

```
In [303...]
from qiskit import QuantumCircuit, transpile, QuantumRegister
from qiskit.visualization import plot_bloch_multivector, plot_state_city
from qiskit.quantum_info import Statevector
from qiskit_aer import AerSimulator
import numpy as np
from qiskit.circuit.library import QFT
import matplotlib.pyplot as plt
from qiskit import QuantumCircuit, ClassicalRegister
from qiskit.quantum_info import Statevector
from qiskit.visualization import plot_bloch_multivector, plot_state_city
import numpy as np
import matplotlib.pyplot as plt
from qiskit.visualization import plot_histogram
%matplotlib inline
```

```
In [304...]
def show_statevector(sv, title=""):
    print("\n" + "="*70)
    print(title)
    print("-"*70)

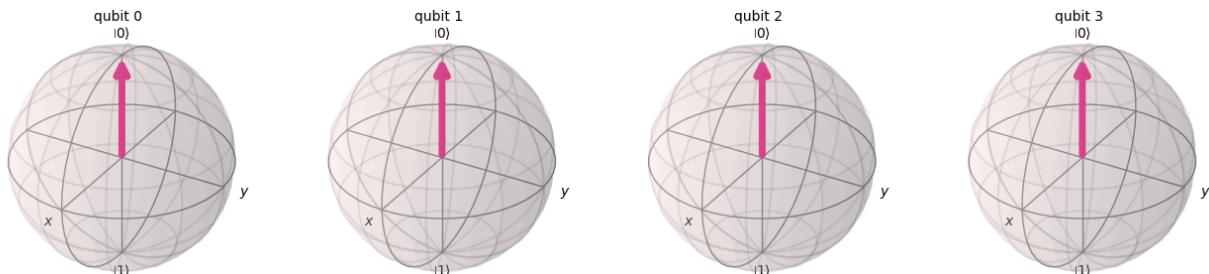
    # Print non-zero amplitudes
    for i, amp in enumerate(sv.data):
        if abs(amp) > 1e-6:
            print(f"||{i:010b}> {amp}")

    # Plot Bloch vectors
    fig = plot_bloch_multivector(sv)
    fig.suptitle(title)
    plt.show()
```

```
In [305...]
# Create 4-qubit circuit
qc = QuantumCircuit(4, 2) # 4 qubits, 2 classical bits (final key)
#q0 q1 → User1 chips
#q2 q3 → User2 chips

#Now we have encoded 4-qubit state.

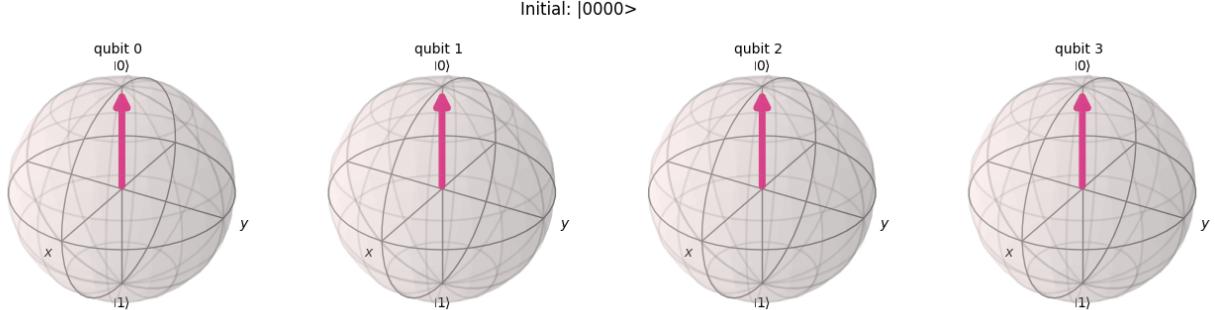
# Simulate initial state
state_vector= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector, "Initial: |0000>")
```



Figure(162.08x451.5)

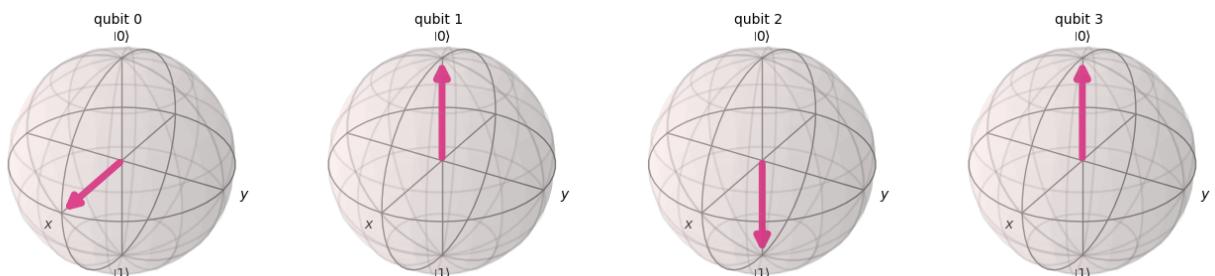
 q_0 — q_1 — q_2 — q_3 — C $\frac{2}{\sqrt{2}}$

```
=====
Initial: |0000>
=====
|000000000> (1+0j)
```

