1)^T

2D vector = (x, y, 0)^T

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

对点会进行平移改变，对向量则不会改变其位置，向量只有方向和长度没有位置

Valid operation if w-coordinate of result is 1 or 0

- vector + vector = vector
- point – point = vector
- point + vector = point
- point + point = ??

在齐次坐标中，两个点相加相当于这两个点的中心点

In homogeneous coordinates,

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} \text{ is the 2D point } \begin{pmatrix} x/w \\ y/w \\ 1 \end{pmatrix}, \quad w \neq 0$$

仿射变换

Affine map = linear map + translation 仿射变换

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

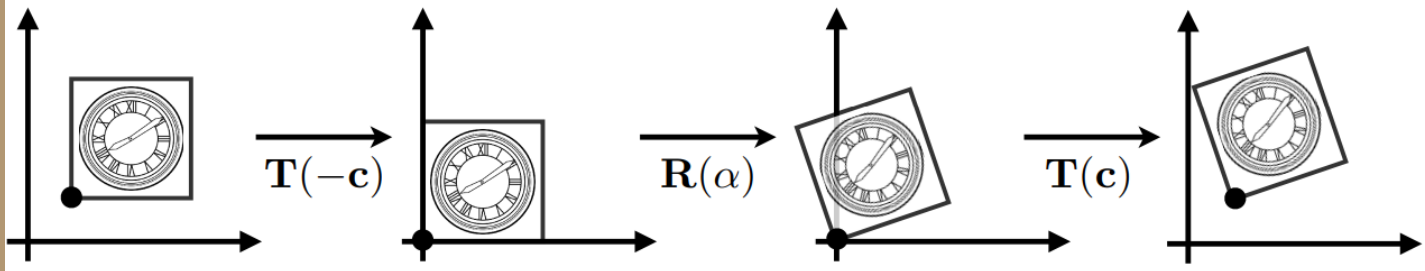
Using homogenous coordinates: 齐次坐标

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

先做线性变换后做平移变换

How to rotate around a given point c?

1. Translate center to origin
2. Rotate
3. Translate back



Matrix representation?

以任意点为旋转中心

移动至原点后旋转后平移

$$\mathbf{T}(\mathbf{c}) \cdot \mathbf{R}(\alpha) \cdot \mathbf{T}(-\mathbf{c})$$