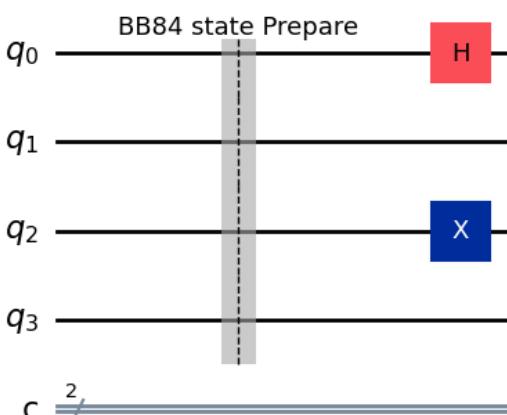


```
In [306...]
# -----
# STEP 1: BB84 prep
# -----
# Example:  $|+\rangle$  for user1,  $|1\rangle$  for user2
qc.barrier(label="BB84 state Prepare")
qc.h(0)          # user1 =  $|+\rangle$ 
qc.x(2)          # user2 =  $|1\rangle$ 
# ----- User 2 -----
# Already  $|0\rangle$ 
#Now the State is  $|00\rangle=|+\rangle\otimes|1\rangle$ 
```

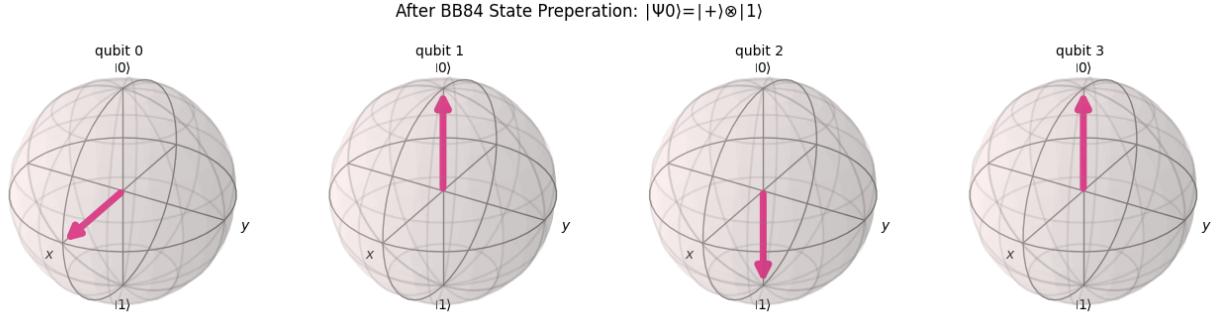
```
# Simulate initial state
state_vector_after_BB84= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector_after_BB84)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector_after_BB84, "After BB84 State Preparation:  $|00\rangle=|+\rangle\otimes|1\rangle$ ")
```



Figure(538.33x451.5)



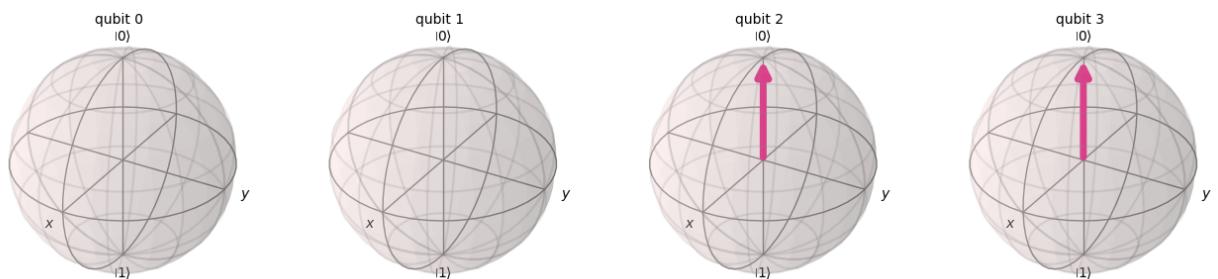
```
=====
After BB84 State Preparation:  $|00\rangle=|+\rangle\otimes|1\rangle$ 
=====
|000000100> (0.7071067811865475+0j)
|000000101> (0.7071067811865475+0j)
```



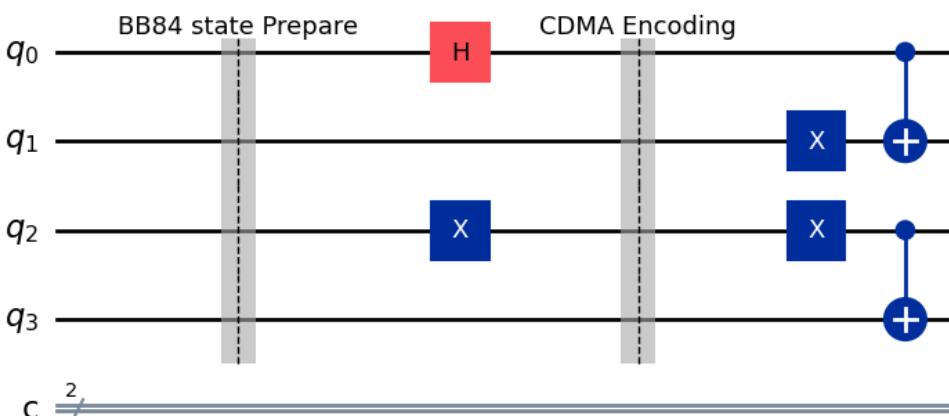
```
In [307... # STEP 2: CDMA Encode
# -----
qc.barrier(label="CDMA Encoding")
# User 1 C1=01
qc.x(1)
qc.cx(0,1)

# User 2 C2=10
qc.x(2)
qc.cx(2,3)

# Simulate initial state
state_vector_before_qft= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector_before_qft)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector, "After CDMA Encode:")
```

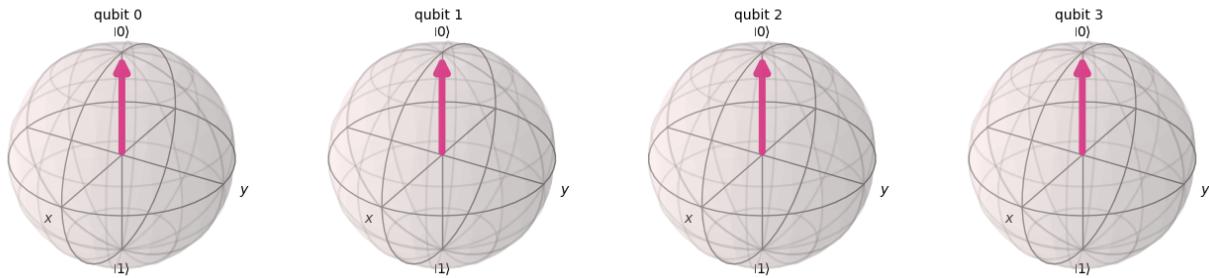


Figure(956.385x451.5)

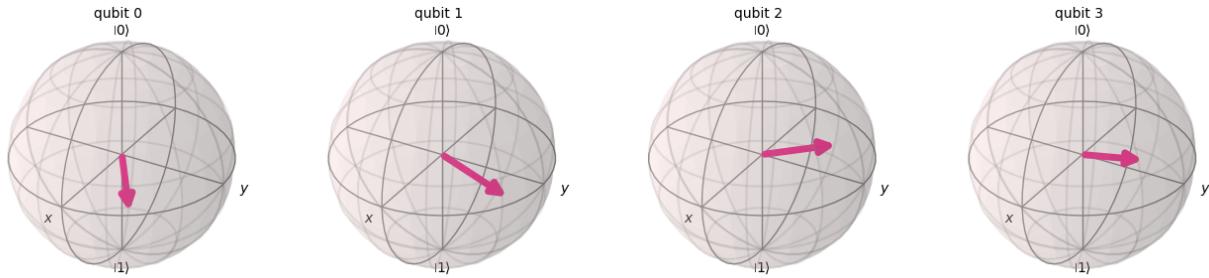


```
=====
After CDMA Encode:
=====
|0000000000> (1+0j)
```

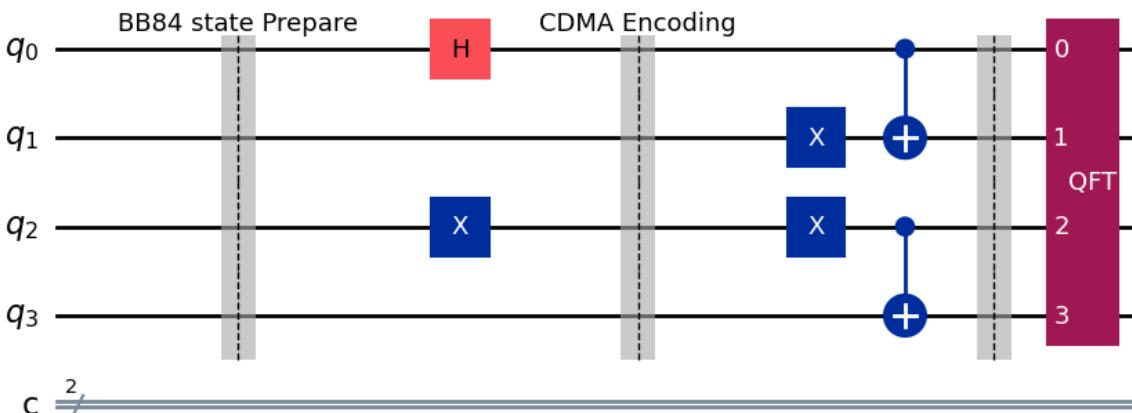
After CDMA Encode:



```
In [308]: qc.barrier(label="")
# STEP 3: QFT
# -----
qc.append(QFT(4), [0,1,2,3])
# Simulate initial state
state_vector= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector, "After QFT Multiplexing:")
#Now system is in Fourier multiplexed state.
#qc.barrier(Label="QFT*IQFT=Identity=I")
```



Figure(1123.61x451.5)

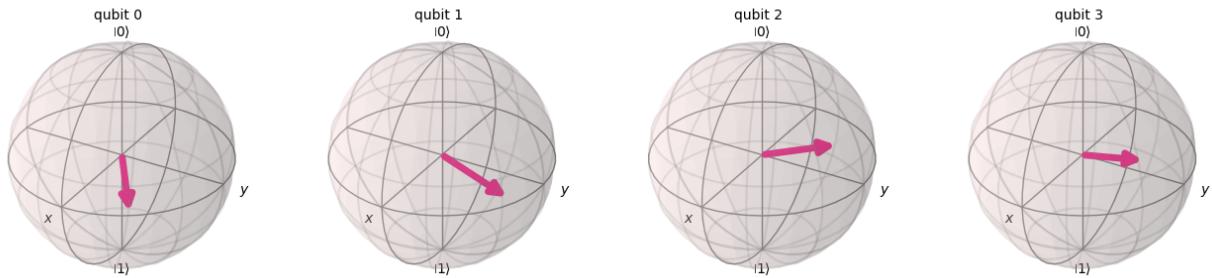


=====

After QFT Multiplexing:

```
=====
|000000000> (0.3535533905932736+0j)
|000000001> (0.28832037060954696+0.19264951251827456j)
|000000010> (0.1249999999999997+0.30177669529663675j)
|000000011> (-0.057350487481725346+0.28832037060954696j)
|000000100> (-0.1767766952966368j+0.1767766952966368j)
|000000101> (-0.19264951251827453+0.03832037060954703j)
|000000110> (-0.1249999999999997-0.05177669529663685j)
|000000111> (-0.038320370609547044-0.05735048748172535j)
|000001001> (-0.03832037060954703+0.05735048748172537j)
|000001010> (-0.1249999999999994+0.05177669529663685j)
|000001011> (-0.19264951251827456-0.03832037060954703j)
|000001100> (-0.1767766952966368-0.1767766952966368j)
|000001101> (-0.05735048748172538-0.28832037060954696j)
|000001110> (0.1249999999999994-0.30177669529663675j)
|000001111> (0.28832037060954696-0.19264951251827456j)
```

After QFT Multiplexing:

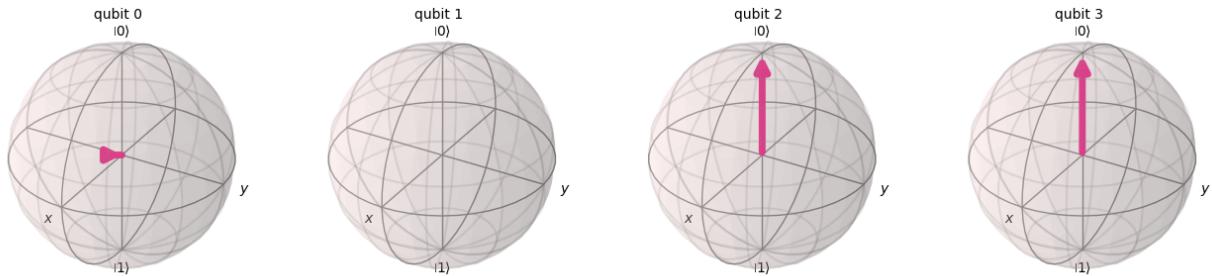


In [309...]

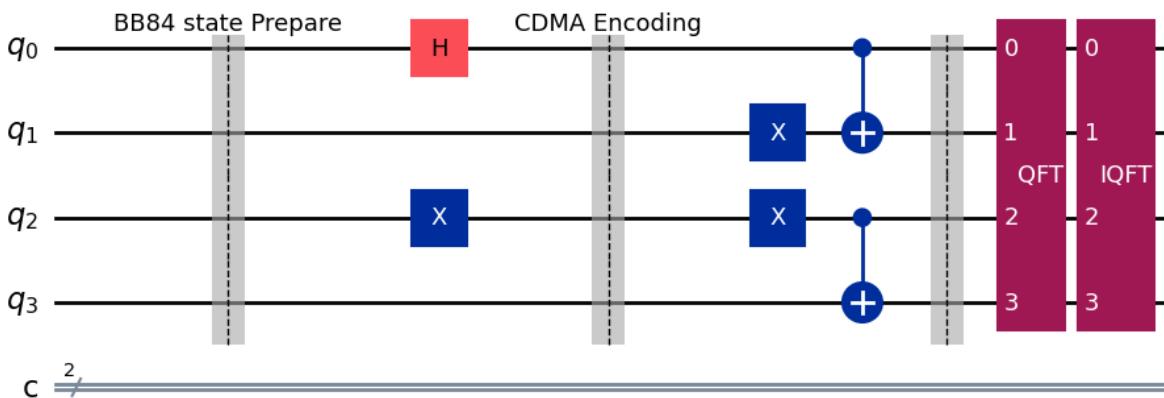
```
# -----
# STEP 4: Channel
# (Loss handled classically)
# -----
```

In [310...]

```
# -----
# STEP 5: IQFT
# -----
qc.append(QFT(4).inverse(), [0,1,2,3])
# Simulate initial state
state_vector_after_iqft= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector_after_iqft)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector_after_iqft, "After IQFT De-Multiplexing:")
#This restores encoded computational state.
qc.barrier(label="")
qc.barrier(label="CDMA Decoding")
```



Figure(1207.22x451.5)

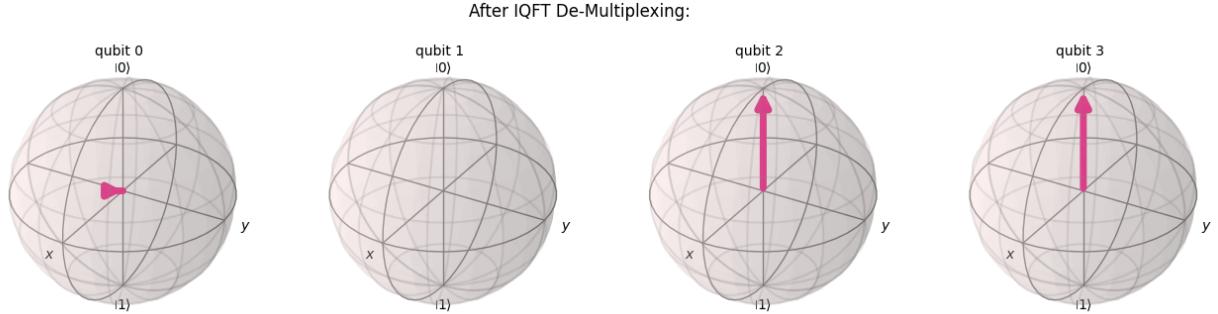


=====

After IQFT De-Multiplexing:

=====

```
|0000000001> (0.707106781186547-6.378200895168973e-18j)
|0000000010> (0.707106781186547-9.205408607085307e-18j)
```

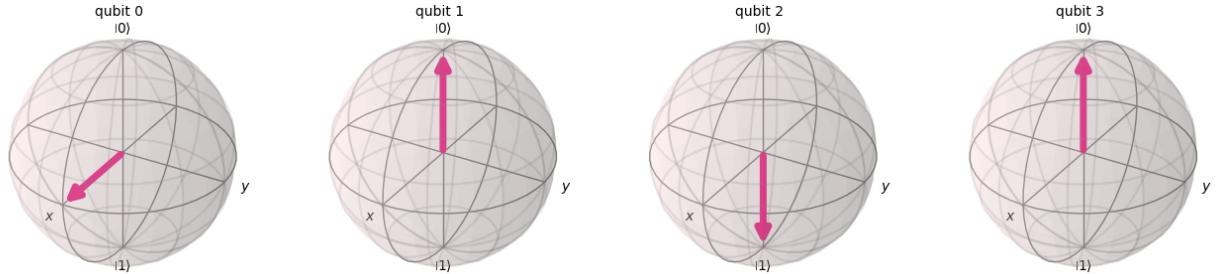


```
Out[310]: CircuitInstruction(operation=Instruction(name='barrier', num_qubits=4, num_clbits=0, params=[]), qubits=(Qubit(QuantumRegister(4, 'q'), 0), Qubit(QuantumRegister(4, 'q'), 1), Qubit(QuantumRegister(4, 'q'), 2), Qubit(QuantumRegister(4, 'q'), 3)), clbits=())
```

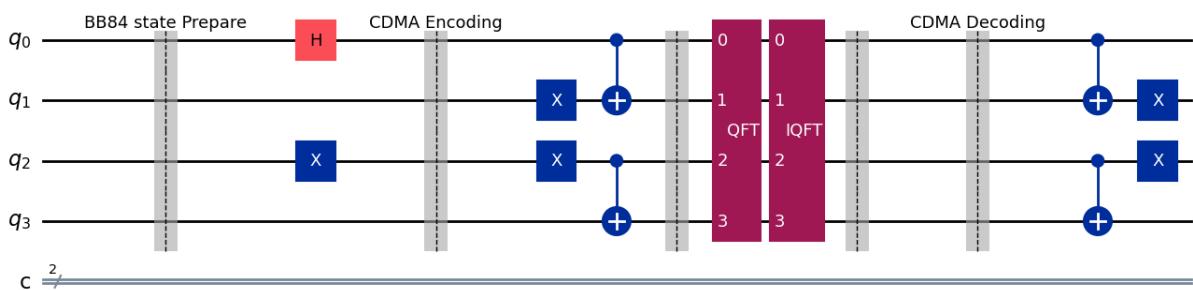
```
In [311]: # Check equality
if np.allclose(state_vector_before_qft.data, state_vector_after_iqft.data):
    print("\n✓ Circuit proof: QFT * IQFT = Identity")
else:
    print("\n✗ Circuit proof: QFT * IQFT != Identity")
✓ Circuit proof: QFT * IQFT = Identity
```

```
In [312]: # STEP 6: Decode
# ----

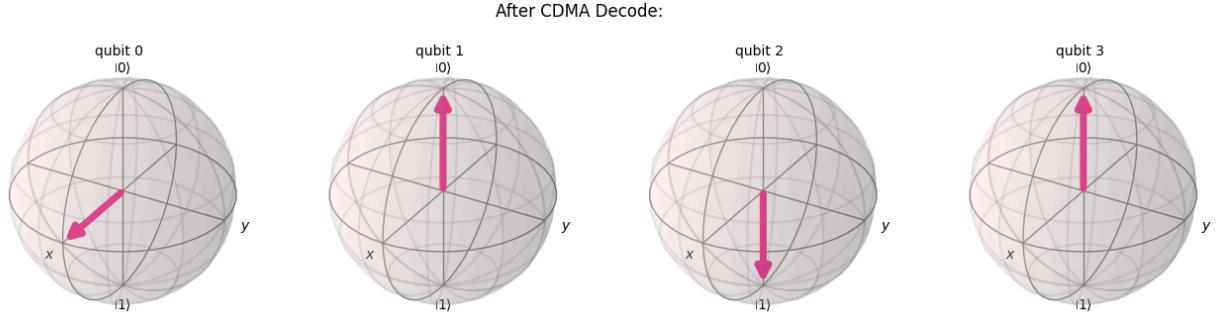
qc.cx(2,3)
qc.x(2)
qc.cx(0,1)
qc.x(1)
# Simulate initial state
state_vector_after_CDMA_Decode= Statevector.from_instruction(qc)
plot_bloch_multivector(state_vector_after_CDMA_Decode)
plt.show()
print(qc.draw('mpl'))
plt.show()
show_statevector(state_vector_after_CDMA_Decode, "After CDMA Decode:")
```



Figure(1708.89x451.5)



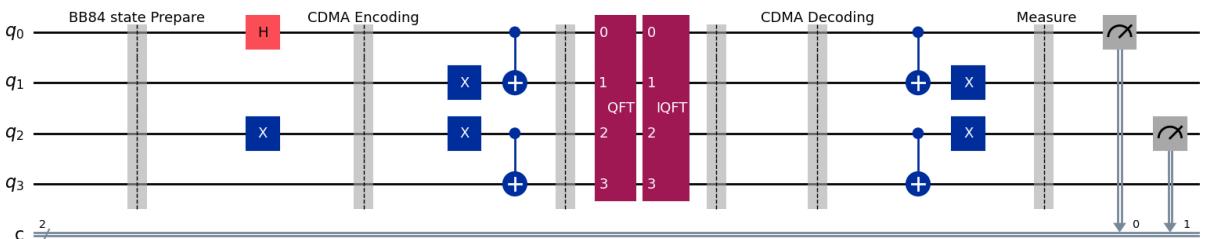
```
=====
After CDMA Decode:
=====
|0000000100> (0.707106781186547-9.205408607085307e-18j)
|0000000101> (0.707106781186547-6.378200895168973e-18j)
```



```
In [313... if np.allclose(state_vector_after_BB84.data, state_vector_after_CDMA_Decode.data):
    print("\n✓ Circuit proof: CDMA_Encoding * CDMA_Decoding = Identity")
else:
    print("\n✗ Circuit proof: CDMA_Encoding * CDMA_Decoding != Identity")
✓ Circuit proof: CDMA_Encoding * CDMA_Decoding = Identity
```

```
In [314... # -----
# STEP 7: Measure
# -----
qc.barrier(label=" Measure")
qc.measure(0,0)
qc.measure(2,1)

qc.draw('mpl')
#print(qc.draw('mpl'))
plt.show()
```



```
In [315... from qiskit_aer import AerSimulator
from qiskit import transpile
# To Simulate and Run
sim = AerSimulator()
job = sim.run(transpile(qc, sim), shots=1024)
counts = job.result().get_counts()

#print("\nMeasurement results of (C1x,C1y,C2x,C2y):")
print(counts)

#Expected result: User1 → always 1 and User2 → always 0 If no noise → zero QBER.
```

```
{'10': 522, '11': 502}
```

```
In [316... eta = 0.1 # survival probability , Change it see the variations

total_shots = 10000
survivals = int(total_shots * eta)

print("Expected detections:", survivals)

Expected detections: 1000
```

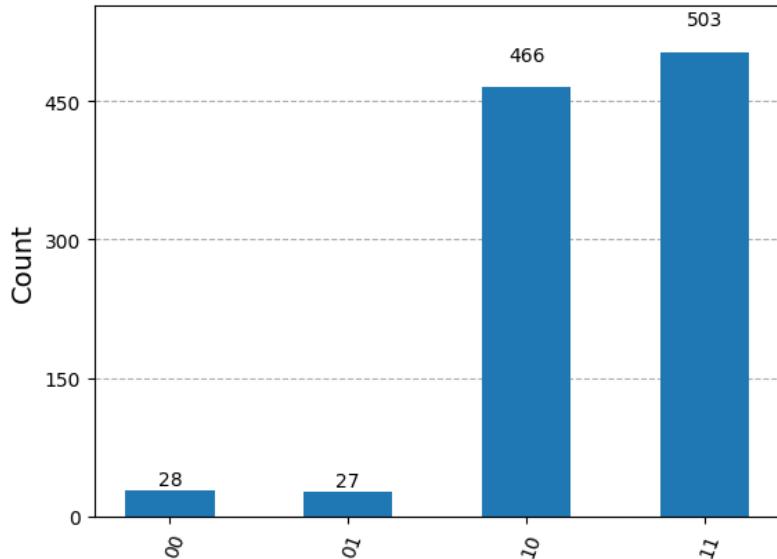
```
In [317... from qiskit_aer.noise import NoiseModel, depolarizing_error

noise_model = NoiseModel()
error = depolarizing_error(0.02, 1)
noise_model.add_all_qubit_quantum_error(error, ['x','h'])

# To Simulate and Run
sim = AerSimulator()
job = sim.run(transpile(qc, sim), shots=1024, noise_model=noise_model)
counts = job.result().get_counts()

#print("\nMeasurement results of (C1x,C1y,C2x,C2y):")
print(counts)
plot_histogram(counts)
plt.show()
```

```
{'11': 503, '10': 466, '01': 27, '00': 28}
```



In [318...]

```
import numpy as np
import matplotlib.pyplot as plt

# ----- PARAMETERS -----
alpha = 0.2           # dB/km
mu = 0.5
eta_d = 0.3
p_d = 1e-7
e_opt = 0.015
G = 64               # spreading gain

# Distance range
L_vals = np.linspace(0, 200, 400)  #(0,200,400)

QBER_vals = []

for L in L_vals:

    # Fiber transmission
    t = 10**(-alpha * L / 10)

    # Signal detection probability
    p_s = 1 - np.exp(-mu * t * eta_d)

    # CDMA interference
    p_MUI = 1 / G

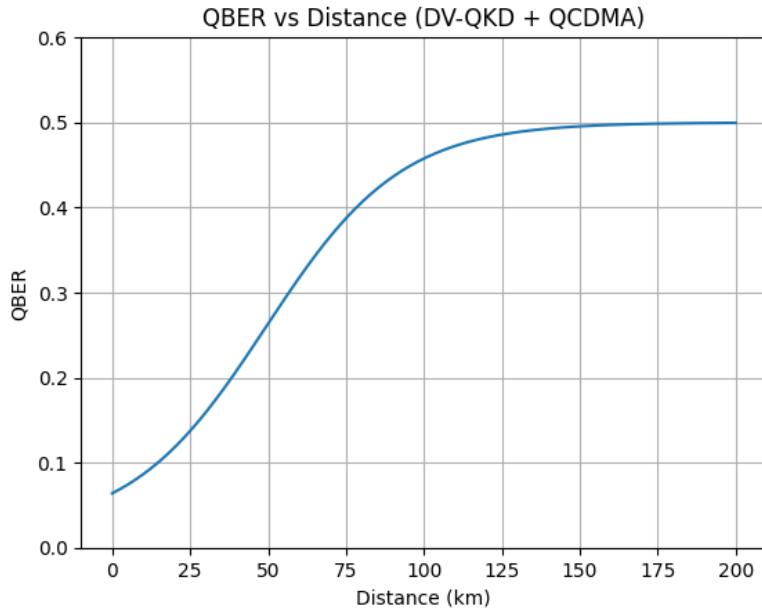
    # Total detection probability
    Q = p_s + p_d + p_MUI

    # QBER
    E = (e_opt * p_s + 0.5 * p_d + 0.5 * p_MUI) / Q

    QBER_vals.append(E)

# ----- PLOT -----

plt.figure()
plt.plot(L_vals, QBER_vals)
plt.xlabel("Distance (km)")
plt.ylabel("QBER")
plt.title("QBER vs Distance (DV-QKD + QCDMA)")
plt.ylim(0, 0.6)
plt.grid(True)
plt.show()
```



```
In [319...]: import numpy as np
import matplotlib.pyplot as plt

# ----- PARAMETERS -----
alpha = 0.2          # dB/km
mu = 0.5
eta_d = 0.3
p_d = 1e-7
e_opt = 0.015

G_list = [16, 32, 64, 128, 256, 512, 1024]

# Distance range
L_vals = np.linspace(0, 200, 400)

plt.figure()

for G in G_list:
    QBER_vals = []
    for L in L_vals:
        # Fiber transmission
        t = 10**(-alpha * L / 10)

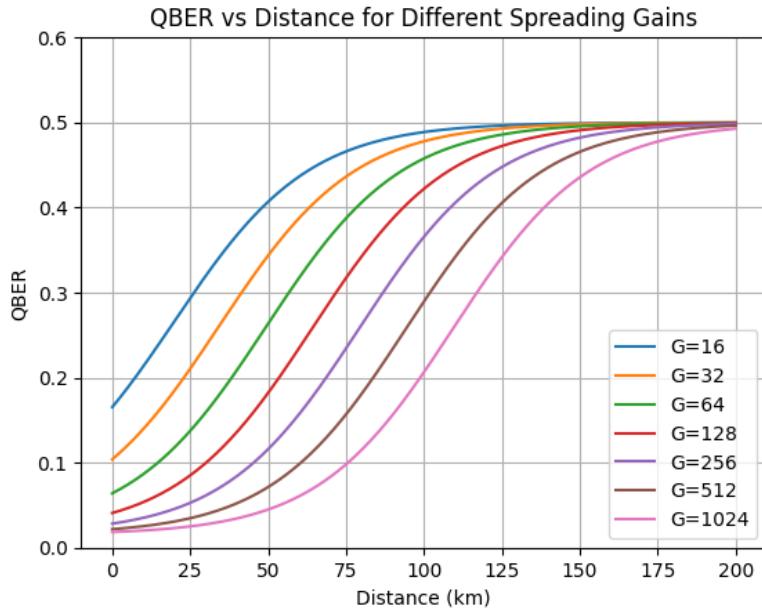
        # Signal detection probability
        p_s = 1 - np.exp(-mu * t * eta_d)

        # CDMA interference
        p_MUI = 1 / G

        # Total detection probability
        Q = p_s + p_d + p_MUI

        # QBER
        E = (e_opt * p_s + 0.5 * p_d + 0.5 * p_MUI) / Q
        QBER_vals.append(E)
    plt.plot(L_vals, QBER_vals)

# ----- PLOT -----
plt.xlabel("Distance (km)")
plt.ylabel("QBER")
plt.title("QBER vs Distance for Different Spreading Gains")
plt.ylim(0, 0.6)
plt.legend(["G=16", "G=32", "G=64", "G=128", "G=256", "G=512", "G=1024"])
plt.grid(True)
plt.show()
```



```
In [320]: import numpy as np
import matplotlib.pyplot as plt

# -----
# Physical parameters
# -----
alpha = 0.2
eta_d = 0.3
p_d = 1e-7
p_bg = 5e-7           # background photons
e0 = 0.01              # intrinsic optical error
e_vis = 0.04            # visibility degradation strength
L0 = 80                # degradation length scale
f = 1.16
mu = 0.5
N = 8 # Change it to see the variations

G_list = [32, 64, 128, 256, 512, 1024]
L_vals = np.linspace(0, 250, 400)

def H2(x):
    x = np.clip(x, 1e-12, 1-1e-12)
    return -x*np.log2(x) - (1-x)*np.log2(1-x)

results_QBER = {}
results_R = {}

for G in G_list:
    QBER_vals = []
    R_vals = []

    for L in L_vals:
        # Channel transmission
        t = 10**(-alpha * L / 10)

        # Signal probability
        p_s = 1 - np.exp(-mu * t * eta_d)

        # Distance-dependent optical error
        e_opt_L = e0 + e_vis * (1 - np.exp(-L / L0))

        # Modified MUI (nonlinear scaling)
        p_MUI = ((N - 1) / G) * (p_s**0.8)

        # Total detection probability
        Q = p_s + p_d + p_bg + p_MUI

        # QBER
        E = (e_opt_L * p_s + 0.5*p_d + 0.5*p_bg + 0.5*p_MUI) / Q

        # Secret key rate
        R = Q * (1 - f*H2(E) - H2(E))
        R = max(R, 0)

        QBER_vals.append(E)
        R_vals.append(R)

    results_QBER[G] = QBER_vals
    results_R[G] = R_vals
```

```

results_QBER[G] = QBER_vals
results_R[G] = R_vals

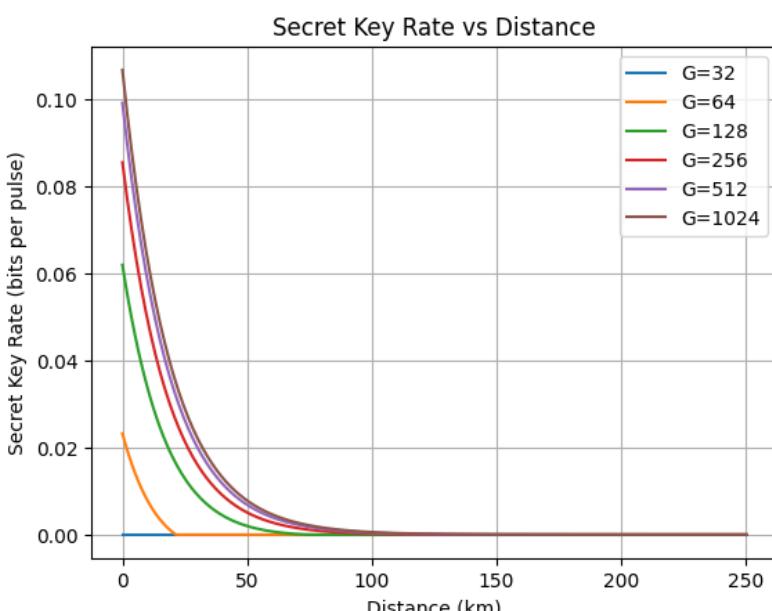
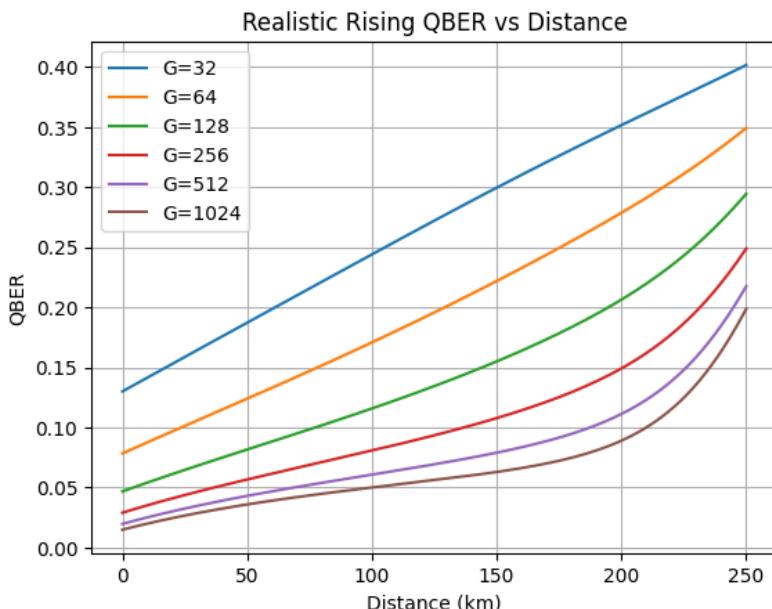
# -----
# Plot QBER
# -----
plt.figure()
for G in G_list:
    plt.plot(L_vals, results_QBER[G])

plt.xlabel("Distance (km)")
plt.ylabel("QBER")
plt.title("Realistic Rising QBER vs Distance")
plt.legend(["G=32", "G=64", "G=128", "G=256", "G=512", "G=1024"])
plt.grid(True)
plt.show()

# -----
# Plot Secret Key Rate
# -----
plt.figure()
for G in G_list:
    plt.plot(L_vals, results_R[G])

plt.xlabel("Distance (km)")
plt.ylabel("Secret Key Rate (bits per pulse)")
plt.title("Secret Key Rate vs Distance")
plt.legend(["G=32", "G=64", "G=128", "G=256", "G=512", "G=1024"])
plt.grid(True)
plt.show()

```



In []: