Python code for Artificial Intelligence: Foundations of Computational Agents

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http://aipython.org http://artint.info

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Chapter 1

Python for Artificial Intelligence

AIPython contains runnable code for the book *Artificial Intelligence, foundations of computational agents, 3rd Edition* [Poole and Mackworth, 2023]. It has the following design goals:

- Readability is more important than efficiency, although the asymptotic complexity is not compromised. AIPython is not a replacement for well-designed libraries, or optimized tools. Think of it like a model of an engine made of glass, so you can see the inner workings; don't expect it to power a big truck, but it lets you see how a metal engine can power a truck.

- It uses as few libraries as possible. A reader only needs to understand Python. Libraries hide details that we make explicit. The only library used is matplotlib for plotting and drawing.

1.1 Why Python?

We use Python because Python programs can be close to pseudo-code. It is designed for humans to read.

Python is reasonably efficient. Efficiency is usually not a problem for small examples. If your Python code is not efficient enough, a general procedure to improve it is to find out what is taking most of the time, and implement just that part more efficiently in some lower-level language. Most of these lower-level languages interoperate with Python nicely. This will result in much less programming and more efficient code (because you will have more time to optimize) than writing everything in a low-level language. You will not have to do that for the code here if you are using it for larger projects.

1.2 Getting Python

You need Python 3.9 or later[1] (`https://python.org/`) and a compatible version of matplotlib (`https://matplotlib.org/`). This code is *not* compatible with Python 2 (e.g., with Python 2.7).

Download and install the latest Python 3 release from `https://python.org/` or `https://www.anaconda.com/download`. This should also install *pip*3. You can install matplotlib using

```
pip3 install matplotlib
```

in a terminal shell (not in Python). That should "just work". If not, try using `pip` instead of `pip3`.

The command `python` or `python3` should then start the interactive python shell. You can quit Python with a control-D or with `quit()`.

To upgrade matplotlib to the latest version (which you should do if you install a new version of Python) do:

```
pip3 install --upgrade matplotlib
```

We recommend using the enhanced interactive python **ipython** (`https://ipython.org/`) [Pérez and Granger, 2007]. To install ipython after you have installed python do:

```
pip3 install ipython
```

1.3 Running Python

We assume that everything is done with an interactive Python shell. You can either do this with an IDE, such as IDLE that comes with standard Python distributions, or just running ipython3 or python3 (or perhaps just ipython or python) from a shell.

Here we describe the most simple version that uses no IDE. If you download the zip file, and cd to the "`aipython`" folder where the .py files are, you should be able to do the following, with user input in bold. The first `python` command is in the operating system shell; the `-i` is important to enter interactive mode.

```
python -i searchGeneric.py
Testing problem 1:
7 paths have been expanded and 4 paths remain in the frontier
Path found: A --> C --> B --> D --> G
Passed unit test
>>> searcher2 = AStarSearcher(searchProblem.acyclic_delivery_problem) #A*
>>> searcher2.search()  # find first path
```

[1] The only feature of 3.9 used is dictionary union. The feature of 3.8 used is `:=`. To use earlier versions 3.8, replace `|` with `dict_union` defined in Section 1.7.4.

```
16 paths have been expanded and 5 paths remain in the frontier
o103 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find first path
21 paths have been expanded and 6 paths remain in the frontier
o103 --> b3 --> b4 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find first path
28 paths have been expanded and 5 paths remain in the frontier
o103 --> b3 --> b1 --> b2 --> b4 --> o109 --> o119 --> o123 --> r123
>>> searcher2.search()  # find first path
No (more) solutions. Total of 33 paths expanded.
>>>
```

You can then interact at the last prompt.

There are many textbooks for Python. The best source of information about python is `https://www.python.org/`. The documentation is at `https://docs.python.org/3/`.

The rest of this chapter is about what is special about the code for AI tools. We will only use the standard Python library and matplotlib. All of the exercises can be done (and should be done) without using other libraries; the aim is for you to spend your time thinking about how to solve the problem rather than searching for pre-existing solutions.

1.4 Pitfalls

It is important to know when side effects occur. Often AI programs consider what would/might happen given certain conditions. In many such cases, we don't want side effects. When an agent acts in the world, side effects are appropriate.

In Python, you need to be careful to understand side effects. For example, the inexpensive function to add an element to a list, namely *append*, changes the list. In a functional language like Haskell or Lisp, adding a new element to a list, without changing the original list, is a cheap operation. For example if x is a list containing n elements, adding an extra element to the list in Python (using *append*) is fast, but it has the side effect of changing the list x. To construct a new list that contains the elements of x plus a new element, without changing the value of x, entails copying the list, or using a different representation for lists. In the searching code, we will use a different representation for lists for this reason.

1.5 Features of Python

1.5.1 f-strings

Python can use matching `'`, `"`, `'''` or `"""`, the latter two respecting line breaks in the string. We use the convention that when the string denotes a unique symbol, we use single quotes, and when it is a designed to be for printing, we use double quotes.

We make extensive use of f-strings `https://docs.python.org/3/tutorial/inputoutput.html`. In its simplest form

```
"str1{e1}str2{e2}str3"
```

where `e1` and `e2` are expressions, is an abbreviation for

```
"str1"+str(e2)+"str2"+str(e2)+"str3"
```

where `+` is string concatenation, and `str` is the function that returns a string representation of its expression argument.

1.5.2 Lists, Tuples, Sets, Dictionaries and Comprehensions

We make extensive uses of lists, tuples, sets and dictionaries (dicts). See `https://docs.python.org/3/library/stdtypes.html`

One of the nice features of Python is the use of **comprehensions**[2] (and also list, tuple, set and dictionary comprehensions). A generator expression is of the form

(*fe* for *e* in *iter* if *cond*)

enumerates the values *fe* for each *e* in *iter* for which *cond* is true. The "if cond" part is optional, but the "for" and "in" are not optional. Here *e* is a variable (or a pattern that can be on the left side of =), *iter* is an iterator, which can generate a stream of data, such as a list, a set, a range object (to enumerate integers between ranges) or a file. *cond* is an expression that evaluates to either True or False for each *e*, and *fe* is an expression that will be evaluated for each value of *e* for which *cond* returns *True*.

The result can go in a list or used in another iteration, or can be called directly using *next*. The procedure *next* takes an iterator returns the next element (advancing the iterator) and raises a StopIteration exception if there is no next element. The following shows a simple example, where user input is prepended with >>>

```
>>> [e*e for e in range(20) if e%2==0]
[0, 4, 16, 36, 64, 100, 144, 196, 256, 324]
>>> a = (e*e for e in range(20) if e%2==0)
>>> next(a)
```

[2] `https://docs.python.org/3/reference/expressions.html#displays-for-lists-sets-and-dictionaries`

```
0
>>> next(a)
4
>>> next(a)
16
>>> list(a)
[36, 64, 100, 144, 196, 256, 324]
>>> next(a)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
StopIteration
```

Notice how *list*(*a*) continued on the enumeration, and got to the end of it.

Comprehensions can also be used for dictionaries. The following code creates an index for list *a*:

```
>>> a = ["a","f","bar","b","a","aaaaa"]
>>> ind = {a[i]:i for i in range(len(a))}
>>> ind
{'a': 4, 'f': 1, 'bar': 2, 'b': 3, 'aaaaa': 5}
>>> ind['b']
3
```

which means that `'b'` is the 3rd element of the list.

The assignment of *ind* could have also be written as:

```
>>> ind = {val:i for (i,val) in enumerate(a)}
```

where *enumerate* is a built-in function that, given a dictionary, returns an iterator of (*index*, *value*) pairs.

1.5.3 Functions as first-class objects

Python can create lists and other data structures that contain functions. There is an issue that tricks many newcomers to Python. For a local variable in a function, the function uses the last value of the variable when the function is *called*, not the value of the variable when the function was defined (this is called "late binding"). This means if you want to use the value a variable has when the function is created, you need to save the current value of that variable. Whereas Python uses "late binding" by default, the alternative that newcomers often expect is "early binding", where a function uses the value a variable had when the function was defined, can be easily implemented.

Consider the following programs designed to create a list of 5 functions, where the *i*th function in the list is meant to add *i* to its argument:[3]

[3]Numbered lines are Python code available in the code-directory, `aipython`. The name of the file is given in the gray text above the listing. The numbers correspond to the line numbers in that file.

pythonDemo.py — Some tricky examples

```
fun_list1 = []
for i in range(5):
    def fun1(e):
        return e+i
    fun_list1.append(fun1)

fun_list2 = []
for i in range(5):
    def fun2(e,iv=i):
        return e+iv
    fun_list2.append(fun2)

fun_list3 = [lambda e: e+i for i in range(5)]

fun_list4 = [lambda e,iv=i: e+iv for i in range(5)]

i=56
```

Try to predict, and then test to see the output, of the output of the following calls, remembering that the function uses the latest value of any variable that is not bound in the function call:

pythonDemo.py — (continued)

```
# in Shell do
## ipython -i pythonDemo.py
# Try these (copy text after the comment symbol and paste in the Python
    prompt):
# print([f(10) for f in fun_list1])
# print([f(10) for f in fun_list2])
# print([f(10) for f in fun_list3])
# print([f(10) for f in fun_list4])
```

In the first for-loop, the function *fun* uses *i*, whose value is the last value it was assigned. In the second loop, the function *fun*2 uses *iv*. There is a separate *iv* variable for each function, and its value is the value of *i* when the function was defined. Thus *fun*1 uses late binding, and *fun*2 uses early binding. *fun_list*3 and *fun_list*4 are equivalent to the first two (except *fun_list*4 uses a different *i* variable).

One of the advantages of using the embedded definitions (as in *fun*1 and *fun*2 above) over the lambda is that is it possible to add a __doc__ string, which is the standard for documenting functions in Python, to the embedded definitions.

1.5.4 Generators

Python has generators which can be used for a form of lazy evaluation – only computing values when needed.

The `yield` command returns a value that is obtained with `next`. It is typically used to enumerate the values for a `for` loop or in generators. (The `yield` command can also be used for coroutines, but AIPython only uses it for generators.)

A version of the built-in *range*, with 2 or 3 arguments (and positive steps) can be implemented as:

pythonDemo.py — (continued)

```
37 def myrange(start, stop, step=1):
38     """enumerates the values from start in steps of size step that are
39     less than stop.
40     """
41     assert step>0, f"only positive steps implemented in myrange: {step}"
42     i = start
43     while i<stop:
44         yield i
45         i += step
46
47 print("list(myrange(2,30,3)):",list(myrange(2,30,3)))
```

Note that the built-in *range* is unconventional in how it handles a single argument, as the single argument acts as the second argument of the function. Note also that the built-in range also allows for indexing (e.g., $range(2,30,3)[2]$ returns 8), which the above implementation does not. However *myrange* also works for floats, which the built-in range does not.

Exercise 1.1 Implement a version of *myrange* that acts like the built-in version when there is a single argument. (Hint: make the second argument have a default value that can be recognized in the function.) There is not need to make it with indexing.

Yield can be used to generate the same sequence of values as in the example of Section 1.5.2:

pythonDemo.py — (continued)

```
49 def ga(n):
50     """generates square of even nonnegative integers less than n"""
51     for e in range(n):
52         if e%2==0:
53             yield e*e
54 a = ga(20)
```

The sequence of next(a), and list(a) gives exactly the same results as the comprehension in Section 1.5.2.

It is straightforward to write a version of the built-in *enumerate* called *myenumerate*:

pythonDemo.py — (continued)

```
56 def myenumerate(enum):
57     for i in range(len(enum)):
58         yield i,enum[i]
```

Exercise 1.2 Write a version of *enumerate* where the only iteration is "for val in enum". Hint: keep track of the index.

1.6 Useful Libraries

1.6.1 Timing Code

In order to compare algorithms, we often want to compute how long a program takes; this is called the **run time** of the program. The most straightforward way to compute run time is to use *time.perf_counter()*, as in:

```
import time
start_time = time.perf_counter()
compute_for_a_while()
end_time = time.perf_counter()
print("Time:", end_time - start_time, "seconds")
```

Note that `time.perf_counter()` measures clock time; so this should be done without user interaction between the calls. On the interactive python shell, you should do:

```
start_time = time.perf_counter(); compute_for_a_while(); end_time = time.perf_counter()
```

If this time is very small (say less than 0.2 second), it is probably very inaccurate, and it may be better to run your code many times to get a more accurate count. For this you can use *timeit* (`https://docs.python.org/3/library/timeit.html`). To use timeit to time the call to *foo.bar(aaa)* use:

```
import timeit
time = timeit.timeit("foo.bar(aaa)",
                     setup="from __main__ import foo,aaa", number=100)
```

The setup is needed so that Python can find the meaning of the names in the string that is called. This returns the number of seconds to execute *foo.bar(aaa)* 100 times. The variable *number* should be set so that the run time is at least 0.2 seconds.

You should not trust a single measurement as that can be confounded by interference from other processes. *timeit.repeat* can be used for running *timit* a few (say 3) times. When reporting the time of any computation, you should be explicit and explain what you are reporting. Usually the minimum time is the one to report.

1.6.2 Plotting: Matplotlib

The standard plotting for Python is matplotlib (`https://matplotlib.org/`). We will use the most basic plotting using the pyplot interface.

Here is a simple example that uses everything we will use.

pythonDemo.py — (continued)

```
60 import matplotlib.pyplot as plt
61
62 def myplot(minv,maxv,step,fun1,fun2):
63     plt.ion() # make it interactive
64     plt.xlabel("The x axis")
65     plt.ylabel("The y axis")
66     plt.xscale('linear') # Makes a 'log' or 'linear' scale
67     xvalues = range(minv,maxv,step)
68     plt.plot(xvalues,[fun1(x) for x in xvalues],
69              label="The first fun")
70     plt.plot(xvalues,[fun2(x) for x in xvalues], linestyle='--',color='k',
71              label=fun2.__doc__) # use the doc string of the function
72     plt.legend(loc="upper right") # display the legend
73
74 def slin(x):
75     """y=2x+7"""
76     return 2*x+7
77 def sqfun(x):
78     """y=(x-40)^2/10-20"""
79     return (x-40)**2/10-20
80
81 # Try the following:
82 # from pythonDemo import myplot, slin, sqfun
83 # import matplotlib.pyplot as plt
84 # myplot(0,100,1,slin,sqfun)
85 # plt.legend(loc="best")
86 # import math
87 # plt.plot([41+40*math.cos(th/10) for th in range(50)],
88 #          [100+100*math.sin(th/10) for th in range(50)])
89 # plt.text(40,100,"ellipse?")
90 # plt.xscale('log')
```

At the end of the code are some commented-out commands you should try in interactive mode. Cut from the file and paste into Python (and remember to remove the comments symbol and leading space).

1.7 Utilities

1.7.1 Display

In this distribution, to keep things simple and to only use standard Python, we use a text-oriented tracing of the code. A graphical depiction of the code could override the definition of *display* (but we leave it as a project).

The method *self.display* is used to trace the program. Any call

$$self.display(level, to\_print \ldots)$$

where the level is less than or equal to the value for *max_display_level* will be printed. The *to_print* ... can be anything that is accepted by the built-in *print* (including any keyword arguments).

The definition of *display* is:

display.py — A simple way to trace the intermediate steps of algorithms.

```
class Displayable(object):
    """Class that uses 'display'.
    The amount of detail is controlled by max_display_level
    """
    max_display_level = 1 # can be overridden in subclasses or instances

    def display(self,level,*args,**nargs):
        """print the arguments if level is less than or equal to the
        current max_display_level.
        level is an integer.
        the other arguments are whatever arguments print can take.
        """
        if level <= self.max_display_level:
            print(*args, **nargs) ##if error you are using Python2 not
                Python3
```

Note that *args* gets a tuple of the positional arguments, and *nargs* gets a dictionary of the keyword arguments). This will not work in Python 2, and will give an error.

Any class that wants to use *display* can be made a subclass of *Displayable*.

To change the maximum display level to say 3, for a class do:

$$Classname.max\_display\_level = 3$$

which will make calls to *display* in that class print when the value of *level* is less than-or-equal to 3. The default display level is 1. It can also be changed for individual objects (the object value overrides the class value).

The value of *max_display_level* by convention is:

0 display nothing

1 display solutions (nothing that happens repeatedly)

2 also display the values as they change (little detail through a loop)

3 also display more details

4 and above even more detail

In order to implement more sophisticated visualizations of the algorithm, we add a **visualize** "decorator" to the methods to be visualized. The following code ignores the decorator:

display.py — (continued)

```
26 def visualize(func):
27     """A decorator for algorithms that do interactive visualization.
28     Ignored here.
29     """
30     return func
```

1.7.2 Argmax

Python has a built-in *max* function that takes a generator (or a list or set) and returns the maximum value. The *argmax* method returns the index of an element that has the maximum value. If there are multiple elements with the maximum value, one if the indexes to that value is returned at random. *argmaxe* assumes an enumeration; a generator of $(element, value)$ pairs, as for example is generated by the built-in $enumerate(list)$ for lists or $dict.items()$ for dicts.

utilities.py — AIPython useful utilities

```
11 import random
12 import math
13
14 def argmaxall(gen):
15     """gen is a generator of (element,value) pairs, where value is a real.
16     argmaxall returns a list of all of the elements with maximal value.
17     """
18     maxv = -math.inf     # negative infinity
19     maxvals = []    # list of maximal elements
20     for (e,v) in gen:
21         if v>maxv:
22             maxvals,maxv = [e], v
23         elif v==maxv:
24             maxvals.append(e)
25     return maxvals
26
27 def argmaxe(gen):
28     """gen is a generator of (element,value) pairs, where value is a real.
29     argmaxe returns an element with maximal value.
30     If there are multiple elements with the max value, one is returned at
           random.
31     """
32     return random.choice(argmaxall(gen))
33
34 def argmax(lst):
35     """returns maximum index in a list"""
36     return argmaxe(enumerate(lst))
37 # Try:
38 # argmax([1,6,3,77,3,55,23])
39
40 def argmaxd(dct):
41    """returns the arg max of a dictionary dct"""
```

```
42      return argmaxe(dct.items())
43  # Try:
44  # arxmaxd({2:5,5:9,7:7})
```

Exercise 1.3 Change argmax to have an optional argument that specifies whether you want the "first", "last" or a "random" index of the maximum value returned. If you want the first or the last, you don't need to keep a list of the maximum elements.

1.7.3 Probability

For many of the simulations, we want to make a variable True with some probability. $flip(p)$ returns True with probability p, and otherwise returns False.

utilities.py — (continued)

```
45  def flip(prob):
46      """return true with probability prob"""
47      return random.random() < prob
```

The *pick_from_dist* method takes in a *item* : *probability* dictionary, and returns one of the items in proportion to its probability.

utilities.py — (continued)

```
49  def pick_from_dist(item_prob_dist):
50      """ returns a value from a distribution.
51      item_prob_dist is an item:probability dictionary, where the
52          probabilities sum to 1.
53      returns an item chosen in proportion to its probability
54      """
55      ranreal = random.random()
56      for (it,prob) in item_prob_dist.items():
57          if ranreal < prob:
58              return it
59          else:
60              ranreal -= prob
61      raise RuntimeError(f"{item_prob_dist} is not a probability
            distribution")
```

1.7.4 Dictionary Union

This is now | in Python 3.9, has been be replaced in the code. Use this if you want to back-port to an older version of Python.

The function $dict\_union(d1, d2)$ returns the union of dictionaries $d1$ and $d2$. If the values for the keys conflict, the values in $d2$ are used. This is similar to $dict(d1, **d2)$, but that only works when the keys of $d2$ are strings.

utilities.py — (continued)

```
63  def dict_union(d1,d2):
64      """returns a dictionary that contains the keys of d1 and d2.
```

```
65      The value for each key that is in d2 is the value from d2,
66      otherwise it is the value from d1.
67      This does not have side effects.
68      """
69      d = dict(d1)  # copy d1
70      d.update(d2)
71      return d
```

1.8 Testing Code

It is important to test code early and test it often. We include a simple form of **unit test**. The value of the current module is in `__name__` and if the module is run at the top-level, it's value is `"__main__"`. See `https://docs.python.org/3/library/__main__.html`.

The following code tests `argmax` and `dict_union`, but only when if `utilities` is loaded in the top-level. If it is loaded in a module the test code is not run.

In your code, you should do more substantial testing than done here. In particular, you should also test boundary cases.

utilities.py — (continued)

```
73  def test():
74      """Test part of utilities"""
75      assert argmax(enumerate([1,6,55,3,55,23])) in [2,4]
76      assert dict_union({1:4, 2:5, 3:4},{5:7, 2:9}) == {1:4, 2:9, 3:4, 5:7}
77      print("Passed unit test in utilities")
78
79  if __name__ == "__main__":
80      test()
```

Chapter 2

Agent Architectures and Hierarchical Control

This implements the controllers described in Chapter 2 of Poole and Mackworth [2023].

These provide sequential implementations of the control. More sophisticated version may have them run concurrently (either as coroutines or in parallel).

In this version the higher-levels call the lower-levels. The higher-levels calling the lower-level works in simulated environments when there is a single agent, and where the lower-level are written to make sure they return (and don't go on forever), and the higher level doesn't take too long (as the lower-levels will wait until called again).

2.1 Representing Agents and Environments

In the initial implementation, both agents and the envoronment are treated as objects in the send of object-oriented programs: they can have an internal state they maintain, and can evaluate methods that can provide answers. This is the same representation used for the reinforcement learning algorithms (Chapter 13).

An **environment** takes in actions of the agents, updates it internal state and returns the next percept, using the method `do`.

An **agent** takes the precept, updates its internal state, and output it next action. An agent implements the method `select_action` that takes percept and returns its next action.

The methods do and `select_action` are chained together to build a simulator. In order to start this, we need either an action or a percept. There are two variants used:

- An agent implements the `initial_action()` method which is used initially. This is the method used in the reinforcement learning chapter (page 273).
- The environment implements the `initial_percept()` method which gives the initial percept. This is the method used in this chapter.

In this implementation, the state of the agent and the state of the environment are represented using standard Python variables, which are updated as the state changes. The percept and the actions are represented as variable-value dictionaries. When agent has only a limited number of actions, the action can be a single value.

In the following code `raise NotImplementedError()` is a way to specify an abstract method that needs to be overridden in any implemented agent or environment.

agents.py — Agent and Controllers

```
from display import Displayable

class Agent(Displayable):

    def initial_action(self, percept):
        """return the initial action"""
        raise NotImplementedError("go") # abstract method

    def select_action(self, percept):
        """return the next action (and update internal state) given percept
        percept is variable:value dictionary
        """
        raise NotImplementedError("go") # abstract method
```

The environment implements a *do*(*action*) method where *action* is a variable-value dictionary. This returns a percept, which is also a variable-value dictionary. The use of dictionaries allows for structured actions and percepts.

Note that *Environment* is a subclass of *Displayable* so that it can use the *display* method described in Section 1.7.1.

agents.py — (continued)

```
class Environment(Displayable):
    def initial_percept(self):
        """returns the initial percept for the agent"""
        raise NotImplementedError("initial_percept") # abstract method

    def do(self, action):
        """does the action in the environment
```

```
32        returns the next percept """
33        raise NotImplementedError("Environment.do") # abstract method
```

The simulator lets the agent and the environment take turns in updating their states and returning the action and the percept.

The first implementation is a simple procedure to carry out n steps of the simulation and return the agent state and the environment state at the end.

agents.py — (continued)

```
35 class Simulate(Displayable):
36     """simulate the interaction between the agent and the environment
37     for n time steps.
38     Returns a pair of the agent state and the environment state.
39     """
40     def __init__(self,agent, environment):
41         self.agent = agent
42         self.env = environment
43         self.percept = self.env.initial_percept()
44         self.percept_history = [self.percept]
45         self.action_history = []
46 
47     def go(self, n):
48         for i in range(n):
49             action = self.agent.select_action(self.percept)
50             self.display(2,f"i={i} action={action}")
51             self.percept = self.env.do(action)
52             self.display(2,f"    percept={self.percept}")
```

2.2 Paper buying agent and environment

To run the demo, in folder "aipython", load "agents.py", using e.g., ipython -i agentBuying.py, and copy and paste the commented-out commands at the bottom of that file.

This is an implementation of Example 2.1 of Poole and Mackworth [2023]. You might get different plots to Fugures 2.2 and 2.3 as there is randomness in the environment.

2.2.1 The Environment

The environment state is given in terms of the *time* and the amount of paper in *stock*. It also remembers the in-stock history and the price history. The percept consistes of the price and the amount of paper in stock. The action of the agent is the number to buy.

Here we assume that the prices are obtained from the *prices* list (which cycles) plus a random integer in range $[0, max\_price\_addon)$ plus a linear "infla-

tion". The agent cannot access the price model; it just observes the prices and the amount in stock.

agentBuying.py — Paper-buying agent

```
import random
from agents import Agent, Environment, Simulate
from utilities import pick_from_dist

class TP_env(Environment):
    prices = [234, 234, 234, 234, 255, 255, 275, 275, 211, 211, 211,
    234, 234, 234, 234, 199, 199, 275, 275, 234, 234, 234, 234, 255,
    255, 260, 260, 265, 265, 265, 265, 270, 270, 255, 255, 260, 260,
    265, 265, 150, 150, 265, 265, 270, 270, 255, 255, 260, 260, 265,
    265, 265, 265, 270, 270, 211, 211, 255, 255, 260, 260, 265, 265,
    260, 265, 270, 270, 205, 255, 255, 260, 260, 265, 265, 265, 265,
    270, 270]
    max_price_addon = 20 # maximum of random value added to get price

    def __init__(self):
        """paper buying agent"""
        self.time=0
        self.stock=20
        self.stock_history = [] # memory of the stock history
        self.price_history = [] # memory of the price history

    def initial_percept(self):
        """return initial percept"""
        self.stock_history.append(self.stock)
        price = self.prices[0]+random.randrange(self.max_price_addon)
        self.price_history.append(price)
        return {'price': price,
                'instock': self.stock}

    def do(self, action):
        """does action (buy) and returns percept consiting of price and
            instock"""
        used = pick_from_dist({6:0.1, 5:0.1, 4:0.1, 3:0.3, 2:0.2, 1:0.2})
        # used = pick_from_dist({7:0.1, 6:0.2, 5:0.2, 4:0.3, 3:0.1, 2:0.1})
            # uses more paper
        bought = action['buy']
        self.stock = self.stock+bought-used
        self.stock_history.append(self.stock)
        self.time += 1
        price = (self.prices[self.time%len(self.prices)] # repeating pattern
                 +random.randrange(self.max_price_addon) # plus randomness
                 +self.time//2)                      # plus inflation
        self.price_history.append(price)
        return {'price': price,
                'instock': self.stock}
```

2.2.2 The Agent

The agent does not have access to the price model but can only observe the current price and the amount in stock. It has to decide how much to buy.

The belief state of the agent is an estimate of the average price of the paper, and the total amount of money the agent has spent.

agentBuying.py — (continued)

```
55 class TP_agent(Agent):
56     def __init__(self):
57         self.spent = 0
58         percept = env.initial_percept()
59         self.ave = self.last_price = percept['price']
60         self.instock = percept['instock']
61         self.buy_history = []
62 
63     def select_action(self, percept):
64         """return next action to caurry out
65         """
66         self.last_price = percept['price']
67         self.ave = self.ave+(self.last_price-self.ave)*0.05
68         self.instock = percept['instock']
69         if self.last_price < 0.9*self.ave and self.instock < 60:
70             tobuy = 48
71         elif self.instock < 12:
72             tobuy = 12
73         else:
74             tobuy = 0
75         self.spent += tobuy*self.last_price
76         self.buy_history.append(tobuy)
77         return {'buy': tobuy}
```

Set up an environment and an agent. Uncomment the last lines to run the agent for 90 steps, and determine the average amount spent.

agentBuying.py — (continued)

```
79 env = TP_env()
80 ag = TP_agent()
81 sim = Simulate(ag,env)
82 #sim.go(90)
83 #ag.spent/env.time ## average spent per time period
```

2.2.3 Plotting

The following plots the price and number in stock history:

agentBuying.py — (continued)

```
85 import matplotlib.pyplot as plt
86 
87 class Plot_history(object):
```

```
88      """Set up the plot for history of price and number in stock"""
89      def __init__(self, ag, env):
90          self.ag = ag
91          self.env = env
92          plt.ion()
93          plt.xlabel("Time")
94          plt.ylabel("Value")
95
96
97      def plot_env_hist(self):
98          """plot history of price and instock"""
99          num = len(env.stock_history)
100         plt.plot(range(num),env.price_history,label="Price")
101         plt.plot(range(num),env.stock_history,label="In stock")
102         plt.legend()
103         #plt.draw()
104
105     def plot_agent_hist(self):
106         """plot history of buying"""
107         num = len(ag.buy_history)
108         plt.bar(range(1,num+1), ag.buy_history, label="Bought")
109         plt.legend()
110         #plt.draw()
111
112 # pl = Plot_history(ag,env)
113 # sim.go(90)
114 #pl.plot_env_hist()
115 #pl.plot_agent_hist()
```

Figure 2.1 shows the result of the plotting in the previous code.

Exercise 2.1 Design a better controller for a paper-buying agent.

- Justify a performance measure that is a fair comparison. Note that minimizing the total amount of money spent may be unfair to agents who have built up a stockpile, and favors agents that end up with no paper.
- Give a controller that can work for many different price histories. An agent can use other local state variables, but does not have access to the environment model.
- Is it worthwhile trying to infer the amount of paper that the home uses? (Try your controller with the different paper consumption commented out in `TP_env.do`.)

2.3 Hierarchical Controller

To run the hierarchical controller, in folder "`aipython`", load "`agentTop.py`", using e.g., `ipython -i agentTop.py`, and copy and paste the commands near the bottom of that file.

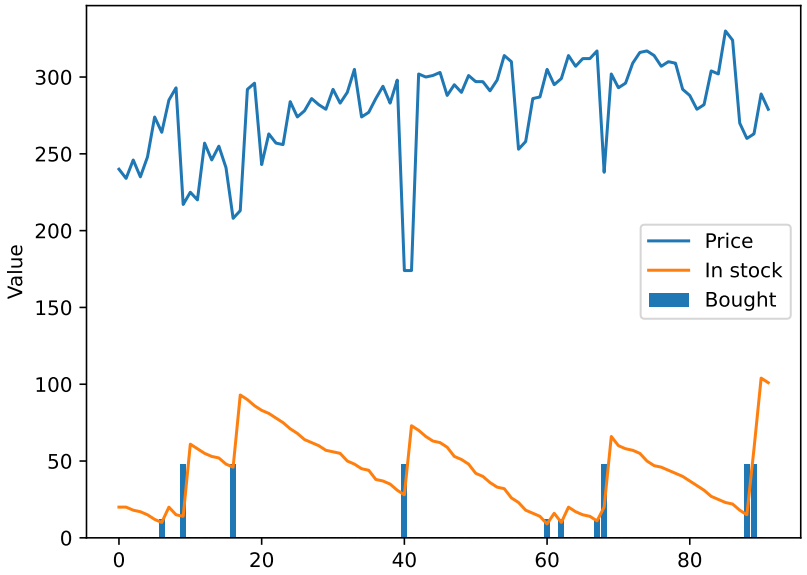


Figure 2.1: Percept and command traces for the paper-buying agent

In this implementation, each layer, including the top layer, implements the environment class, because each layer is seen as an environment from the layer above.

We arbitrarily divide the environment and the body, so that the environment just defines the walls, and the body includes everything to do with the agent. Note that the named locations are part of the (top-level of the) agent, not part of the environment, although they could have been.

2.3.1 Environment

The environment defines the walls.

agentEnv.py — Agent environment

```
11 import math
12 from agents import Environment
13
14 class Rob_env(Environment):
15     def __init__(self,walls = {}):
16         """walls is a set of line segments
17               where each line segment is of the form ((x0,y0),(x1,y1))
18         """
19         self.walls = walls
```

2.3.2 Body

The body defines everything about the agent body.

agentEnv.py — (continued)

```
import math
from agents import Environment
import matplotlib.pyplot as plt
import time

class Rob_body(Environment):
    def __init__(self, env, init_pos=(0,0,90)):
        """ env is the current environment
        init_pos is a triple of (x-position, y-position, direction)
            direction is in degrees; 0 is to right, 90 is straight-up, etc
        """
        self.env = env
        self.rob_x, self.rob_y, self.rob_dir = init_pos
        self.turning_angle = 18 # degrees that a left makes
        self.whisker_length = 6 # length of the whisker
        self.whisker_angle = 30 # angle of whisker relative to robot
        self.crashed = False
        # The following control how it is plotted
        self.plotting = True    # whether the trace is being plotted
        self.sleep_time = 0.05  # time between actions (for real-time
            plotting)
        # The following are data structures maintained:
        self.history = [(self.rob_x, self.rob_y)] # history of (x,y)
            positions
        self.wall_history = []  # history of hitting the wall

    def percept(self):
        return {'rob_x_pos':self.rob_x, 'rob_y_pos':self.rob_y,
                'rob_dir':self.rob_dir, 'whisker':self.whisker(),
                    'crashed':self.crashed}
    initial_percept = percept # use percept function for initial percept too

    def do(self,action):
        """ action is {'steer':direction}
        direction is 'left', 'right' or 'straight'
        """
        if self.crashed:
            return self.percept()
        direction = action['steer']
        compass_deriv =
            {'left':1,'straight':0,'right':-1}[direction]*self.turning_angle
        self.rob_dir = (self.rob_dir + compass_deriv +360)%360 # make in
            range [0,360)
        rob_x_new = self.rob_x + math.cos(self.rob_dir*math.pi/180)
        rob_y_new = self.rob_y + math.sin(self.rob_dir*math.pi/180)
        path = ((self.rob_x,self.rob_y),(rob_x_new,rob_y_new))
        if any(line_segments_intersect(path,wall) for wall in
            self.env.walls):
            self.crashed = True
```

```
64            if self.plotting:
65                plt.plot([self.rob_x],[self.rob_y],"r*",markersize=20.0)
66                plt.draw()
67        self.rob_x, self.rob_y = rob_x_new, rob_y_new
68        self.history.append((self.rob_x, self.rob_y))
69        if self.plotting and not self.crashed:
70            plt.plot([self.rob_x],[self.rob_y],"go")
71            plt.draw()
72            plt.pause(self.sleep_time)
73        return self.percept()
```

The Boolean whisker method returns True when the whisker and the wall intersect.

agentEnv.py — (continued)

```
75    def whisker(self):
76        """returns true whenever the whisker sensor intersects with a wall
77        """
78        whisk_ang_world = (self.rob_dir-self.whisker_angle)*math.pi/180
79            # angle in radians in world coordinates
80        wx = self.rob_x + self.whisker_length * math.cos(whisk_ang_world)
81        wy = self.rob_y + self.whisker_length * math.sin(whisk_ang_world)
82        whisker_line = ((self.rob_x,self.rob_y),(wx,wy))
83        hit = any(line_segments_intersect(whisker_line,wall)
84                  for wall in self.env.walls)
85        if hit:
86            self.wall_history.append((self.rob_x, self.rob_y))
87            if self.plotting:
88                plt.plot([self.rob_x],[self.rob_y],"ro")
89                plt.draw()
90        return hit
91
92 def line_segments_intersect(linea,lineb):
93    """returns true if the line segments, linea and lineb intersect.
94    A line segment is represented as a pair of points.
95    A point is represented as a (x,y) pair.
96    """
97    ((x0a,y0a),(x1a,y1a)) = linea
98    ((x0b,y0b),(x1b,y1b)) = lineb
99    da, db = x1a-x0a, x1b-x0b
100   ea, eb = y1a-y0a, y1b-y0b
101   denom = db*ea-eb*da
102   if denom==0:  # line segments are parallel
103       return False
104   cb = (da*(y0b-y0a)-ea*(x0b-x0a))/denom # position along line b
105   if cb<0 or cb>1:
106       return False
107   ca = (db*(y0b-y0a)-eb*(x0b-x0a))/denom # position along line a
108   return 0<=ca<=1
109
110 # Test cases:
```

```
# assert line_segments_intersect(((0,0),(1,1)),((1,0),(0,1)))
# assert not line_segments_intersect(((0,0),(1,1)),((1,0),(0.6,0.4)))
# assert line_segments_intersect(((0,0),(1,1)),((1,0),(0.4,0.6)))
```

2.3.3 Middle Layer

The middle layer acts like both a controller (for the environment layer) and an environment for the upper layer. It has to tell the environment how to steer. Thus it calls $env.do(\cdot)$. It also is told the position to go to and the timeout. Thus it also has to implement $do(\cdot)$.

agentMiddle.py — Middle Layer

```
from agents import Environment
import math

class Rob_middle_layer(Environment):
    def __init__(self,env):
        self.env=env
        self.percept = env.initial_percept()
        self.straight_angle = 11 # angle that is close enough to straight
            ahead
        self.close_threshold = 2 # distance that is close enough to arrived
        self.close_threshold_squared = self.close_threshold**2 # just
            compute it once

    def initial_percept(self):
        return {}

    def do(self, action):
        """action is {'go_to':target_pos,'timeout':timeout}
        target_pos is (x,y) pair
        timeout is the number of steps to try
        returns {'arrived':True} when arrived is true
             or {'arrived':False} if it reached the timeout
        """
        if 'timeout' in action:
            remaining = action['timeout']
        else:
            remaining = -1  # will never reach 0
        target_pos = action['go_to']
        arrived = self.close_enough(target_pos)
        while not arrived and remaining != 0:
            self.percept = self.env.do({"steer":self.steer(target_pos)})
            remaining -= 1
            arrived = self.close_enough(target_pos)
        return {'arrived':arrived}
```

The following method determines how to steer depending on whether the goal is to the right or the left of where the robot is facing.

agentMiddle.py — (continued)

```
44      def steer(self,target_pos):
45          if self.percept['whisker']:
46              self.display(3,'whisker on', self.percept)
47              return "left"
48          else:
49              gx,gy = target_pos
50              rx,ry = self.percept['rob_x_pos'],self.percept['rob_y_pos']
51              goal_dir = math.acos((gx-rx)/math.sqrt((gx-rx)*(gx-rx)
52                                                     +(gy-ry)*(gy-ry)))*180/math.pi
53              if ry>gy:
54                  goal_dir = -goal_dir
55              goal_from_rob = (goal_dir - self.percept['rob_dir']+540)%360-180
56              assert -180 < goal_from_rob <= 180
57              if goal_from_rob > self.straight_angle:
58                  return "left"
59              elif goal_from_rob < -self.straight_angle:
60                  return "right"
61              else:
62                  return "straight"
63
64      def close_enough(self,target_pos):
65          gx,gy = target_pos
66          rx,ry = self.percept['rob_x_pos'],self.percept['rob_y_pos']
67          return (gx-rx)**2 + (gy-ry)**2 <= self.close_threshold_squared
```

2.3.4 Top Layer

The top layer treats the middle layer as its environment. Note that the top layer is an environment for us to tell it what to visit.

agentTop.py — Top Layer

```
11  from agentMiddle import Rob_middle_layer
12  from agents import Environment
13
14  class Rob_top_layer(Environment):
15      def __init__(self, middle, timeout=200, locations = {'mail':(-5,10),
16                           'o103':(50,10), 'o109':(100,10),'storage':(101,51)}
                                 ):
17          """middle is the middle layer
18          timeout is the number of steps the middle layer goes before giving
                up
19          locations is a loc:pos dictionary
20              where loc is a named location, and pos is an (x,y) position.
21          """
22          self.middle = middle
23          self.timeout = timeout # number of steps before the middle layer
                should give up
24          self.locations = locations
```

```
25
26      def do(self,plan):
27          """carry out actions.
28          actions is of the form {'visit':list_of_locations}
29          It visits the locations in turn.
30          """
31          to_do = plan['visit']
32          for loc in to_do:
33              position = self.locations[loc]
34              arrived = self.middle.do({'go_to':position,
                    'timeout':self.timeout})
35              self.display(1,"Arrived at",loc,arrived)
```

2.3.5 Plotting

The following is used to plot the locations, the walls and (eventually) the movement of the robot. It can either plot the movement if the robot as it is going (with the default *env.plotting* = *True*), or not plot it as it is going (setting *env.plotting* = *False*; in this case the trace can be plotted using *pl.plot_run*()).

agentTop.py — (continued)

```
37  import matplotlib.pyplot as plt
38
39  class Plot_env(object):
40      def __init__(self, body,top):
41          """sets up the plot
42          """
43          self.body = body
44          plt.ion()
45          plt.clf()
46          plt.axes().set_aspect('equal')
47          for wall in body.env.walls:
48              ((x0,y0),(x1,y1)) = wall
49              plt.plot([x0,x1],[y0,y1],"-k",linewidth=3)
50          for loc in top.locations:
51              (x,y) = top.locations[loc]
52              plt.plot([x],[y],"k<")
53              plt.text(x+1.0,y+0.5,loc) # print the label above and to the
                    right
54          plt.plot([body.rob_x],[body.rob_y],"go")
55          plt.draw()
56
57      def plot_run(self):
58          """plots the history after the agent has finished.
59          This is typically only used if body.plotting==False
60          """
61          xs,ys = zip(*self.body.history)
62          plt.plot(xs,ys,"go")
63          wxs,wys = zip(*self.body.wall_history)
```

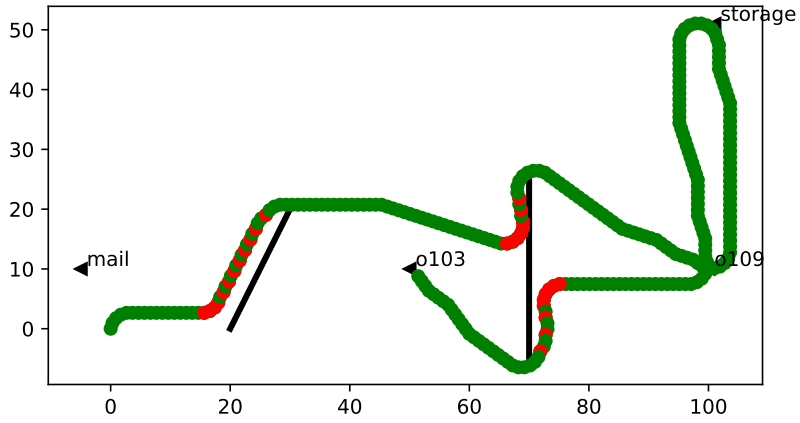


Figure 2.2: A trace of the trajectory of the agent. Red dots correspond to the whisker sensor being on; the green dot to the whisker sensor being off. The agent starts at position $(0, 0)$ facing up.

```
64        plt.plot(wxs,wys,"ro")
65        #plt.draw()
```

The following code plots the agent as it acts in the world. Figure 2.2 shows the result of the `top.do`

agentTop.py — (continued)

```
67  from agentEnv import Rob_body, Rob_env
68
69  env = Rob_env({((20,0),(30,20)), ((70,-5),(70,25))})
70  body = Rob_body(env)
71  middle = Rob_middle_layer(body)
72  top = Rob_top_layer(middle)
73
74  # try:
75  # pl=Plot_env(body,top)
76  # top.do({'visit':['o109','storage','o109','o103']})
77  # You can directly control the middle layer:
78  # middle.do({'go_to':(30,-10), 'timeout':200})
79  # Can you make it crash?
```

Exercise 2.2 The following code implements a robot trap (Figure 2.3). Write a controller that can escape the "trap" and get to the goal. See Exercise 2.4 in the textbook for hints.

agentTop.py — (continued)

```
81  # Robot Trap for which the current controller cannot escape:
82  trap_env = Rob_env({((10,-21),(10,0)), ((10,10),(10,31)),
        ((30,-10),(30,0)),
83                      ((30,10),(30,20)), ((50,-21),(50,31)),
                            ((10,-21),(50,-21)),
```

Figure 2.3: Robot trap

```
                  ((10,0),(30,0)), ((10,10),(30,10)), ((10,31),(50,31))})
trap_body = Rob_body(trap_env,init_pos=(-1,0,90))
trap_middle = Rob_middle_layer(trap_body)
trap_top = Rob_top_layer(trap_middle,locations={'goal':(71,0)})

# Robot trap exercise:
# pl=Plot_env(trap_body,trap_top)
# trap_top.do({'visit':['goal']})
```

Chapter 3

Searching for Solutions

3.1 Representing Search Problems

A search problem consists of:

- a start node
- a *neighbors* function that given a node, returns an enumeration of the arcs from the node
- a specification of a goal in terms of a Boolean function that takes a node and returns true if the node is a goal
- a (optional) heuristic function that, given a node, returns a non-negative real number. The heuristic function defaults to zero.

As far as the searcher is concerned a node can be anything. If multiple-path pruning is used, a node must be *hashable*. In the simple examples, it is a string, but in more complicated examples (in later chapters) it can be a tuple, a frozen set, or a Python object.

In the following code, "`raise NotImplementedError()`" is a way to specify that this is an abstract method that needs to be overridden to define an actual search problem.

searchProblem.py — representations of search problems

```
11 class Search_problem(object):
12     """A search problem consists of:
13     * a start node
14     * a neighbors function that gives the neighbors of a node
15     * a specification of a goal
16     * a (optional) heuristic function.
```

```
    The methods must be overridden to define a search problem."""

    def start_node(self):
        """returns start node"""
        raise NotImplementedError("start_node") # abstract method

    def is_goal(self,node):
        """is True if node is a goal"""
        raise NotImplementedError("is_goal") # abstract method

    def neighbors(self,node):
        """returns a list (or enumeration) of the arcs for the neighbors of
            node"""
        raise NotImplementedError("neighbors") # abstract method

    def heuristic(self,n):
        """Gives the heuristic value of node n.
        Returns 0 if not overridden."""
        return 0
```

The neighbors is a list of arcs. A (directed) arc consists of a *from_node* node and a *to_node* node. The arc is the pair $\langle$*from_node*, *to_node*$\rangle$, but can also contain a non-negative *cost* (which defaults to 1) and can be labeled with an *action*.

searchProblem.py — (continued)

```
class Arc(object):
    """An arc has a from_node and a to_node node and a (non-negative)
        cost"""
    def __init__(self, from_node, to_node, cost=1, action=None):
        self.from_node = from_node
        self.to_node = to_node
        self.action = action
        self.cost=cost
        assert cost >= 0, (f"Cost cannot be negative: {self}, cost={cost}")

    def __repr__(self):
        """string representation of an arc"""
        if self.action:
            return f"{self.from_node} --{self.action}--> {self.to_node}"
        else:
            return f"{self.from_node} --> {self.to_node}"
```

3.1.1 Explicit Representation of Search Graph

The first representation of a search problem is from an explicit graph (as opposed to one that is generated as needed).

An **explicit graph** consists of

- a list or set of nodes

- a list or set of arcs
- a start node
- a list or set of goal nodes
- (optionally) a dictionary that maps a node to a heuristic value for that node

To define a search problem, we need to define the start node, the goal predicate, the neighbors function and the heuristic function.

searchProblem.py — (continued)

```
52 class Search_problem_from_explicit_graph(Search_problem):
53     """A search problem consists of:
54     * a list or set of nodes
55     * a list or set of arcs
56     * a start node
57     * a list or set of goal nodes
58     * a dictionary that maps each node into its heuristic value.
59     * a dictionary that maps each node into its (x,y) position
60     """
61
62     def __init__(self, nodes, arcs, start=None, goals=set(), hmap={},
           positions={}):
63         self.neighs = {}
64         self.nodes = nodes
65         for node in nodes:
66             self.neighs[node]=[]
67         self.arcs = arcs
68         for arc in arcs:
69             self.neighs[arc.from_node].append(arc)
70         self.start = start
71         self.goals = goals
72         self.hmap = hmap
73         self.positions = positions
74
75     def start_node(self):
76         """returns start node"""
77         return self.start
78
79     def is_goal(self,node):
80         """is True if node is a goal"""
81         return node in self.goals
82
83     def neighbors(self,node):
84         """returns the neighbors of node (a list of arcs)"""
85         return self.neighs[node]
86
87     def heuristic(self,node):
88         """Gives the heuristic value of node n.
```

```
89          Returns 0 if not overridden in the hmap."""
90          if node in self.hmap:
91              return self.hmap[node]
92          else:
93              return 0
94
95      def __repr__(self):
96          """returns a string representation of the search problem"""
97          res=""
98          for arc in self.arcs:
99              res += f"{arc}. "
100         return res
```

3.1.2 Paths

A searcher will return a path from the start node to a goal node. A Python list is not a suitable representation for a path, as many search algorithms consider multiple paths at once, and these paths should share initial parts of the path. If we wanted to do this with Python lists, we would need to keep copying the list, which can be expensive if the list is long. An alternative representation is used here in terms of a recursive data structure that can share subparts.

A path is either:

- a node (representing a path of length 0) or
- a path, *initial* and an arc, where the *from_node* of the arc is the node at the end of *initial*.

These cases are distinguished in the following code by having $arc = None$ if the path has length 0, in which case *initial* is the node of the path. Nore that we only use the most basic form of Python's `yield` for enumerations (Section 1.5.4).

searchProblem.py — (continued)

```
102 class Path(object):
103     """A path is either a node or a path followed by an arc"""
104
105     def __init__(self,initial,arc=None):
106         """initial is either a node (in which case arc is None) or
107         a path (in which case arc is an object of type Arc)"""
108         self.initial = initial
109         self.arc=arc
110         if arc is None:
111             self.cost=0
112         else:
113             self.cost = initial.cost+arc.cost
114
115     def end(self):
116         """returns the node at the end of the path"""
```

```
        if self.arc is None:
            return self.initial
        else:
            return self.arc.to_node

    def nodes(self):
        """enumerates the nodes for the path.
        This enumerates the nodes in the path from the last elements
            backwards.
        """
        current = self
        while current.arc is not None:
            yield current.arc.to_node
            current = current.initial
        yield current.initial

    def initial_nodes(self):
        """enumerates the nodes for the path before the end node.
        This calls nodes() for the initial part of the path.
        """
        if self.arc is not None:
            yield from self.initial.nodes()

    def __repr__(self):
        """returns a string representation of a path"""
        if self.arc is None:
            return str(self.initial)
        elif self.arc.action:
            return f"{self.initial}\n --{self.arc.action}-->
                {self.arc.to_node}"
        else:
            return f"{self.initial} --> {self.arc.to_node}"
```

3.1.3 Example Search Problems

The first search problem is one with 5 nodes where the least-cost path is one with many arcs. See Figure 3.1. Note that this example is used for the unit tests, so the test (in `searchGeneric`) will need to be changed if this is changed.

searchProblem.py — (continued)

```
problem1 = Search_problem_from_explicit_graph(
    {'A','B','C','D','G'},
    [Arc('A','B',3), Arc('A','C',1), Arc('B','D',1), Arc('B','G',3),
         Arc('C','B',1), Arc('C','D',3), Arc('D','G',1)],
    start = 'A',
    goals = {'G'},
    positions={'A': (0, 2), 'B': (1, 1), 'C': (0,1), 'D': (1,0), 'G':
        (2,0)})
```

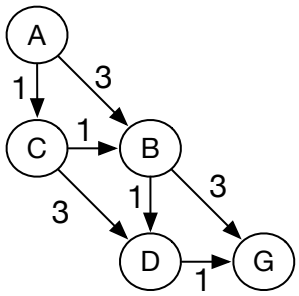


Figure 3.1: problem1

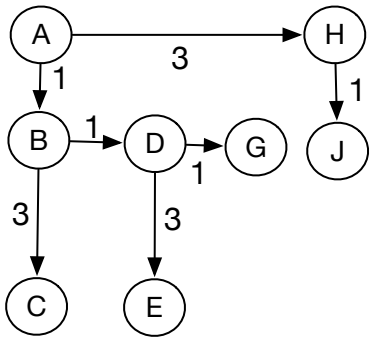


Figure 3.2: problem2

The second search problem is one with 8 nodes where many paths do not lead to the goal. See Figure 3.2.

searchProblem.py — (continued)

```
problem2 = Search_problem_from_explicit_graph(
    {'a','b','c','d','e','g','h','j'},
    [Arc('a','b',1), Arc('b','c',3), Arc('b','d',1), Arc('d','e',3),
        Arc('d','g',1), Arc('a','h',3), Arc('h','j',1)],
    start = 'a',
    goals = {'g'},
    positions={'a': (0, 0), 'b': (0, 1), 'c': (0,4), 'd': (1,1), 'e': (1,4),
                  'g': (2,1), 'h': (3,0), 'j': (3,1)})
```

The third search problem is a disconnected graph (contains no arcs), where the start node is a goal node. This is a boundary case to make sure that weird cases work.

searchProblem.py — (continued)

```
problem3 = Search_problem_from_explicit_graph(
```

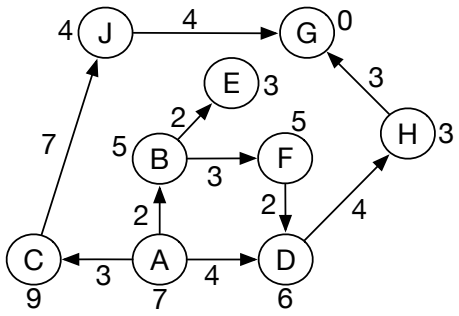


Figure 3.3: simp_delivery_graph with arc costs and h values of nodes

```
    {'a','b','c','d','e','g','h','j'},
    [],
    start = 'g',
    goals = {'k','g'})
```

The simp_delivery_graph is the graph shown Figure 3.3. This is Figure 3.3 with the heuristics of Figure 3.1 as shown in Fugure 3.13 of [Poole and Mackworth, 2023],

searchProblem.py — (continued)

```
simp_delivery_graph = Search_problem_from_explicit_graph(
    {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'J'},
    [   Arc('A', 'B', 2),
        Arc('A', 'C', 3),
        Arc('A', 'D', 4),
        Arc('B', 'E', 2),
        Arc('B', 'F', 3),
        Arc('C', 'J', 7),
        Arc('D', 'H', 4),
        Arc('F', 'D', 2),
        Arc('H', 'G', 3),
        Arc('J', 'G', 4)],
   start = 'A',
   goals = {'G'},
   hmap = {
        'A': 7,
        'B': 5,
        'C': 9,
        'D': 6,
        'E': 3,
        'F': 5,
        'G': 0,
        'H': 3,
        'J': 4,
    })
```

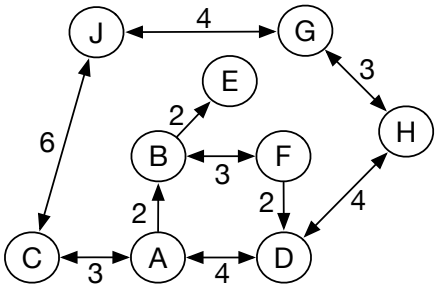


Figure 3.4: cyclic_simp_delivery_graph with arc costs

cyclic_simp_delivery_graph is the graph shown Figure 3.4. This is the graph of Figure 3.10 of [Poole and Mackworth, 2023]. The heuristic values are the same as in simp_delivery_graph.

searchProblem.py — (continued)

```
cyclic_simp_delivery_graph = Search_problem_from_explicit_graph(
    {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'J'},
    [    Arc('A', 'B', 2),
         Arc('A', 'C', 3),
         Arc('A', 'D', 4),
         Arc('B', 'A', 2),
         Arc('B', 'E', 2),
         Arc('B', 'F', 3),
         Arc('C', 'A', 3),
         Arc('C', 'J', 7),
         Arc('D', 'A', 4),
         Arc('D', 'H', 4),
         Arc('F', 'B', 3),
         Arc('F', 'D', 2),
         Arc('G', 'H', 3),
         Arc('G', 'J', 4),
         Arc('H', 'D', 4),
         Arc('H', 'G', 3),
         Arc('J', 'C', 6),
         Arc('J', 'G', 4)],
    start = 'A',
    goals = {'G'},
    hmap = {
        'A': 7,
        'B': 5,
        'C': 9,
        'D': 6,
        'E': 3,
        'F': 5,
        'G': 0,
```

```
        'H': 3,
        'J': 4,
    })
```

3.2 Generic Searcher and Variants

To run the search demos, in folder "aipython", load "searchGeneric.py" , using e.g., ipython -i searchGeneric.py, and copy and paste the example queries at the bottom of that file.

3.2.1 Searcher

A *Searcher* for a problem can be asked repeatedly for the next path. To solve a problem, you can construct a *Searcher* object for the problem and then repeatedly ask for the next path using *search*. If there are no more paths, *None* is returned.

searchGeneric.py — Generic Searcher, including depth-first and A*

```
from display import Displayable, visualize

class Searcher(Displayable):
    """returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    This does depth-first search unless overridden
    """
    def __init__(self, problem):
        """creates a searcher from a problem
        """
        self.problem = problem
        self.initialize_frontier()
        self.num_expanded = 0
        self.add_to_frontier(Path(problem.start_node()))
        super().__init__()

    def initialize_frontier(self):
        self.frontier = []

    def empty_frontier(self):
        return self.frontier == []

    def add_to_frontier(self,path):
        self.frontier.append(path)

    @visualize
    def search(self):
        """returns (next) path from the problem's start node
        to a goal node.
```

```
40          Returns None if no path exists.
41          """
42          while not self.empty_frontier():
43              path = self.frontier.pop()
44              self.display(2, "Expanding:",path,"(cost:",path.cost,")")
45              self.num_expanded += 1
46              if self.problem.is_goal(path.end()): # solution found
47                  self.display(1, self.num_expanded, "paths have been expanded
                        and",
48                               len(self.frontier), "paths remain in the
                                   frontier")
49                  self.solution = path # store the solution found
50                  return path
51              else:
52                  neighs = self.problem.neighbors(path.end())
53                  self.display(3,"Neighbors are", neighs)
54                  for arc in reversed(list(neighs)):
55                      self.add_to_frontier(Path(path,arc))
56                  self.display(3,"Frontier:",self.frontier)
57          self.display(1,"No (more) solutions. Total of",
58                       self.num_expanded,"paths expanded.")
```

Note that this reverses the neighbors so that it implements depth-first search in an intuitive manner (expanding the first neighbor first). The call to *list* is for the case when the neighbors are generated (and not already in a list). Reversing the neighbors might not be required for other methods. The calls to *reversed* and *list* can be removed, and the algorithm still implements depth-fist search.

To use depth-first search to find multiple paths for `problem1` and `simp_delivery_graph`, copy and paste the following into Python's read-evaluate-print loop; keep finding next solutions until there are no more:

searchGeneric.py — (continued)

```
60 # Depth-first search for problem1; do the following:
61 # searcher1 = Searcher(searchProblem.problem1)
62 # searcher1.search() # find first solution
63 # searcher1.search() # find next solution (repeat until no solutions)
64 # searcher_sdg = Searcher(searchProblem.simp_delivery_graph)
65 # searcher_sdg.search() # find first or next solution
```

Exercise 3.1 Implement breadth-first search. Only *add_to_frontier* and/or *pop* need to be modified to implement a first-in first-out queue.

3.2.2 Frontier as a Priority Queue

In many of the search algorithms, such as A^* and other best-first searchers, the frontier is implemented as a priority queue. The following code uses the Python's built-in priority queue implementations, `heapq`.

Following the lead of the Python documentation, `http://docs.python.org/3.9/library/heapq.html`, a frontier is a list of triples. The first element of each

triple is the value to be minimized. The second element is a unique index which specifies the order that the elemnets were added to the queue, and the third element is the path that is on the queue. The use of the unique index ensures that the priority queue implementation does not compare paths; whether one path is less than another is not defined. It also lets us control what sort of search (e.g., depth-first or breadth-first) occurs when the value to be minimized does not give a unique next path.

The variable *frontier_index* is the total number of elements of the frontier that have been created. As well as being used as the unique index, it is useful for statistics, particularly in conjunction with the current size of the frontier.

searchGeneric.py — (continued)

```
67 import heapq      # part of the Python standard library
68 from searchProblem import Path
69
70 class FrontierPQ(object):
71     """A frontier consists of a priority queue (heap), frontierpq, of
72         (value, index, path) triples, where
73     * value is the value we want to minimize (e.g., path cost + h).
74     * index is a unique index for each element
75     * path is the path on the queue
76     Note that the priority queue always returns the smallest element.
77     """
78
79     def __init__(self):
80         """constructs the frontier, initially an empty priority queue
81         """
82         self.frontier_index = 0 # the number of items added to the frontier
83         self.frontierpq = [] # the frontier priority queue
84
85     def empty(self):
86         """is True if the priority queue is empty"""
87         return self.frontierpq == []
88
89     def add(self, path, value):
90         """add a path to the priority queue
91         value is the value to be minimized"""
92         self.frontier_index += 1 # get a new unique index
93         heapq.heappush(self.frontierpq,(value, -self.frontier_index, path))
94
95     def pop(self):
96         """returns and removes the path of the frontier with minimum value.
97         """
98         (_,_,path) = heapq.heappop(self.frontierpq)
99         return path
```

The following methods are used for finding and printing information about the frontier.

searchGeneric.py — (continued)

```
    def count(self,val):
        """returns the number of elements of the frontier with value=val"""
        return sum(1 for e in self.frontierpq if e[0]==val)

    def __repr__(self):
        """string representation of the frontier"""
        return str([(n,c,str(p)) for (n,c,p) in self.frontierpq])

    def __len__(self):
        """length of the frontier"""
        return len(self.frontierpq)

    def __iter__(self):
        """iterate through the paths in the frontier"""
        for (_,_,path) in self.frontierpq:
            yield path
```

3.2.3 A^* Search

For an A^* **Search** the frontier is implemented using the FrontierPQ class.

searchGeneric.py — (continued)

```
class AStarSearcher(Searcher):
    """returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    """

    def __init__(self, problem):
        super().__init__(problem)

    def initialize_frontier(self):
        self.frontier = FrontierPQ()

    def empty_frontier(self):
        return self.frontier.empty()

    def add_to_frontier(self,path):
        """add path to the frontier with the appropriate cost"""
        value = path.cost+self.problem.heuristic(path.end())
        self.frontier.add(path, value)
```

Code should always be tested. The following provides a simple **unit test**, using problem1 as the the default problem.

searchGeneric.py — (continued)

```
import searchProblem as searchProblem

def test(SearchClass, problem=searchProblem.problem1,
    solutions=[['G','D','B','C','A']] ):
    """Unit test for aipython searching algorithms.
```

```
141     SearchClass is a class that takes a problem and implements search()
142     problem is a search problem
143     solutions is a list of optimal solutions
144     """
145     print("Testing problem 1:")
146     schr1 = SearchClass(problem)
147     path1 = schr1.search()
148     print("Path found:",path1)
149     assert path1 is not None, "No path is found in problem1"
150     assert list(path1.nodes()) in solutions, "Shortest path not found in
            problem1"
151     print("Passed unit test")
152 
153 if __name__ == "__main__":
154     #test(Searcher)    # what needs to be changed to make this succeed?
155     test(AStarSearcher)
156 
157 # example queries:
158 # searcher1 = Searcher(searchProblem.simp_delivery_graph) # DFS
159 # searcher1.search() # find first path
160 # searcher1.search() # find next path
161 # searcher2 = AStarSearcher(searchProblem.simp_delivery_graph) # A*
162 # searcher2.search() # find first path
163 # searcher2.search() # find next path
164 # searcher3 = Searcher(searchProblem.cyclic_simp_delivery_graph) # DFS
165 # searcher3.search() # find first path with DFS. What do you expect to
        happen?
166 # searcher4 = AStarSearcher(searchProblem.cyclic_simp_delivery_graph) # A*
167 # searcher4.search() # find first path
```

Exercise 3.2 Change the code so that it implements (i) best-first search and (ii) lowest-cost-first search. For each of these methods compare it to A^* in terms of the number of paths expanded, and the path found.

Exercise 3.3 In the *add* method in *FrontierPQ* what does the "-" in front of *frontier_index* do? When there are multiple paths with the same f-value, which search method does this act like? What happens if the "-" is removed? When there are multiple paths with the same value, which search method does this act like? Does it work better with or without the "-"? What evidence did you base your conclusion on?

Exercise 3.4 The searcher acts like a Python iterator, in that it returns one value (here a path) and then returns other values (paths) on demand, but does not implement the iterator interface. Change the code so it implements the iterator interface. What does this enable us to do?

3.2.4 Multiple Path Pruning

To run the multiple-path pruning demo, in folder "aipython", load "searchMPP.py" , using e.g., `ipython -i searchMPP.py`, and copy and paste the example queries at the bottom of that file.

The following implements A^* with multiple-path pruning. It overrides *search*() in *Searcher*.

searchMPP.py — Searcher with multiple-path pruning

```
from searchGeneric import AStarSearcher, visualize
from searchProblem import Path

class SearcherMPP(AStarSearcher):
    """returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    """
    def __init__(self, problem):
        super().__init__(problem)
        self.explored = set()

    @visualize
    def search(self):
        """returns next path from an element of problem's start nodes
        to a goal node.
        Returns None if no path exists.
        """
        while not self.empty_frontier():
            path = self.frontier.pop()
            if path.end() not in self.explored:
                self.display(2, "Expanding:",path,"(cost:",path.cost,")")
                self.explored.add(path.end())
                self.num_expanded += 1
                if self.problem.is_goal(path.end()):
                    self.display(1, self.num_expanded, "paths have been
                        expanded and",
                            len(self.frontier), "paths remain in the
                                frontier")
                    self.solution = path # store the solution found
                    return path
                else:
                    neighs = self.problem.neighbors(path.end())
                    self.display(3,"Neighbors are", neighs)
                    for arc in neighs:
                        self.add_to_frontier(Path(path,arc))
                    self.display(3,"Frontier:",self.frontier)
        self.display(1,"No (more) solutions. Total of",
                     self.num_expanded,"paths expanded.")

from searchGeneric import test
if __name__ == "__main__":
    test(SearcherMPP)

import searchProblem
# searcherMPPcdp = SearcherMPP(searchProblem.cyclic_simp_delivery_graph)
# searcherMPPcdp.search() # find first path
```

Exercise 3.5 Implement a searcher that implements cycle pruning instead of multiple-path pruning. You need to decide whether to check for cycles when paths are added to the frontier or when they are removed. (Hint: either method can be implemented by only changing one or two lines in SearcherMPP. Hint: there is a cycle if `path.end() in path.initial_nodes()`) Compare no pruning, multiple path pruning and cycle pruning for the cyclic delivery problem. Which works better in terms of number of paths expanded, computational time or space?

3.3 Branch-and-bound Search

To run the demo, in folder "aipython", load "searchBranchAndBound.py", and copy and paste the example queries at the bottom of that file.

Depth-first search methods do not need an a priority queue, but can use a list as a stack. In this implementation of branch-and-bound search, we call *search* to find an optimal solution with cost less than bound. This uses depth-first search to find a path to a goal that extends *path* with cost less than the bound. Once a path to a goal has been found, that path is remembered as the *best_path*, the bound is reduced, and the search continues.

searchBranchAndBound.py — Branch and Bound Search

```
11 from searchProblem import Path
12 from searchGeneric import Searcher
13 from display import Displayable, visualize
14
15 class DF_branch_and_bound(Searcher):
16     """returns a branch and bound searcher for a problem.
17     An optimal path with cost less than bound can be found by calling
           search()
18     """
19     def __init__(self, problem, bound=float("inf")):
20         """creates a searcher than can be used with search() to find an
               optimal path.
21         bound gives the initial bound. By default this is infinite -
               meaning there
22         is no initial pruning due to depth bound
23         """
24         super().__init__(problem)
25         self.best_path = None
26         self.bound = bound
27
28     @visualize
29     def search(self):
30         """returns an optimal solution to a problem with cost less than
               bound.
31         returns None if there is no solution with cost less than bound."""
32         self.frontier = [Path(self.problem.start_node())]
```

```
33          self.num_expanded = 0
34          while self.frontier:
35              path = self.frontier.pop()
36              if path.cost+self.problem.heuristic(path.end()) < self.bound:
37                  # if path.end() not in path.initial_nodes(): # for cycle
                        pruning
38                  self.display(3,"Expanding:",path,"cost:",path.cost)
39                  self.num_expanded += 1
40                  if self.problem.is_goal(path.end()):
41                      self.best_path = path
42                      self.bound = path.cost
43                      self.display(2,"New best path:",path," cost:",path.cost)
44                  else:
45                      neighs = self.problem.neighbors(path.end())
46                      self.display(3,"Neighbors are", neighs)
47                      for arc in reversed(list(neighs)):
48                          self.add_to_frontier(Path(path, arc))
49          self.display(1,"Number of paths expanded:",self.num_expanded,
50                         "(optimal" if self.best_path else "(no", "solution
                               found)")
51          self.solution = self.best_path
52          return self.best_path
```

Note that this code used *reversed* in order to expand the neighbors of a node in the left-to-right order one might expect. It does this because $pop()$ removes the rightmost element of the list. The call to *list* is there because reversed only works on lists and tuples, but the neighbors can be generated.

Here is a unit test and some queries:

searchBranchAndBound.py — (continued)

```
54  from searchGeneric import test
55  if __name__ == "__main__":
56      test(DF_branch_and_bound)
57
58  # Example queries:
59  import searchProblem
60  # searcherb1 = DF_branch_and_bound(searchProblem.simp_delivery_graph)
61  # searcherb1.search()     # find optimal path
62  # searcherb2 =
        DF_branch_and_bound(searchProblem.cyclic_simp_delivery_graph,
        bound=100)
63  # searcherb2.search()     # find optimal path
```

Exercise 3.6 In searcherb2, in the code above, what happens if the bound is smaller, say 10? What if it larger, say 1000?

Exercise 3.7 Implement a branch-and-bound search uses recursion. Hint: you don't need an explicit frontier, but can do a recursive call for the children.

Exercise 3.8 After the branch-and-bound search found a solution, Sam ran search again, and noticed a different count. Sam hypothesized that this count was related

to the number of nodes that an $A*$ search would use (either expand or be added to the frontier). Or maybe, Sam thought, the count for a number of nodes when the bound is slightly above the optimal path case is related to how A^* would work. Is there relationship between these counts? Are there different things that it could count so they are related? Try to find the most specific statement that is true, and explain why it is true.

To test the hypothesis, Sam wrote the following code, but isn't sure it is helpful:

searchTest.py — code that may be useful to compare A* and branch-and-bound

```
from searchGeneric import Searcher, AStarSearcher
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP

DF_branch_and_bound.max_display_level = 1
Searcher.max_display_level = 1

def run(problem,name):
    print("\n\n*******",name)

    print("\nA*:")
    asearcher = AStarSearcher(problem)
    print("Path found:",asearcher.search()," cost=",asearcher.solution.cost)
    print("there are",asearcher.frontier.count(asearcher.solution.cost),
         "elements remaining on the queue with
             f-value=",asearcher.solution.cost)

    print("\nA* with MPP:"),
    msearcher = SearcherMPP(problem)
    print("Path found:",msearcher.search()," cost=",msearcher.solution.cost)
    print("there are",msearcher.frontier.count(msearcher.solution.cost),
         "elements remaining on the queue with
             f-value=",msearcher.solution.cost)

    bound = asearcher.solution.cost+0.01
    print("\nBranch and bound (with too-good initial bound of", bound,")")
    tbb = DF_branch_and_bound(problem,bound) # cheating!!!!
    print("Path found:",tbb.search()," cost=",tbb.solution.cost)
    print("Rerunning B&B")
    print("Path found:",tbb.search())

    bbound = asearcher.solution.cost*2+10
    print("\nBranch and bound (with not-very-good initial bound of",
        bbound, ")")
    tbb2 = DF_branch_and_bound(problem,bbound) # cheating!!!!
    print("Path found:",tbb2.search()," cost=",tbb2.solution.cost)
    print("Rerunning B&B")
    print("Path found:",tbb2.search())

    print("\nDepth-first search: (Use ^C if it goes on forever)")
```

```
48      tsearcher = Searcher(problem)
49      print("Path found:",tsearcher.search()," cost=",tsearcher.solution.cost)
50
51
52  import searchProblem
53  from searchTest import run
54  if __name__ == "__main__":
55      run(searchProblem.problem1,"Problem 1")
56  #   run(searchProblem.simp_delivery_graph,"Acyclic Delivery")
57  #   run(searchProblem.cyclic_simp_delivery_graph,"Cyclic Delivery")
58  # also test some graphs with cycles, and some with multiple least-cost
        paths
```

Chapter 4

Reasoning with Constraints

4.1 Constraint Satisfaction Problems

4.1.1 Variables

A **variable** consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of constraints.

variable.py — Representations of a variable in CSPs and probabilistic models

```
11 import random
12 import matplotlib.pyplot as plt
13
14 class Variable(object):
15     """A random variable.
16     name (string) - name of the variable
17     domain (list) - a list of the values for the variable.
18     Variables are ordered according to their name.
19     """
20
21     def __init__(self, name, domain, position=None):
22         """Variable
23         name a string
24         domain a list of printable values
25         position of form (x,y)
26         """
27         self.name = name # string
28         self.domain = domain # list of values
29         self.position = position if position else (random.random(),
               random.random())
30         self.size = len(domain)
31
```

```
32      def __str__(self):
33          return self.name
34
35      def __repr__(self):
36          return self.name # f"Variable({self.name})"
```

4.1.2 Constraints

A **constraint** consists of:

- A tuple (or list) of variables is called the **scope**.
- A **condition** is a Boolean function that takes the same number of arguments as there are variables in the scope. The condition must have a `__name__` property that gives a printable name of the function; built-in functions and functions that are defined using *def* have such a property; for other functions you may need to define this property.
- An optional name
- An optional (x, y) position

cspProblem.py — Representations of a Constraint Satisfaction Problem

```
11  from variable import Variable
12
13  class Constraint(object):
14      """A Constraint consists of
15      * scope: a tuple of variables
16      * condition: a Boolean function that can applied to a tuple of values
            for variables in scope
17      * string: a string for printing the constraints. All of the strings
            must be unique.
18      for the variables
19      """
20      def __init__(self, scope, condition, string=None, position=None):
21          self.scope = scope
22          self.condition = condition
23          if string is None:
24              self.string = f"{self.condition.__name__}({self.scope})"
25          else:
26              self.string = string
27          self.position = position
28
29      def __repr__(self):
30          return self.string
```

An **assignment** is a *variable*:*value* dictionary.

If *con* is a constraint, *con.holds*(*assignment*) returns True or False depending on whether the condition is true or false for that assignment. The assignment

assignment must assigns a value to every variable in the scope of the constraint *con* (and could also assign values other variables); *con.holds* gives an error if not all variables in the scope of *con* are assigned in the assignment. It ignores variables in *assignment* that are not in the scope of the constraint.

In Python, the $*$ notation is used for unpacking a tuple. For example, $F(*(1,2,3))$ is the same as $F(1,2,3)$. So if t has value $(1,2,3)$, then $F(*t)$ is the same as $F(1,2,3)$.

cspProblem.py — (continued)

```
32      def can_evaluate(self, assignment):
33          """
34          assignment is a variable:value dictionary
35          returns True if the constraint can be evaluated given assignment
36          """
37          return all(v in assignment for v in self.scope)
38
39      def holds(self,assignment):
40          """returns the value of Constraint con evaluated in assignment.
41
42          precondition: all variables are assigned in assignment, ie
                self.can_evaluate(assignment) is true
43          """
44          return self.condition(*tuple(assignment[v] for v in self.scope))
```

4.1.3 CSPs

A constraint satisfaction problem (CSP) requires:

- *variables*: a list or set of variables
- *constraints*: a set or list of constraints.

Other properties are inferred from these:

- *var_to_const* is a mapping from variables to set of constraints, such that *var_to_const*[*var*] is the set of constraints with *var* in the scope.

cspProblem.py — (continued)

```
46  class CSP(object):
47      """A CSP consists of
48      * a title (a string)
49      * variables, a set of variables
50      * constraints, a list of constraints
51      * var_to_const, a variable to set of constraints dictionary
52      """
53      def __init__(self, title, variables, constraints):
54          """title is a string
55          variables is set of variables
56          constraints is a list of constraints
```

```
57          """
58          self.title = title
59          self.variables = variables
60          self.constraints = constraints
61          self.var_to_const = {var:set() for var in self.variables}
62          for con in constraints:
63              for var in con.scope:
64                  self.var_to_const[var].add(con)
65
66      def __str__(self):
67          """string representation of CSP"""
68          return str(self.title)
69
70      def __repr__(self):
71          """more detailed string representation of CSP"""
72          return f"CSP({self.title}, {self.variables}, {([str(c) for c in
                self.constraints])})"
```

csp.consistent(*assignment*) returns true if the assignment is consistent with each of the constraints in *csp* (i.e., all of the constraints that can be evaluated evaluate to true). Note that this is a local consistency with each constraint; it does *not* imply the CSP is consistent or has a solution.

cspProblem.py — (continued)

```
74      def consistent(self,assignment):
75          """assignment is a variable:value dictionary
76          returns True if all of the constraints that can be evaluated
77                          evaluate to True given assignment.
78          """
79          return all(con.holds(assignment)
80                     for con in self.constraints
81                     if con.can_evaluate(assignment))
```

The **show** method uses matplotlib to show the graphical structure of a constraint network. If the node positions are not specified, this gives different positions each time it is run; if you don't like the graph, try again.

cspProblem.py — (continued)

```
83      def show(self):
84          plt.ion()  # interactive
85          ax = plt.figure().gca()
86          ax.set_axis_off()
87          plt.title(self.title)
88          var_bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
89          con_bbox = dict(boxstyle="square,pad=1.0",color="green")
90          for var in self.variables:
91              if var.position is None:
92                  var.position = (random.random(), random.random())
93          for con in self.constraints:
94              if con.position is None:
```

```
            con.position = tuple(sum(var.position[i] for var in
                con.scope)/len(con.scope)
                                 for i in range(2))
        bbox = dict(boxstyle="square,pad=1.0",color="green")
        for var in con.scope:
            ax.annotate(con.string, var.position, xytext=con.position,
                        arrowprops={'arrowstyle':'-'},bbox=con_bbox,
                        ha='center')
    for var in self.variables:
        x,y = var.position
        plt.text(x,y,var.name,bbox=var_bbox,ha='center')
```

4.1.4 Examples

In the following code *ne_*, when given a number, returns a function that is true when its argument is not that number. For example, if $f = ne\_(3)$, then $f(2)$ is True and $f(3)$ is False. That is, $ne\_(x)(y)$ is true when $x \neq y$. Allowing a function of multiple arguments to use its arguments one at a time is called **currying**, after the logician Haskell Curry. Functions used as conditions in constraints require names (so they can be printed).

cspExamples.py — Example CSPs

```
from cspProblem import Variable, CSP, Constraint
from operator import lt,ne,eq,gt

def ne_(val):
    """not equal value"""
    # nev = lambda x: x != val # alternative definition
    # nev = partial(neq,val)  # another alternative definition
    def nev(x):
        return val != x
    nev.__name__ = f"{val} != "   # name of the function
    return nev
```

Similarly $is\_(x)(y)$ is true when $x = y$.

cspExamples.py — (continued)

```
def is_(val):
    """is a value"""
    # isv = lambda x: x == val # alternative definition
    # isv = partial(eq,val)   # another alternative definition
    def isv(x):
        return val == x
    isv.__name__ = f"{val} == "
    return isv
```

The CSP, *csp*0 has variables X, Y and Z, each with domain $\{1,2,3\}$. The constraints are $X < Y$ and $Y < Z$.

cspExamples.py — (continued)

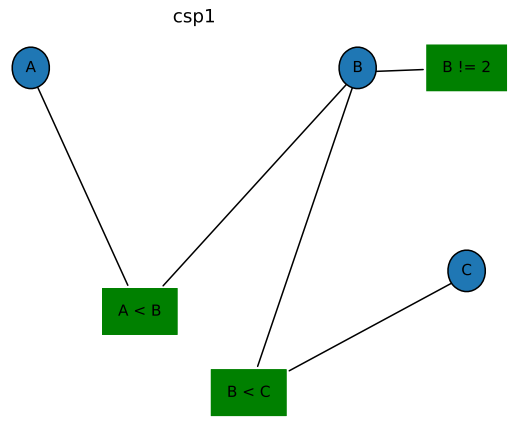


Figure 4.1: csp1.show()

```
X = Variable('X', {1,2,3})
Y = Variable('Y', {1,2,3})
Z = Variable('Z', {1,2,3})
csp0 = CSP("csp0", {X,Y,Z},
          [ Constraint([X,Y],lt),
            Constraint([Y,Z],lt)])
```

The CSP, *csp*1 has variables A, B and C, each with domain $\{1,2,3,4\}$. The constraints are $A < B$, $B \neq 2$, and $B < C$. This is slightly more interesting than *csp*0 as it has more solutions. This example is used in the unit tests, and so if it is changed, the unit tests need to be changed. The CSP *csp*1*s* is the same, but with only the constraints $A < B$ and $B < C$

cspExamples.py — (continued)

```
A = Variable('A', {1,2,3,4}, position=(0.2,0.9))
B = Variable('B', {1,2,3,4}, position=(0.8,0.9))
C = Variable('C', {1,2,3,4}, position=(1,0.4))
C0 = Constraint([A,B], lt, "A < B", position=(0.4,0.3))
C1 = Constraint([B], ne_(2), "B != 2", position=(1,0.9))
C2 = Constraint([B,C], lt, "B < C", position=(0.6,0.1))
csp1 = CSP("csp1", {A, B, C},
          [C0, C1, C2])

csp1s = CSP("csp1s", {A, B, C},
          [C0, C2]) # A<B, B<C
```

The next CSP, *csp*2 is Example 4.9 of Poole and Mackworth [2023]; the domain consistent network (after applying the unary constraints) is shown in Figure 4.2. Note that we use the same variables as the previous example and add

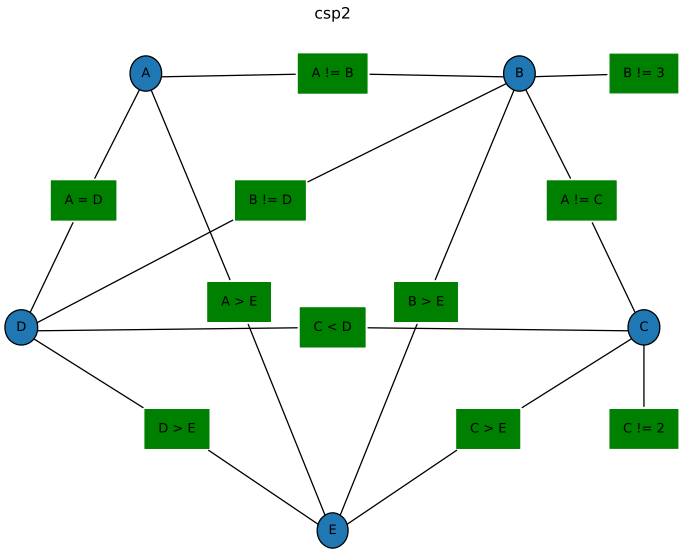


Figure 4.2: csp2.show()

two more.

cspExamples.py — (continued)

```
51 D = Variable('D', {1,2,3,4}, position=(0,0.4))
52 E = Variable('E', {1,2,3,4}, position=(0.5,0))
53 csp2 = CSP("csp2", {A,B,C,D,E},
54           [ Constraint([B], ne_(3), "B != 3", position=(1,0.9)),
55            Constraint([C], ne_(2), "C != 2", position=(1,0.2)),
56            Constraint([A,B], ne, "A != B"),
57            Constraint([B,C], ne, "A != C"),
58            Constraint([C,D], lt, "C < D"),
59            Constraint([A,D], eq, "A = D"),
60            Constraint([E,A], lt, "E < A"),
61            Constraint([E,B], lt, "E < B"),
62            Constraint([E,C], lt, "E < C"),
63            Constraint([E,D], lt, "E < D"),
64            Constraint([B,D], ne, "B != D")])
```

The following example is another scheduling problem (but with multiple answers). This is the same a scheduling 2 in the original AIspace.org consistency app.

cspExamples.py — (continued)

```
66 csp3 = CSP("csp3", {A,B,C,D,E},
67           [Constraint([A,B], ne, "A != B"),
68            Constraint([A,D], lt, "A < D"),
```

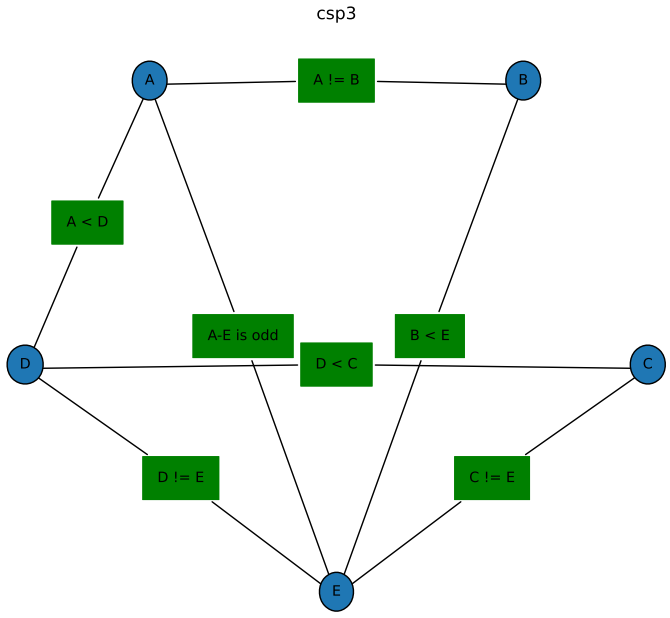


Figure 4.3: csp3.show()

```
69            Constraint([A,E], lambda a,e: (a-e)%2 == 1, "A-E is odd"),
70            Constraint([B,E], lt, "B < E"),
71            Constraint([D,C], lt, "D < C"),
72            Constraint([C,E], ne, "C != E"),
73            Constraint([D,E], ne, "D != E")])
```

The following example is another abstract scheduling problem. What are the solutions?

cspExamples.py — (continued)

```
75 def adjacent(x,y):
76    """True when x and y are adjacent numbers"""
77    return abs(x-y) == 1
78
79 csp4 = CSP("csp4", {A,B,C,D,E},
80           [Constraint([A,B], adjacent, "adjacent(A,B)"),
81            Constraint([B,C], adjacent, "adjacent(B,C)"),
82            Constraint([C,D], adjacent, "adjacent(C,D)"),
83            Constraint([D,E], adjacent, "adjacent(D,E)"),
84            Constraint([A,C], ne, "A != C"),
85            Constraint([B,D], ne, "B != D"),
86            Constraint([C,E], ne, "C != E")])
```

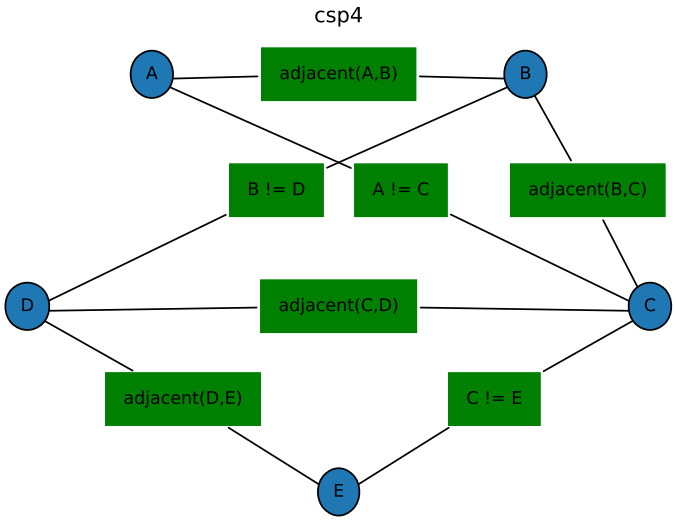
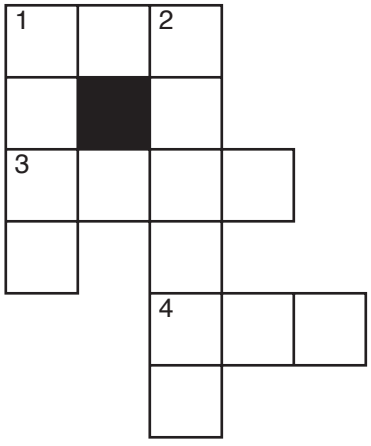


Figure 4.4: csp4.show()



Words:
ant, big, bus, car, has, book, buys, hold, lane, year, ginger, search, symbol, syntax.

Figure 4.5: crossword1: a crossword puzzle to be solved

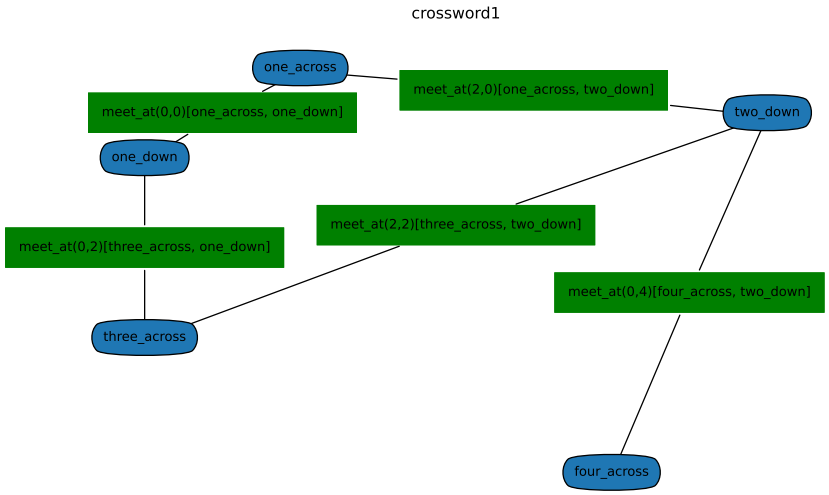


Figure 4.6: crossword1.show()

The following examples represent the crossword shown in Figure 4.5.

In the first representation, the variables represent words. The constraint imposed by the crossword is that where two words intersect, the letter at the intersection must be the same. The method `meet_at` is used to test whether two words intersect with the same letter. For example, the constraint `meet_at(2,0)` means that the third letter (at position 2) of the first argument is the same as the first letter of the second argument. This is shown in Figure 4.6.

cspExamples.py — (continued)

```
88  def meet_at(p1,p2):
89      """returns a function of two words that is true
90                  when the words intersect at positions p1, p2.
91      The positions are relative to the words; starting at position 0.
92      meet_at(p1,p2)(w1,w2) is true if the same letter is at position p1 of
            word w1
93           and at position p2 of word w2.
94      """
95      def meets(w1,w2):
96          return w1[p1] == w2[p2]
97      meets.__name__ = f"meet_at({p1},{p2})"
98      return meets
99
100 one_across = Variable('one_across', {'ant', 'big', 'bus', 'car', 'has'},
        position=(0.3,0.9))
101 one_down = Variable('one_down', {'book', 'buys', 'hold', 'lane', 'year'},
        position=(0.1,0.7))
102 two_down = Variable('two_down', {'ginger', 'search', 'symbol', 'syntax'},
```

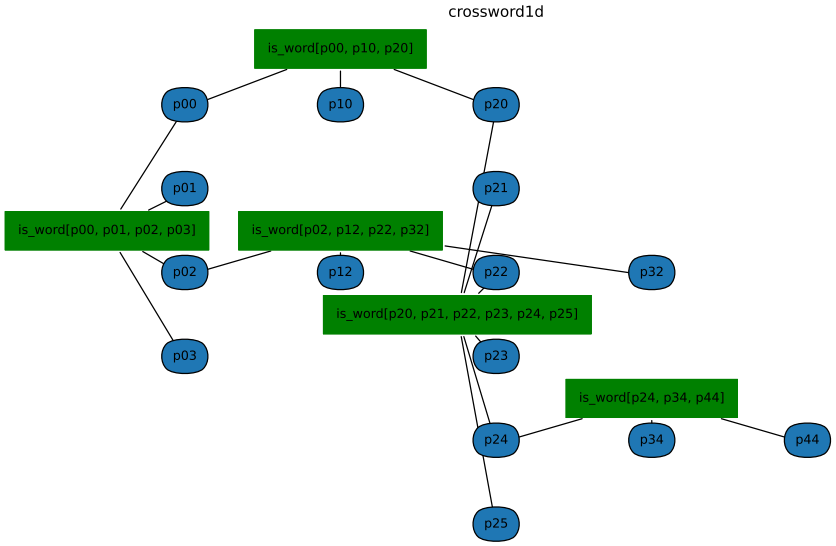


Figure 4.7: crossword1d.show()

```
    position=(0.9,0.8))
three_across = Variable('three_across', {'book', 'buys', 'hold', 'land',
    'year'}, position=(0.1,0.3))
four_across = Variable('four_across',{'ant', 'big', 'bus', 'car', 'has'},
    position=(0.7,0.0))
crossword1 = CSP("crossword1",
                 {one_across, one_down, two_down, three_across,
                     four_across},
                 [Constraint([one_across,one_down], meet_at(0,0)),
                  Constraint([one_across,two_down], meet_at(2,0)),
                  Constraint([three_across,two_down], meet_at(2,2)),
                  Constraint([three_across,one_down], meet_at(0,2)),
                  Constraint([four_across,two_down], meet_at(0,4))])
```

In an alternative representation of a crossword (the "dual" representation), the variables represent letters, and the constraints are that adjacent sequences of letters form words. This is shown in Figure 4.7.

cspExamples.py — (continued)

```
words = {'ant', 'big', 'bus', 'car', 'has','book', 'buys', 'hold',
        'lane', 'year', 'ginger', 'search', 'symbol', 'syntax'}

def is_word(*letters, words=words):
```

```
117     """is true if the letters concatenated form a word in words"""
118     return "".join(letters) in words
119
120 letters = {"a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k", "l",
121   "m", "n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y",
122   "z"}
123
124 # pij is the variable representing the letter i from the left and j down
        (starting from 0)
125 p00 = Variable('p00', letters, position=(0.1,0.85))
126 p10 = Variable('p10', letters, position=(0.3,0.85))
127 p20 = Variable('p20', letters, position=(0.5,0.85))
128 p01 = Variable('p01', letters, position=(0.1,0.7))
129 p21 = Variable('p21', letters, position=(0.5,0.7))
130 p02 = Variable('p02', letters, position=(0.1,0.55))
131 p12 = Variable('p12', letters, position=(0.3,0.55))
132 p22 = Variable('p22', letters, position=(0.5,0.55))
133 p32 = Variable('p32', letters, position=(0.7,0.55))
134 p03 = Variable('p03', letters, position=(0.1,0.4))
135 p23 = Variable('p23', letters, position=(0.5,0.4))
136 p24 = Variable('p24', letters, position=(0.5,0.25))
137 p34 = Variable('p34', letters, position=(0.7,0.25))
138 p44 = Variable('p44', letters, position=(0.9,0.25))
139 p25 = Variable('p25', letters, position=(0.5,0.1))
140
141 crossword1d = CSP("crossword1d",
142                 {p00, p10, p20, # first row
143                  p01, p21, # second row
144                  p02, p12, p22, p32, # third row
145                  p03, p23, #fourth row
146                  p24, p34, p44, # fifth row
147                  p25 # sixth row
148                  },
149                 [Constraint([p00, p10, p20], is_word,
                        position=(0.3,0.95)), #1-across
150                  Constraint([p00, p01, p02, p03], is_word,
                         position=(0,0.625)), # 1-down
151                  Constraint([p02, p12, p22, p32], is_word,
                         position=(0.3,0.625)), # 3-across
152                  Constraint([p20, p21, p22, p23, p24, p25], is_word,
                         position=(0.45,0.475)), # 2-down
153                  Constraint([p24, p34, p44], is_word,
                         position=(0.7,0.325)) # 4-across
154                  ])
```

Exercise 4.1 How many assignments of a value to each variable are there for each of the representations of the above crossword? Do you think an exhaustive enumeration will work for either one?

The queens problem is a puzzle on a chess board, where the idea is to place a queen on each column so the queens cannot take each other: there are no

two queens on the same row, column or diagonal. The **n-queens problem** is a generalization where the size of the board is an $n \times n$, and n queens have to be placed.

Here is a representation of the n-queens problem, where the variables are the columns and the values are the rows in which the queen is placed. The original queens problem on a standard (8×8) chess board is `n_queens(8)`

cspExamples.py — (continued)

```
156 def queens(ri,rj):
157     """ri and rj are different rows, return the condition that the queens
            cannot take each other"""
158     def no_take(ci,cj):
159         """is true if queen at (ri,ci) cannot take a queen at (rj,cj)"""
160         return ci != cj and abs(ri-ci) != abs(rj-cj)
161     return no_take
162 
163 def n_queens(n):
164     """returns a CSP for n-queens"""
165     columns = list(range(n))
166     variables = [Variable(f"R{i}",columns) for i in range(n)]
167     return CSP("n-queens",
168                variables,
169                 [Constraint([variables[i], variables[j]], queens(i,j))
170                      for i in range(n) for j in range(n) if i != j])
171 
172 # try the CSP n_queens(8) in one of the solvers.
173 # What is the smallest n for which there is a solution?
```

Exercise 4.2 How many constraints does this representation of the n-queens problem produce? Can it be done with fewer constraints? Either explain why it can't be done with fewer constraints, or give a solution using fewer constraints.

Unit tests

The following defines a **unit test** for csp solvers, by default using example csp1.

cspExamples.py — (continued)

```
175 def test_csp(CSP_solver, csp=csp1,
176                solutions=[{A: 1, B: 3, C: 4}, {A: 2, B: 3, C: 4}]):
177     """CSP_solver is a solver that takes a csp and returns a solution
178     csp is a constraint satisfaction problem
179     solutions is the list of all solutions to csp
180     This tests whether the solution returned by CSP_solver is a solution.
181     """
182     print("Testing csp with",CSP_solver.__doc__)
183     sol0 = CSP_solver(csp)
184     print("Solution found:",sol0)
185     assert sol0 in solutions, f"Solution not correct for {csp}"
186     print("Passed unit test")
```

Exercise 4.3 Modify *test* so that instead of taking in a list of solutions, it checks whether the returned solution actually is a solution.

Exercise 4.4 Propose a test that is appropriate for CSPs with no solutions. Assume that the test designer knows there are no solutions. Consider what a CSP solver should return if there are no solutions to the CSP.

Exercise 4.5 Write a unit test that checks whether all solutions (e.g., for the search algorithms that can return multiple solutions) are correct, and whether all solutions can be found.

4.2 A Simple Depth-first Solver

The first solver carries out a depth-first search through the space of partial assignments. This takes in a CSP problem and an optional variable ordering (a list of the variables in the CSP). It returns a generator of the solutions (see Section 1.5.4 on `yield` for enumerations).

cspDFS.py — Solving a CSP using depth-first search.

```
from cspExamples import csp1,csp1s,csp2,test_csp, crossword1, crossword1d

def dfs_solver(constraints, context, var_order):
    """generator for all solutions to csp.
    context is an assignment of values to some of the variables.
    var_order is  a list of the variables in csp that are not in context.
    """
    to_eval = {c for c in constraints if c.can_evaluate(context)}
    if all(c.holds(context) for c in to_eval):
        if var_order == []:
            yield context
        else:
            rem_cons = [c for c in constraints if c not in to_eval]
            var = var_order[0]
            for val in var.domain:
                yield from dfs_solver(rem_cons, context|{var:val},
                    var_order[1:])

def dfs_solve_all(csp, var_order=None):
    """depth-first CSP solver to return a list of all solutions to csp.
    """
    if var_order == None:  # use an arbitrary variable order
        var_order = list(csp.variables)
    return list( dfs_solver(csp.constraints, {}, var_order))

def dfs_solve1(csp, var_order=None):
    """depth-first CSP solver to find single solution or None if there are
        no solutions.
    """
    if var_order == None:  # use an arbitrary variable order
        var_order = list(csp.variables)
```

```
40      gen = dfs_solver(csp.constraints, {}, var_order)
41      try:    # Python generators raise an exception if there are no more
            elements.
42          return next(gen)
43      except StopIteration:
44          return None
45
46  if __name__ == "__main__":
47      test_csp(dfs_solve1)
48
49  #Try:
50  # dfs_solve_all(csp1)
51  # dfs_solve_all(csp2)
52  # dfs_solve_all(crossword1)
53  # dfs_solve_all(crossword1d) # warning: may take a *very* long time!
```

Exercise 4.6 Instead of testing all constraints at every node, change it so each constraint is only tested when all of it variables are assigned. Given an elimination ordering, it is possible to determine when each constraint needs to be tested. Implement this. Hint: create a parallel list of sets of constraints, where at each position i in the list, the constraints at position i can be evaluated when the variable at position i has been assigned.

Exercise 4.7 Estimate how long `dfs_solve_all(crossword1d)` will take on your computer. To do this, reduce the number of variables that need to be assigned, so that the simplifies problem can be solved in a reasonable time (between 0.1 second and 10 seconds). This can be done by reducing the number of variables in `var_order`, as the program only splits on these. How much more time will it take if the number of variables is increased by 1? (Try it!) Then extrapolate to all of the variables. See Section 1.6.1 for how to time your code. Would making the code 100 times faster or using a computer 100 times faster help?

4.3 Converting CSPs to Search Problems

To run the demo, in folder "`aipython`", load "`cspSearch.py`", and copy and paste the example queries at the bottom of that file.

The next solver constructs a search space that can be solved using the search methods of the previous chapter. This takes in a CSP problem and an optional variable ordering, which is a list of the variables in the CSP. In this search space:

- A node is a *variable* : *value* dictionary which does not violate any constraints (so that dictionaries that violate any conmtratints are not added).
- An arc corresponds to an assignment of a value to the next variable. This assumes a static ordering; the next variable chosen to split does not depend on the context. If no variable ordering is given, this makes no attempt to choose a good ordering.

cspSearch.py — Representations of a Search Problem from a CSP.

```
from cspProblem import CSP, Constraint
from searchProblem import Arc, Search_problem

class Search_from_CSP(Search_problem):
    """A search problem directly from the CSP.

    A node is a variable:value dictionary"""
    def __init__(self, csp, variable_order=None):
        self.csp=csp
        if variable_order:
            assert set(variable_order) == set(csp.variables)
            assert len(variable_order) == len(csp.variables)
            self.variables = variable_order
        else:
            self.variables = list(csp.variables)

    def is_goal(self, node):
        """returns whether the current node is a goal for the search
        """
        return len(node)==len(self.csp.variables)

    def start_node(self):
        """returns the start node for the search
        """
        return {}
```

The *neighbors*(*node*) method uses the fact that the length of the node, which is the number of variables already assigned, is the index of the next variable to split on. Note that we do no need to check whether there are no more variables to split on, as the nodes are all consistent, by construction, and so when there are no more variables we have a solution, and so don't need the neighbors.

cspSearch.py — (continued)

```
    def neighbors(self, node):
        """returns a list of the neighboring nodes of node.
        """
        var = self.variables[len(node)] # the next variable
        res = []
        for val in var.domain:
            new_env = node|{var:val} #dictionary union
            if self.csp.consistent(new_env):
                res.append(Arc(node,new_env))
        return res
```

The unit tests relies on a solver. The following procedure creates a solver using search that can be tested.

cspSearch.py — (continued)

```
from cspExamples import csp1,csp1a,csp2,test_csp, crossword1, crossword1d
from searchGeneric import Searcher
```

```
50
51 def solver_from_searcher(csp):
52     """depth-first search solver"""
53     path = Searcher(Search_from_CSP(csp)).search()
54     if path is not None:
55         return path.end()
56     else:
57         return None
58
59 if __name__ == "__main__":
60     test_csp(solver_from_searcher)
61
62 ## Test Solving CSPs with Search:
63 searcher1 = Searcher(Search_from_CSP(csp1))
64 #print(searcher1.search()) # get next solution
65 searcher2 = Searcher(Search_from_CSP(csp2))
66 #print(searcher2.search()) # get next solution
67 searcher3 = Searcher(Search_from_CSP(crossword1))
68 #print(searcher3.search()) # get next solution
69 searcher4 = Searcher(Search_from_CSP(crossword1d))
70 #print(searcher4.search()) # get next solution (warning: slow)
```

Exercise 4.8 What would happen if we constructed the new assignment by assigning $node[var] = val$ (with side effects) instead of using dictionary union? Give an example of where this could give a wrong answer. How could the algorithm be changed to work with side effects? (Hint: think about what information needs to be in a node).

Exercise 4.9 Change neighbors so that it returns an iterator of values rather than a list. (Hint: use *yield*.)

4.4 Consistency Algorithms

> To run the demo, in folder "aipython", load "cspConsistency.py", and copy and paste the commented-out example queries at the bottom of that file.

A *Con_solver* is used to simplify a CSP using arc consistency.

cspConsistency.py — Arc Consistency and Domain splitting for solving a CSP

```
11 from display import Displayable
12
13 class Con_solver(Displayable):
14     """Solves a CSP with arc consistency and domain splitting
15     """
16     def __init__(self, csp, **kwargs):
17         """a CSP solver that uses arc consistency
18         * csp is the CSP to be solved
19         * kwargs is the keyword arguments for Displayable superclass
```

```
20          """
21          self.csp = csp
22          super().__init__(**kwargs) # Or Displayable.__init__(self,**kwargs)
```

The following implementation of arc consistency maintains the set *to_do* of (variable, constraint) pairs that are to be checked. It takes in a domain dictionary and returns a new domain dictionary. It needs to be careful to avoid side effects (by copying the *domains* dictionary and the *to_do* set).

cspConsistency.py — (continued)

```
24      def make_arc_consistent(self, orig_domains=None, to_do=None):
25          """Makes this CSP arc-consistent using generalized arc consistency
26          orig_domains is the original domains
27          to_do is a set of (variable,constraint) pairs
28          returns the reduced domains (an arc-consistent variable:domain
                dictionary)
29          """
30          if orig_domains is None:
31              orig_domains = {var:var.domain for var in self.csp.variables}
32          if to_do is None:
33              to_do = {(var, const) for const in self.csp.constraints
34                       for var in const.scope}
35          else:
36              to_do = to_do.copy() # use a copy of to_do
37          domains = orig_domains.copy()
38          self.display(2,"Performing AC with domains", domains)
39          while to_do:
40              var, const = self.select_arc(to_do)
41              self.display(3, "Processing arc (", var, ",", const, ")")
42              other_vars = [ov for ov in const.scope if ov != var]
43              new_domain = {val for val in domains[var]
44                            if self.any_holds(domains, const, {var: val},
                                  other_vars)}
45              if new_domain != domains[var]:
46                  self.display(4, "Arc: (", var, ",", const, ") is
                        inconsistent")
47                  self.display(3, "Domain pruned", "dom(", var, ") =",
                        new_domain,
48                               " due to ", const)
49                  domains[var] = new_domain
50                  add_to_do = self.new_to_do(var, const) - to_do
51                  to_do |= add_to_do    # set union
52                  self.display(3, " adding", add_to_do if add_to_do else
                        "nothing", "to to_do.")
53              self.display(4, "Arc: (", var, ",", const, ") now consistent")
54          self.display(2, "AC done. Reduced domains", domains)
55          return domains
56
57      def new_to_do(self, var, const):
58          """returns new elements to be added to to_do after assigning
59          variable var in constraint const.
```

```
60          """
61          return {(nvar, nconst) for nconst in self.csp.var_to_const[var]
62                  if nconst != const
63                  for nvar in nconst.scope
64                  if nvar != var}
```

The following selects an arc. Any element of *to_do* can be selected. The selected element needs to be removed from *to_do*. The default implementation just selects which ever element *pop* method for sets returns. For pedagogical purposes, a user interface could allow the user to select an arc. Alternatively a more sophisticated selection could be employed.

cspConsistency.py — (continued)

```
66      def select_arc(self, to_do):
67          """Selects the arc to be taken from to_do .
68          * to_do is a set of arcs, where an arc is a (variable,constraint)
                pair
69          the element selected must be removed from to_do.
70          """
71          return to_do.pop()
```

The value of `new_domain` is the subset of the domain of `var` that is consistent with the assignment to the other variables. To make it easier to understand, the following code treats unary (with no other variables in the constraint) and binary (with one other variables in the constraint) constraints as special cases. These cases are not strictly necessary; the last case covers the first two cases, but is more difficult to understand without seeing the first two cases.

```
            if len(other_vars)==0:          # unary constraint
                new_domain = {val for val in domains[var]
                              if const.holds({var:val})}
            elif len(other_vars)==1:        # binary constraint
                other = other_vars[0]
                new_domain = {val for val in domains[var]
                        if any(const.holds({var: val,other:other_val})
                                    for other_val in domains[other])}
            else:                           # general case
                new_domain = {val for val in domains[var]
                    if self.any_holds(domains, const, {var: val}, other_vars)}
```

any_holds is a recursive function that tries to finds an assignment of values to the other variables (*other_vars*) that satisfies constraint *const* given the assignment in *env*. The integer variable *ind* specifies which index to *other_vars* needs to be checked next. As soon as one assignment returns *True*, the algorithm returns *True*.

cspConsistency.py — (continued)

```
73      def any_holds(self, domains, const, env, other_vars, ind=0):
74          """returns True if Constraint const holds for an assignment
```

```
75          that extends env with the variables in other_vars[ind:]
76          env is a dictionary
77          """
78          if ind == len(other_vars):
79              return const.holds(env)
80          else:
81              var = other_vars[ind]
82              for val in domains[var]:
83                  if self.any_holds(domains, const, env|{var:val}, other_vars,
                        ind + 1):
84                      return True
85              return False
```

4.4.1 Direct Implementation of Domain Splitting

The following is a direct implementation of domain splitting with arc consistency that uses recursion. It finds one solution if one exists or returns False if there are no solutions.

cspConsistency.py — (continued)

```
87      def solve_one(self, domains=None, to_do=None):
88          """return a solution to the current CSP or False if there are no
                solutions
89          to_do is the list of arcs to check
90          """
91          new_domains = self.make_arc_consistent(domains, to_do)
92          if any(len(new_domains[var]) == 0 for var in new_domains):
93              return False
94          elif all(len(new_domains[var]) == 1 for var in new_domains):
95              self.display(2, "solution:", {var: select(
96                  new_domains[var]) for var in new_domains})
97              return {var: select(new_domains[var]) for var in new_domains}
98          else:
99              var = self.select_var(x for x in self.csp.variables if
                    len(new_domains[x]) > 1)
100             if var:
101                 dom1, dom2 = partition_domain(new_domains[var])
102                 self.display(3, "...splitting", var, "into", dom1, "and",
                        dom2)
103                 new_doms1 = copy_with_assign(new_domains, var, dom1)
104                 new_doms2 = copy_with_assign(new_domains, var, dom2)
105                 to_do = self.new_to_do(var, None)
106                 self.display(3, " adding", to_do if to_do else "nothing",
                        "to to_do.")
107                 return self.solve_one(new_doms1, to_do) or
                        self.solve_one(new_doms2, to_do)
108
109     def select_var(self, iter_vars):
110         """return the next variable to split"""
111         return select(iter_vars)
```

```
112
113 def partition_domain(dom):
114     """partitions domain dom into two.
115     """
116     split = len(dom) // 2
117     dom1 = set(list(dom)[:split])
118     dom2 = dom - dom1
119     return dom1, dom2
```

The domains are implemented as a dictionary that maps each variables to its domain. Assigning a value in Python has side effects which we want to avoid. *copy_with_assign* takes a copy of the domains dictionary, perhaps allowing for a new domain for a variable. It creates a copy of the CSP with an (optional) assignment of a new domain to a variable. Only the domains are copied.

cspConsistency.py — (continued)

```
121 def copy_with_assign(domains, var=None, new_domain={True, False}):
122     """create a copy of the domains with an assignment var=new_domain
123     if var==None then it is just a copy.
124     """
125     newdoms = domains.copy()
126     if var is not None:
127         newdoms[var] = new_domain
128     return newdoms
```

cspConsistency.py — (continued)

```
130 def select(iterable):
131     """select an element of iterable. Returns None if there is no such
            element.
132
133     This implementation just picks the first element.
134     For many of the uses, which element is selected does not affect
            correctness,
135     but may affect efficiency.
136     """
137     for e in iterable:
138         return e # returns first element found
```

Exercise 4.10 Implement of *solve_all* that is like *solve_one* but returns the set of all solutions.

Exercise 4.11 Implement *solve_enum* that enumerates the solutions. It should use Python's *yield* (and perhaps *yield from*).

Unit test:

cspConsistency.py — (continued)

```
140 from cspExamples import test_csp
141 def ac_solver(csp):
142     "arc consistency (solve_one)"
```

```
143     return Con_solver(csp).solve_one()
144 
145 if __name__ == "__main__":
146     test_csp(ac_solver)
```

4.4.2 Domain Splitting as an interface to graph searching

An alternative implementation is to implement domain splitting in terms of the search abstraction of Chapter 3.

A node is domains dictionary.

cspConsistency.py — (continued)

```
148 from searchProblem import Arc, Search_problem
149 
150 class Search_with_AC_from_CSP(Search_problem,Displayable):
151     """A search problem with arc consistency and domain splitting
152 
153     A node is a CSP """
154     def __init__(self, csp):
155         self.cons = Con_solver(csp) #copy of the CSP
156         self.domains = self.cons.make_arc_consistent()
157 
158     def is_goal(self, node):
159         """node is a goal if all domains have 1 element"""
160         return all(len(node[var])==1 for var in node)
161 
162     def start_node(self):
163         return self.domains
164 
165     def neighbors(self,node):
166         """returns the neighboring nodes of node.
167         """
168         neighs = []
169         var = select(x for x in node if len(node[x])>1)
170         if var:
171             dom1, dom2 = partition_domain(node[var])
172             self.display(2,"Splitting", var, "into", dom1, "and", dom2)
173             to_do = self.cons.new_to_do(var,None)
174             for dom in [dom1,dom2]:
175                 newdoms = copy_with_assign(node,var,dom)
176                 cons_doms = self.cons.make_arc_consistent(newdoms,to_do)
177                 if all(len(cons_doms[v])>0 for v in cons_doms):
178                     # all domains are non-empty
179                     neighs.append(Arc(node,cons_doms))
180                 else:
181                     self.display(2,"...",var,"in",dom,"has no solution")
182         return neighs
```

Exercise 4.12 When splitting a domain, this code splits the domain into half, approximately in half (without any effort to make a sensible choice). Does it work better to split one element from a domain?

Unit test:

cspConsistency.py — (continued)

```
from cspExamples import test_csp
from searchGeneric import Searcher

def ac_search_solver(csp):
    """arc consistency (search interface)"""
    sol = Searcher(Search_with_AC_from_CSP(csp)).search()
    if sol:
        return {v:select(d) for (v,d) in sol.end().items()}

if __name__ == "__main__":
    test_csp(ac_search_solver)
```

Testing:

cspConsistency.py — (continued)

```
from cspExamples import csp1, csp1s, csp2, csp3, csp4, crossword1,
    crossword1d

## Test Solving CSPs with Arc consistency and domain splitting:
#Con_solver.max_display_level = 4 # display details of AC (0 turns off)
#Con_solver(csp1).solve_one()
#searcher1d = Searcher(Search_with_AC_from_CSP(csp1))
#print(searcher1d.search())
#Searcher.max_display_level = 2 # display search trace (0 turns off)
#searcher2c = Searcher(Search_with_AC_from_CSP(csp2))
#print(searcher2c.search())
#searcher3c = Searcher(Search_with_AC_from_CSP(crossword1))
#print(searcher3c.search())
#searcher4c = Searcher(Search_with_AC_from_CSP(crossword1d))
#print(searcher4c.search())
```

4.5 Solving CSPs using Stochastic Local Search

To run the demo, in folder "aipython", load "cspSLS.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3. Some of the queries require matplotlib.

The following code implements the two-stage choice (select one of the variables that are involved in the most constraints that are violated, then a value), the any-conflict algorithm (select a variable that participates in a violated constraint) and a random choice of variable, as well as a probabilistic mix of the three.

Given a CSP, the stochastic local searcher (*SLSearcher*) creates the data structures:

- *variables_to_select* is the set of all of the variables with domain-size greater than one. For a variable not in this set, we cannot pick another value from that variable.
- *var_to_constraints* maps from a variable into the set of constraints it is involved in. Note that the inverse mapping from constraints into variables is part of the definition of a constraint.

cspSLS.py — Stochastic Local Search for Solving CSPs

```
from cspProblem import CSP, Constraint
from searchProblem import Arc, Search_problem
from display import Displayable
import random
import heapq

class SLSearcher(Displayable):
    """A search problem directly from the CSP..

    A node is a variable:value dictionary"""
    def __init__(self, csp):
        self.csp = csp
        self.variables_to_select = {var for var in self.csp.variables
                                    if len(var.domain) > 1}
        # Create assignment and conflicts set
        self.current_assignment = None # this will trigger a random restart
        self.number_of_steps = 0 #number of steps after the initialization
```

restart creates a new total assignment, and constructs the set of conflicts (the constraints that are false in this assignment).

cspSLS.py — (continued)

```
    def restart(self):
        """creates a new total assignment and the conflict set
        """
        self.current_assignment = {var:random_choice(var.domain) for
                                   var in self.csp.variables}
        self.display(2,"Initial assignment",self.current_assignment)
        self.conflicts = set()
        for con in self.csp.constraints:
            if not con.holds(self.current_assignment):
                self.conflicts.add(con)
        self.display(2,"Number of conflicts",len(self.conflicts))
        self.variable_pq = None
```

The *search* method is the top-level searching algorithm. It can either be used to start the search or to continue searching. If there is no current assignment, it must create one. Note that, when counting steps, a restart is counted as one

step, which is not appropriate for CSPs with many variables, as it is a relatively expensive operation for these cases.

This method selects one of two implementations. The argument *pob_best* is the probability of selecting a best variable (one involving the most conflicts). When the value of *prob_best* is positive, the algorithm needs to maintain a priority queue of variables and the number of conflicts (using *search_with_var_pq*). If the probability of selecting a best variable is zero, it does not need to maintain this priority queue (as implemented in *search_with_any_conflict*).

The argument *prob_anycon* is the probability that the any-conflict strategy is used (which selects a variable at random that is in a conflict), assuming that it is not picking a best variable. Note that for the probability parameters, any value less that zero acts like probability zero and any value greater than 1 acts like probability 1. This means that when *prob_anycon* $= 1.0$, a best variable is chosen with probability *prob_best*, otherwise a variable in any conflict is chosen. A variable is chosen at random with probability $1 - \mathit{prob\_anycon} - \mathit{prob\_best}$ as long as that is positive.

This returns the number of steps needed to find a solution, or *None* if no solution is found. If there is a solution, it is in *self.current_assignment*.

cspSLS.py — (continued)

```
42      def search(self,max_steps, prob_best=0, prob_anycon=1.0):
43          """
44          returns the number of steps or None if these is no solution.
45          If there is a solution, it can be found in self.current_assignment
46
47          max_steps is the maximum number of steps it will try before giving
                up
48          prob_best is the probability that a best variable (one in most
                conflict) is selected
49          prob_anycon is the probability that a variable in any conflict is
                selected
50          (otherwise a variable is chosen at random)
51          """
52          if self.current_assignment is None:
53              self.restart()
54              self.number_of_steps += 1
55              if not self.conflicts:
56                  self.display(1,"Solution found:", self.current_assignment,
                        "after restart")
57                  return self.number_of_steps
58          if prob_best > 0: # we need to maintain a variable priority queue
59              return self.search_with_var_pq(max_steps, prob_best,
                    prob_anycon)
60          else:
61              return self.search_with_any_conflict(max_steps, prob_anycon)
```

Exercise 4.13 This does an initial random assignment but does not do any random restarts. Implement a searcher that takes in the maximum number of walk

steps (corresponding to existing *max_steps*) and the maximum number of restarts, and returns the total number of steps for the first solution found. (As in *search*, the solution found can be extracted from the variable *self.current_assignment*).

4.5.1 Any-conflict

If the probability of picking a best variable is zero, the implementation need to keeps track of which variables are in conflicts.

cspSLS.py — (continued)

```
63    def search_with_any_conflict(self, max_steps, prob_anycon=1.0):
64        """Searches with the any_conflict heuristic.
65        This relies on just maintaining the set of conflicts;
66        it does not maintain a priority queue
67        """
68        self.variable_pq = None # we are not maintaining the priority queue.
69                                # This ensures it is regenerated if
70                                #   we call search_with_var_pq.
71        for i in range(max_steps):
72            self.number_of_steps +=1
73            if  random.random() < prob_anycon:
74                con = random_choice(self.conflicts) # pick random conflict
75                var = random_choice(con.scope) # pick variable in conflict
76            else:
77                var = random_choice(self.variables_to_select)
78            if len(var.domain) > 1:
79                val = random_choice([val for val in var.domain
80                                     if val is not
                                         self.current_assignment[var]])
81                self.display(2,self.number_of_steps,":
                      Assigning",var,"=",val)
82                self.current_assignment[var]=val
83                for varcon in self.csp.var_to_const[var]:
84                    if varcon.holds(self.current_assignment):
85                        if varcon in self.conflicts:
86                            self.conflicts.remove(varcon)
87                    else:
88                        if varcon not in self.conflicts:
89                            self.conflicts.add(varcon)
90                self.display(2,"   Number of conflicts",len(self.conflicts))
91            if not self.conflicts:
92                self.display(1,"Solution found:", self.current_assignment,
93                             "in", self.number_of_steps,"steps")
94                return self.number_of_steps
95        self.display(1,"No solution in",self.number_of_steps,"steps",
96                    len(self.conflicts),"conflicts remain")
97        return None
```

Exercise 4.14 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value the reduces

the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

4.5.2 Two-Stage Choice

This is the top-level searching algorithm that maintains a priority queue of variables ordered by (the negative of) the number of conflicts, so that the variable with the most conflicts is selected first. If there is no current priority queue of variables, one is created.

The main complexity here is to maintain the priority queue. When a variable `var` is assigned a value `val`, for each constraint that has become satisfied or unsatisfied, each variable involved in the constraint need to have it's count updates. The change is recorded in the dictionary *var_differential*, which is used to update the priority queue (see Section 4.5.3).

cspSLS.py — (continued)

```
99      def search_with_var_pq(self,max_steps, prob_best=1.0, prob_anycon=1.0):
100         """search with a priority queue of variables.
101         This is used to select a variable with the most conflicts.
102         """
103         if not self.variable_pq:
104             self.create_pq()
105         pick_best_or_con = prob_best + prob_anycon
106         for i in range(max_steps):
107             self.number_of_steps +=1
108             randnum = random.random()
109             ## Pick a variable
110             if randnum < prob_best: # pick best variable
111                 var,oldval = self.variable_pq.top()
112             elif randnum < pick_best_or_con: # pick a variable in a conflict
113                 con = random_choice(self.conflicts)
114                 var = random_choice(con.scope)
115             else: #pick any variable that can be selected
116                 var = random_choice(self.variables_to_select)
117             if len(var.domain) > 1: # var has other values
118                 ## Pick a value
119                 val = random_choice([val for val in var.domain if val is not
120                                      self.current_assignment[var]])
121                 self.display(2,"Assigning",var,val)
122                 ## Update the priority queue
123                 var_differential = {}
124                 self.current_assignment[var]=val
125                 for varcon in self.csp.var_to_const[var]:
126                     self.display(3,"Checking",varcon)
127                     if varcon.holds(self.current_assignment):
128                         if varcon in self.conflicts: #was incons, now consis
129                             self.display(3,"Became consistent",varcon)
130                             self.conflicts.remove(varcon)
```

```
                        for v in varcon.scope: # v is in one fewer
                            conflicts
                            var_differential[v] =
                                var_differential.get(v,0)-1
                    else:
                        if varcon not in self.conflicts: # was consis, not now
                            self.display(3,"Became inconsistent",varcon)
                            self.conflicts.add(varcon)
                            for v in varcon.scope: # v is in one more
                                conflicts
                                var_differential[v] =
                                    var_differential.get(v,0)+1
                self.variable_pq.update_each_priority(var_differential)
                self.display(2,"Number of conflicts",len(self.conflicts))
            if not self.conflicts: # no conflicts, so solution found
                self.display(1,"Solution found:",
                    self.current_assignment,"in",
                            self.number_of_steps,"steps")
                return self.number_of_steps
        self.display(1,"No solution in",self.number_of_steps,"steps",
                    len(self.conflicts),"conflicts remain")
        return None
```

create_pq creates an updatable priority queue of the variables, ordered by the number of conflicts they participate in. The priority queue only includes variables in conflicts and the value of a variable is the *negative* of the number of conflicts the variable is in. This ensures that the priority queue, which picks the minimum value, picks a variable with the most conflicts.

cspSLS.py — (continued)

```
    def create_pq(self):
        """Create the variable to number-of-conflicts priority queue.
        This is needed to select the variable in the most conflicts.

        The value of a variable in the priority queue is the negative of the
        number of conflicts the variable appears in.
        """
        self.variable_pq = Updatable_priority_queue()
        var_to_number_conflicts = {}
        for con in self.conflicts:
            for var in con.scope:
                var_to_number_conflicts[var] =
                    var_to_number_conflicts.get(var,0)+1
        for var,num in var_to_number_conflicts.items():
            if num>0:
                self.variable_pq.add(var,-num)
```

cspSLS.py — (continued)

```
def random_choice(st):
    """selects a random element from set st.
```

```
167     It would be more efficient to convert to a tuple or list only once
168     (left as exercise)."""
169     return random.choice(tuple(st))
```

Exercise 4.15 This makes no attempt to find the best alternative value for a variable. Modify the code so that after selecting a variable it selects a value the reduces the number of conflicts by the most. Have a parameter that specifies the probability that the best value is chosen.

Exercise 4.16 These implementations always select a value for the variable selected that is different from its current value (if that is possible). Change the code so that it does not have this restriction (so it can leave the value the same). Would you expect this code to be faster? Does it work worse (or better)?

4.5.3 Updatable Priority Queues

An **updatable priority queue** is a priority queue, where key-value pairs can be stored, and the pair with the smallest key can be found and removed quickly, and where the values can be updated. This implementation follows the idea of `http://docs.python.org/3.9/library/heapq.html`, where the updated elements are marked as removed. This means that the priority queue can be used unmodified. However, this might be expensive if changes are more common than popping (as might happen if the probability of choosing the best is close to zero).

In this implementation, the equal values are sorted randomly. This is achieved by having the elements of the heap being $[val, rand, elt]$ triples, where the second element is a random number. Note that Python requires this to be a list, not a tuple, as the tuple cannot be modified.

cspSLS.py — (continued)

```
171 class Updatable_priority_queue(object):
172     """A priority queue where the values can be updated.
173     Elements with the same value are ordered randomly.
174 
175     This code is based on the ideas described in
176     http://docs.python.org/3.3/library/heapq.html
177     It could probably be done more efficiently by
178     shuffling the modified element in the heap.
179     """
180     def __init__(self):
181         self.pq = []  # priority queue of [val,rand,elt] triples
182         self.elt_map = {} # map from elt to [val,rand,elt] triple in pq
183         self.REMOVED = "*removed*" # a string that won't be a legal element
184         self.max_size=0
185 
186     def add(self,elt,val):
187         """adds elt to the priority queue with priority=val.
188         """
189         assert val <= 0,val
```

```
190         assert elt not in self.elt_map, elt
191         new_triple = [val, random.random(),elt]
192         heapq.heappush(self.pq, new_triple)
193         self.elt_map[elt] = new_triple
194 
195     def remove(self,elt):
196         """remove the element from the priority queue"""
197         if elt in self.elt_map:
198             self.elt_map[elt][2] = self.REMOVED
199             del self.elt_map[elt]
200 
201     def update_each_priority(self,update_dict):
202         """update values in the priority queue by subtracting the values in
203         update_dict from the priority of those elements in priority queue.
204         """
205         for elt,incr in update_dict.items():
206             if incr != 0:
207                 newval = self.elt_map.get(elt,[0])[0] - incr
208                 assert newval <= 0, f"{elt}:{newval+incr}-{incr}"
209                 self.remove(elt)
210                 if newval != 0:
211                     self.add(elt,newval)
212 
213     def pop(self):
214         """Removes and returns the (elt,value) pair with minimal value.
215         If the priority queue is empty, IndexError is raised.
216         """
217         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
218         triple = heapq.heappop(self.pq)
219         while triple[2] == self.REMOVED:
220             triple = heapq.heappop(self.pq)
221         del self.elt_map[triple[2]]
222         return triple[2], triple[0] # elt, value
223 
224     def top(self):
225         """Returns the (elt,value) pair with minimal value, without
                removing it.
226         If the priority queue is empty, IndexError is raised.
227         """
228         self.max_size = max(self.max_size, len(self.pq)) # keep statistics
229         triple = self.pq[0]
230         while triple[2] == self.REMOVED:
231             heapq.heappop(self.pq)
232             triple = self.pq[0]
233         return triple[2], triple[0] # elt, value
234 
235     def empty(self):
236         """returns True iff the priority queue is empty"""
237         return all(triple[2] == self.REMOVED for triple in self.pq)
```

4.5.4 Plotting Run-Time Distributions

Runtime_distribution uses matplotlib to plot run time distributions. Here the run time is a misnomer as we are only plotting the number of steps, not the time. Computing the run time is non-trivial as many of the runs have a very short run time. To compute the time accurately would require running the same code, with the same random seed, multiple times to get a good estimate of the run time. This is left as an exercise.

cspSLS.py — (continued)

```
239 import matplotlib.pyplot as plt
240 # plt.style.use('grayscale')
241
242 class Runtime_distribution(object):
243     def __init__(self, csp, xscale='log'):
244         """Sets up plotting for csp
245         xscale is either 'linear' or 'log'
246         """
247         self.csp = csp
248         plt.ion()
249         plt.xlabel("Number of Steps")
250         plt.ylabel("Cumulative Number of Runs")
251         plt.xscale(xscale) # Makes a 'log' or 'linear' scale
252
253     def plot_runs(self,num_runs=100,max_steps=1000, prob_best=1.0,
            prob_anycon=1.0):
254         """Plots num_runs of SLS for the given settings.
255         """
256         stats = []
257         SLSearcher.max_display_level, temp_mdl = 0,
                SLSearcher.max_display_level # no display
258         for i in range(num_runs):
259             searcher = SLSearcher(self.csp)
260             num_steps = searcher.search(max_steps, prob_best, prob_anycon)
261             if num_steps:
262                 stats.append(num_steps)
263         stats.sort()
264         if prob_best >= 1.0:
265             label = "P(best)=1.0"
266         else:
267             p_ac = min(prob_anycon, 1-prob_best)
268             label = "P(best)=%.2f, P(ac)=%.2f" % (prob_best, p_ac)
269         plt.plot(stats,range(len(stats)),label=label)
270         plt.legend(loc="upper left")
271         SLSearcher.max_display_level= temp_mdl #restore display
```

Figure 4.8 gives run-time distributions for 3 algorithms. It is also useful to compare the distributions of different runs of the same algorithms and settings.

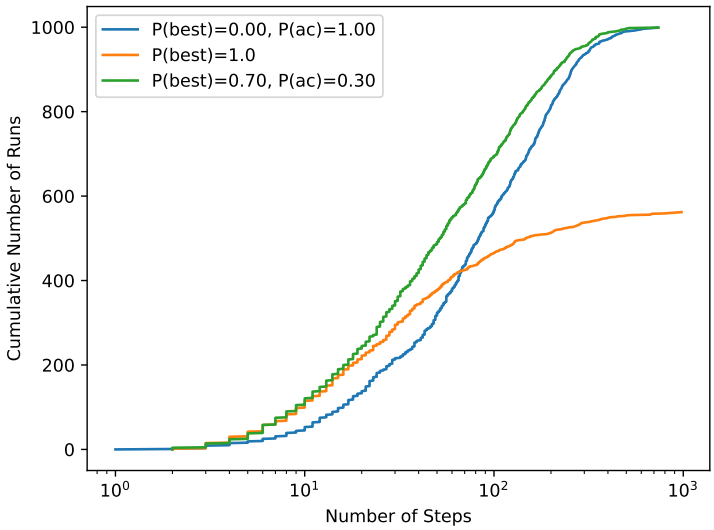


Figure 4.8: Run-time distributions for three algorithms on *csp*2.

4.5.5 Testing

cspSLS.py — (continued)

```
273 from cspExamples import test_csp
274 def sls_solver(csp,prob_best=0.7):
275     """stochastic local searcher (prob_best=0.7)"""
276     se0 = SLSearcher(csp)
277     se0.search(1000,prob_best)
278     return se0.current_assignment
279 def any_conflict_solver(csp):
280     """stochastic local searcher (any-conflict)"""
281     return sls_solver(csp,0)
282
283 if __name__ == "__main__":
284     test_csp(sls_solver)
285     test_csp(any_conflict_solver)
286
287 from cspExamples import csp1, csp1s, csp2, crossword1, crossword1d
288
289 ## Test Solving CSPs with Search:
290 #se1 = SLSearcher(csp1); print(se1.search(100))
291 #se2 = SLSearcher(csp2); print(se2.search(1000,1.0)) # greedy
292 #se2 = SLSearcher(csp2); print(se2.search(1000,0)) # any_conflict
293 #se2 = SLSearcher(csp2); print(se2.search(1000,0.7)) # 70% greedy; 30%
        any_conflict
294 #SLSearcher.max_display_level=2 #more detailed display
```

```
#se3 = SLSearcher(crossword1); print(se3.search(100),0.7)
#p = Runtime_distribution(csp2)
#p.plot_runs(1000,1000,0) # any_conflict
#p.plot_runs(1000,1000,1.0) # greedy
#p.plot_runs(1000,1000,0.7) # 70% greedy; 30% any_conflict
```

Exercise 4.17 Modify this to plot the run time, instead of the number of steps. To measure run time use *timeit* (`https://docs.python.org/3.9/library/timeit.html`). Small run times are inaccurate, so timeit can run the same code multiple times. Stochastic local algorithms give different run times each time called. To make the timing meaningful, you need to make sure the random seed is the same for each repeated call (see `random.getstate` and `random.setstate` in `https://docs.python.org/3.9/library/random.html`). Because the run time for different seeds can vary a great deal, for each seed, you should start with 1 iteration and multiplying it by, say 10, until the time is greater than 0.2 seconds. Make sure you plot the average time for each run. Before you start, try to estimate the total run time, so you will be able to tell if there is a problem with the algorithm stopping.

4.6 Discrete Optimization

A `SoftConstraint` is a constraint, but where the condition is a real-valued function. Because the definition of the constraint class did not force the condition to be Boolean, you can use the `Constraint` class for soft constraints too.

cspSoft.py — Representations of Soft Constraints

```
from cspProblem import Variable, Constraint, CSP
class SoftConstraint(Constraint):
    """A Constraint consists of
    * scope: a tuple of variables
    * function: a real-valued function that can applied to a tuple of values
    * string: a string for printing the constraints. All of the strings
        must be unique.
    for the variables
    """
    def __init__(self, scope, function, string=None, position=None):
        Constraint.__init__(self, scope, function, string, position)

    def value(self,assignment):
        return self.holds(assignment)
```

cspSoft.py — (continued)

```
A = Variable('A', {1,2}, position=(0.2,0.9))
B = Variable('B', {1,2,3}, position=(0.8,0.9))
C = Variable('C', {1,2}, position=(0.5,0.5))
D = Variable('D', {1,2}, position=(0.8,0.1))

def c1fun(a,b):
    if a==1: return (5 if b==1 else 2)
    else: return (0 if b==1 else 4 if b==2 else 3)
```

```
c1 = SoftConstraint([A,B],c1fun,"c1")
def c2fun(b,c):
    if b==1: return (5 if c==1 else 2)
    elif b==2: return (0 if c==1 else 4)
    else: return (2 if c==1 else 0)
c2 = SoftConstraint([B,C],c2fun,"c2")
def c3fun(b,d):
    if b==1: return (3 if d==1 else 0)
    elif b==2: return 2
    else: return (2 if d==1 else 4)
c3 = SoftConstraint([B,D],c3fun,"c3")

def penalty_if_same(pen):
    "returns a function that gives a penalty of pen if the arguments are
        the same"
    return lambda x,y: (pen if (x==y) else 0)

c4 = SoftConstraint([C,A],penalty_if_same(3),"c4")

scsp1 = CSP("scsp1", {A,B,C,D}, [c1,c2,c3,c4])

### The second soft CSP has an extra variable, and 2 constraints
E = Variable('E', {1,2}, position=(0.1,0.1))

c5 = SoftConstraint([C,E],penalty_if_same(3),"c5")
c6 = SoftConstraint([D,E],penalty_if_same(2),"c6")
scsp2 = CSP("scsp1", {A,B,C,D,E}, [c1,c2,c3,c4,c5,c6])
```

4.6.1 Branch-and-bound Search

Here we specialize the branch-and-bound algorithm (Section 3.3 on page 51) to solve soft CSP problems.

cspSoft.py — (continued)

```
from display import Displayable, visualize
import math

class DF_branch_and_bound_opt(Displayable):
    """returns a branch and bound searcher for a problem.
    An optimal assignment with cost less than bound can be found by calling
        search()
    """
    def __init__(self, csp, bound=math.inf):
        """creates a searcher than can be used with search() to find an
            optimal path.
        bound gives the initial bound. By default this is infinite -
            meaning there
        is no initial pruning due to depth bound
        """
        super().__init__()
```

```
73          self.csp = csp
74          self.best_asst = None
75          self.bound = bound
76
77      def optimize(self):
78          """returns an optimal solution to a problem with cost less than
                bound.
79          returns None if there is no solution with cost less than bound."""
80          self.num_expanded=0
81          self.cbsearch({}, 0, self.csp.constraints)
82          self.display(1,"Number of paths expanded:",self.num_expanded)
83          return self.best_asst, self.bound
84
85      def cbsearch(self, asst, cost, constraints):
86          """finds the optimal solution that extends path and is less the
                bound"""
87          self.display(2,"cbsearch:",asst,cost,constraints)
88          can_eval = [c for c in constraints if c.can_evaluate(asst)]
89          rem_cons = [c for c in constraints if c not in can_eval]
90          newcost = cost + sum(c.value(asst) for c in can_eval)
91          self.display(2,"Evaluaing:",can_eval,"cost:",newcost)
92          if newcost < self.bound:
93              self.num_expanded += 1
94              if rem_cons==[]:
95                  self.best_asst = asst
96                  self.bound = newcost
97                  self.display(1,"New best assignment:",asst," cost:",newcost)
98              else:
99                  var = next(var for var in self.csp.variables if var not in
                        asst)
100                 for val in var.domain:
101                     self.cbsearch({var:val}|asst, newcost, rem_cons)
102
103 # bnb = DF_branch_and_bound_opt(scsp1)
104 # bnb.max_display_level=3 # show more detail
105 # bnb.optimize()
```

Exercise 4.18 Change the stochastic-local search algorithms to work for soft constraints. Hint: The analog of a conflict is a soft constraint that is not at its lowest value. Instead of the number of constraints violated, consider how much a change in a variable affects the objective function. Instead of returning a solution, return the best assignment found.

Chapter 5

Propositions and Inference

5.1 Representing Knowledge Bases

A clause consists of a head (an atom) and a body. A body is represented as a list of atoms. Atoms are represented as strings.

logicProblem.py — Representations Logics

```
11 class Clause(object):
12     """A definite clause"""
13
14     def __init__(self,head,body=[]):
15         """clause with atom head and lost of atoms body"""
16         self.head=head
17         self.body = body
18
19     def __repr__(self):
20         """returns the string representation of a clause.
21         """
22         if self.body:
23             return self.head + " <- " + " & ".join(str(a) for a in
                   self.body) + "\n"
24         else:
25             return self.head + "."
```

An askable atom can be asked of the user. The user can respond in English or French or just with a "y".

logicProblem.py — (continued)

```
27 class Askable(object):
28     """An askable atom"""
29
30     def __init__(self,atom):
```

```
31          """clause with atom head and lost of atoms body"""
32          self.atom=atom
33
34      def __str__(self):
35          """returns the string representation of a clause."""
36          return "askable " + self.atom + "."
37
38  def yes(ans):
39      """returns true if the answer is yes in some form"""
40      return ans.lower() in ['yes', 'yes.', 'oui', 'oui.', 'y', 'y.'] #
            bilingual
```

A knowledge base is a list of clauses and askables. In order to make top-down inference faster, this creates a dictionary that maps each atoms into the set of clauses with that atom in the head.

logicProblem.py — (continued)

```
42  from display import Displayable
43
44  class KB(Displayable):
45      """A knowledge base consists of a set of clauses.
46      This also creates a dictionary to give fast access to the clauses with
            an atom in head.
47      """
48      def __init__(self, statements=[]):
49          self.statements = statements
50          self.clauses = [c for c in statements if isinstance(c, Clause)]
51          self.askables = [c.atom for c in statements if isinstance(c,
                Askable)]
52          self.atom_to_clauses = {} # dictionary giving clauses with atom as
                head
53          for c in self.clauses:
54              self.add_clause(c)
55
56      def add_clause(self, c):
57          if c.head in self.atom_to_clauses:
58              self.atom_to_clauses[c.head].add(c)
59          else:
60              self.atom_to_clauses[c.head] = {c}
61
62      def clauses_for_atom(self,a):
63          """returns set of clauses with atom a as the head"""
64          if a in self.atom_to_clauses:
65              return self.atom_to_clauses[a]
66          else:
67              return set()
68
69      def __str__(self):
70          """returns a string representation of this knowledge base.
71          """
72          return '\n'.join([str(c) for c in self.statements])
```

Here is a trivial example (I think therefore I am) using in the unit tests:

logicProblem.py — (continued)

```
triv_KB = KB([
    Clause('i_am', ['i_think']),
    Clause('i_think'),
    Clause('i_smell', ['i_exist'])
    ])
```

Here is a representation of the electrical domain of the textbook:

logicProblem.py — (continued)

```
elect = KB([
    Clause('light_l1'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','live_w1']),
    Clause('live_w0', ['down_s2','live_w2']),
    Clause('live_w1', ['up_s1', 'live_w3']),
    Clause('live_w2', ['down_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2')
    ])

# print(kb)
```

The following knowledge base is false of the intended interpretation. One of the clauses is wrong; can you see which one? We will show how to debug it.

logicProblem.py — (continued)

```
elect_bug = KB([
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
```

```
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('light_l1'),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','live_w1']),
    Clause('live_w0', ['down_s2','live_w2']),
    Clause('live_w1', ['up_s3', 'live_w3']),
    Clause('live_w2', ['down_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('light_l2'),
    Clause('ok_l1'),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('ok_l2'),
    Clause('ok_cb1'),
    Clause('ok_cb2'),
    Clause('live_outside'),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2')
    ])

# print(kb)
```

5.2 Bottom-up Proofs (with askables)

fixed_point computes the fixed point of the knowledge base *kb*.

logicBottomUp.py — Bottom-up Proof Procedure for Definite Clauses

```
from logicProblem import yes

def fixed_point(kb):
    """Returns the fixed point of knowledge base kb.
    """
    fp = ask_askables(kb)
    added = True
    while added:
        added = False  # added is true when an atom was added to fp this
            iteration
        for c in kb.clauses:
            if c.head not in fp and all(b in fp for b in c.body):
                fp.add(c.head)
                added = True
                kb.display(2,c.head,"added to fp due to clause",c)
    return fp

def ask_askables(kb):
    return {at for at in kb.askables if yes(input("Is "+at+" true? "))}
```

The following provides a trivial **unit test**, by default using the knowledge base `triv_KB`:

logicBottomUp.py — (continued)

```
from logicProblem import triv_KB
def test(kb=triv_KB, fixedpt = {'i_am','i_think'}):
    fp = fixed_point(kb)
    assert fp == fixedpt, f"kb gave result {fp}"
    print("Passed unit test")
if __name__ == "__main__":
    test()

from logicProblem import elect
# elect.max_display_level=3 # give detailed trace
# fixed_point(elect)
```

Exercise 5.1 It is not very user-friendly to ask all of the askables up-front. Implement ask-the-user so that questions are only asked if useful, and are not re-asked. For example, if there is a clause $h \leftarrow a \wedge b \wedge c \wedge d \wedge e$, where c and e are askable, c and e only need to be asked if a, b, d are all in fp and they have not been asked before. Askable e only needs to be asked if the user says "yes" to c. Askable c doesn't need to be asked if the user previously replied "no" to e.

This form of ask-the-user can ask a different set of questions than the top-down interpreter that asks questions when encountered. Give an example where they ask different questions (neither set of questions asked is a subset of the other).

Exercise 5.2 This algorithm runs in time $O(n^2)$, where n is the number of clauses, for a bounded number of elements in the body; each iteration goes through each of the clauses, and in the worst case, it will do an iteration for each clause. It is possible to implement this in time $O(n)$ time by creating an index that maps an atom to the set of clauses with that atom in the body. Implement this. What is its

complexity as a function of n and b, the maximum number of atoms in the body of a clause?

Exercise 5.3 It is possible to be asymptotically more efficient (in terms of the number of elements in a body) than the method in the previous question by noticing that each element of the body of clause only needs to be checked once. For example, the clause $a \leftarrow b \wedge c \wedge d$, needs only be considered when b is added to fp. Once b is added to fp, if c is already in pf, we know that a can be added as soon as d is added. Implement this. What is its complexity as a function of n and b, the maximum number of atoms in the body of a clause?

5.3 Top-down Proofs (with askables)

$prove(kb, goal)$ is used to prove $goal$ from a knowledge base, kb, where a $goal$ is a list of atoms. It returns *True* if $kb \vdash goal$. The *indent* is used when displaying the code (and doesn't need to be called initially with a non-default value).

logicTopDown.py — Top-down Proof Procedure for Definite Clauses

```
from logicProblem import yes

def prove(kb, ans_body, indent=""):
    """returns True if kb |- ans_body
    ans_body is a list of atoms to be proved
    """
    kb.display(2,indent,'yes <-',' & '.join(ans_body))
    if ans_body:
        selected = ans_body[0] # select first atom from ans_body
        if selected in kb.askables:
            return (yes(input("Is "+selected+" true? "))
                    and  prove(kb,ans_body[1:],indent+" "))
        else:
            return any(prove(kb,cl.body+ans_body[1:],indent+" ")
                       for cl in kb.clauses_for_atom(selected))
    else:
        return True   # empty body is true
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicTopDown.py — (continued)

```
from logicProblem import triv_KB
def test():
    a1 = prove(triv_KB,['i_am'])
    assert a1, f"triv_KB proving i_am gave {a1}"
    a2 = prove(triv_KB,['i_smell'])
    assert not a2, f"triv_KB proving i_smell gave {a2}"
    print("Passed unit tests")
if __name__ == "__main__":
    test()
# try
from logicProblem import elect
```

```
# elect.max_display_level=3 # give detailed trace
# prove(elect,['live_w6'])
# prove(elect,['lit_l1'])
```

Exercise 5.4 This code can re-ask a question multiple times. Implement this code so that it only asks a question once and remembers the answer. Also implement a function to forget the answers.

Exercise 5.5 What search method is this using? Implement the search interface so that it can use A^* or other searching methods. Define an admissible heuristic that is not always 0.

5.4 Debugging and Explanation

Here we modify the top-down procedure to build a proof tree than can be traversed for explanation and debugging.

`prove_atom(kb,atom)` returns a proof for *atom* from a knowledge base *kb*, where a proof is a pair of the atom and the proofs for the elements of the body of the clause used to prove the atom. `prove_body(kb,body)` returns a list of proofs for list *body* from a knowledge base, *kb*. The *indent* is used when displaying the code (and doesn't need to have a non-default value).

logicExplain.py — Explaining Proof Procedure for Definite Clauses

```
from logicProblem import yes # for asking the user

def prove_atom(kb, atom, indent=""):
    """returns a pair (atom,proofs) where proofs is the list of proofs
       of the elements of a body of a clause used to prove atom.
    """
    kb.display(2,indent,'proving',atom)
    if atom in kb.askables:
        if yes(input("Is "+atom+" true? ")):
            return (atom,"answered")
        else:
            return "fail"
    else:
        for cl in kb.clauses_for_atom(atom):
            kb.display(2,indent,"trying",atom,'<-',' & '.join(cl.body))
            pr_body = prove_body(kb, cl.body, indent)
            if pr_body != "fail":
                return (atom, pr_body)
        return "fail"

def prove_body(kb, ans_body, indent=""):
    """returns proof tree if kb |- ans_body or "fail" if there is no proof
    ans_body is a list of atoms in a body to be proved
    """
    proofs = []
    for atom in ans_body:
```

```
37          proof_at = prove_atom(kb, atom, indent+" ")
38          if proof_at == "fail":
39              return "fail" # fail if any proof fails
40          else:
41              proofs.append(proof_at)
42      return proofs
```

The following provides a simple **unit test** that is hard wired for `triv_KB`:

logicExplain.py — (continued)

```
44  from logicProblem import triv_KB
45  def test():
46      a1 = prove_atom(triv_KB,'i_am')
47      assert a1, f"triv_KB proving i_am gave {a1}"
48      a2 = prove_atom(triv_KB,'i_smell')
49      assert a2=="fail", "triv_KB proving i_smell gave {a2}"
50      print("Passed unit tests")
51  if __name__ == "__main__":
52      test()
53  # try
54  from logicProblem import elect, elect_bug
55  # elect.max_display_level=3 # give detailed trace
56  # prove_atom(elect, 'live_w6')
57  # prove_atom(elect, 'lit_l1')
```

The `interact(kb)` provides an interactive interface to explore proofs for knowledge base `kb`. The user can ask to prove atoms and can ask `how` an atom was proved.

To ask `how`, there must be a current atom for which there is a proof. This starts as the atom asked. When the user asks "how n" the current atom becomes the n-th element of the body of the clause used to prove the (previous) current atom. The command "up" makes the current atom the atom in the head of the rule containing the (previous) current atom. Thus "how n" moves down the proof tree and "up" moves up the proof tree, allowing the user to explore the full proof.

logicExplain.py — (continued)

```
59  helptext = """Commands are:
60  ask atom    ask is there is a proof for atom (atom should not be in quotes)
61  how         show the clause that was used to prove atom
62  how n       show the clause used to prove the nth element of the body
63  up          go back up proof tree to explore other parts of the proof tree
64  kb          print the knowledge base
65  quit        quit this interaction (and go back to Python)
66  help        print this text
67  """
68
69  def interact(kb):
70      going = True
71      ups = []   # stack for going up
```

```
72      proof="fail" # there is no proof to start
73      while going:
74          inp = input("logicExplain: ")
75          inps = inp.split(" ")
76          try:
77              command = inps[0]
78              if command == "quit":
79                  going = False
80              elif command == "ask":
81                  proof = prove_atom(kb, inps[1])
82                  if proof == "fail":
83                      print("fail")
84                  else:
85                      print("yes")
86              elif command == "how":
87                  if proof=="fail":
88                      print("there is no proof")
89                  elif len(inps)==1:
90                     print_rule(proof)
91                  else:
92                      try:
93                          ups.append(proof)
94                          proof = proof[1][int(inps[1])] #nth argument of rule
95                          print_rule(proof)
96                      except:
97                          print('In "how n", n must be a number between 0
                                and',len(proof[1])-1,"inclusive.")
98              elif command == "up":
99                  if ups:
100                     proof = ups.pop()
101                 else:
102                     print("No rule to go up to.")
103                 print_rule(proof)
104             elif command == "kb":
105                  print(kb)
106             elif command == "help":
107                 print(helptext)
108             else:
109                 print("unknown command:", inp)
110                 print("use help for help")
111         except:
112             print("unknown command:", inp)
113             print("use help for help")
114
115 def print_rule(proof):
116     (head,body) = proof
117     if body == "answered":
118         print(head,"was answered yes")
119     elif body == []:
120             print(head,"is a fact")
```

```
121     else:
122             print(head,"<-")
123             for i,a in enumerate(body):
124                 print(i,":",a[0])
125 
126 # try
127 # interact(elect)
128 # Which clause is wrong in elect_bug? Try:
129 # interact(elect_bug)
130 # logicExplain: ask lit_l1
```

The following shows an interaction for the knowledge base `elect`:

```
>>> interact(elect)
logicExplain: ask lit_l1
Is up_s2 true? no
Is down_s2 true? yes
Is down_s1 true? yes
yes
logicExplain: how
lit_l1 <-
0 : light_l1
1 : live_l1
2 : ok_l1
logicExplain: how 1
live_l1 <-
0 : live_w0
logicExplain: how 0
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 0
down_s2 was answered yes
logicExplain: up
live_w0 <-
0 : down_s2
1 : live_w2
logicExplain: how 1
live_w2 <-
0 : down_s1
1 : live_w3
logicExplain: quit
>>>
```

Exercise 5.6 The above code only ever explores one proof – the first proof found. Change the code to enumerate the proof trees (by returning a list all proof trees, or preferably using yield). Add the command "retry" to the user interface to try another proof.

5.5 Assumables

Atom a can be made assumable by including $Assumable(a)$ in the knowledge base. A knowledge base that can include assumables is declared with *KBA*.

logicAssumables.py — Definite clauses with assumables

```
from logicProblem import Clause, Askable, KB, yes

class Assumable(object):
    """An askable atom"""

    def __init__(self,atom):
        """clause with atom head and lost of atoms body"""
        self.atom = atom

    def __str__(self):
        """returns the string representation of a clause.
        """
        return "assumable " + self.atom + "."

class KBA(KB):
    """A knowledge base that can include assumables"""
    def __init__(self,statements):
        self.assumables = [c.atom for c in statements if isinstance(c,
            Assumable)]
        KB.__init__(self,statements)
```

The top-down Horn clause interpreter, *prove_all_ass* returns a list of the sets of assumables that imply *ans_body*. This list will contain all of the minimal sets of assumables, but can also find non-minimal sets, and repeated sets, if they can be generated with separate proofs. The set *assumed* is the set of assumables already assumed.

logicAssumables.py — (continued)

```
    def prove_all_ass(self, ans_body, assumed=set()):
        """returns a list of sets of assumables that extends assumed
        to imply ans_body from self.
        ans_body is a list of atoms (it is the body of the answer clause).
        assumed is a set of assumables already assumed
        """
        if ans_body:
            selected = ans_body[0] # select first atom from ans_body
            if selected in self.askables:
                if yes(input("Is "+selected+" true? ")):
                    return self.prove_all_ass(ans_body[1:],assumed)
                else:
                    return []  # no answers
            elif selected in self.assumables:
                return self.prove_all_ass(ans_body[1:],assumed|{selected})
            else:
                return [ass
```

```
                    for cl in self.clauses_for_atom(selected)
                    for ass in
                        self.prove_all_ass(cl.body+ans_body[1:],assumed)
                      ]  # union of answers for each clause with
                         head=selected
        else:             # empty body
            return [assumed]  # one answer

    def conflicts(self):
        """returns a list of minimal conflicts"""
        return minsets(self.prove_all_ass(['false']))
```

Given a list of sets, *minsets* returns a list of the minimal sets in the list. For example, *minsets*($[\{2,3,4\},\{2,3\},\{6,2,3\},\{2,3\},\{2,4,5\}]$) returns $[\{2,3\},\{2,4,5\}]$.

logicAssumables.py — (continued)

```
def minsets(ls):
    """ls is a list of sets
    returns a list of minimal sets in ls
    """
    ans = []    # elements known to be minimal
    for c in ls:
        if not any(c1<c for c1 in ls) and not any(c1 <= c for c1 in ans):
            ans.append(c)
    return ans

# minsets([{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

Warning: *minsets* works for a list of sets or for a set of (frozen) sets, but it does not work for a generator of sets (because `ls` is references in the loop). For example, try to predict and then test:

```
minsets(e for e in [{2, 3, 4}, {2, 3}, {6, 2, 3}, {2, 3}, {2, 4, 5}])
```

The diagnoses can be constructed from the (minimal) conflicts as follows. This also works if there are non-minimal conflicts, but is not as efficient.

logicAssumables.py — (continued)

```
def diagnoses(cons):
    """cons is a list of (minimal) conflicts.
    returns a list of diagnoses."""
    if cons == []:
        return [set()]
    else:
        return minsets([{e}|d)            # | is set union
                    for e in cons[0]
                    for d in diagnoses(cons[1:])])
```

Test cases:

logicAssumables.py — (continued)

```
electa = KBA([
```

```
    Clause('light_l1'),
    Clause('light_l2'),
    Assumable('ok_l1'),
    Assumable('ok_l2'),
    Assumable('ok_s1'),
    Assumable('ok_s2'),
    Assumable('ok_s3'),
    Assumable('ok_cb1'),
    Assumable('ok_cb2'),
    Assumable('live_outside'),
    Clause('live_l1', ['live_w0']),
    Clause('live_w0', ['up_s2','ok_s2','live_w1']),
    Clause('live_w0', ['down_s2','ok_s2','live_w2']),
    Clause('live_w1', ['up_s1', 'ok_s1', 'live_w3']),
    Clause('live_w2', ['down_s1', 'ok_s1','live_w3' ]),
    Clause('live_l2', ['live_w4']),
    Clause('live_w4', ['up_s3','ok_s3','live_w3' ]),
    Clause('live_p_1', ['live_w3']),
    Clause('live_w3', ['live_w5', 'ok_cb1']),
    Clause('live_p_2', ['live_w6']),
    Clause('live_w6', ['live_w5', 'ok_cb2']),
    Clause('live_w5', ['live_outside']),
    Clause('lit_l1', ['light_l1', 'live_l1', 'ok_l1']),
    Clause('lit_l2', ['light_l2', 'live_l2', 'ok_l2']),
    Askable('up_s1'),
    Askable('down_s1'),
    Askable('up_s2'),
    Askable('down_s2'),
    Askable('up_s3'),
    Askable('down_s2'),
    Askable('dark_l1'),
    Askable('dark_l2'),
    Clause('false', ['dark_l1', 'lit_l1']),
    Clause('false', ['dark_l2', 'lit_l2'])
    ])
# electa.prove_all_ass(['false'])
# cs=electa.conflicts()
# print(cs)
# diagnoses(cs)      # diagnoses from conflicts
```

Exercise 5.7 To implement a version of *conflicts* that never generates non-minimal conflicts, modify *prove_all_ass* to implement iterative deepening on the number of assumables used in a proof, and prune any set of assumables that is a superset of a conflict.

Exercise 5.8 Implement *explanations*(*self*, *body*), where *body* is a list of atoms, that returns the a list of the minimal explanations of the body. This does not require modification of *prove_all_ass*.

Exercise 5.9 Implement *explanations*, as in the previous question, so that it never generates non-minimal explanations. Hint: modify *prove_all_ass* to implement iter-

ative deepening on the number of assumptions, generating conflicts and explanations together, and pruning as early as possible.

5.6 Negation-as-failure

The negation af an atom a is written as `Not(a)` in a body.

logicNegation.py — Propositional negation-as-failure

```
from logicProblem import KB, Clause, Askable, yes

class Not(object):
    def __init__(self, atom):
        self.theatom = atom

    def atom(self):
        return self.theatom

    def __repr__(self):
        return f"Not({self.theatom})"
```

Prove with negation-as-failure (`prove_naf`) is like prove, but with the extra case to cover `Not`:

logicNegation.py — (continued)

```
def prove_naf(kb, ans_body, indent=""):
    """ prove with negation-as-failure and askables
    returns True if kb |- ans_body
    ans_body is a list of atoms to be proved
    """
    kb.display(2,indent,'yes <-',' & '.join(str(e) for e in ans_body))
    if ans_body:
        selected = ans_body[0] # select first atom from ans_body
        if isinstance(selected, Not):
            kb.display(2,indent,f"proving {selected.atom()}")
            if prove_naf(kb, [selected.atom()], indent):
                kb.display(2,indent,f"{selected.atom()} succeeded so
                    Not({selected.atom()}) fails")
                return False
            else:
                kb.display(2,indent,f"{selected.atom()} fails so
                    Not({selected.atom()}) succeeds")
                return prove_naf(kb, ans_body[1:],indent+" ")
        if selected in kb.askables:
            return (yes(input("Is "+selected+" true? "))
                    and  prove_naf(kb,ans_body[1:],indent+" "))
        else:
            return any(prove_naf(kb,cl.body+ans_body[1:],indent+" ")
                       for cl in kb.clauses_for_atom(selected))
    else:
        return True  # empty body is true
```

Test cases:

logicNegation.py — (continued)

```
triv_KB_naf = KB([
    Clause('i_am', ['i_think']),
    Clause('i_think'),
    Clause('i_smell', ['i_am', Not('dead')]),
    Clause('i_bad', ['i_am', Not('i_think')])
    ])

triv_KB_naf.max_display_level = 4
def test():
    a1 = prove_naf(triv_KB_naf,['i_smell'])
    assert a1, f"triv_KB_naf proving i_smell gave {a1}"
    a2 = prove_naf(triv_KB_naf,['i_bad'])
    assert not a2, f"triv_KB_naf proving i_bad gave {a2}"
    print("Passed unit tests")
if __name__ == "__main__":
    test()
```

Default reasoning about beaches at resorts (Example 5.28 of Poole and Mackworth [2023]):

logicNegation.py — (continued)

```
beach_KB = KB([
   Clause('away_from_beach', [Not('on_beach')]),
   Clause('beach_access', ['on_beach', Not('ab_beach_access')]),
   Clause('swim_at_beach', ['beach_access', Not('ab_swim_at_beach')]),
   Clause('ab_swim_at_beach', ['enclosed_bay', 'big_city',
       Not('ab_no_swimming_near_city')]),
   Clause('ab_no_swimming_near_city', ['in_BC', Not('ab_BC_beaches')])
    ])

# prove_naf(beach_KB, ['away_from_beach'])
# prove_naf(beach_KB, ['beach_access'])
# beach_KB.add_clause(Clause('on_beach',[]))
# prove_naf(beach_KB, ['away_from_beach'])
# prove_naf(beach_KB, ['swim_at_beach'])
# beach_KB.add_clause(Clause('enclosed_bay',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
# beach_KB.add_clause(Clause('big_city',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
# beach_KB.add_clause(Clause('in_BC',[]))
# prove_naf(beach_KB, ['swim_at_beach'])
```

Chapter 6

Deterministic Planning

6.1 Representing Actions and Planning Problems

The STRIPS representation of an action consists of:

- the name of the action
- preconditions: a dictionary of *feature:value* pairs that specifies that the feature must have this value for the action to be possible
- effects: a dictionary of *feature:value* pairs that are made true by this action. In particular, a feature in the dictionary has the corresponding value (and not its previous value) after the action, and a feature not in the dictionary keeps its old value.

stripsProblem.py — STRIPS Representations of Actions

```
11 class Strips(object):
12     def __init__(self, name, preconds, effects, cost=1):
13         """
14         defines the STRIPS representation for an action:
15         * name is the name of the action
16         * preconds, the preconditions, is feature:value dictionary that
               must hold
17         for the action to be carried out
18         * effects is a feature:value map that this action makes
19         true. The action changes the value of any feature specified
20         here, and leaves other features unchanged.
21         * cost is the cost of the action
22         """
```

```
23          self.name = name
24          self.preconds = preconds
25          self.effects = effects
26          self.cost = cost
27
28      def __repr__(self):
29          return self.name
```

A STRIPS domain consists of:

- A set of actions.
- A dictionary that maps each feature into a set of possible values for the feature.
- A list of the actions

stripsProblem.py — (continued)

```
31  class STRIPS_domain(object):
32      def __init__(self, feature_domain_dict, actions):
33          """Problem domain
34          feature_domain_dict is a feature:domain dictionary,
35                 mapping each feature to its domain
36          actions
37          """
38          self.feature_domain_dict = feature_domain_dict
39          self.actions = actions
```

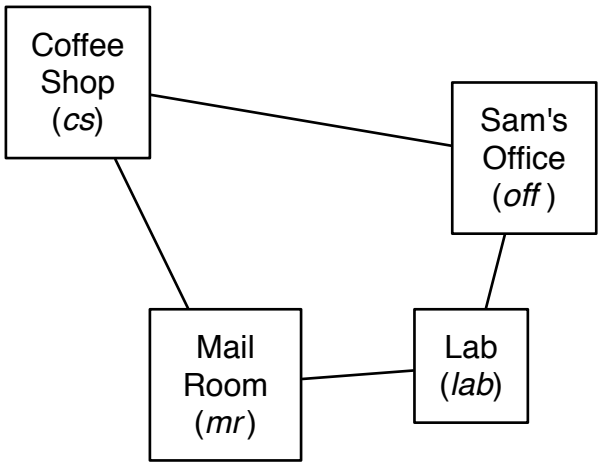
A planning problem consists of a planning domain, an initial state, and a goal. The goal does not need to fully specify the final state.

stripsProblem.py — (continued)

```
41  class Planning_problem(object):
42      def __init__(self, prob_domain, initial_state, goal):
43          """
44          a planning problem consists of
45          * a planning domain
46          * the initial state
47          * a goal
48          """
49          self.prob_domain = prob_domain
50          self.initial_state = initial_state
51          self.goal = goal
```

6.1.1 Robot Delivery Domain

The following specifies the robot delivery domain of Section 6.1, shown in Figure 6.1.



Features to describe states

RLoc – Rob's location
RHC – Rob has coffee
SWC – Sam wants coffee
MW – Mail is waiting
RHM – Rob has mail

Actions

mc – move clockwise
mcc – move counterclockwise
puc – pickup coffee
dc – deliver coffee
pum – pickup mail
dm – deliver mail

Figure 6.1: Robot Delivery Domain

stripsProblem.py — (continued)

```
boolean = {True, False}
delivery_domain = STRIPS_domain(
   {'RLoc':{'cs', 'off', 'lab', 'mr'}, 'RHC':boolean, 'SWC':boolean,
    'MW':boolean, 'RHM':boolean},       #feature:values dictionary
   { Strips('mc_cs', {'RLoc':'cs'}, {'RLoc':'off'}),
    Strips('mc_off', {'RLoc':'off'}, {'RLoc':'lab'}),
    Strips('mc_lab', {'RLoc':'lab'}, {'RLoc':'mr'}),
    Strips('mc_mr', {'RLoc':'mr'}, {'RLoc':'cs'}),
    Strips('mcc_cs', {'RLoc':'cs'}, {'RLoc':'mr'}),
    Strips('mcc_off', {'RLoc':'off'}, {'RLoc':'cs'}),
    Strips('mcc_lab', {'RLoc':'lab'}, {'RLoc':'off'}),
    Strips('mcc_mr', {'RLoc':'mr'}, {'RLoc':'lab'}),
    Strips('puc', {'RLoc':'cs', 'RHC':False}, {'RHC':True}),
    Strips('dc', {'RLoc':'off', 'RHC':True}, {'RHC':False, 'SWC':False}),
    Strips('pum', {'RLoc':'mr','MW':True}, {'RHM':True,'MW':False}),
    Strips('dm', {'RLoc':'off', 'RHM':True}, {'RHM':False})
   } )
```

stripsProblem.py — (continued)

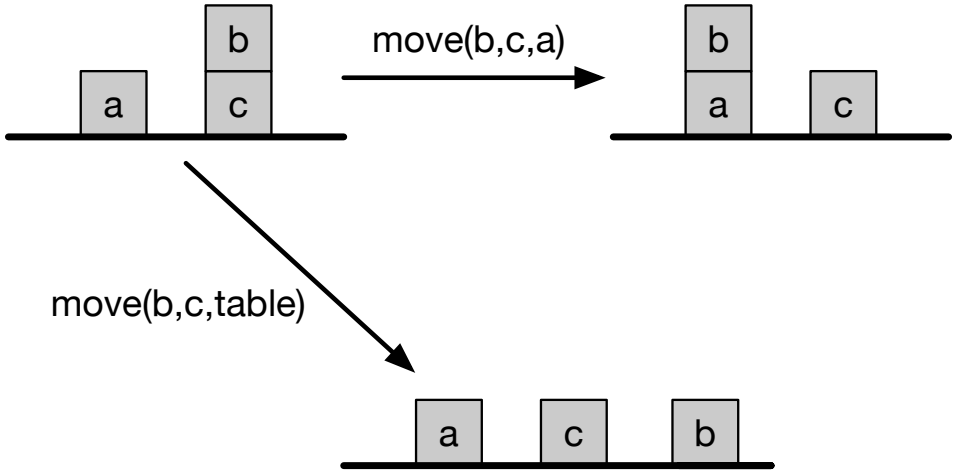


Figure 6.2: Blocks world with two actions

```
problem0 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'RLoc':'off'})
problem1 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'SWC':False})
problem2 = Planning_problem(delivery_domain,
                            {'RLoc':'lab', 'MW':True, 'SWC':True, 'RHC':False,
                             'RHM':False},
                            {'SWC':False, 'MW':False, 'RHM':False})
```

6.1.2 Blocks World

The blocks world consist of blocks and a table. Each block can be on the table or on another block. A block can only have one other block on top of it. Figure 6.2 shows 3 states with some of the actions between them.

A state is defined by the two features:

- *on* where $on(x) = y$ when block x is on block or table y
- *clear* where $clear(x) = True$ when block x has nothing on it.

There is one parameterized action

- $move(x, y, z)$ move block x from y to z, where y and z could be a block or the table.

To handle parameterized actions (which depend on the blocks involved), the actions and the features are all strings, created for the all combinations of the blocks. Note that we treat moving to a block separately from moving to the table, because the blocks needs to be clear, but the table always has room for another block.

stripsProblem.py — (continued)

```
### blocks world
def move(x,y,z):
    """string for the 'move' action"""
    return 'move_'+x+'_from_'+y+'_to_'+z
def on(x):
    """string for the 'on' feature"""
    return x+'_is_on'
def clear(x):
    """string for the 'clear' feature"""
    return 'clear_'+x
def create_blocks_world(blocks = {'a','b','c','d'}):
    blocks_and_table = blocks | {'table'}
    stmap = {Strips(move(x,y,z),{on(x):y, clear(x):True, clear(z):True},
                                {on(x):z, clear(y):True, clear(z):False})
                    for x in blocks
                    for y in blocks_and_table
                    for z in blocks
                    if x!=y and y!=z and z!=x}
    stmap.update({Strips(move(x,y,'table'), {on(x):y, clear(x):True},
                                {on(x):'table', clear(y):True})
                    for x in blocks
                    for y in blocks
                    if x!=y})
    feature_domain_dict = {on(x):blocks_and_table-{x} for x in blocks}
    feature_domain_dict.update({clear(x):boolean for x in blocks_and_table})
    return STRIPS_domain(feature_domain_dict, stmap)
```

The problem *blocks*1 is a classic example, with 3 blocks, and the goal consists of two conditions. See Figure 6.3. Note that this example is challenging because we can't achieve one of the goals and then the other; whichever one we achieve first has to be undone to achieve the second.

stripsProblem.py — (continued)

```
blocks1dom = create_blocks_world({'a','b','c'})
blocks1 = Planning_problem(blocks1dom,
    {on('a'):'table', clear('a'):True,
     on('b'):'c', clear('b'):True,
     on('c'):'table', clear('c'):False}, # initial state
    {on('a'):'b', on('c'):'a'}) #goal
```

The problem *blocks*2 is one to invert a tower of size 4.

stripsProblem.py — (continued)

```
blocks2dom = create_blocks_world({'a','b','c','d'})
```

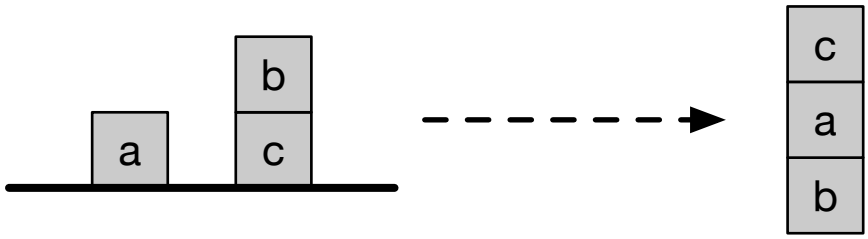


Figure 6.3: Blocks problem blocks1

```
tower4 = {clear('a'):True, on('a'):'b',
         clear('b'):False, on('b'):'c',
         clear('c'):False, on('c'):'d',
         clear('d'):False, on('d'):'table'}
blocks2 = Planning_problem(blocks2dom,
    tower4, # initial state
    {on('d'):'c',on('c'):'b',on('b'):'a'}) #goal
```

The problem *blocks*3 is to move the bottom block to the top of a tower of size 4.

stripsProblem.py — (continued)

```
blocks3 = Planning_problem(blocks2dom,
    tower4, # initial state
    {on('d'):'a', on('a'):'b', on('b'):'c'}) #goal
```

Exercise 6.1 Represent the problem of given a tower of 4 blocks (a on b on c on d on table), the goal is to have a tower with the previous top block on the bottom (b on c on d on a). Do not include the table in your goal (the goal does not care whether a is on the table). [Before you run the program, estimate how many steps it will take to solve this.] How many steps does an optimal planner take?

Exercise 6.2 Represent the domain so that $on(x, y)$ is a Boolean feature that is True when x is on y, Does the representation of the state need to not include negative *on* facts? Why or why not? (Note that this may depend on the planner; write your answer with respect to particular planners.)

Exercise 6.3 It is possible to write the representation of the problem without using *clear*, where $clear(x)$ means nothing is on x. Change the definition of the blocks world so that it does not use *clear* but uses *on* being false instead. Does this work better for any of the planners?

6.2 Forward Planning

To run the demo, in folder "aipython", load "stripsForwardPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a forward planner, a node is a state. A state consists of an assignment, which is a variable:value dictionary. In order to be able to do multiple-path pruning, we need to define a hash function, and equality between states.

stripsForwardPlanner.py — Forward Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem
from stripsProblem import Strips, STRIPS_domain

class State(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

In order to define a search problem (page 37), we need to define the goal condition, the start nodes, the neighbours, and (optionally) a heuristic function. Here *zero* is the default heuristic function.

stripsForwardPlanner.py — (continued)

```
def zero(*args,**nargs):
    """always returns 0"""
    return 0

class Forward_STRIPS(Search_problem):
    """A search problem from a planning problem where:
    * a node is a state object.
    * the dynamics are specified by the STRIPS representation of actions
    """
    def __init__(self, planning_problem, heur=zero):
        """creates a forward search space from a planning problem.
        heur(state,goal) is a heuristic function,
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries.
        """
        self.prob_domain = planning_problem.prob_domain
        self.initial_state = State(planning_problem.initial_state)
        self.goal = planning_problem.goal
        self.heur = heur

    def is_goal(self, state):
        """is True if node is a goal.

        Every goal feature has the same value in the state and the goal."""
        return all(state.assignment[prop]==self.goal[prop]
```

```
                for prop in self.goal)

    def start_node(self):
        """returns start node"""
        return self.initial_state

    def neighbors(self,state):
        """returns neighbors of state in this problem"""
        return [ Arc(state, self.effect(act,state.assignment), act.cost,
            act)
                for act in self.prob_domain.actions
                if self.possible(act,state.assignment)]

    def possible(self,act,state_asst):
        """True if act is possible in state.
        act is possible if all of its preconditions have the same value in
            the state"""
        return all(state_asst[pre] == act.preconds[pre]
                   for pre in act.preconds)

    def effect(self,act,state_asst):
        """returns the state that is the effect of doing act given
            state_asst
        Python 3.9: return state_asst | act.effects"""
        new_state_asst = state_asst.copy()
        new_state_asst.update(act.effects)
        return State(new_state_asst)

    def heuristic(self,state):
        """in the forward planner a node is a state.
        the heuristic is an (under)estimate of the cost
        of going from the state to the top-level goal.
        """
        return self.heur(state.assignment, self.goal)
```

Here are some test cases to try.

stripsForwardPlanner.py — (continued)

```
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

# SearcherMPP(Forward_STRIPS(problem1)).search() #A* with MPP
# DF_branch_and_bound(Forward_STRIPS(problem1),10).search() #B&B
# To find more than one plan:
# s1 = SearcherMPP(Forward_STRIPS(problem1)) #A*
# s1.search() #find another plan
```

6.2.1 Defining Heuristics for a Planner

Each planning domain requires its own heuristics. If you change the actions, you will need to reconsider the heuristic function, as there might then be a lower-cost path, which might make the heuristic non-admissible.

Here is an example of defining heuristics for the coffee delivery planning domain.

First we define the distance between two locations, which is used for the heuristics.

stripsHeuristic.py — Planner with Heuristic Function

```
11 def dist(loc1, loc2):
12     """returns the distance from location loc1 to loc2
13     """
14     if loc1==loc2:
15         return 0
16     if {loc1,loc2} in [{'cs','lab'},{'mr','off'}]:
17         return 2
18     else:
19         return 1
```

Note that the current state is a complete description; there is a value for every feature. However the goal need not be complete; it does not need to define a value for every feature. Before checking the value for a feature in the goal, a heuristic needs to define whether the feature is defined in the goal.

stripsHeuristic.py — (continued)

```
21 def h1(state,goal):
22     """ the distance to the goal location, if there is one"""
23     if 'RLoc' in goal:
24         return dist(state['RLoc'], goal['RLoc'])
25     else:
26         return 0
27
28 def h2(state,goal):
29     """ the distance to the coffee shop plus getting coffee and delivering
           it
30     if the robot needs to get coffee
31     """
32     if ('SWC' in goal and goal['SWC']==False
33             and state['SWC']==True
34             and state['RHC']==False):
35         return dist(state['RLoc'],'cs')+3
36     else:
37         return 0
```

The maximum of the values of a set of admissible heuristics is also an admissible heuristic. The function `maxh` takes a number of heuristic functions as arguments, and returns a new heuristic function that takes the maximum of the values of the heuristics. For example, `h1` and `h2` are heuristic functions and so `maxh(h1,h2)` is also. `maxh` can take an arbitrary number of arguments.

stripsHeuristic.py — (continued)

```
def maxh(*heuristics):
    """Returns a new heuristic function that is the maximum of the
        functions in heuristics.
    heuristics is the list of arguments which must be heuristic functions.
    """
    # return lambda state,goal: max(h(state,goal) for h in heuristics)
    def newh(state,goal):
        return max(h(state,goal) for h in heuristics)
    return newh
```

The following runs the example with and without the heuristic.

stripsHeuristic.py — (continued)

```
##### Forward Planner #####
from searchMPP import SearcherMPP
from stripsForwardPlanner import Forward_STRIPS
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

def test_forward_heuristic(thisproblem=problem1):
    print("\n***** FORWARD NO HEURISTIC")
    print(SearcherMPP(Forward_STRIPS(thisproblem)).search())

    print("\n***** FORWARD WITH HEURISTIC h1")
    print(SearcherMPP(Forward_STRIPS(thisproblem,h1)).search())

    print("\n***** FORWARD WITH HEURISTIC h2")
    print(SearcherMPP(Forward_STRIPS(thisproblem,h2)).search())

    print("\n***** FORWARD WITH HEURISTICs h1 and h2")
    print(SearcherMPP(Forward_STRIPS(thisproblem,maxh(h1,h2))).search())

if __name__ == "__main__":
    test_forward_heuristic()
```

Exercise 6.4 For more than one start-state/goal combination, test the forward planner with a heuristic function of just $h1$, with just $h2$ and with both. Explain why each one prunes or doesn't prune the search space.

Exercise 6.5 Create a better heuristic than $maxh(h1, h2)$. Try it for a number of different problems. In particular, try and include the following costs:

i) $h3$ is like $h2$ but also takes into account the case when *Rloc* is in goal.

ii) $h4$ uses the distance to the mail room plus getting mail and delivering it if the robot needs to get need to deliver mail.

iii) $h5$ is for getting mail when goal is for the robot to have mail, and then getting to the goal destination (if there is one).

Exercise 6.6 Create an admissible heuristic for the blocks world.

6.3 Regression Planning

To run the demo, in folder "aipython", load "stripsRegressionPlanner.py", and copy and paste the commented-out example queries at the bottom of that file.

In a regression planner a node is a subgoal that need to be achieved.

A *Subgoal* object consists of an assignment, which is *variable:value* dictionary. We make it hashable so that multiple path pruning can work. The hash is only computed when necessary (and only once).

stripsRegressionPlanner.py — Regression Planner with STRIPS actions

```
from searchProblem import Arc, Search_problem

class Subgoal(object):
    def __init__(self,assignment):
        self.assignment = assignment
        self.hash_value = None
    def __hash__(self):
        if self.hash_value is None:
            self.hash_value = hash(frozenset(self.assignment.items()))
        return self.hash_value
    def __eq__(self,st):
        return self.assignment == st.assignment
    def __str__(self):
        return str(self.assignment)
```

A regression search has subgoals as nodes. The initial node is the top-level goal of the planner. The goal for the search (when the search can stop) is a subgoal that holds in the initial state.

stripsRegressionPlanner.py — (continued)

```
from stripsForwardPlanner import zero

class Regression_STRIPS(Search_problem):
    """A search problem where:
    * a node is a goal to be achieved, represented by a set of propositions.
    * the dynamics are specified by the STRIPS representation of actions
    """

    def __init__(self, planning_problem, heur=zero):
        """creates a regression search space from a planning problem.
        heur(state,goal) is a heuristic function;
           an underestimate of the cost from state to goal, where
           both state and goals are feature:value dictionaries
        """
        self.prob_domain = planning_problem.prob_domain
        self.top_goal = Subgoal(planning_problem.goal)
        self.initial_state = planning_problem.initial_state
        self.heur = heur
```

```
44
45      def is_goal(self, subgoal):
46          """if subgoal is true in the initial state, a path has been found"""
47          goal_asst = subgoal.assignment
48          return all(self.initial_state[g]==goal_asst[g]
49                     for g in goal_asst)
50
51      def start_node(self):
52          """the start node is the top-level goal"""
53          return self.top_goal
54
55      def neighbors(self,subgoal):
56          """returns a list of the arcs for the neighbors of subgoal in this
                problem"""
57          goal_asst = subgoal.assignment
58          return [ Arc(subgoal, self.weakest_precond(act,goal_asst),
                    act.cost, act)
59                   for act in self.prob_domain.actions
60                   if self.possible(act,goal_asst)]
61
62      def possible(self,act,goal_asst):
63          """True if act is possible to achieve goal_asst.
64
65          the action achieves an element of the effects and
66          the action doesn't delete something that needs to be achieved and
67          the preconditions are consistent with other subgoals that need to
                be achieved
68          """
69          return ( any(goal_asst[prop] == act.effects[prop]
70                       for prop in act.effects if prop in goal_asst)
71                   and all(goal_asst[prop] == act.effects[prop]
72                           for prop in act.effects if prop in goal_asst)
73                   and all(goal_asst[prop]== act.preconds[prop]
74                           for prop in act.preconds if prop not in act.effects
                                and prop in goal_asst)
75                   )
76
77      def weakest_precond(self,act,goal_asst):
78          """returns the subgoal that must be true so goal_asst holds after
                act
79          should be:  act.preconds | (goal_asst - act.effects)
80          """
81          new_asst = act.preconds.copy()
82          for g in goal_asst:
83              if g not in act.effects:
84                  new_asst[g] = goal_asst[g]
85          return Subgoal(new_asst)
86
87      def heuristic(self,subgoal):
88          """in the regression planner a node is a subgoal.
```

```
89          the heuristic is an (under)estimate of the cost of going from the
                initial state to subgoal.
90          """
91          return self.heur(self.initial_state, subgoal.assignment)
```

stripsRegressionPlanner.py — (continued)

```
93  from searchBranchAndBound import DF_branch_and_bound
94  from searchMPP import SearcherMPP
95  from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
        blocks3
96
97  # SearcherMPP(Regression_STRIPS(problem1)).search() #A* with MPP
98  # DF_branch_and_bound(Regression_STRIPS(problem1),10).search() #B&B
```

Exercise 6.7 Multiple path pruning could be used to prune more than the current code. In particular, if the current node contains more conditions than a previously visited node, it can be pruned. For example, if $\{a : True, b : False\}$ has been visited, then any node that is a superset, e.g., $\{a : True, b : False, d : True\}$, need not be expanded. If the simpler subgoal does not lead to a solution, the more complicated one wont either. Implement this more severe pruning. (Hint: This may require modifications to the searcher.)

Exercise 6.8 It is possible that, as knowledge of the domain, that some assignment of values to variables can never be achieved. For example, the robot cannot be holding mail when there is mail waiting (assuming it isn't holding mail initially). An assignment of values to (some of the) variables is incompatible if no possible (reachable) state can include that assignment. For example, $\{'MW' : True, 'RHM' : True\}$ is an incompatible assignment. This information may be useful information for a planner; there is no point in trying to achieve these together. Define a subclass of *STRIPS_domain* that can accept a list of incompatible assignments. Modify the regression planner code to use such a list of incompatible assignments. Give an example where the search space is smaller.

Exercise 6.9 After completing the previous exercise, design incompatible assignments for the blocks world. (This should result in dramatic search improvements.)

6.3.1 Defining Heuristics for a Regression Planner

The regression planner can use the same heuristic function as the forward planner. However, just because a heuristic is useful for a forward planner does not mean it is useful for a regression planner, and vice versa. you should experiment with whether the same heuristic works well for both a a regression planner and a forward planner.

The following runs the same example as the forward planner with and without the heuristic defined for the forward planner:

stripsHeuristic.py — (continued)

```
69  ##### Regression Planner
70  from stripsRegressionPlanner import Regression_STRIPS
```

```
71
72 def test_regression_heuristic(thisproblem=problem1):
73     print("\n***** REGRESSION NO HEURISTIC")
74     print(SearcherMPP(Regression_STRIPS(thisproblem)).search())
75
76     print("\n***** REGRESSION WITH HEURISTICs h1 and h2")
77     print(SearcherMPP(Regression_STRIPS(thisproblem,maxh(h1,h2))).search())
78
79 if __name__ == "__main__":
80     test_regression_heuristic()
```

Exercise 6.10 Try the regression planner with a heuristic function of just $h1$ and with just $h2$ (defined in Section 6.2.1). Explain how each one prunes or doesn't prune the search space.

Exercise 6.11 Create a better heuristic than *heuristic_fun* defined in Section 6.2.1.

6.4 Planning as a CSP

> To run the demo, in folder "aipython", load "stripsCSPPlanner.py", and copy and paste the commented-out example queries at the bottom of that file. This assumes Python 3.

Here we implement the CSP planner assuming there is a single action at each step. This creates a CSP that can use any of the CSP algorithms to solve (e.g., stochastic local search or arc consistency with domain splitting).

This assumes the same action representation as before; we do not consider factored actions (action features), nor do we implement state constraints.

stripsCSPPlanner.py — CSP planner where actions are represented using STRIPS

```
11 from cspProblem import Variable, CSP, Constraint
12
13 class CSP_from_STRIPS(CSP):
14     """A CSP where:
15     * CSP variables are constructed for each feature and time, and each
           action and time
16     * the dynamics are specified by the STRIPS representation of actions
17     """
18
19     def __init__(self, planning_problem, number_stages=2):
20         prob_domain = planning_problem.prob_domain
21         initial_state = planning_problem.initial_state
22         goal = planning_problem.goal
23         # self.action_vars[t] is the action variable for time t
24         self.action_vars = [Variable(f"Action{t}", prob_domain.actions)
25                             for t in range(number_stages)]
26         # feat_time_var[f][t] is the variable for feature f at time t
27         feat_time_var = {feat: [Variable(f"{feat}_{t}",dom)
28                                 for t in range(number_stages+1)]
```

```
                        for (feat,dom) in
                            prob_domain.feature_domain_dict.items()}

        # initial state constraints:
        constraints = [Constraint((feat_time_var[feat][0],), is_(val))
                       for (feat,val) in initial_state.items()]

        # goal constraints on the final state:
        constraints += [Constraint((feat_time_var[feat][number_stages],),
                                   is_(val))
                        for (feat,val) in goal.items()]

        # precondition constraints:
        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t==val if action@t==act
                        for act in prob_domain.actions
                        for (feat,val) in act.preconds.items()
                        for t in range(number_stages)]

        # effect constraints:
        constraints += [Constraint((feat_time_var[feat][t+1],
            self.action_vars[t]),
                                   if_(val,act)) # feat@t+1==val if
                                       action@t==act
                        for act in prob_domain.actions
                        for feat,val in act.effects.items()
                        for t in range(number_stages)]
        # frame constraints:

        constraints += [Constraint((feat_time_var[feat][t],
            self.action_vars[t], feat_time_var[feat][t+1]),
                                   eq_if_not_in_({act for act in
                                       prob_domain.actions
                                                  if feat in act.effects}))
                        for feat in prob_domain.feature_domain_dict
                        for t in range(number_stages) ]
        variables = set(self.action_vars) | {feat_time_var[feat][t]
                                             for feat in
                                                 prob_domain.feature_domain_dict
                                             for t in range(number_stages+1)}
        CSP.__init__(self, variables, constraints)

    def extract_plan(self,soln):
        return [soln[a] for a in self.action_vars]
```

The following methods return methods which can be applied to the particular environment.

For example, $is\_(3)$ returns a function that when applied to 3, returns True and when applied to any other value returns False. So $is\_(3)(3)$ returns *True*

and $is\_(3)(7)$ returns *False*.

Note that the underscore ('_') is part of the name; here we use it as the convention that it is a function that returns a function. This uses two different styles to define *is_* and *if_*; returning a function defined by *lambda* is equivalent to returning the embedded function, except that the embedded function has a name. The embedded function can also be given a docstring.

stripsCSPPlanner.py — (continued)

```
68 def is_(val):
69     """returns a function that is true when it is it applied to val.
70     """
71     #return lambda x: x == val
72     def is_fun(x):
73         return x == val
74     is_fun.__name__ = f"value_is_{val}"
75     return is_fun
76 
77 def if_(v1,v2):
78     """if the second argument is v2, the first argument must be v1"""
79     #return lambda x1,x2: x1==v1 if x2==v2 else True
80     def if_fun(x1,x2):
81         return x1==v1 if x2==v2 else True
82     if_fun.__name__ = f"if x2 is {v2} then x1 is {v1}"
83     return if_fun
84 
85 def eq_if_not_in_(actset):
86     """first and third arguments are equal if action is not in actset"""
87     # return lambda x1, a, x2: x1==x2 if a not in actset else True
88     def eq_if_not_fun(x1, a, x2):
89         return x1==x2 if a not in actset else True
90     eq_if_not_fun.__name__ = f"first and third arguments are equal if
           action is not in {actset}"
91     return eq_if_not_fun
```

Putting it together, this returns a list of actions that solves the problem *prob* for a given horizon. If you want to do more than just return the list of actions, you might want to get it to return the solution. Or even enumerate the solutions (by using *Search_with_AC_from_CSP*).

stripsCSPPlanner.py — (continued)

```
93 def con_plan(prob,horizon):
94     """finds a plan for problem prob given horizon.
95     """
96     csp = CSP_from_STRIPS(prob, horizon)
97     sol = Con_solver(csp).solve_one()
98     return csp.extract_plan(sol) if sol else sol
```

The following are some example queries.

stripsCSPPlanner.py — (continued)

```
100 from searchGeneric import Searcher
```

```
from stripsProblem import delivery_domain
from cspConsistency import Search_with_AC_from_CSP, Con_solver
from stripsProblem import Planning_problem, problem0, problem1, problem2,
    blocks1, blocks2, blocks3

# Problem 0
# con_plan(problem0,1) # should it succeed?
# con_plan(problem0,2) # should it succeed?
# con_plan(problem0,3) # should it succeed?
# To use search to enumerate solutions
#searcher0a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem0,
    1)))
#print(searcher0a.search()) # returns path to solution

## Problem 1
# con_plan(problem1,5) # should it succeed?
# con_plan(problem1,4) # should it succeed?
## To use search to enumerate solutions:
#searcher15a = Searcher(Search_with_AC_from_CSP(CSP_from_STRIPS(problem1,
    5)))
#print(searcher15a.search()) # returns path to solution

## Problem 2
#con_plan(problem2, 6) # should fail??
#con_plan(problem2, 7) # should succeed???

## Example 6.13
problem3 = Planning_problem(delivery_domain,
                            {'SWC':True, 'RHC':False}, {'SWC':False})
#con_plan(problem3,2) # Horizon of 2
#con_plan(problem3,3) # Horizon of 3

problem4 = Planning_problem(delivery_domain,{'SWC':True},
                              {'SWC':False, 'MW':False, 'RHM':False})

# For the stochastic local search:
#from cspSLS import SLSearcher, Runtime_distribution
# cspplanning15 = CSP_from_STRIPS(problem1, 5) # should succeed
#se0 = SLSearcher(cspplanning15); print(se0.search(100000,0.5))
#p = Runtime_distribution(cspplanning15)
#p.plot_runs(1000,1000,0.7) # warning will take a few minutes
```

6.5 Partial-Order Planning

To run the demo, in folder "aipython", load "stripsPOP.py", and copy and paste the commented-out example queries at the bottom of that file.

A partial order planner maintains a partial order of action instances. An action instance consists of a name and an index. We need action instances because the same action could be carried out at different times.

stripsPOP.py — Partial-order Planner using STRIPS representation

```
from searchProblem import Arc, Search_problem
import random

class Action_instance(object):
    next_index = 0
    def __init__(self,action,index=None):
        if index is None:
            index = Action_instance.next_index
            Action_instance.next_index += 1
        self.action = action
        self.index = index

    def __str__(self):
        return f"{self.action}#{self.index}"

    __repr__ = __str__ # __repr__ function is the same as the __str__
        function
```

A node (as in the abstraction of search space) in a partial-order planner consists of:

- *actions*: a set of action instances.
- *constraints*: a set of (a_1, a_2) pairs, where a_1 and a_2 are action instances, which represents that a_1 must come before a_2 in the partial order. There are a number of ways that this could be represented. Here we represent the set of pairs that are in transitive closure of the *before* relation. This lets us quickly determine whether some before relation is consistent with the current constraints.
- *agenda*: a list of (s, a) pairs, where s is a (var, val) pair and a is an action instance. This means that variable *var* must have value *val* before a can occur.
- *causal_links*: a set of $(a0, g, a1)$ triples, where a_1 and a_2 are action instances and g is a (var, val) pair. This holds when action a_0 makes g true for action a_1.

stripsPOP.py — (continued)

```
class POP_node(object):
    """a (partial) partial-order plan. This is a node in the search
        space."""
    def __init__(self, actions, constraints, agenda, causal_links):
        """
```

```
        * actions is a set of action instances
        * constraints a set of (a0,a1) pairs, representing a0<a1,
          closed under transitivity
        * agenda list of (subgoal,action) pairs to be achieved, where
          subgoal is a (variable,value) pair
        * causal_links is a set of (a0,g,a1) triples,
          where ai are action instances, and g is a (variable,value) pair
        """
        self.actions = actions # a set of action instances
        self.constraints = constraints # a set of (a0,a1) pairs
        self.agenda = agenda  # list of (subgoal,action) pairs to be
            achieved
        self.causal_links = causal_links # set of (a0,g,a1) triples

    def __str__(self):
        return ("actions: "+str({str(a) for a in self.actions})+
                "\nconstraints: "+
                str({(str(a1),str(a2)) for (a1,a2) in self.constraints})+
                "\nagenda: "+
                str([(str(s),str(a)) for (s,a) in self.agenda])+
                "\ncausal_links:"+
                str({(str(a0),str(g),str(a2)) for (a0,g,a2) in
                    self.causal_links}) )
```

extract_plan constructs a total order of action instances that is consistent with the partial order.

stripsPOP.py — (continued)

```
    def extract_plan(self):
        """returns a total ordering of the action instances consistent
        with the constraints.
        raises IndexError if there is no choice.
        """
        sorted_acts = []
        other_acts = set(self.actions)
        while other_acts:
            a = random.choice([a for a in other_acts if
                    all(((a1,a) not in self.constraints) for a1 in
                        other_acts)])
            sorted_acts.append(a)
            other_acts.remove(a)
        return sorted_acts
```

POP_search_from_STRIPS is an instance of a search problem. As such, we need to define the start nodes, the goal, and the neighbors of a node.

stripsPOP.py — (continued)

```
from display import Displayable

class POP_search_from_STRIPS(Search_problem, Displayable):
    def __init__(self,planning_problem):
```

```
72          Search_problem.__init__(self)
73          self.planning_problem = planning_problem
74          self.start = Action_instance("start")
75          self.finish = Action_instance("finish")
76
77      def is_goal(self, node):
78          return node.agenda == []
79
80      def start_node(self):
81          constraints = {(self.start, self.finish)}
82          agenda = [(g, self.finish) for g in
                self.planning_problem.goal.items()]
83          return POP_node([self.start,self.finish], constraints, agenda, [] )
```

The *neighbors* method is a coroutine that enumerates the neighbors of a given node.

stripsPOP.py — (continued)

```
85      def neighbors(self, node):
86          """enumerates the neighbors of node"""
87          self.display(3,"finding neighbors of\n",node)
88          if node.agenda:
89              subgoal,act1 = node.agenda[0]
90              self.display(2,"selecting",subgoal,"for",act1)
91              new_agenda = node.agenda[1:]
92              for act0 in node.actions:
93                  if (self.achieves(act0, subgoal) and
94                     self.possible((act0,act1),node.constraints)):
95                      self.display(2," reusing",act0)
96                      consts1 =
                            self.add_constraint((act0,act1),node.constraints)
97                      new_clink = (act0,subgoal,act1)
98                      new_cls = node.causal_links + [new_clink]
99                      for consts2 in
                            self.protect_cl_for_actions(node.actions,consts1,new_clink):
100                         yield Arc(node,
101                                 POP_node(node.actions,consts2,new_agenda,new_cls),
102                                 cost=0)
103             for a0 in self.planning_problem.prob_domain.actions: #a0 is an
                    action
104                 if self.achieves(a0, subgoal):
105                     #a0 acheieves subgoal
106                     new_a = Action_instance(a0)
107                     self.display(2," using new action",new_a)
108                     new_actions = node.actions + [new_a]
109                     consts1 =
                            self.add_constraint((self.start,new_a),node.constraints)
110                     consts2 = self.add_constraint((new_a,act1),consts1)
111                     new_agenda1 = new_agenda + [(pre,new_a) for pre in
                            a0.preconds.items()]
112                     new_clink = (new_a,subgoal,act1)
```

```
                new_cls = node.causal_links + [new_clink]
                for consts3 in
                    self.protect_all_cls(node.causal_links,new_a,consts2):
                    for consts4 in
                        self.protect_cl_for_actions(node.actions,consts3,new_clink):
                        yield Arc(node,
                                  POP_node(new_actions,consts4,new_agenda1,new_cls),
                                  cost=1)
```

Given a casual link $(a0, subgoal, a1)$, the following method protects the causal link from each action in *actions*. Whenever an action deletes *subgoal*, the action needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal link from all actions.

stripsPOP.py — (continued)

```
    def protect_cl_for_actions(self, actions, constrs, clink):
        """yields constraints that extend constrs and
        protect causal link (a0, subgoal, a1)
        for each action in actions
        """
        if actions:
            a = actions[0]
            rem_actions = actions[1:]
            a0, subgoal, a1 = clink
            if a != a0 and a != a1 and self.deletes(a,subgoal):
                if self.possible((a,a0),constrs):
                    new_const = self.add_constraint((a,a0),constrs)
                    for e in
                        self.protect_cl_for_actions(rem_actions,new_const,clink):
                        yield e # could be "yield from"
                if self.possible((a1,a),constrs):
                    new_const = self.add_constraint((a1,a),constrs)
                    for e in
                        self.protect_cl_for_actions(rem_actions,new_const,clink):
                        yield e
            else:
                for e in
                    self.protect_cl_for_actions(rem_actions,constrs,clink):
                    yield e
        else:
            yield constrs
```

Given an action *act*, the following method protects all the causal links in *clinks* from *act*. Whenever *act* deletes *subgoal* from some causal link $(a0, subgoal, a1)$, the action *act* needs to be before $a0$ or after $a1$. This method enumerates all constraints that result from protecting the causal links from *act*.

stripsPOP.py — (continued)

```
    def protect_all_cls(self, clinks, act, constrs):
        """yields constraints that protect all causal links from act"""
        if clinks:
```

```
144             (a0,cond,a1) = clinks[0] # select a causal link
145             rem_clinks = clinks[1:] # remaining causal links
146             if act != a0 and act != a1 and self.deletes(act,cond):
147                 if self.possible((act,a0),constrs):
148                     new_const = self.add_constraint((act,a0),constrs)
149                     for e in self.protect_all_cls(rem_clinks,act,new_const):
                            yield e
150                 if self.possible((a1,act),constrs):
151                     new_const = self.add_constraint((a1,act),constrs)
152                     for e in self.protect_all_cls(rem_clinks,act,new_const):
                            yield e
153             else:
154                 for e in self.protect_all_cls(rem_clinks,act,constrs): yield
                        e
155         else:
156             yield constrs
```

The following methods check whether an action (or action instance) achieves or deletes some subgoal.

stripsPOP.py — (continued)

```
158     def achieves(self,action,subgoal):
159         var,val = subgoal
160         return var in self.effects(action) and self.effects(action)[var] ==
                val
161
162     def deletes(self,action,subgoal):
163         var,val = subgoal
164         return var in self.effects(action) and self.effects(action)[var] !=
                val
165
166     def effects(self,action):
167         """returns the variable:value dictionary of the effects of action.
168         works for both actions and action instances"""
169         if isinstance(action, Action_instance):
170             action = action.action
171         if action == "start":
172             return self.planning_problem.initial_state
173         elif action == "finish":
174             return {}
175         else:
176             return action.effects
```

The constraints are represented as a set of pairs closed under transitivity. Thus if (a, b) and (b, c) are the list, then (a, c) must also be in the list. This means that adding a new constraint means adding the implied pairs, but querying whether some order is consistent is quick.

stripsPOP.py — (continued)

```
178     def add_constraint(self, pair, const):
179         if pair in const:
```

```
            return const
        todo = [pair]
        newconst = const.copy()
        while todo:
            x0,x1 = todo.pop()
            newconst.add((x0,x1))
            for x,y in newconst:
                if x==x1 and (x0,y) not in newconst:
                    todo.append((x0,y))
                if y==x0 and (x,x1) not in newconst:
                    todo.append((x,x1))
        return newconst

    def possible(self,pair,constraint):
        (x,y) = pair
        return (y,x) not in constraint
```

Some code for testing:

stripsPOP.py — (continued)

```
from searchBranchAndBound import DF_branch_and_bound
from searchMPP import SearcherMPP
from stripsProblem import problem0, problem1, problem2, blocks1, blocks2,
    blocks3

rplanning0 = POP_search_from_STRIPS(problem0)
rplanning1 = POP_search_from_STRIPS(problem1)
rplanning2 = POP_search_from_STRIPS(problem2)
searcher0 = DF_branch_and_bound(rplanning0,5)
searcher0a = SearcherMPP(rplanning0)
searcher1 = DF_branch_and_bound(rplanning1,10)
searcher1a = SearcherMPP(rplanning1)
searcher2 = DF_branch_and_bound(rplanning2,10)
searcher2a = SearcherMPP(rplanning2)
# Try one of the following searchers
# a = searcher0.search()
# a = searcher0a.search()
# a.end().extract_plan() # print a plan found
# a.end().constraints  # print the constraints
# SearcherMPP.max_display_level = 0 # less detailed display
# DF_branch_and_bound.max_display_level = 0 # less detailed display
# a = searcher1.search()
# a = searcher1a.search()
# a = searcher2.search()
# a = searcher2a.search()
```

Chapter 7

Supervised Machine Learning

This chapter is the first on machine learning. It covers the following topics:

- Data: how to load it, training and test sets
- Features: many of the features come directly from the data. Sometimes it is useful to construct features, e.g. $height > 1.9m$ might be a Boolean feature constructed from the real-values feature *height*. The next chapter is about neural networdks and how to learn features; in this chapter we construct explicitly in what is often known a **feature engineering**.
- Learning with no input features: this is the base case of many methods. What should we predict if we have no input features? This provides the base cases for many algorithms (e.g., decision tree algorithm) and baselines that more sophisticated algorithms need to beat. It also provides ways to test various predictors.
- Decision tree learning: one of the classic and simplest learning algorithms, which is the basis of many other algorithms.
- Cross validation and parameter tuning: methods to prevent overfitting.
- Linear regression and classification: other classic and simple techniques that often work well (particularly combined with feature learning or engineering).
- Boosting: combining simpler learning methods to make even better learners.

A good source of classic datasets is the UCI Machine Learning Repository [Lichman, 2013] [Dua and Graff, 2017]. The SPECT, IRIS, and car datasets (carbook is a Boolean version of the car dataset) are from this repository.

Dataset	# Examples	#Columns	Input Types	Target Type
SPECT	267	23	Boolean	Boolean
IRIS	150	5	numeric	categorical
carbool	1728	7	categorical/numeric	numeric
holiday	32	6	Boolean	Boolean
mail_reading	28	5	Boolean	Boolean
tv_likes	12	5	Boolean	Boolean
simp_regr	7	2	numeric	numeric

Figure 7.1: Some of the datasets used here.

7.1 Representations of Data and Predictions

The code uses the following definitions and conventions:

- A **dataset** is an enumeration of examples.
- An **example** is a list (or tuple) of values. The values can be numbers or strings.
- A **feature** is a function from examples into the range of the feature. Each feature `f` also has the following attributes:

  `f.ftype`, the type of `f`, one of: `"boolean"`, `"categorical"`, `"numeric"`

  `f.frange`, the set of values of `f` seen in the dataset, represented as a list. The `ftype` is inferred from the `frange` if not given explicitly.

  `f.__doc__`, the docstring, a string description of `f` (for printing).

  Thus for example, a **Boolean feature** is a function from the examples into $\{False, True\}$. So, if f is a Boolean feature, $f.frange == [False, True]$, and if e is an example, $f(e)$ is either *True* or *False*.

learnProblem.py — A Learning Problem

```
11 import math, random, statistics
12 import csv
13 from display import Displayable
14 from utilities import argmax
15
16 boolean = [False, True]
```

When creating a dataset, we partition the data into a training set (*train*) and a test set (*test*). The target feature is the feature that we are making a prediction of. A dataset `ds` has the following attributes

`ds.train` a list of the training examples

`ds.test` a list of the test examples

`ds.target_index` the index of the target

`ds.target` the feature corresponding to the target (a function as described above)

`ds.input_features` a list of the input features

learnProblem.py — (continued)

```
18 class Data_set(Displayable):
19     """ A dataset consists of a list of training data and a list of test
           data.
20     """
21
22     def __init__(self, train, test=None, prob_test=0.20, target_index=0,
23                  header=None, target_type= None, seed=None): #12345):
24         """A dataset for learning.
25         train is a list of tuples representing the training examples
26         test is the list of tuples representing the test examples
27         if test is None, a test set is created by selecting each
28             example with probability prob_test
29         target_index is the index of the target.
30             If negative, it counts from right.
31             If target_index is larger than the number of properties,
32             there is no target (for unsupervised learning)
33         header is a list of names for the features
34         target_type is either None for automatic detection of target type
35              or one of "numeric", "boolean", "cartegorical"
36         seed is for random number; None gives a different test set each time
37         """
38         if seed: # given seed makes partition consistent from run-to-run
39             random.seed(seed)
40         if test is None:
41             train,test = partition_data(train, prob_test)
42         self.train = train
43         self.test = test
44
45         self.display(1,"Training set has",len(train),"examples. Number of
               columns: ",{len(e) for e in train})
46         self.display(1,"Test set has",len(test),"examples. Number of
               columns: ",{len(e) for e in test})
47         self.prob_test = prob_test
48         self.num_properties = len(self.train[0])
49         if target_index < 0: #allows for -1, -2, etc.
50             self.target_index = self.num_properties + target_index
51         else:
52             self.target_index = target_index
53         self.header = header
54         self.domains = [set() for i in range(self.num_properties)]
55         for example in self.train:
56             for ind,val in enumerate(example):
```

```
57              self.domains[ind].add(val)
58      self.conditions_cache = {} # cache for computed conditions
59      self.create_features()
60      if target_type:
61          self.target.ftype = target_type
62      self.display(1,"There are",len(self.input_features),"input
            features")
63
64  def __str__(self):
65      if self.train and len(self.train)>0:
66          return ("Data: "+str(len(self.train))+" training examples, "
67                  +str(len(self.test))+" test examples, "
68                  +str(len(self.train[0]))+" features.")
69      else:
70          return ("Data: "+str(len(self.train))+" training examples, "
71                  +str(len(self.test))+" test examples.")
```

A **feature** is a function that takes an example and returns a value in the range of the feature. Each feature has a **frange**, which gives the range of the feature, and an **ftype** that gives the type, one of "boolean", "numeric" or "categorical".

learnProblem.py — (continued)

```
73  def create_features(self):
74      """create the set of features
75      """
76      self.target = None
77      self.input_features = []
78      for i in range(self.num_properties):
79          def feat(e,index=i):
80              return e[index]
81          if self.header:
82              feat.__doc__ = self.header[i]
83          else:
84              feat.__doc__ = "e["+str(i)+"]"
85          feat.frange = list(self.domains[i])
86          feat.ftype = self.infer_type(feat.frange)
87          if i == self.target_index:
88              self.target = feat
89          else:
90              self.input_features.append(feat)
```

We try to infer the type of each feature. Sometimes this can be wrong, (e.g., when the numbers are really categorical) and may need to be set explicitly.

learnProblem.py — (continued)

```
92  def infer_type(self,domain):
93      """Infers the type of a feature with domain
94      """
95      if all(v in {True,False} for v in domain):
96          return "boolean"
```

```
97          if all(isinstance(v,(float,int)) for v in domain):
98              return "numeric"
99          else:
100             return "categorical"
```

7.1.1 Creating Boolean Conditions from Features

Some of the algorithms require Boolean input features or features with range $\{0, 1\}$. In order to be able to use these algorithms on datasets that allow for arbitrary domains of input variables, we construct Boolean conditions from the attributes.

There are 3 cases:

- When the range only has two values, we designate one to be the "true" value.
- When the values are all numeric, we assume they are ordered (as opposed to just being some classes that happen to be labelled with numbers) and construct Boolean features for splits of the data. That is, the feature is $e[ind] < cut$ for some value cut. We choose a number of cut values, up to a maximum number of cuts, given by max_num_cuts.
- When the values are not all numeric, we create an indicator function for each value. An indicator function for a value returns true when that value is given and false otherwise. Note that we can't create an indicator function for values that appear in the test set but not in the training set because we haven't seen the test set. For the examples in the test set with a value that doesn't appear in the training set for that feature, the indicator functions all return false.

There is also an option `categorical_only` to only create Boolean features for categorical input features, and not to make cuts for numerical values.

learnProblem.py — (continued)

```
102     def conditions(self, max_num_cuts=8, categorical_only = False):
103         """returns a set of boolean conditions from the input features
104         max_num_cuts is the maximum number of cute for numeric features
105         categorical_only is true if only categorical features are made
                binary
106         """
107         if (max_num_cuts, categorical_only) in self.conditions_cache:
108             return self.conditions_cache[(max_num_cuts, categorical_only)]
109         conds = []
110         for ind,frange in enumerate(self.domains):
111             if ind != self.target_index and len(frange)>1:
112                 if len(frange) == 2:
113                     # two values, the feature is equality to one of them.
114                     true_val = list(frange)[1] # choose one as true
```

```
                    def feat(e, i=ind, tv=true_val):
                        return e[i]==tv
                    if self.header:
                        feat.__doc__ = f"{self.header[ind]}=={true_val}"
                    else:
                        feat.__doc__ = f"e[{ind}]=={true_val}"
                    feat.frange = boolean
                    feat.ftype = "boolean"
                    conds.append(feat)
                elif all(isinstance(val,(int,float)) for val in frange):
                    if categorical_only: # numeric, don't make cuts
                        def feat(e, i=ind):
                            return e[i]
                        feat.__doc__ = f"e[{ind}]"
                        conds.append(feat)
                    else:
                        # all numeric, create cuts of the data
                        sorted_frange = sorted(frange)
                        num_cuts = min(max_num_cuts,len(frange))
                        cut_positions = [len(frange)*i//num_cuts for i in
                            range(1,num_cuts)]
                        for cut in cut_positions:
                            cutat = sorted_frange[cut]
                            def feat(e, ind_=ind, cutat=cutat):
                                return e[ind_] < cutat

                            if self.header:
                                feat.__doc__ = self.header[ind]+"<"+str(cutat)
                            else:
                                feat.__doc__ = "e["+str(ind)+"]<"+str(cutat)
                            feat.frange = boolean
                            feat.ftype = "boolean"
                            conds.append(feat)
                else:
                    # create an indicator function for every value
                    for val in frange:
                        def feat(e, ind_=ind, val_=val):
                            return e[ind_] == val_
                        if self.header:
                            feat.__doc__ = self.header[ind]+"=="+str(val)
                        else:
                            feat.__doc__= "e["+str(ind)+"]=="+str(val)
                        feat.frange = boolean
                        feat.ftype = "boolean"
                        conds.append(feat)
        self.conditions_cache[(max_num_cuts, categorical_only)] = conds
        return conds
```

Exercise 7.1 Change the code so that it splits using $e[ind] \leq cut$ instead of $e[ind] < cut$. Check boundary cases, such as 3 elements with 2 cuts. As a test case, make sure that when the range is the 30 integers from 100 to 129, and you want 2 cuts,

the resulting Boolean features should be $e[ind] \leq 109$ and $e[ind] \leq 119$ to make sure that each of the resulting domains is of equal size.

Exercise 7.2 This splits on whether the feature is less than one of the values in the training set. Sam suggested it might be better to split between the values in the training set, and suggested using

$$cutat = (sorted\_frange[cut] + sorted\_frange[cut-1])/2$$

Why might Sam have suggested this? Does this work better? (Try it on a few datasets).

7.1.2 Evaluating Predictions

A **predictor** is a function that takes an example and makes a prediction on the values of the target features.

A **loss** takes a prediction and the actual value and returns a non-negative real number; lower is better. The **error** for a dataset is either the mean loss, or sometimes the sum of the losses. When reporting results the mean is usually used. When it is the sum, this will be made explicit.

The function *evaluate_dataset* returns the average error for each example, where the error for each example depends on the evaluation criteria. Here we consider three evaluation criteria, the squared error (average of the square of the difference between the actual and predicted values), absolute errors(average of the absolute difference between the actual and predicted values) and the log loss (the a average negative log-likelihood, which can be interpreted as the number of bits to describe an example using a code based on the prediction treated as a probability).

learnProblem.py — (continued)

```
162     def evaluate_dataset(self, data, predictor, error_measure):
163         """Evaluates predictor on data according to the error_measure
164         predictor is a function that takes an example and returns a
165                 prediction for the target features.
166         error_measure(prediction,actual) -> non-negative real
167         """
168         if data:
169             try:
170                 value = statistics.mean(error_measure(predictor(e),
                        self.target(e))
171                         for e in data)
172             except ValueError: # if error_measure gives an error
173                 return float("inf") # infinity
174             return value
175         else:
176             return math.nan # not a number
```

The following evaluation criteria are defined. This is defined using a class, `Evaluate` but no instances will be created. Just use `Evaluate.squared_loss` etc.

(Please keep the __doc__ strings a consistent length as they are used in tables.) The prediction is either a real value or a $\{value : probability\}$ dictionary or a list. The actual is either a real number or a key of the prediction.

learnProblem.py — (continued)

```
class Evaluate(object):
    """A container for the evaluation measures"""

    def squared_loss(prediction, actual):
        "squared loss "
        if isinstance(prediction, (list,dict)):
            return (1-prediction[actual])**2 # the correct value is 1
        else:
            return (prediction-actual)**2

    def absolute_loss(prediction, actual):
        "absolute loss "
        if isinstance(prediction, (list,dict)):
            return abs(1-prediction[actual]) # the correct value is 1
        else:
            return abs(prediction-actual)

    def log_loss(prediction, actual):
        "log loss (bits)"
        try:
            if isinstance(prediction, (list,dict)):
                return -math.log2(prediction[actual])
            else:
                return -math.log2(prediction) if actual==1 else
                    -math.log2(1-prediction)
        except ValueError:
            return float("inf") # infinity

    def accuracy(prediction, actual):
        "accuracy    "
        if isinstance(prediction, dict):
            prev_val = prediction[actual]
            return 1 if all(prev_val >= v for v in prediction.values())
                else 0
        if isinstance(prediction, list):
            prev_val = prediction[actual]
            return 1 if all(prev_val >= v for v in prediction) else 0
        else:
            return 1 if abs(actual-prediction) <= 0.5 else 0

    all_criteria = [accuracy, absolute_loss, squared_loss, log_loss]
```

7.1.3 Creating Test and Training Sets

The following method partitions the data into a training set and a test set. Note that this does not guarantee that the test set will contain exactly a proportion of the data equal to *prob_test*.

[An alternative is to use *random.sample*() which can guarantee that the test set will contain exactly a particular proportion of the data. However this would require knowing how many elements are in the dataset, which we may not know, as *data* may just be a generator of the data (e.g., when reading the data from a file).]

learnProblem.py — (continued)

```
218 def partition_data(data, prob_test=0.30):
219     """partitions the data into a training set and a test set, where
220     prob_test is the probability of each example being in the test set.
221     """
222     train = []
223     test = []
224     for example in data:
225         if random.random() < prob_test:
226             test.append(example)
227         else:
228             train.append(example)
229     return train, test
```

7.1.4 Importing Data From File

A dataset is typically loaded from a file. The default here is that it loaded from a CSV (comma separated values) file, although the separator can be changed. This assumes that all lines that contain the separator are valid data (so we only include those data items that contain more than one element). This allows for blank lines and comment lines that do not contain the separator. However, it means that this method is not suitable for cases where there is only one feature.

Note that *data_all* and *data_tuples* are generators. *data_all* is a generator of a list of list of strings. This version assumes that CSV files are simple. The standard *csv* package, that allows quoted arguments, can be used by uncommenting the line for *data_all* and commenting out the following line. *data_tuples* contains only those lines that contain the delimiter (others lines are assumed to be empty or comments), and tries to convert the elements to numbers whenever possible.

This allows for some of the columns to be included; specified by *include_only*. Note that if *include_only* is specified, the target index is the index for the included columns, not the original columns.

learnProblem.py — (continued)

```
231 class Data_from_file(Data_set):
232     def __init__(self, file_name, separator=',', num_train=None,
        prob_test=0.3,
```

```
            has_header=False, target_index=0, boolean_features=True,
            categorical=[], target_type= None, include_only=None,
                seed=None): #seed=12345):
        """create a dataset from a file
        separator is the character that separates the attributes
        num_train is a number specifying the first num_train tuples are
            training, or None
        prob_test is the probability an example should in the test set (if
            num_train is None)
        has_header is True if the first line of file is a header
        target_index specifies which feature is the target
        boolean_features specifies whether we want to create Boolean
            features
            (if False, it uses the original features).
        categorical is a set (or list) of features that should be treated
            as categorical
        target_type is either None for automatic detection of target type
             or one of "numeric", "boolean", "cartegorical"
        include_only is a list or set of indexes of columns to include
        """
        self.boolean_features = boolean_features
        with open(file_name,'r',newline='') as csvfile:
            self.display(1,"Loading",file_name)
            # data_all = csv.reader(csvfile,delimiter=separator) # for more
                complicated CSV files
            data_all = (line.strip().split(separator) for line in csvfile)
            if include_only is not None:
                data_all = ([v for (i,v) in enumerate(line) if i in
                    include_only]
                            for line in data_all)
            if has_header:
                header = next(data_all)
            else:
                header = None
            data_tuples = (interpret_elements(d) for d in data_all if
                len(d)>1)
            if num_train is not None:
                # training set is divided into training then text examples
                # the file is only read once, and the data is placed in
                    appropriate list
                train = []
                for i in range(num_train):  # will give an error if
                    insufficient examples
                    train.append(next(data_tuples))
                test = list(data_tuples)
                Data_set.__init__(self,train, test=test,
                    target_index=target_index,header=header)
            else:    # randomly assign training and test examples
                Data_set.__init__(self,data_tuples, test=None,
                    prob_test=prob_test,
```

```
271                                 target_index=target_index, header=header,
                                        seed=seed, target_type=target_type)
```

The following class is used for datasets where the training and test are in different files

learnProblem.py — (continued)

```
273 class Data_from_files(Data_set):
274     def __init__(self, train_file_name, test_file_name, separator=',',
275                  has_header=False, target_index=0, boolean_features=True,
276                  categorical=[], target_type= None, include_only=None):
277         """create a dataset from separate training and file
278         separator is the character that separates the attributes
279         num_train is a number specifying the first num_train tuples are
                training, or None
280         prob_test is the probability an example should in the test set (if
                num_train is None)
281         has_header is True if the first line of file is a header
282         target_index specifies which feature is the target
283         boolean_features specifies whether we want to create Boolean
                features
284            (if False, it uses the original features).
285         categorical is a set (or list) of features that should be treated
                as categorical
286         target_type is either None for automatic detection of target type
287             or one of "numeric", "boolean", "cartegorical"
288         include_only is a list or set of indexes of columns to include
289         """
290         self.boolean_features = boolean_features
291         with open(train_file_name,'r',newline='') as train_file:
292           with open(test_file_name,'r',newline='') as test_file:
293             # data_all = csv.reader(csvfile,delimiter=separator) # for more
                    complicated CSV files
294             train_data = (line.strip().split(separator) for line in
                    train_file)
295             test_data = (line.strip().split(separator) for line in
                    test_file)
296             if include_only is not None:
297                 train_data = ([v for (i,v) in enumerate(line) if i in
                        include_only]
298                               for line in train_data)
299                 test_data = ([v for (i,v) in enumerate(line) if i in
                        include_only]
300                               for line in test_data)
301             if has_header: # this assumes the training file has a header
                    and the test file doesn't
302                 header = next(train_data)
303             else:
304                 header = None
305             train_tuples = [interpret_elements(d) for d in train_data if
                    len(d)>1]
```

```
306             test_tuples = [interpret_elements(d) for d in test_data if
                    len(d)>1]
307             Data_set.__init__(self,train_tuples, test_tuples,
308                               target_index=target_index, header=header)
```

When reading from a file all of the values are strings. This next method tries to convert each values into a number (an int or a float) or Boolean, if it is possible.

learnProblem.py — (continued)

```
310 def interpret_elements(str_list):
311     """make the elements of string list str_list numeric if possible.
312     Otherwise remove initial and trailing spaces.
313     """
314     res = []
315     for e in str_list:
316         try:
317             res.append(int(e))
318         except ValueError:
319             try:
320                 res.append(float(e))
321             except ValueError:
322                 se = e.strip()
323                 if se in ["True","true","TRUE"]:
324                     res.append[True]
325                 if se in ["False","false","FALSE"]:
326                     res.append[False]
327                 else:
328                     res.append(e.strip())
329     return res
```

7.1.5 Augmented Features

Sometimes we want to augment the features with new features computed from the old features (eg. the product of features). Here we allow the creation of a new dataset from an old dataset but with new features. Note that special cases of these are **kernel**s; mapping the original feature space into a new space, which allow a neat way to do learning in the augmented space for many mappings (the "kernel trick"). This is beyond the scope of AIPython; those interested should read about support vector machines.

A feature is a function of examples. A unary feature constructor takes a feature and returns a new feature. A binary feature combiner takes two features and returns a new feature.

learnProblem.py — (continued)

```
331 class Data_set_augmented(Data_set):
332     def __init__(self, dataset, unary_functions=[], binary_functions=[],
            include_orig=True):
333         """creates a dataset like dataset but with new features
```

```
334         unary_function is a list of unary feature constructors
335         binary_functions is a list of binary feature combiners.
336         include_orig specifies whether the original features should be
                included
337         """
338         self.orig_dataset = dataset
339         self.unary_functions = unary_functions
340         self.binary_functions = binary_functions
341         self.include_orig = include_orig
342         self.target = dataset.target
343         Data_set.__init__(self,dataset.train, test=dataset.test,
344                           target_index = dataset.target_index)
345
346     def create_features(self):
347         if self.include_orig:
348             self.input_features = self.orig_dataset.input_features.copy()
349         else:
350             self.input_features = []
351         for u in self.unary_functions:
352             for f in self.orig_dataset.input_features:
353                 self.input_features.append(u(f))
354         for b in self.binary_functions:
355             for f1 in self.orig_dataset.input_features:
356                 for f2 in self.orig_dataset.input_features:
357                     if f1 != f2:
358                         self.input_features.append(b(f1,f2))
```

The following are useful unary feature constructors and binary feature combiner.

learnProblem.py — (continued)

```
360 def square(f):
361     """a unary feature constructor to construct the square of a feature
362     """
363     def sq(e):
364         return f(e)**2
365     sq.__doc__ = f.__doc__+"**2"
366     return sq
367
368 def power_feat(n):
369     """given n returns a unary feature constructor to construct the nth
            power of a feature.
370     e.g., power_feat(2) is the same as square, defined above
371     """
372     def fn(f,n=n):
373         def pow(e,n=n):
374             return f(e)**n
375         pow.__doc__ = f.__doc__+"**"+str(n)
376         return pow
377     return fn
378
```

```
def prod_feat(f1,f2):
    """a new feature that is the product of features f1 and f2
    """
    def feat(e):
        return f1(e)*f2(e)
    feat.__doc__ = f1.__doc__+"*"+f2.__doc__
    return feat

def eq_feat(f1,f2):
    """a new feature that is 1 if f1 and f2 give same value
    """
    def feat(e):
        return 1 if f1(e)==f2(e) else 0
    feat.__doc__ = f1.__doc__+"=="+f2.__doc__
    return feat

def neq_feat(f1,f2):
    """a new feature that is 1 if f1 and f2 give different values
    """
    def feat(e):
        return 1 if f1(e)!=f2(e) else 0
    feat.__doc__ = f1.__doc__+"!="+f2.__doc__
    return feat
```

Example:

learnProblem.py — (continued)

```
# from learnProblem import Data_set_augmented,prod_feat
# data = Data_from_file('data/holiday.csv', has_header=True, num_train=19,
    target_index=-1)
# data = Data_from_file('data/iris.data', prob_test=1/3, target_index=-1)
## Data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0)
# dataplus = Data_set_augmented(data,[],[prod_feat])
# dataplus = Data_set_augmented(data,[],[prod_feat,neq_feat])
```

Exercise 7.3 For symmetric properties, such as product, we don't need both $f1 * f2$ as well as $f2 * f1$ as extra properties. Allow the user to be able to declare feature constructors as symmetric (by associating a Boolean feature with them). Change *construct_features* so that it does not create both versions for symmetric combiners.

7.2 Generic Learner Interface

A **learner** takes a dataset (and possibly other arguments specific to the method). To get it to learn, we call the *learn*() method. This implements *Displayable* so that we can display traces at multiple levels of detail (and perhaps with a GUI).

learnProblem.py — (continued)

```
409 from display import Displayable
410
411 class Learner(Displayable):
412     def __init__(self, dataset):
413         raise NotImplementedError("Learner.__init__") # abstract method
414
415     def learn(self):
416         """returns a predictor, a function from a tuple to a value for the
                target feature
417         """
418         raise NotImplementedError("learn") # abstract method
```

7.3 Learning With No Input Features

If we make the same prediction for each example, what prediction should we make? This can be used as a naive baseline; if a more sophisticated method does not do better than this, it is not useful. This also provides the base case for some methods, such as decision-tree learning.

> To run demo to compare different prediction methods on various evaluation criteria, in folder "aipython", load "learnNoInputs.py", using e.g., `ipython -i learnNoInputs.py`, and it prints some test results.

There are a few alternatives as to what could be allowed in a prediction:

- a point prediction, where we are only allowed to predict one of the values of the feature. For example, if the values of the feature are $\{0, 1\}$ we are only allowed to predict 0 or 1 or of the values are ratings in $\{1, 2, 3, 4, 5\}$, we can only predict one of these integers.

- a point prediction, where we are allowed to predict any value. For example, if the values of the feature are $\{0, 1\}$ we may be allowed to predict 0.3, 1, or even 1.7. For all of the criteria we can imagine, there is no point in predicting a value greater than 1 or less that zero (but that doesn't mean we can't), but it is often useful to predict a value between 0 and 1. If the values are ratings in $\{1, 2, 3, 4, 5\}$, we may want to predict 3.4.

- a probability distribution over the values of the feature. For each value v, we predict a non-negative number p_v, such that the sum over all predictions is 1.

For regression, we do the first of these. For classification, we do the second. The third can be implemented by having multiple indicator functions for the target.

Here are some prediction functions that take in an enumeration of values, a domain, and returns a value or dictionary of $\{value : prediction\}$. Note that

cmedian returns one of middle values when there are an even number of examples, whereas median gives the average of them (and so cmedian is applicable for ordinals that cannot be considered cardinal values). Similarly, cmode picks one of the values when more than one value has the maximum number of elements.

learnNoInputs.py — Learning ignoring all input features

```
from learnProblem import Evaluate
import math, random, collections, statistics
import utilities # argmax for (element,value) pairs

class Predict(object):
    """The class of prediction methods for a list of values.
    Please make the doc strings the same length, because they are used in tables.
    Note that we don't need self argument, as we are creating Predict objects,
    To use call Predict.laplace(data) etc."""

    ### The following return a distribution over values (for classification)
    def empirical(data, domain=[0,1], icount=0):
        "empirical dist "
        # returns a distribution over values
        counts = {v:icount for v in domain}
        for e in data:
            counts[e] += 1
        s = sum(counts.values())
        return {k:v/s for (k,v) in counts.items()}

    def bounded_empirical(data, domain=[0,1], bound=0.01):
        "bounded empirical"
        return {k:min(max(v,bound),1-bound) for (k,v) in
            Predict.empirical(data, domain).items()}

    def laplace(data, domain=[0,1]):
        "Laplace       "  # for categorical data
        return Predict.empirical(data, domain, icount=1)

    def cmode(data, domain=[0,1]):
        "mode          " # for categorical data
        md = statistics.mode(data)
        return {v: 1 if v==md else 0 for v in domain}

    def cmedian(data, domain=[0,1]):
        "median        " # for categorical data
        md = statistics.median_low(data) # always return one of the values
        return {v: 1 if v==md else 0 for v in domain}

    ### The following return a single prediction (for regression). domain is ignored.
```

```
50
51      def mean(data, domain=[0,1]):
52          "mean          "
53          # returns a real number
54          return statistics.mean(data)
55
56      def rmean(data, domain=[0,1], mean0=0, pseudo_count=1):
57          "regularized mean"
58          # returns a real number.
59          # mean0 is the mean to be used for 0 data points
60          # With mean0=0.5, pseudo_count=2, same as laplace for [0,1] data
61          # this works for enumerations as well as lists
62          sum = mean0 * pseudo_count
63          count = pseudo_count
64          for e in data:
65              sum += e
66              count += 1
67          return sum/count
68
69      def mode(data, domain=[0,1]):
70          "mode          "
71          return statistics.mode(data)
72
73      def median(data, domain=[0,1]):
74          "median        "
75          return statistics.median(data)
76
77      all = [empirical, mean, rmean, bounded_empirical, laplace, cmode, mode,
            median,cmedian]
78
79      # The following suggests appropriate predictions as a function of the
            target type
80      select = {"boolean": [empirical, bounded_empirical, laplace, cmode,
            cmedian],
81                "categorical": [empirical, bounded_empirical, laplace, cmode,
                      cmedian],
82                "numeric": [mean, rmean, mode, median]}
```

7.3.1 Evaluation

To evaluate a point prediction, we first generate some data from a simple (Bernoulli) distribution, where there are two possible values, 0 and 1 for the target feature. Given *prob*, a number in the range $[0, 1]$, this generate some training and test data where *prob* is the probability of each example being 1. To generate a 1 with probability *prob*, we generate a random number in range [0,1] and return 1 if that number is less than *prob*. A prediction is computed by applying the predictor to the training data, which is evaluated on the test set. This is repeated `num_samples` times.

Let's evaluate the predictions of the possible selections according to the different evaluation criteria, for various training sizes.

learnNoInputs.py — (continued)

```
def test_no_inputs(error_measures = Evaluate.all_criteria,
    num_samples=10000, test_size=10 ):
    for train_size in [1,2,3,4,5,10,20,100,1000]:
        results = {predictor: {error_measure: 0 for error_measure in
            error_measures}
                   for predictor in Predict.all}
        for sample in range(num_samples):
            prob = random.random()
            training = [1 if random.random()<prob else 0 for i in
                range(train_size)]
            test = [1 if random.random()<prob else 0 for i in
                range(test_size)]
            for predictor in Predict.all:
                prediction = predictor(training)
                for error_measure in error_measures:
                    results[predictor][error_measure] += sum(
                        error_measure(prediction,actual) for actual in
                        test)/test_size
        print(f"For training size {train_size}:")
        print("  Predictor\t","\t".join(error_measure.__doc__ for
                                        error_measure in
                                            error_measures),sep="\t")
        for predictor in Predict.all:
            print(f"  {predictor.__doc__}",
                  "\t".join("{:.7f}".format(results[predictor][error_measure]/num_samples)
                                for error_measure in
                                    error_measures),sep="\t")

if __name__ == "__main__":
    test_no_inputs()
```

Exercise 7.4 Which predictor works best for low counts when the error is

(a) Squared error

(b) Absolute error

(c) Log loss

You may need to try this a few times to make sure your answer is supported by the evidence. Does the difference from the other methods get more or less as the number of examples grow?

Exercise 7.5 Suggest some other predictions that only take the training data. Does your method do better than the given methods? A simple way to get other predictors is to vary the threshold of bounded average, or to change the pseodo-counts of the Laplace method (use other numbers instead of 1 and 2).

7.4 Decision Tree Learning

To run the decision tree learning demo, in folder "aipython", load "learnDT.py", using e.g., `ipython -i learnDT.py`, and it prints some test results. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

The decision tree algorithm does binary splits, and assumes that all input features are binary functions of the examples. It stops splitting if there are no input features, the number of examples is less than a specified number of examples or all of the examples agree on the target feature.

learnDT.py — Learning a binary decision tree

```
11 from learnProblem import Learner, Evaluate
12 from learnNoInputs import Predict
13 import math
14
15 class DT_learner(Learner):
16     def __init__(self,
17                  dataset,
18                  split_to_optimize=Evaluate.log_loss, # to minimize for at
                        each split
19                  leaf_prediction=Predict.empirical, # what to use for value
                        at leaves
20                  train=None,                  # used for cross validation
21                  max_num_cuts=8, # maximum number of conditions to split a
                        numeric feature into
22                  gamma=1e-7 , # minimum improvement needed to expand a node
23                  min_child_weight=10):
24         self.dataset = dataset
25         self.target = dataset.target
26         self.split_to_optimize = split_to_optimize
27         self.leaf_prediction = leaf_prediction
28         self.max_num_cuts = max_num_cuts
29         self.gamma = gamma
30         self.min_child_weight = min_child_weight
31         if train is None:
32             self.train = self.dataset.train
33         else:
34             self.train = train
35
36     def learn(self, max_num_cuts=8):
37         """learn a decision tree"""
38         return self.learn_tree(self.dataset.conditions(self.max_num_cuts),
                    self.train)
```

The main recursive algorithm, takes in a set of input features and a set of training data. It first decides whether to split. If it doesn't split, it makes a point prediction, ignoring the input features.

It only splits if the best split increases the error by at least gamma. This implies it does not split when:

- there are no more input features
- there are fewer examples than *min_number_examples*,
- all the examples agree on the value of the target, or
- the best split makes all examples in the same partition.

If it splits, it selects the best split according to the evaluation criterion (assuming that is the only split it gets to do), and returns the condition to split on (in the variable *split*) and the corresponding partition of the examples.

learnDT.py — (continued)

```
40      def learn_tree(self, conditions, data_subset):
41          """returns a decision tree
42          conditions is a set of possible conditions
43          data_subset is a subset of the data used to build this (sub)tree
44
45          where a decision tree is a function that takes an example and
46          makes a prediction on the target feature
47          """
48          self.display(2,f"learn_tree with {len(conditions)} features and
                {len(data_subset)} examples")
49          split, partn = self.select_split(conditions, data_subset)
50          if split is None: # no split; return a point prediction
51              prediction = self.leaf_value(data_subset, self.target.frange)
52              self.display(2,f"leaf prediction for {len(data_subset)}
                    examples is {prediction}")
53              def leaf_fun(e):
54                  return prediction
55              leaf_fun.__doc__ = str(prediction)
56              leaf_fun.num_leaves = 1
57              return leaf_fun
58          else:  # a split succeeded
59              false_examples, true_examples = partn
60              rem_features = [fe for fe in conditions if fe != split]
61              self.display(2,"Splitting on",split.__doc__,"with examples
                    split",
62                           len(true_examples),":",len(false_examples))
63              true_tree = self.learn_tree(rem_features,true_examples)
64              false_tree = self.learn_tree(rem_features,false_examples)
65              def fun(e):
66                  if split(e):
67                      return true_tree(e)
68                  else:
69                      return false_tree(e)
70              #fun = lambda e: true_tree(e) if split(e) else false_tree(e)
71              fun.__doc__ = (f"(if {split.__doc__} then {true_tree.__doc__}"
```

```
                        f" else {false_tree.__doc__})")
            fun.num_leaves = true_tree.num_leaves + false_tree.num_leaves
            return fun
```

learnDT.py — (continued)

```
    def leaf_value(self, egs, domain):
        return self.leaf_prediction((self.target(e) for e in egs), domain)

    def select_split(self, conditions, data_subset):
        """finds best feature to split on.

        conditions is a non-empty list of features.
        returns feature, partition
        where feature is an input feature with the smallest error as
              judged by split_to_optimize or
              feature==None if there are no splits that improve the error
        partition is a pair (false_examples, true_examples) if feature is
            not None
        """
        best_feat = None # best feature
        # best_error = float("inf") # infinity - more than any error
        best_error = self.sum_losses(data_subset) - self.gamma
        self.display(3," no split has
            error=",best_error,"with",len(conditions),"conditions")
        best_partition = None
        for feat in conditions:
            false_examples, true_examples = partition(data_subset,feat)
            if
                min(len(false_examples),len(true_examples))>=self.min_child_weight:
                err = (self.sum_losses(false_examples)
                       + self.sum_losses(true_examples))
                self.display(3," split on",feat.__doc__,"has error=",err,
                             "splits
                                 into",len(true_examples),":",len(false_examples),"gamma=",self.gamma)
                if err < best_error:
                    best_feat = feat
                    best_error=err
                    best_partition = false_examples, true_examples
        self.display(2,"best split is on",best_feat.__doc__,
                               "with err=",best_error)
        return best_feat, best_partition

    def sum_losses(self, data_subset):
        """returns sum of losses for dataset (with no more splits)
        There a single prediction for all leaves using leaf_prediction
        It is evaluated using split_to_optimize
        """
        prediction = self.leaf_value(data_subset, self.target.frange)
        error = sum(self.split_to_optimize(prediction, self.target(e))
                    for e in data_subset)
```

```
117         return error
118
119 def partition(data_subset,feature):
120     """partitions the data_subset by the feature"""
121     true_examples = []
122     false_examples = []
123     for example in data_subset:
124         if feature(example):
125             true_examples.append(example)
126         else:
127             false_examples.append(example)
128     return false_examples, true_examples
```

Test cases:

learnDT.py — (continued)

```
131 from learnProblem import Data_set, Data_from_file
132
133 def testDT(data, print_tree=True, selections = None, **tree_args):
134     """Prints errors and the trees for various evaluation criteria and ways
            to select leaves.
135     """
136     if selections == None: # use selections suitable for target type
137         selections = Predict.select[data.target.ftype]
138     evaluation_criteria = Evaluate.all_criteria
139     print("Split Choice","Leaf Choice\t","#leaves",'\t'.join(ecrit.__doc__
140                                             for ecrit in
                                                    evaluation_criteria),sep="\t")
141     for crit in evaluation_criteria:
142         for leaf in selections:
143             tree = DT_learner(data, split_to_optimize=crit,
                        leaf_prediction=leaf,
144                               **tree_args).learn()
145             print(crit.__doc__, leaf.__doc__, tree.num_leaves,
146                   "\t".join("{:.7f}".format(data.evaluate_dataset(data.test,
                              tree, ecrit))
147                             for ecrit in evaluation_criteria),sep="\t")
148             if print_tree:
149                 print(tree.__doc__)
150
151 #DT_learner.max_display_level = 4
152 if __name__ == "__main__":
153     # Choose one of the data files
154     #data=Data_from_file('data/SPECT.csv', target_index=0);
            print("SPECT.csv")
155     #data=Data_from_file('data/iris.data', target_index=-1);
            print("iris.data")
156     data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
157     #data = Data_from_file('data/mail_reading.csv', target_index=-1);
            print("mail_reading.csv")
```

```
158     #data = Data_from_file('data/holiday.csv', has_header=True,
            num_train=19, target_index=-1); print("holiday.csv")
159     testDT(data, print_tree=False)
```

Note that different runs may provide different values as they split the training and test sets differently. So if you have a hypothesis about what works better, make sure it is true for different runs.

Exercise 7.6 The current algorithm does not have a very sophisticated stopping criterion. What is the current stopping criterion? (Hint: you need to look at both *learn_tree* and *select_split*.)

Exercise 7.7 Extend the current algorithm to include in the stopping criterion

(a) A minimum child size; don't use a split if one of the children has fewer elements that this.

(b) A depth-bound on the depth of the tree.

(c) An improvement bound such that a split is only carried out if error with the split is better than the error without the split by at least the improvement bound.

Which values for these parameters make the prediction errors on the test set the smallest? Try it on more than one dataset.

Exercise 7.8 Without any input features, it is often better to include a pseudo-count that is added to the counts from the training data. Modify the code so that it includes a pseudo-count for the predictions. When evaluating a split, including pseudo counts can make the split worse than no split. Does pruning with an improvement bound and pseudo-counts make the algorithm work better than with an improvement bound by itself?

Exercise 7.9 Some people have suggested using information gain (which is equivalent to greedy optimization of log loss) as the measure of improvement when building the tree, even in they want to have non-probabilistic predictions in the final tree. Does this work better than myopically choosing the split that is best for the evaluation criteria we will use to judge the final prediction?

7.5 Cross Validation and Parameter Tuning

To run the cross validation demo, in folder "aipython", load "learnCrossValidation.py", using e.g., `ipython -i learnCrossValidation.py`. Run the examples at the end to produce a graph like Figure 7.15. Note that different runs will produce different graphs, so your graph will not look like the one in the textbook. To try more examples, copy and paste the commented-out commands at the bottom of that file. This requires Python 3 with `matplotlib`.

The above decision tree overfits the data. One way to determine whether the prediction is overfitting is by cross validation. The code below implements k-fold cross validation, which can be used to choose the value of parameters to best fit the training data. If we want to use parameter tuning to improve predictions on a particular dataset, we can only use the training data (and not the test data) to tune the parameter.

In k-fold cross validation, we partition the training set into k approximately equal-sized folds (each fold is an enumeration of examples). For each fold, we train on the other examples, and determine the error of the prediction on that fold. For example, if there are 10 folds, we train on 90% of the data, and then test on remaining 10% of the data. We do this 10 times, so that each example gets used as a test set once, and in the training set 9 times.

The code below creates one copy of the data, and multiple views of the data. For each fold, *fold* enumerates the examples in the fold, and *fold_complement* enumerates the examples not in the fold.

learnCrossValidation.py — Cross Validation for Parameter Tuning

```
from learnProblem import Data_set, Data_from_file, Evaluate
from learnNoInputs import Predict
from learnDT import DT_learner
import matplotlib.pyplot as plt
import random

class K_fold_dataset(object):
    def __init__(self, training_set, num_folds):
        self.data = training_set.train.copy()
        self.target = training_set.target
        self.input_features = training_set.input_features
        self.num_folds = num_folds
        self.conditions = training_set.conditions

        random.shuffle(self.data)
        self.fold_boundaries = [(len(self.data)*i)//num_folds
                                for i in range(0,num_folds+1)]

    def fold(self, fold_num):
        for i in range(self.fold_boundaries[fold_num],
                       self.fold_boundaries[fold_num+1]):
            yield self.data[i]

    def fold_complement(self, fold_num):
        for i in range(0,self.fold_boundaries[fold_num]):
            yield self.data[i]
        for i in range(self.fold_boundaries[fold_num+1],len(self.data)):
            yield self.data[i]
```

The validation error is the average error for each example, where we test on each fold, and learn on the other folds.

learnCrossValidation.py — (continued)

```
40      def validation_error(self, learner, error_measure, **other_params):
41          error = 0
42          try:
43              for i in range(self.num_folds):
44                  predictor = learner(self,
                        train=list(self.fold_complement(i)),
45                                  **other_params).learn()
46                  error += sum( error_measure(predictor(e), self.target(e))
47                              for e in self.fold(i))
48          except ValueError:
49              return float("inf") #infinity
50          return error/len(self.data)
```

The *plot_error* method plots the average error as a function of a the minimum number of examples in decision-tree search, both for the validation set and for the test set. The error on the validation set can be used to tune the parameter — choose the value of the parameter that minimizes the error. The error on the test set cannot be used to tune the parameters; if is were to be used this way then it cannot be used to test.

learnCrossValidation.py — (continued)

```
52  def plot_error(data, criterion=Evaluate.squared_loss,
        leaf_prediction=Predict.empirical,
53                  num_folds=5, maxx=None, xscale='linear'):
54      """Plots the error on the validation set and the test set
55      with respect to settings of the minimum number of examples.
56      xscale should be 'log' or 'linear'
57      """
58      plt.ion()
59      plt.xscale(xscale) # change between log and linear scale
60      plt.xlabel("min_child_weight")
61      plt.ylabel("average "+criterion.__doc__)
62      folded_data = K_fold_dataset(data, num_folds)
63      if maxx == None:
64          maxx = len(data.train)//2+1
65      verrors = [] # validation errors
66      terrors = [] # test set errors
67      for mcw in range(1,maxx):
68          verrors.append(folded_data.validation_error(DT_learner,criterion,leaf_prediction=leaf_predi
69                                                      min_child_weight=mcw))
70          tree = DT_learner(data, criterion, leaf_prediction=leaf_prediction,
              min_child_weight=mcw).learn()
71          terrors.append(data.evaluate_dataset(data.test,tree,criterion))
72      plt.plot(range(1,maxx), verrors, ls='-',color='k',
73                  label="validation for "+criterion.__doc__)
74      plt.plot(range(1,maxx), terrors, ls='--',color='k',
75                  label="test set for "+criterion.__doc__)
76      plt.legend()
77      plt.draw()
78
```

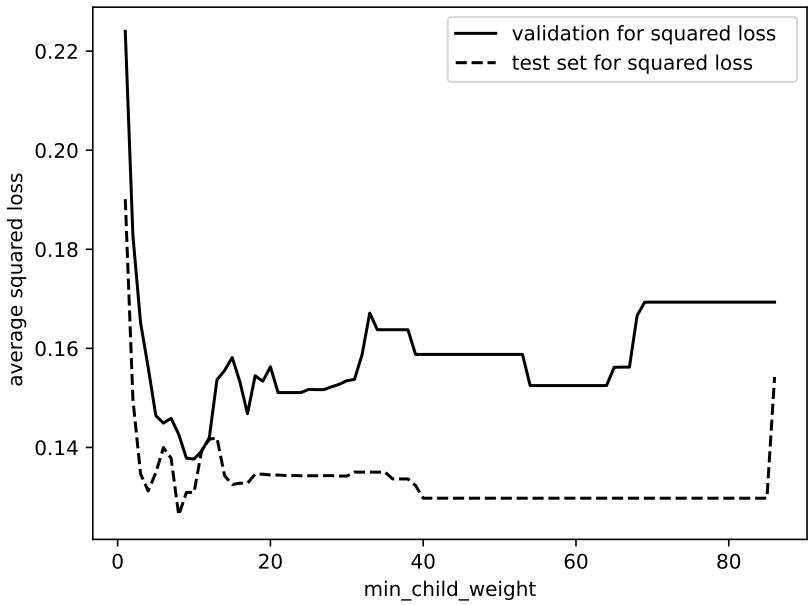


Figure 7.2: plot_error for SPECT dataset

```
79 # The following produces the graphs of Figure 7.18 of Poole and Mackworth
       [2023]
80 # data = Data_from_file('data/SPECT.csv',target_index=0, seed=123)
81 # plot_error(data, criterion=Evaluate.log_loss,
       leaf_prediction=Predict.laplace)
82
83 #also try:
84 # plot_error(data)
85 # data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
```

Figure 7.2 shows the average squared loss in the validation and test sets as a function of the min_child_weight in the decision-tree learning algorithm. (SPECT data with seed 12345 followed by plot_error(data)). Different seeds will produce different graphs. The assumption behind cross vaildation is that the parameter that minimizes the loss on the validation set, will be a good parameter for the test set.

Note that different runs for the same data will have the same test error, but different validation error. If you rerun the Data_from_file, with a different seed, you will get the new test and training sets, and so the graph will change.

Exercise 7.10 Change the error plot so that it can evaluate the stopping criteria of the exercise of Section 7.6. Which criteria makes the most difference?

7.6 Linear Regression and Classification

Here is a stochastic gradient descent searcher for linear regression and classification.

learnLinear.py — Linear Regression and Classification

```
from learnProblem import Learner
import random, math

class Linear_learner(Learner):
    def __init__(self, dataset, train=None,
                 learning_rate=0.1, max_init = 0.2,
                 squashed=True, batch_size=10):
        """Creates a gradient descent searcher for a linear classifier.
        The main learning is carried out by learn()

        dataset provides the target and the input features
        train provides a subset of the training data to use
        number_iterations is the default number of steps of gradient descent
        learning_rate is the gradient descent step size
        max_init is the maximum absolute value of the initial weights
        squashed specifies whether the output is a squashed linear function
        """
        self.dataset = dataset
        self.target = dataset.target
        if train==None:
            self.train = self.dataset.train
        else:
            self.train = train
        self.learning_rate = learning_rate
        self.squashed = squashed
        self.batch_size = batch_size
        self.input_features = [one]+dataset.input_features # one is defined
            below
        self.weights = {feat:random.uniform(-max_init,max_init)
                        for feat in self.input_features}
```

predictor predicts the value of an example from the current parameter settings. *predictor_string* gives a string representation of the predictor.

learnLinear.py — (continued)

```

    def predictor(self,e):
        """returns the prediction of the learner on example e"""
        linpred = sum(w*f(e) for f,w in self.weights.items())
        if self.squashed:
            return sigmoid(linpred)
        else:
            return linpred

    def predictor_string(self, sig_dig=3):
```

```
        """returns the doc string for the current prediction function
        sig_dig is the number of significant digits in the numbers"""
        doc = "+".join(str(round(val,sig_dig))+"*"+feat.__doc__
                       for feat,val in self.weights.items())
        if self.squashed:
            return "sigmoid("+ doc+")"
        else:
            return doc
```

learn is the main algorithm of the learner. It does *num_iter* steps of stochastic gradient descent. Only the number of iterations is specified; the other parameters it gets from the class.

learnLinear.py — (continued)

```
    def learn(self,num_iter=100):
        batch_size = min(self.batch_size, len(self.train))
        d = {feat:0 for feat in self.weights}
        for it in range(num_iter):
            self.display(2,"prediction=",self.predictor_string())
            for e in random.sample(self.train, batch_size):
                error = self.predictor(e) - self.target(e)
                update = self.learning_rate*error
                for feat in self.weights:
                    d[feat] += update*feat(e)
            for feat in self.weights:
                self.weights[feat] -= d[feat]
                d[feat]=0
        return self.predictor
```

one is a function that always returns 1. This is used for one of the input properties.

learnLinear.py — (continued)

```
def one(e):
    "1"
    return 1
```

$sigmoid(x)$ is the function

$$\frac{1}{1+e^{-x}}$$

The inverse of *sigmoid* is the *logit* function

learnLinear.py — (continued)

```
def sigmoid(x):
    return 1/(1+math.exp(-x))

def logit(x):
    return -math.log(1/x-1)
```

$sigmoid([x_0, v_2, \ldots])$ returns $[v_0, v_2, \ldots]$ where

$$v_i = \frac{exp(x_i)}{\sum_j exp(x_j)}$$

The inverse of *sigmoid* is the *logit* function

learnLinear.py — (continued)

```
def softmax(xs,domain=None):
    """xs is a list of values, and
    domain is the domain (a list) or None if the list should be returned
    returns a distribution over the domain (a dict)
    """
    m = max(xs) # use of m prevents overflow (and all values underflowing)
    exps = [math.exp(x-m) for x in xs]
    s = sum(exps)
    if domain:
        return {d:v/s for (d,v) in zip(domain,exps)}
    else:
        return [v/s for v in exps]

def indicator(v, domain):
    return [1 if v==dv else 0 for dv in domain]
```

The following tests the learner on a datasets. Uncomment the other datasets for different examples.

learnLinear.py — (continued)

```
from learnProblem import Data_set, Data_from_file, Evaluate
from learnProblem import Evaluate
import matplotlib.pyplot as plt

def test(**args):
    data = Data_from_file('data/SPECT.csv', target_index=0)
    # data = Data_from_file('data/mail_reading.csv', target_index=-1)
    # data = Data_from_file('data/carbool.csv', target_index=-1)
    learner = Linear_learner(data,**args)
    learner.learn()
    print("function learned is", learner.predictor_string())
    for ecrit in Evaluate.all_criteria:
        test_error = data.evaluate_dataset(data.test, learner.predictor,
            ecrit)
        print("   Average", ecrit.__doc__, "is", test_error)
```

The following plots the errors on the training and test sets as a function of the number of steps of gradient descent.

learnLinear.py — (continued)

```
def plot_steps(learner=None,
               data = None,
               criterion=Evaluate.squared_loss,
```

```
            step=1,
            num_steps=1000,
            log_scale=True,
            legend_label=""):
    """
    plots the training and test error for a learner.
    data is the
    learner_class is the class of the learning algorithm
    criterion gives the evaluation criterion plotted on the y-axis
    step specifies how many steps are run for each point on the plot
    num_steps is the number of points to plot

    """
    if legend_label != "": legend_label+=" "
    plt.ion()
    plt.xlabel("step")
    plt.ylabel("Average "+criterion.__doc__)
    if log_scale:
        plt.xscale('log') #plt.semilogx() #Makes a log scale
    else:
        plt.xscale('linear')
    if data is None:
        data = Data_from_file('data/holiday.csv', has_header=True,
            num_train=19, target_index=-1)
        #data = Data_from_file('data/SPECT.csv', target_index=0)
        # data = Data_from_file('data/mail_reading.csv', target_index=-1)
        # data = Data_from_file('data/carbool.csv', target_index=-1)
    #random.seed(None)  # reset seed
    if learner is None:
        learner = Linear_learner(data)
    train_errors = []
    test_errors = []
    for i in range(1,num_steps+1,step):
        test_errors.append(data.evaluate_dataset(data.test,
            learner.predictor, criterion))
        train_errors.append(data.evaluate_dataset(data.train,
            learner.predictor, criterion))
        learner.display(2, "Train error:",train_errors[-1],
                        "Test error:",test_errors[-1])
        learner.learn(num_iter=step)
    plt.plot(range(1,num_steps+1,step),train_errors,ls='-',label=legend_label+"training")
    plt.plot(range(1,num_steps+1,step),test_errors,ls='--',label=legend_label+"test")
    plt.legend()
    plt.draw()
    learner.display(1, "Train error:",train_errors[-1],
                        "Test error:",test_errors[-1])

if __name__ == "__main__":
    test()

```

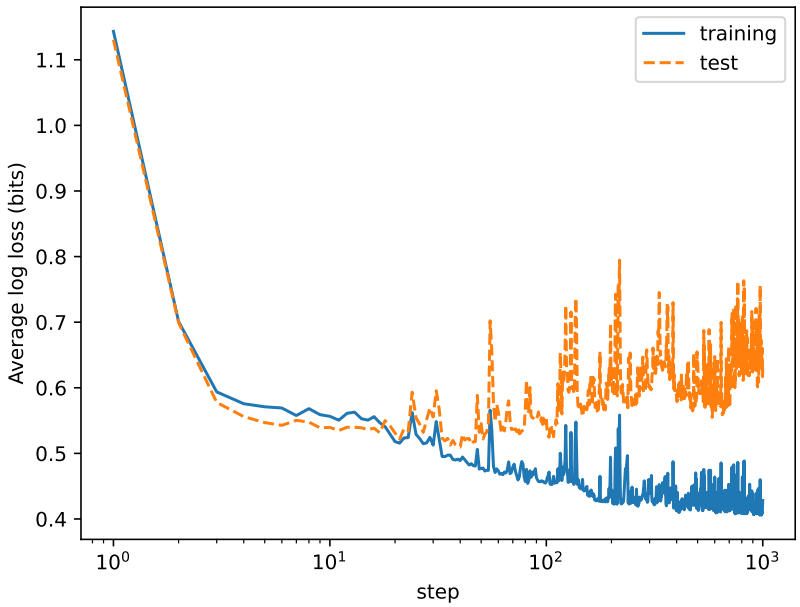


Figure 7.3: plot_steps for SPECT dataset

```
# This generates the figure
# from learnProblem import Data_set_augmented, prod_feat
# data = Data_from_file('data/SPECT.csv', prob_test=0.5, target_index=0,
    seed=123)
# dataplus = Data_set_augmented(data, [], [prod_feat])
# plot_steps(data=data, num_steps=1000)
# plot_steps(data=dataplus, num_steps=1000) # warning very slow
```

Figure 7.3 shows the result of `plot_steps(data=data, num_steps=1000)` in the code above. What would you expect to happen with the augmented data (with extra features)? Hint: think about underitting and overfitting.

Exercise 7.11 The squashed learner only makes predictions in the range $(0,1)$. If the output values are $\{1,2,3,4\}$ there is no use prediction less than 1 or greater than 4. Change the squashed learner so that it can learn values in the range $(1,4)$. Test it on the file `'data/car.csv'`.

The following plots the prediction as a function of the function of the number of steps of gradient descent. We first define a version of *range* that allows for real numbers (integers and floats).

learnLinear.py — (continued)

```
def arange(start,stop,step):
    """returns enumeration of values in the range [start,stop) separated by
        step.
```

```
    like the built-in range(start,stop,step) but allows for integers and
        floats.
    Note that rounding errors are expected with real numbers. (or use
        numpy.arange)
    """
    while start<stop:
        yield start
        start += step

def plot_prediction(data,
                learner = None,
                minx = 0,
                maxx = 5,
                step_size = 0.01,  # for plotting
                label = "function"):
    plt.ion()
    plt.xlabel("x")
    plt.ylabel("y")
    if learner is None:
        learner = Linear_learner(data, squashed=False)
    learner.learning_rate=0.001
    learner.learn(100)
    learner.learning_rate=0.0001
    learner.learn(1000)
    learner.learning_rate=0.00001
    learner.learn(10000)
    learner.display(1,"function learned is", learner.predictor_string(),
              "error=",data.evaluate_dataset(data.train, learner.predictor,
                  Evaluate.squared_loss))
    plt.plot([e[0] for e in data.train],[e[-1] for e in
        data.train],"bo",label="data")
    plt.plot(list(arange(minx,maxx,step_size)),[learner.predictor([x])
                                        for x in
                                            arange(minx,maxx,step_size)],
                                      label=label)
    plt.legend()
    plt.draw()
```

learnLinear.py — (continued)

```
from learnProblem import Data_set_augmented, power_feat
def plot_polynomials(data,
                learner_class = Linear_learner,
                max_degree = 5,
                minx = 0,
                maxx = 5,
                num_iter = 1000000,
                learning_rate = 0.00001,
                step_size = 0.01, # for plotting
                ):
    plt.ion()
```

```
218     plt.xlabel("x")
219     plt.ylabel("y")
220     plt.plot([e[0] for e in data.train],[e[-1] for e in
            data.train],"ko",label="data")
221     x_values = list(arange(minx,maxx,step_size))
222     line_styles = ['-','--','-.',':']
223     colors = ['0.5','k','k','k','k']
224     for degree in range(max_degree):
225         data_aug = Data_set_augmented(data,[power_feat(n) for n in
                range(1,degree+1)],
226                                       include_orig=False)
227         learner = learner_class(data_aug,squashed=False)
228         learner.learning_rate = learning_rate
229         learner.learn(num_iter)
230         learner.display(1,"For degree",degree,
231                         "function learned is", learner.predictor_string(),
232                         "error=",data.evaluate_dataset(data.train,
                            learner.predictor, Evaluate.squared_loss))
233         ls = line_styles[degree % len(line_styles)]
234         col = colors[degree % len(colors)]
235         plt.plot(x_values,[learner.predictor([x]) for x in x_values],
                linestyle=ls, color=col,
236                          label="degree="+str(degree))
237         plt.legend(loc='upper left')
238         plt.draw()
239
240 # Try:
241 # data0 = Data_from_file('data/simp_regr.csv', prob_test=0,
        boolean_features=False, target_index=-1)
242 # plot_prediction(data0)
243 # plot_polynomials(data0)
244 # What if the step size was bigger?
245 #datam = Data_from_file('data/mail_reading.csv', target_index=-1)
246 #plot_prediction(datam)
```

7.7 Boosting

The following code implements functional gradient boosting for regression.

A Boosted dataset is created from a base dataset by subtracting the prediction of the offset function from each example. This does not save the new dataset, but generates it as needed. The amount of space used is constant, independent on the size of the dataset.

learnBoosting.py — Functional Gradient Boosting

```
11 from learnProblem import Data_set, Learner, Evaluate
12 from learnNoInputs import Predict
13 from learnLinear import sigmoid
14 import statistics
15 import random
```

```
16
17 class Boosted_dataset(Data_set):
18     def __init__(self, base_dataset, offset_fun, subsample=1.0):
19         """new dataset which is like base_dataset,
20            but offset_fun(e) is subtracted from the target of each example e
21         """
22         self.base_dataset = base_dataset
23         self.offset_fun = offset_fun
24         self.train =
               random.sample(base_dataset.train,int(subsample*len(base_dataset.train)))
25         self.test = base_dataset.test
26         #Data_set.__init__(self, base_dataset.train, base_dataset.test,
27         #                  base_dataset.prob_test, base_dataset.target_index)
28
29         #def create_features(self):
30         """creates new features - called at end of Data_set.init()
31         defines a new target
32         """
33         self.input_features = self.base_dataset.input_features
34         def newout(e):
35             return self.base_dataset.target(e) - self.offset_fun(e)
36         newout.frange = self.base_dataset.target.frange
37         newout.ftype = self.infer_type(newout.frange)
38         self.target = newout
39
40     def conditions(self, *args, colsample_bytree=0.5, **nargs):
41         conds = self.base_dataset.conditions(*args, **nargs)
42         return random.sample(conds, int(colsample_bytree*len(conds)))
```

A boosting learner takes in a dataset and a base learner, and returns a new predictor. The base learner, takes a dataset, and returns a Learner object.

learnBoosting.py — (continued)

```
44 class Boosting_learner(Learner):
45     def __init__(self, dataset, base_learner_class, subsample=0.8):
46         self.dataset = dataset
47         self.base_learner_class = base_learner_class
48         self.subsample = subsample
49         mean = sum(self.dataset.target(e)
50                    for e in self.dataset.train)/len(self.dataset.train)
51         self.predictor = lambda e:mean  # function that returns mean for
               each example
52         self.predictor.__doc__ = "lambda e:"+str(mean)
53         self.offsets = [self.predictor] # list of base learners
54         self.predictors = [self.predictor] # list of predictors
55         self.errors = [data.evaluate_dataset(data.test, self.predictor,
               Evaluate.squared_loss)]
56         self.display(1,"Predict mean test set mean squared loss=",
               self.errors[0] )
57
58
```

```
59      def learn(self, num_ensembles=10):
60          """adds num_ensemble learners to the ensemble.
61          returns a new predictor.
62          """
63          for i in range(num_ensembles):
64              train_subset = Boosted_dataset(self.dataset, self.predictor,
                    subsample=self.subsample)
65              learner = self.base_learner_class(train_subset)
66              new_offset = learner.learn()
67              self.offsets.append(new_offset)
68              def new_pred(e, old_pred=self.predictor, off=new_offset):
69                  return old_pred(e)+off(e)
70              self.predictor = new_pred
71              self.predictors.append(new_pred)
72              self.errors.append(data.evaluate_dataset(data.test,
                    self.predictor, Evaluate.squared_loss))
73              self.display(1,f"Iteration {len(self.offsets)-1},treesize =
                    {new_offset.num_leaves}. mean squared
                    loss={self.errors[-1]}")
74          return self.predictor
```

For testing, *sp_DT_learner* returns a learner that predicts the mean at the leaves and is evaluated using squared loss. It can also take arguments to change the default arguments for the trees.

learnBoosting.py — (continued)

```
76  # Testing
77
78  from learnDT import DT_learner
79  from learnProblem import Data_set, Data_from_file
80
81  def sp_DT_learner(split_to_optimize=Evaluate.squared_loss,
82                            leaf_prediction=Predict.mean,**nargs):
83      """Creates a learner with different default arguments replaced by
            **nargs
84      """
85      def new_learner(dataset):
86          return DT_learner(dataset,split_to_optimize=split_to_optimize,
87                                leaf_prediction=leaf_prediction, **nargs)
88      return new_learner
89
90  #data = Data_from_file('data/car.csv', target_index=-1) regression
91  data = Data_from_file('data/student/student-mat-nq.csv',
        separator=';',has_header=True,target_index=-1,seed=13,include_only=list(range(30))+[32])
        #2.0537973790924946
92  #data = Data_from_file('data/SPECT.csv', target_index=0, seed=62) #123)
93  #data = Data_from_file('data/mail_reading.csv', target_index=-1)
94  #data = Data_from_file('data/holiday.csv', has_header=True, num_train=19,
        target_index=-1)
95  #learner10 = Boosting_learner(data,
        sp_DT_learner(split_to_optimize=Evaluate.squared_loss,
```

```
    leaf_prediction=Predict.mean, min_child_weight=10))
#learner7 = Boosting_learner(data, sp_DT_learner(0.7))
#learner5 = Boosting_learner(data, sp_DT_learner(0.5))
#predictor9 =learner9.learn(10)
#for i in learner9.offsets: print(i.__doc__)
import matplotlib.pyplot as plt

def plot_boosting_trees(data, steps=10, mcws=[30,20,20,10], gammas=
    [100,200,300,500]):
    # to reduce clutter uncomment one of following two lines
    #mcws=[10]
    #gammas=[200]
    learners = [(mcw, gamma, Boosting_learner(data,
        sp_DT_learner(min_child_weight=mcw, gamma=gamma)))
                for gamma in gammas for mcw in mcws
                ]
    plt.ion()
    plt.xscale('linear') # change between log and linear scale
    plt.xlabel("number of trees")
    plt.ylabel("mean squared loss")
    markers = (m+c for c in ['k','g','r','b','m','c','y'] for m in
        ['-','--','-.',':'])
    for (mcw,gamma,learner) in learners:
        data.display(1,f"min_child_weight={mcw}, gamma={gamma}")
        learner.learn(steps)
        plt.plot(range(steps+1), learner.errors, next(markers),
                 label=f"min_child_weight={mcw}, gamma={gamma}")
    plt.legend()
    plt.draw()

# plot_boosting_trees(data)
```

7.7.1 Gradient Tree Boosting

The following implements gradient Boosted trees for classification. If you want to use this gradient tree boosting for a real problem, we recommend using **XGBoost** [Chen and Guestrin, 2016] or **LightGBM** [Ke, Meng, Finley, Wang, Chen, Ma, Ye, and Liu, 2017].

`GTB_learner` subclasses `DT-learner`. The method `learn_tree` is used unchanged. `DT-learner` assumes that the value at the leaf is the prediction of the leaf, thus `leaf_value` needs to be overridden. It also assumes that all nodes at a leaf have the same prediction, but in GBT the elements of a leaf can have different values, depending on the previous trees. Thus `sum_losses` also needs to be overridden.

learnBoosting.py — (continued)

```
class GTB_learner(DT_learner):
    def __init__(self, dataset, number_trees, lambda_reg=1, gamma=0,
        **dtargs):
```

```
126         DT_learner.__init__(self, dataset,
                split_to_optimize=Evaluate.log_loss, **dtargs)
127         self.number_trees = number_trees
128         self.lambda_reg = lambda_reg
129         self.gamma = gamma
130         self.trees = []
131
132     def learn(self):
133         for i in range(self.number_trees):
134             tree =
                    self.learn_tree(self.dataset.conditions(self.max_num_cuts),
                    self.train)
135             self.trees.append(tree)
136             self.display(1,f"""Iteration {i} treesize = {tree.num_leaves}
                    train logloss={
137                 self.dataset.evaluate_dataset(self.dataset.train,
                        self.gtb_predictor, Evaluate.log_loss)
138                     } test logloss={
139                 self.dataset.evaluate_dataset(self.dataset.test,
                        self.gtb_predictor, Evaluate.log_loss)}""")
140         return self.gtb_predictor
141
142     def gtb_predictor(self, example, extra=0):
143         """prediction for example,
144         extras is an extra contribution for this example being considered
145         """
146         return sigmoid(sum(t(example) for t in self.trees)+extra)
147
148     def leaf_value(self, egs, domain=[0,1]):
149         """value at the leaves for examples egs
150         domain argument is ignored"""
151         pred_acts = [(self.gtb_predictor(e),self.target(e)) for e in egs]
152         return sum(a-p for (p,a) in pred_acts) /(sum(p*(1-p) for (p,a) in
                pred_acts)+self.lambda_reg)
153
154
155     def sum_losses(self, data_subset):
156         """returns sum of losses for dataset (assuming a leaf is formed
                with no more splits)
157         """
158         leaf_val = self.leaf_value(data_subset)
159         error = sum(Evaluate.log_loss(self.gtb_predictor(e,leaf_val),
                    self.target(e))
160                     for e in data_subset) + self.gamma
161         return error
```

Testing

learnBoosting.py — (continued)

```
163 # data = Data_from_file('data/carbool.csv', target_index=-1, seed=123)
164 # gtb_learner = GTB_learner(data, 10)
```

```
165 # gtb_learner.learn()
```

Chapter 8

Neural Networks and Deep Learning

Warning: this is not meant to be an efficient implementation of deep learning. If you want to do serious machine learning on meduim-sized or large data, we recommend Keras (`https://keras.io`) [Chollet, 2021] or PyTorch (`https://pytorch.org`), which are very efficient, particularly on GPUs. They are, however, black boxes. The AIPython neural network code should be seen like a car engine made of glass; you can see exactly how it works, even if it is not fast.

The parameters that are the same as in Keras have the same names.

8.1 Layers

A neural network is built from layers.

This provides a modular implementation of layers. Layers can easily be stacked in many configurations. A layer needs to implement a function to compute the output values from the inputs, a way to back-propagate the error, and perhaps update its parameters.

learnNN.py — Neural Network Learning

```
11 from learnProblem import Learner, Data_set, Data_from_file,
       Data_from_files, Evaluate
12 from learnLinear import sigmoid, one, softmax, indicator
13 import random, math, time
14
15 class Layer(object):
16     def __init__(self, nn, num_outputs=None):
17         """Given a list of inputs, outputs will produce a list of length
               num_outputs.
18         nn is the neural network this layer is part of
```

```
19          num outputs is the number of outputs for this layer.
20          """
21          self.nn = nn
22          self.num_inputs = nn.num_outputs # output of nn is the input to
                this layer
23          if num_outputs:
24              self.num_outputs = num_outputs
25          else:
26              self.num_outputs = nn.num_outputs # same as the inputs
27
28      def output_values(self,input_values, training=False):
29          """Return the outputs for this layer for the given input values.
30          input_values is a list of the inputs to this layer (of length
                num_inputs)
31          returns a list of length self.num_outputs.
32          It can act differently when training and when predicting.
33          """
34          raise NotImplementedError("output_values") # abstract method
35
36      def backprop(self,errors):
37          """Backpropagate the errors on the outputs
38          errors is a list of errors for the outputs (of length
                self.num_outputs).
39          Returns the errors for the inputs to this layer (of length
                self.num_inputs).
40
41          You can assume that this is only called after corresponding
                output_values,
42             which can remember information information required for the
                   back-propagation.
43          """
44          raise NotImplementedError("backprop") # abstract method
45
46      def update(self):
47          """updates parameters after a batch.
48          overridden by layers that have parameters
49          """
50          pass
```

A linear layer maintains an array of weights. $self.weights[o][i]$ is the weight between input i and output o. A 1 is added to the end of the inputs. The default initialization is the Glorot uniform initializer [Glorot and Bengio, 2010], which is the default in Keras. An alternative is to provide a limit, in which case the values are selected uniformly in the range $[-limit, limit]$. Keras treats the bias separately, and defaults to zero.

learnNN.py — (continued)

```
52  class Linear_complete_layer(Layer):
53      """a completely connected layer"""
54      def __init__(self, nn, num_outputs, limit=None):
55          """A completely connected linear layer.
```

```
56          nn is a neural network that the inputs come from
57          num_outputs is the number of outputs
58          the random initialization of parameters is in range [-limit,limit]
59          """
60          Layer.__init__(self, nn, num_outputs)
61          if limit is None:
62              limit =math.sqrt(6/(self.num_inputs+self.num_outputs))
63          # self.weights[o][i] is the weight between input i and output o
64          self.weights = [[random.uniform(-limit, limit) if inf <
                self.num_inputs else 0
65                           for inf in range(self.num_inputs+1)]
66                          for outf in range(self.num_outputs)]
67          self.delta = [[0 for inf in range(self.num_inputs+1)]
68                          for outf in range(self.num_outputs)]
69
70      def output_values(self,input_values, training=False):
71          """Returns the outputs for the input values.
72          It remembers the values for the backprop.
73
74          Note in self.weights there is a weight list for every output,
75          so wts in self.weights loops over the outputs.
76          The bias is the *last* value of each list in self.weights.
77          """
78          self.inputs = input_values + [1]
79          return [sum(w*val for (w,val) in zip(wts,self.inputs))
80                      for wts in self.weights]
81
82      def backprop(self,errors):
83          """Backpropagate the errors, updating the weights and returning the
                error in its inputs.
84          """
85          input_errors = [0]*(self.num_inputs+1)
86          for out in range(self.num_outputs):
87              for inp in range(self.num_inputs+1):
88                  input_errors[inp] += self.weights[out][inp] * errors[out]
89                  self.delta[out][inp] += self.inputs[inp] * errors[out]
90          return input_errors[:-1] # remove the error for the "1"
91
92      def update(self):
93          """updates parameters after a batch"""
94          batch_step_size = self.nn.learning_rate / self.nn.batch_size
95          for out in range(self.num_outputs):
96              for inp in range(self.num_inputs+1):
97                  self.weights[out][inp] -= batch_step_size *
                        self.delta[out][inp]
98                  self.delta[out][inp] = 0
```

The standard activation function for hidden nodes is the **ReLU**.

learnNN.py — (continued)

```
100 class ReLU_layer(Layer):
```

```
    """Rectified linear unit (ReLU) f(z) = max(0, z).
    The number of outputs is equal to the number of inputs.
    """
    def __init__(self, nn):
        Layer.__init__(self, nn)

    def output_values(self, input_values, training=False):
        """Returns the outputs for the input values.
        It remembers the input values for the backprop.
        """
        self.input_values = input_values
        self.outputs= [max(0,inp) for inp in input_values]
        return self.outputs

    def backprop(self,errors):
        """Returns the derivative of the errors"""
        return [e if inp>0 else 0 for e,inp in zip(errors,
            self.input_values)]
```

One of the old standards for the activation function for hidden layers is the sigmoid. It is included here to experiment with.

learnNN.py — (continued)

```
class Sigmoid_layer(Layer):
    """sigmoids of the inputs.
    The number of outputs is equal to the number of inputs.
    Each output is the sigmoid of its corresponding input.
    """
    def __init__(self, nn):
        Layer.__init__(self, nn)

    def output_values(self, input_values, training=False):
        """Returns the outputs for the input values.
        It remembers the output values for the backprop.
        """
        self.outputs= [sigmoid(inp) for inp in input_values]
        return self.outputs

    def backprop(self,errors):
        """Returns the derivative of the errors"""
        return [e*out*(1-out) for e,out in zip(errors, self.outputs)]
```

8.2 Feedforward Networks

learnNN.py — (continued)

```
class NN(Learner):
    def __init__(self, dataset, validation_proportion = 0.1,
        learning_rate=0.001):
        """Creates a neural network for a dataset,
```

```
141         layers is the list of layers
142         """
143         self.dataset = dataset
144         self.output_type = dataset.target.ftype
145         self.learning_rate = learning_rate
146         self.input_features = dataset.input_features
147         self.num_outputs = len(self.input_features)
148         validation_num = int(len(self.dataset.train)*validation_proportion)
149         if validation_num > 0:
150             random.shuffle(self.dataset.train)
151             self.validation_set = self.dataset.train[-validation_num:]
152             self.training_set = self.dataset.train[:-validation_num]
153         else:
154             self.validation_set = []
155             self.training_set = self.dataset.train
156         self.layers = []
157         self.bn = 0 # number of batches run
158 
159     def add_layer(self,layer):
160         """add a layer to the network.
161         Each layer gets number of inputs from the previous layers outputs.
162         """
163         self.layers.append(layer)
164         self.num_outputs = layer.num_outputs
165 
166     def predictor(self,ex):
167         """Predicts the value of the first output for example ex.
168         """
169         values = [f(ex) for f in self.input_features]
170         for layer in self.layers:
171             values = layer.output_values(values)
172         return sigmoid(values[0]) if self.output_type =="boolean" \
173                else softmax(values, self.dataset.target.frange) if
                        self.output_type == "categorical" \
174                else values[0]
175 
176     def predictor_string(self):
177         return "not implemented"
```

The *learn* method learns a network.

learnNN.py — (continued)

```
179     def learn(self, epochs=5, batch_size=32, num_iter = None,
            report_each=10):
180         """Learns parameters for a neural network using stochastic gradient
                decent.
181         epochs is the number of times through the data (on average)
182         batch_size is the maximum size of each batch
183         num_iter is the number of iterations over the batches
184             - overrides epochs if provided (allows for fractions of epochs)
```

```
185         report_each means give the errors after each multiple of that
                iterations
186         """
187         self.batch_size = min(batch_size, len(self.training_set)) # don't
                have batches bigger than training size
188         if num_iter is None:
189             num_iter = (epochs * len(self.training_set)) // self.batch_size
190         #self.display(0,"Batch\t","\t".join(criterion.__doc__ for criterion
                in Evaluate.all_criteria))
191         for i in range(num_iter):
192             batch = random.sample(self.training_set, self.batch_size)
193             for e in batch:
194                 # compute all outputs
195                 values = [f(e) for f in self.input_features]
196                 for layer in self.layers:
197                     values = layer.output_values(values, training=True)
198                 # backpropagate
199                 predicted = [sigmoid(v) for v in values] if self.output_type
                        == "boolean"\
200                              else softmax(values) if self.output_type ==
                                  "categorical"\
201                              else values
202                 actuals = indicator(self.dataset.target(e),
                        self.dataset.target.frange) \
203                               if self.output_type == "categorical"\
204                               else [self.dataset.target(e)]
205                 errors = [pred-obsd for (obsd,pred) in
                        zip(actuals,predicted)]
206                 for layer in reversed(self.layers):
207                     errors = layer.backprop(errors)
208             # Update all parameters in batch
209             for layer in self.layers:
210                 layer.update()
211             self.bn+=1
212             if (i+1)%report_each==0:
213                 self.display(0,self.bn,"\t",
214                              "\t\t".join("{:.4f}".format(
215                                  self.dataset.evaluate_dataset(self.validation_set,
                                        self.predictor, criterion))
216                                for criterion in Evaluate.all_criteria),
                                    sep="")
```

8.3 Improved Optimization

8.3.1 Momentum

learnNN.py — (continued)

```
218 class Linear_complete_layer_momentum(Linear_complete_layer):
219     """a completely connected layer"""
```

```
220     def __init__(self, nn, num_outputs, limit=None, alpha=0.9, epsilon =
            1e-07, vel0=0):
221         """A completely connected linear layer.
222         nn is a neural network that the inputs come from
223         num_outputs is the number of outputs
224         max_init is the maximum value for random initialization of
                parameters
225         vel0 is the initial velocity for each parameter
226         """
227         Linear_complete_layer.__init__(self, nn, num_outputs, limit=limit)
228         # self.weights[o][i] is the weight between input i and output o
229         self.velocity = [[vel0 for inf in range(self.num_inputs+1)]
230                          for outf in range(self.num_outputs)]
231         self.alpha = alpha
232         self.epsilon = epsilon
233
234     def update(self):
235         """updates parameters after a batch"""
236         batch_step_size = self.nn.learning_rate / self.nn.batch_size
237         for out in range(self.num_outputs):
238             for inp in range(self.num_inputs+1):
239                 self.velocity[out][inp] = self.alpha*self.velocity[out][inp]
                        - batch_step_size * self.delta[out][inp]
240                 self.weights[out][inp] += self.velocity[out][inp]
241                 self.delta[out][inp] = 0
```

8.3.2 RMS-Prop

learnNN.py — (continued)

```
243 class Linear_complete_layer_RMS_Prop(Linear_complete_layer):
244     """a completely connected layer"""
245     def __init__(self, nn, num_outputs, limit=None, rho=0.9, epsilon =
            1e-07):
246         """A completely connected linear layer.
247         nn is a neural network that the inputs come from
248         num_outputs is the number of outputs
249         max_init is the maximum value for random initialization of
                parameters
250         """
251         Linear_complete_layer.__init__(self, nn, num_outputs, limit=limit)
252         # self.weights[o][i] is the weight between input i and output o
253         self.ms = [[0 for inf in range(self.num_inputs+1)]
254                          for outf in range(self.num_outputs)]
255         self.rho = rho
256         self.epsilon = epsilon
257
258     def update(self):
259         """updates parameters after a batch"""
260         for out in range(self.num_outputs):
261             for inp in range(self.num_inputs+1):
```

```
262                 gradient = self.delta[out][inp] / self.nn.batch_size
263                 self.ms[out][inp] = self.rho*self.ms[out][inp]+ (1-self.rho)
                        * gradient**2
264                 self.weights[out][inp] -=
                        self.nn.learning_rate/(self.ms[out][inp]+self.epsilon)**0.5
                        * gradient
265                 self.delta[out][inp] = 0
```

8.4 Dropout

Dropout is implemented as a layer.

learnNN.py — (continued)

```
267 from utilities import flip
268 class Dropout_layer(Layer):
269     """Dropout layer
270     """
271
272     def __init__(self, nn, rate=0):
273         """
274         rate is fraction of the input units to drop. 0 =< rate < 1
275         """
276         self.rate = rate
277         Layer.__init__(self, nn)
278
279     def output_values(self, input_values, training=False):
280         """Returns the outputs for the input values.
281         It remembers the input values for the backprop.
282         """
283         if training:
284             scaling = 1/(1-self.rate)
285             self.mask = [0 if flip(self.rate) else 1
286                          for _ in input_values]
287             return [x*y*scaling for (x,y) in zip(input_values, self.mask)]
288         else:
289             return input_values
290
291     def backprop(self,errors):
292         """Returns the derivative of the errors"""
293         return [x*y for (x,y) in zip(errors, self.mask)]
294
295 class Dropout_layer_0(Layer):
296     """Dropout layer
297     """
298
299     def __init__(self, nn, rate=0):
300         """
301         rate is fraction of the input units to drop. 0 =< rate < 1
302         """
303         self.rate = rate
```

```
304         Layer.__init__(self, nn)
305
306     def output_values(self, input_values, training=False):
307         """Returns the outputs for the input values.
308         It remembers the input values for the backprop.
309         """
310         if training:
311             scaling = 1/(1-self.rate)
312             self.outputs= [0 if flip(self.rate) else inp*scaling # make 0
                    with probability rate
313                            for inp in input_values]
314             return self.outputs
315         else:
316             return input_values
317
318     def backprop(self,errors):
319         """Returns the derivative of the errors"""
320         return errors
```

8.4.1 Examples

The following constructs a neural network with one hidden layer. The output is assumed to be Boolean or Real. If it is categorical, the final layer should have the same number of outputs as the number of cetegories (so it can use a softmax).

learnNN.py — (continued)

```
322 #data = Data_from_file('data/mail_reading.csv', target_index=-1)
323 #data = Data_from_file('data/mail_reading_consis.csv', target_index=-1)
324 data = Data_from_file('data/SPECT.csv', prob_test=0.3, target_index=0,
        seed=12345)
325 #data = Data_from_file('data/iris.data', prob_test=0.2, target_index=-1) #
        150 examples approx 120 test + 30 test
326 #data = Data_from_file('data/if_x_then_y_else_z.csv', num_train=8,
        target_index=-1) # not linearly sep
327 #data = Data_from_file('data/holiday.csv', target_index=-1) #,
        num_train=19)
328 #data = Data_from_file('data/processed.cleveland.data', target_index=-1)
329 #random.seed(None)
330
331 # nn3 is has a single hidden layer of width 3
332 nn3 = NN(data, validation_proportion = 0)
333 nn3.add_layer(Linear_complete_layer(nn3,3))
334 #nn3.add_layer(Sigmoid_layer(nn3))
335 nn3.add_layer(ReLU_layer(nn3))
336 nn3.add_layer(Linear_complete_layer(nn3,1)) # when using
        output_type="boolean"
337 #nn3.learn(epochs = 100)
338
```

```
# nn3do is like nn3 but with dropout on the hidden layer
nn3do = NN(data, validation_proportion = 0)
nn3do.add_layer(Linear_complete_layer(nn3do,3))
#nn3.add_layer(Sigmoid_layer(nn3)) # comment this or the next
nn3do.add_layer(ReLU_layer(nn3do))
nn3do.add_layer(Dropout_layer(nn3do, rate=0.5))
nn3do.add_layer(Linear_complete_layer(nn3do,1))
#nn3do.learn(epochs = 100)

# nn3_rmsp is like nn3 but uses RMS prop
nn3_rmsp = NN(data, validation_proportion = 0)
nn3_rmsp.add_layer(Linear_complete_layer_RMS_Prop(nn3_rmsp,3))
#nn3_rmsp.add_layer(Sigmoid_layer(nn3_rmsp)) # comment this or the next
nn3_rmsp.add_layer(ReLU_layer(nn3_rmsp))
nn3_rmsp.add_layer(Linear_complete_layer_RMS_Prop(nn3_rmsp,1))
#nn3_rmsp.learn(epochs = 100)

# nn3_m is like nn3 but uses momentum
mm1_m = NN(data, validation_proportion = 0)
mm1_m.add_layer(Linear_complete_layer_momentum(mm1_m,3))
#mm1_m.add_layer(Sigmoid_layer(mm1_m)) # comment this or the next
mm1_m.add_layer(ReLU_layer(mm1_m))
mm1_m.add_layer(Linear_complete_layer_momentum(mm1_m,1))
#mm1_m.learn(epochs = 100)

# nn2 has a single a hidden layer of width 2
nn2 = NN(data, validation_proportion = 0)
nn2.add_layer(Linear_complete_layer_RMS_Prop(nn2,2))
nn2.add_layer(ReLU_layer(nn2))
nn2.add_layer(Linear_complete_layer_RMS_Prop(nn2,1))

# nn5 is has a single hidden layer of width 5
nn5 = NN(data, validation_proportion = 0)
nn5.add_layer(Linear_complete_layer_RMS_Prop(nn5,5))
nn5.add_layer(ReLU_layer(nn5))
nn5.add_layer(Linear_complete_layer_RMS_Prop(nn5,1))

# nn0 has no hidden layers, and so is just logistic regression:
nn0 = NN(data, validation_proportion = 0) #learning_rate=0.05)
nn0.add_layer(Linear_complete_layer(nn0,1))
# Or try this for RMS-Prop:
#nn0.add_layer(Linear_complete_layer_RMS_Prop(nn0,1))
```

Plotting. Figure 8.1 shows the training and test performance on the SPECT dataset for the architectures above. Note the `nn5` test has infinite log loss after about 45,000 steps. The noisyness of the predictions might indicate that the step size is too big. This was produced by the code below:

learnNN.py — (continued)

```
from learnLinear import plot_steps
from learnProblem import Evaluate
```

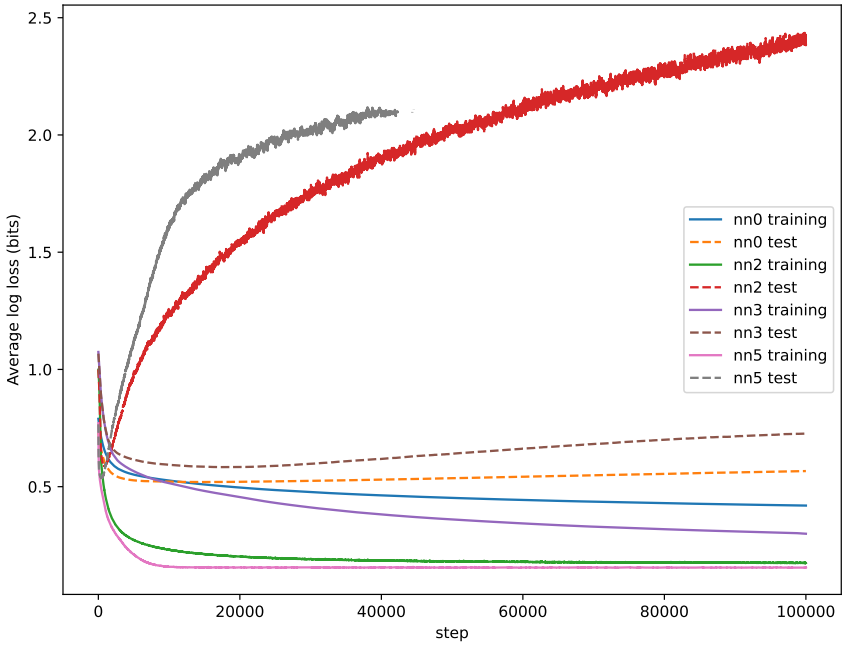


Figure 8.1: Plotting train and test log loss for various algorithms on SPECT dataset

```
384
385 # To show plots first choose a criterion to use
386 # crit = Evaluate.log_loss
387 # crit = Evaluate.accuracy
388 # plot_steps(learner = nn0, data = data, criterion=crit, num_steps=10000,
        log_scale=False, legend_label="nn0")
389 # plot_steps(learner = nn2, data = data, criterion=crit, num_steps=10000,
        log_scale=False, legend_label="nn2")
390 # plot_steps(learner = nn3, data = data, criterion=crit, num_steps=10000,
        log_scale=False, legend_label="nn3")
391 # plot_steps(learner = nn5, data = data, criterion=crit, num_steps=10000,
        log_scale=False, legend_label="nn5")
392
393 # for (nn,nname) in [(nn0,"nn0"),(nn2,"nn2"),(nn3,"nn3"),(nn5,"nn5")]:
        plot_steps(learner = nn, data = data, criterion=crit,
        num_steps=100000, log_scale=False, legend_label=nname)
394
395 # Print some training examples
396 #for eg in random.sample(data.train,10): print(eg,nn3.predictor(eg))
397
398 # Print some test examples
399 #for eg in random.sample(data.test,10): print(eg,nn3.predictor(eg))
```

```
400
401 # To see the weights learned in linear layers
402 # nn3.layers[0].weights
403 # nn3.layers[2].weights
404
405 # Print test:
406 # for e in data.train: print(e,nn0.predictor(e))
407
408 def test(data, hidden_widths = [5], epochs=100,
409             optimizers = [Linear_complete_layer,
410                       Linear_complete_layer_momentum,
                              Linear_complete_layer_RMS_Prop]):
411     data.display(0,"Batch\t","\t".join(criterion.__doc__ for criterion in
            Evaluate.all_criteria))
412     for optimizer in optimizers:
413         nn = NN(data)
414         for width in hidden_widths:
415             nn.add_layer(optimizer(nn,width))
416             nn.add_layer(ReLU_layer(nn))
417         if data.target.ftype == "boolean":
418             nn.add_layer(optimizer(nn,1))
419         else:
420             error(f"Not implemented: {data.output_type}")
421         nn.learn(epochs)
```

The following tests on MNIST. The original files are from `http://yann.lecun.com/exdb/mnist/`. This code assumes you use the csv files from `https://pjreddie.com/projects/mnist-in-csv/`, and put them in the directory `../MNIST/`. Note that this is **very** inefficient; you would be better to use Keras or Pytorch. There are $28 * 28 = 784$ input units and 512 hidden units, which makes 401,408 parameters for the lowest linear layer. So don't be surprised when it takes many hours in AIPython (even if it only takes a few seconds in Keras).

learnNN.py — (continued)

```
423 # Simplified version: (6000 training instances)
424 # data_mnist = Data_from_file('../MNIST/mnist_train.csv', prob_test=0.9,
        target_index=0, boolean_features=False, target_type="categorical")
425
426 # Full version:
427 # data_mnist = Data_from_files('../MNIST/mnist_train.csv',
        '../MNIST/mnist_test.csv', target_index=0, boolean_features=False,
        target_type="categorical")
428
429 # nn_mnist = NN(data_mnist, validation_proportion = 0.02,
        learning_rate=0.001) #validation set = 1200
430 # nn_mnist.add_layer(Linear_complete_layer_RMS_Prop(nn_mnist,512));
        nn_mnist.add_layer(ReLU_layer(nn_mnist));
        nn_mnist.add_layer(Linear_complete_layer_RMS_Prop(nn_mnist,10))
431 # start_time = time.perf_counter();nn_mnist.learn(epochs=1,
        batch_size=128);end_time = time.perf_counter();print("Time:", end_time
```

```
    - start_time,"seconds") #1 epoch
# determine test error:
# data_mnist.evaluate_dataset(data_mnist.test, nn_mnist.predictor,
    Evaluate.accuracy)
# Print some random predictions:
# for eg in random.sample(data_mnist.test,10):
    print(data_mnist.target(eg),nn_mnist.predictor(eg),nn_mnist.predictor(eg)[data_mnist.target(eg
```

Exercise 8.1 In the definition of *nn*3 above, for each of the following, first hypothesize what will happen, then test your hypothesis, then explain whether you testing confirms your hypothesis or not. Test it for more than one data set, and use more than one run for each data set.

(a) Which fits the data better, having a sigmoid layer or a ReLU layer after the first linear layer?

(b) Which is faster, having a sigmoid layer or a ReLU layer after the first linear layer?

(c) What happens if you have both the sigmoid layer and then a ReLU layer after the first linear layer and before the second linear layer?

(d) What happens if you have a ReLU layer then a sigmoid layer after the first linear layer and before the second linear layer?

(e) What happens if you have neither the sigmoid layer nor a ReLU layer after the first linear layer?

Exercise 8.2 Do some

Chapter 9

Reasoning with Uncertainty

9.1 Representing Probabilistic Models

A probabilisitic model uses the same definition of a variable as a CSP (Section 4.1.1, page 55). A variable consists of a name, a domain and an optional (x,y) position (for displaying). The domain of a variable is a list or a tuple, as the ordering will matter in the representation of factors.

9.2 Representing Factors

A **factor** is, mathematically, a function from variables into a number; that is given a value for each of its variable, it gives a number. Factors are used for conditional probabilities, utilities in the next chapter, and are explicitly constructed by some algorithms (in particular variable elimination).

A variable assignment, or just **assignment**, is represented as a $\{variable : value\}$ dictionary. A factor can be evaluated when all of its variables are assigned. The method `get_value` evaluates the factor for an assignment. The assignment can include extra variables not in the factor. This method needs to be defined for every subclass.

probFactors.py — Factors for graphical models

```
11 from display import Displayable
12 import math
13
14 class Factor(Displayable):
15     nextid=0 # each factor has a unique identifier; for printing
16
17     def __init__(self,variables):
18         self.variables = variables # list of variables
```

```
19          self.id = Factor.nextid
20          self.name = f"f{self.id}"
21          Factor.nextid += 1
22
23      def can_evaluate(self,assignment):
24          """True when the factor can be evaluated in the assignment
25          assignment is a {variable:value} dict
26          """
27          return all(v in assignment for v in self.variables)
28
29      def get_value(self,assignment):
30          """Returns the value of the factor given the assignment of values
                to variables.
31          Needs to be defined for each subclass.
32          """
33          assert self.can_evaluate(assignment)
34          raise NotImplementedError("get_value") # abstract method
```

The method `__str__` returns a brief definition (like "`f7(X,Y,Z)`").The method `to_table` returns string representations of a table showing all of the assignments of values to variables, and the corresponding value.

probFactors.py — (continued)

```
36      def __str__(self):
37          """returns a string representing a summary of the factor"""
38          return f"{self.name}({','.join(str(var) for var in
                self.variables)})"
39
40      def to_table(self, variables=None, given={}):
41          """returns a string representation of the factor.
42          Allows for an arbitrary variable ordering.
43          variables is a list of the variables in the factor
44          (can contain other variables)"""
45          if variables==None:
46              variables = [v for v in self.variables if v not in given]
47          else: #enforce ordering and allow for extra variables in ordering
48              variables = [v for v in variables if v in self.variables and v
                    not in given]
49          head = "\t".join(str(v) for v in variables)
50          return head+"\n"+self.ass_to_str(variables, given, variables)
51
52      def ass_to_str(self, vars, asst, allvars):
53          #print(f"ass_to_str({vars}, {asst}, {allvars})")
54          if vars:
55              return "\n".join(self.ass_to_str(vars[1:], {**asst,
                        vars[0]:val}, allvars)
56                               for val in vars[0].domain)
57          else:
58              return ("\t".join(str(asst[var]) for var in allvars)
59                          + "\t"+"{:.6f}".format(self.get_value(asst)) )
60
```

```
61      __repr__ = __str__
```

9.3 Conditional Probability Distributions

A **conditional probability distribution (CPD)** is a type of factor that represents a conditional probability. A CPD representing $P(X \mid Y_1 \ldots Y_k)$ is a type of factor, where given values for X and each Y_i returns a number.

probFactors.py — (continued)

```
63  class CPD(Factor):
64      def __init__(self, child, parents):
65          """represents P(variable | parents)
66          """
67          self.parents = parents
68          self.child = child
69          Factor.__init__(self, parents+[child])
70
71      def __str__(self):
72          """A brief description of a factor using in tracing"""
73          if self.parents:
74              return f"P({self.child}|{','.join(str(p) for p in
                    self.parents)})"
75          else:
76              return f"P({self.child})"
77
78      __repr__ = __str__
```

A constant CPD has no parents, and has probability 1 when the variable has the value specified, and 0 when the variable has a different value.

probFactors.py — (continued)

```
80  class ConstantCPD(CPD):
81      def __init__(self, variable, value):
82          CPD.__init__(self, variable, [])
83          self.value = value
84      def get_value(self, assignment):
85          return 1 if self.value==assignment[self.child] else 0
```

9.3.1 Logistic Regression

A **logistic regression** CPD, for Boolean variable X represents $P(X{=}True \mid Y_1 \ldots Y_k)$, using $k+1$ real-values weights so

$$P(X{=}True \mid Y_1 \ldots Y_k) = sigmoid(w_0 + \sum_i w_i Y_i)$$

where for Boolean Y_i, True is represented as 1 and False as 0.

probFactors.py — (continued)

```
from learnLinear import sigmoid, logit

class LogisticRegression(CPD):
    def __init__(self, child, parents, weights):
        """A logistic regression representation of a conditional
            probability.
        child is the Boolean (or 0/1) variable whose CPD is being defined
        parents is the list of parents
        weights is list of parameters, such that weights[i+1] is the weight
            for parents[i]
        """
        assert len(weights) == 1+len(parents)
        CPD.__init__(self, child, parents)
        self.weights = weights

    def get_value(self,assignment):
        assert self.can_evaluate(assignment)
        prob = sigmoid(self.weights[0]
                       + sum(self.weights[i+1]*assignment[self.parents[i]]
                             for i in range(len(self.parents))))
        if assignment[self.child]: #child is true
            return prob
        else:
            return (1-prob)
```

9.3.2 Noisy-or

A **noisy-or**, for Boolean variable X with Boolean parents $Y_1 \ldots Y_k$ is parametrized by $k+1$ parameters $p_0, p_1, \ldots, p_k$, where each $0 \leq p_i \leq 1$. The sematics is defined as though there are $k+1$ hidden variables $Z_0, Z_1 \ldots Z_k$, where $P(Z_0) = p_0$ and $P(Z_i \mid Y_i) = p_i$ for $i \geq 1$, and where X is true if and only if $Z_0 \vee Z_1 \vee \cdots \vee Z_k$ (where $\vee$ is "or"). Thus X is false if all of the Z_i are false. Intuitively, Z_0 is the probability of X when all Y_i are false and each Z_i is a noisy (probabilistic) measure that Y_i makes X true, and X only needs one to make it true.

probFactors.py — (continued)

```
class NoisyOR(CPD):
    def __init__(self, child, parents, weights):
        """A noisy representation of a conditional probability.
        variable is the Boolean (or 0/1) child variable whose CPD is being
            defined
        parents is the list of Boolean (or 0/1) parents
        weights is list of parameters, such that weights[i+1] is the weight
            for parents[i]
        """
        assert len(weights) == 1+len(parents)
        CPD.__init__(self, child, parents)
        self.weights = weights
```

```
120
121     def get_value(self,assignment):
122         assert self.can_evaluate(assignment)
123         probfalse = (1-self.weights[0])*math.prod(1-self.weights[i+1]
124                                                   for i in
                                                          range(len(self.parents))
125                                                   if
                                                          assignment[self.parents[i]])
126         if assignment[self.child]:
127             return 1-probfalse
128         else:
129             return probfalse
```

9.3.3 Tabular Factors

A **tabular factor** is a factor that represents each assignment of values to variables separately. It is represented by a Python array (or python dict). If the variables are $V_1, V_2, \ldots, V_k$, the value of $f(V_1 = v_1, V_2 = v_1, \ldots, V_k = v_k)$ is stored in $f[v_1][v_2]\ldots[v_k]$.

If the domain of V_i is $[0, \ldots, n_i - 1]$ this can be represented as an array. Otherwise we can use a dictionary. Python is nice in that it doesn't care, whether an array or dict is used **except when enumerating the values**; enumerating a dict gives the keys (the variables) but enumerating an array gives the values. So we have to be careful not to do this.

probFactors.py — (continued)

```
131 from functools import reduce
132
133 class TabFactor(Factor):
134
135     def __init__(self, variables, values):
136         Factor.__init__(self, variables)
137         self.values = values
138
139     def get_value(self, assignment):
140         return self.get_val_rec(self.values, self.variables, assignment)
141
142     def get_val_rec(self, value, variables, assignment):
143         if variables == []:
144            return value
145         else:
146             return self.get_val_rec(value[assignment[variables[0]]],
147                                     variables[1:],assignment)
```

Prob is a factor that represents a conditional probability by enumerating all of the values.

probFactors.py — (continued)

```
149 class Prob(CPD,TabFactor):
```

```
150     """A factor defined by a conditional probability table"""
151     def __init__(self,var,pars,cpt):
152         """Creates a factor from a conditional probability table, cpt
153         The cpt values are assumed to be for the ordering par+[var]
154         """
155         TabFactor.__init__(self,pars+[var],cpt)
156         self.child = var
157         self.parents = pars
```

9.4 Graphical Models

A graphical model consists of a set of variables and a set of factors. A belief network is a graphical model where all of the factors represent conditional probabilities. There are some operations (such as pruning variables) which are applicable to belief networks, but are not applicable to more general models. At the moment, we will treat them as the same.

probGraphicalModels.py — Graphical Models and Belief Networks

```
11 from display import Displayable
12 from probFactors import CPD
13 import matplotlib.pyplot as plt
14
15 class GraphicalModel(Displayable):
16     """The class of graphical models.
17     A graphical model consists of a title, a set of variables and a set of
           factors.
18
19     vars is a set of variables
20     factors is a set of factors
21     """
22     def __init__(self, title, variables=None, factors=None):
23         self.title = title
24         self.variables = variables
25         self.factors = factors
```

A **belief network** (also known as a **Bayesian network**) is a graphical model where all of the factors are conditional probabilities, and every variable has a conditional probability of it given its parents. This only checks the first condition, and builds some useful data structures.

probGraphicalModels.py — (continued)

```
27 class BeliefNetwork(GraphicalModel):
28     """The class of belief networks."""
29
30     def __init__(self, title, variables, factors):
31         """vars is a set of variables
32         factors is a set of factors. All of the factors are instances of
               CPD (e.g., Prob).
33         """
```

```
34        GraphicalModel.__init__(self, title, variables, factors)
35        assert all(isinstance(f,CPD) for f in factors)
36        self.var2cpt = {f.child:f for f in factors}
37        self.var2parents = {f.child:f.parents for f in factors}
38        self.children = {n:[] for n in self.variables}
39        for v in self.var2parents:
40            for par in self.var2parents[v]:
41                self.children[par].append(v)
42        self.topological_sort_saved = None
```

The following creates a topological sort of the nodes, where the parents of a node come before the node in the resulting order. This is based on Kahn's algorithm from 1962.

probGraphicalModels.py — (continued)

```
44    def topological_sort(self):
45        """creates a topological ordering of variables such that the
              parents of
46        a node are before the node.
47        """
48        if self.topological_sort_saved:
49            return self.topological_sort_saved
50        next_vars = {n for n in self.var2parents if not self.var2parents[n]
              }
51        self.display(3,'topological_sort: next_vars',next_vars)
52        top_order=[]
53        while next_vars:
54            var = next_vars.pop()
55            self.display(3,'select variable',var)
56            top_order.append(var)
57            next_vars |= {ch for ch in self.children[var]
58                          if all(p in top_order for p in
                                  self.var2parents[ch])}
59            self.display(3,'var_with_no_parents_left',next_vars)
60        self.display(3,"top_order",top_order)
61        assert
              set(top_order)==set(self.var2parents),(top_order,self.var2parents)
62        self.topologicalsort_saved=top_order
63        return top_order
```

The **show** method uses matplotlib to show the graphical structure of a belief network.

probGraphicalModels.py — (continued)

```
65    def show(self):
66        plt.ion()  # interactive
67        ax = plt.figure().gca()
68        ax.set_axis_off()
69        plt.title(self.title)
70        bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
71        for var in reversed(self.topological_sort()):
```

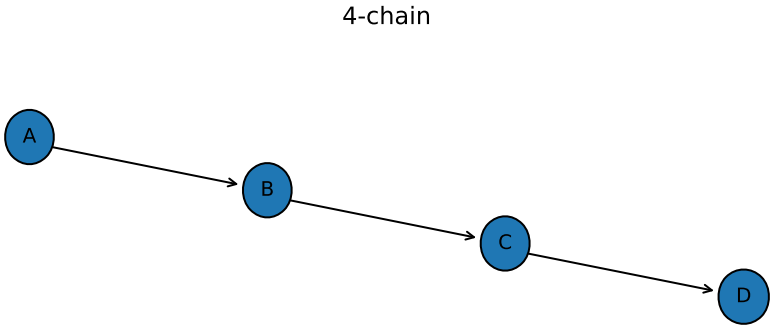


Figure 9.1: bn_4ch.show()

```
72            if self.var2parents[var]:
73                for par in self.var2parents[var]:
74                    ax.annotate(var.name, par.position, xytext=var.position,
75                                arrowprops={'arrowstyle':'<-'},bbox=bbox,
76                                ha='center')
77            else:
78                x,y = var.position
79                plt.text(x,y,var.name,bbox=bbox,ha='center')
```

9.4.1 Example Belief Networks

A Chain of 4 Variables

The first example belief network is a simple chain $A \longrightarrow B \longrightarrow C \longrightarrow D$, shown in Figure 9.1.

Please do not change this, as it is the example used for testing.

probGraphicalModels.py — (continued)

```
81  from variable import Variable
82  from probFactors import Prob, LogisticRegression, NoisyOR
83
84  boolean = [False, True]
85  A = Variable("A", boolean, position=(0,0.8))
86  B = Variable("B", boolean, position=(0.333,0.7))
87  C = Variable("C", boolean, position=(0.666,0.6))
88  D = Variable("D", boolean, position=(1,0.5))
89
90  f_a = Prob(A,[],[0.4,0.6])
91  f_b = Prob(B,[A],[[0.9,0.1],[0.2,0.8]])
92  f_c = Prob(C,[B],[[0.6,0.4],[0.3,0.7]])
93  f_d = Prob(D,[C],[[0.1,0.9],[0.75,0.25]])
94
95  bn_4ch = BeliefNetwork("4-chain", {A,B,C,D}, {f_a,f_b,f_c,f_d})
```

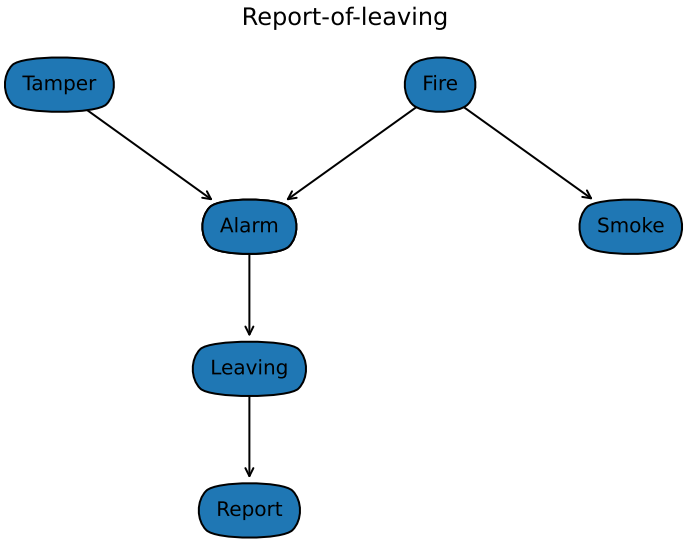


Figure 9.2: The report-of-leaving belief network

Report-of-Leaving Example

The second belief network, `bn_report`, is Example 8.15 of Poole and Mackworth [2017] (`http://artint.info`). The output of `bn_report.show()` is shown in Figure 9.2 of this document.

probGraphicalModels.py — (continued)

```
97  # Belief network report-of-leaving example (Example 8.15 shown in Figure
        8.3) of
98  # Poole and Mackworth, Artificial Intelligence, 2017 http://artint.info
99
100 Alarm =  Variable("Alarm", boolean, position=(0.366,0.5))
101 Fire =   Variable("Fire",  boolean, position=(0.633,0.75))
102 Leaving = Variable("Leaving", boolean, position=(0.366,0.25))
103 Report = Variable("Report", boolean, position=(0.366,0.0))
104 Smoke =  Variable("Smoke", boolean, position=(0.9,0.5))
105 Tamper = Variable("Tamper", boolean, position=(0.1,0.75))
106
107 f_ta = Prob(Tamper,[],[0.98,0.02])
108 f_fi = Prob(Fire,[],[0.99,0.01])
109 f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
110 f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01,
        0.99], [0.5, 0.5]]])
111 f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
112 f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])
```

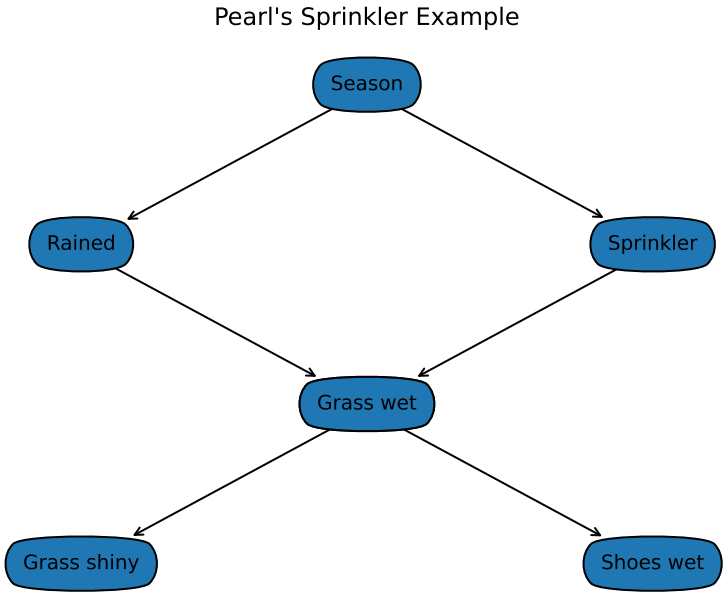


Figure 9.3: The sprinkler belief network

```
113
114 bn_report = BeliefNetwork("Report-of-leaving",
        {Tamper,Fire,Smoke,Alarm,Leaving,Report},
115                             {f_ta,f_fi,f_sm,f_al,f_lv,f_re})
```

Sprinkler Example

The third belief network is the sprinkler example from Pearl. The output of `bn_sprinkler.show()` is shown in Figure 9.3 of this document.

probGraphicalModels.py — (continued)

```
117 Season = Variable("Season", ["summer","winter"], position=(0.5,0.9))
118 Sprinkler = Variable("Sprinkler", ["on","off"], position=(0.9,0.6))
119 Rained = Variable("Rained", boolean, position=(0.1,0.6))
120 Grass_wet = Variable("Grass wet", boolean, position=(0.5,0.3))
121 Grass_shiny = Variable("Grass shiny", boolean, position=(0.1,0))
122 Shoes_wet = Variable("Shoes wet", boolean, position=(0.9,0))
123
124 f_season = Prob(Season,[],{'summer':0.5, 'winter':0.5})
125 f_sprinkler = Prob(Sprinkler,[Season],{'summer':{'on':0.9,'off':0.1},
126                                        'winter':{'on':0.01,'off':0.99}})
127 f_rained = Prob(Rained,[Season],{'summer':[0.9,0.1], 'winter': [0.2,0.8]})
128 f_wet = Prob(Grass_wet,[Sprinkler,Rained], {'on': [[0.1,0.9],[0.01,0.99]],
129                                             'off':[[0.99,0.01],[0.3,0.7]]})
```

```
f_shiny = Prob(Grass_shiny, [Grass_wet], [[0.95,0.05], [0.3,0.7]])
f_shoes = Prob(Shoes_wet, [Grass_wet], [[0.98,0.02], [0.35,0.65]])

bn_sprinkler = BeliefNetwork("Pearl's Sprinkler Example",
                       {Season, Sprinkler, Rained, Grass_wet, Grass_shiny,
                           Shoes_wet},
                       {f_season, f_sprinkler, f_rained, f_wet, f_shiny,
                           f_shoes})

bn_sprinkler_soff = BeliefNetwork("Pearl's Sprinkler Example
    (do(Sprinkler=off))",
                       {Season, Sprinkler, Rained, Grass_wet, Grass_shiny,
                           Shoes_wet},
                       {f_season, f_rained, f_wet, f_shiny, f_shoes,
                            Prob(Sprinkler,[],{'on':0,'off':1})})
```

Bipartite Diagnostic Model with Noisy-or

The belief network bn_no1 is a bipartite diagnostic model, with independent diseases, and the symtoms depend on the diseases, where the CPDs are defined using noisy-or. Bipartite means it is in two parts; the diseases are only connected to the symptoms and the symptoms are only connected to the diseases. The output of bn_no1.show() is shown in Figure 9.4 of this document.

probGraphicalModels.py — (continued)

```
Cough = Variable("Cough", boolean, (0.1,0.1))
Fever = Variable("Fever", boolean, (0.5,0.1))
Sneeze = Variable("Sneeze", boolean, (0.9,0.1))
Cold = Variable("Cold",boolean, (0.1,0.9))
Flu = Variable("Flu",boolean, (0.5,0.9))
Covid = Variable("Covid",boolean, (0.9,0.9))

p_cold_no = Prob(Cold,[],[0.9,0.1])
p_flu_no = Prob(Flu,[],[0.95,0.05])
p_covid_no = Prob(Covid,[],[0.99,0.01])

p_cough_no = NoisyOR(Cough, [Cold,Flu,Covid], [0.1, 0.3, 0.2, 0.7])
p_fever_no = NoisyOR(Fever, [   Flu,Covid], [0.01,    0.6,  0.7])
p_sneeze_no = NoisyOR(Sneeze, [Cold,Flu  ], [0.05, 0.5,  0.2   ])

bn_no1 = BeliefNetwork("Bipartite Diagnostic Network (noisy-or)",
                       {Cough, Fever, Sneeze, Cold, Flu, Covid},
                        {p_cold_no, p_flu_no, p_covid_no, p_cough_no,
                            p_fever_no, p_sneeze_no})

# to see the conditional probability of Noisy-or do:
# print(p_cough_no.to_table())

```

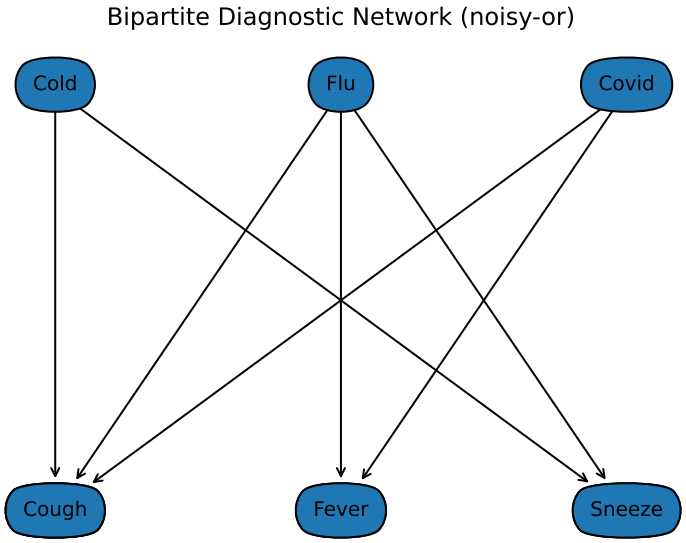


Figure 9.4: A bipartite diagnostic network

```
# example from box "Noisy-or compared to logistic regression"
# X = Variable("X",boolean)
# w0 = 0.01
# print(NoisyOR(X,[A,B,C,D],[w0, 1-(1-0.05)/(1-w0), 1-(1-0.1)/(1-w0),
    1-(1-0.2)/(1-w0), 1-(1-0.2)/(1-w0), ]).to_table(given={X:True}))
```

Bipartite Diagnostic Model with Logistic Regression

The belief network `bn_lr1` is a bipartite diagnostic model, with independent diseases, and the symtoms depend on the diseases, where the CPDs are defined using logistic regression. It has the same graphical structure as the previous example (see Figure 9.4). This has the (approximately) the same conditional probabilities as the previous example when zero or one diseases are present. Note that $sigmoid(-2.2) \approx 0.1$

probGraphicalModels.py — (continued)

```

p_cold_lr = Prob(Cold,[],[0.9,0.1])
p_flu_lr = Prob(Flu,[],[0.95,0.05])
p_covid_lr = Prob(Covid,[],[0.99,0.01])

p_cough_lr = LogisticRegression(Cough, [Cold,Flu,Covid], [-2.2, 1.67,
    1.26, 3.19])
```

```
p_fever_lr = LogisticRegression(Fever, [  Flu,Covid], [-4.6,      5.02,
    5.46])
p_sneeze_lr = LogisticRegression(Sneeze, [Cold,Flu ], [-2.94, 3.04, 1.79
    ])

bn_lr1 = BeliefNetwork("Bipartite Diagnostic Network - logistic
    regression",
                       {Cough, Fever, Sneeze, Cold, Flu, Covid},
                        {p_cold_lr, p_flu_lr, p_covid_lr, p_cough_lr,
                            p_fever_lr, p_sneeze_lr})

# to see the conditional probability of Noisy-or do:
#print(p_cough_lr.to_table())

# example from box "Noisy-or compared to logistic regression"
# from learnLinear import sigmoid, logit
# w0=logit(0.01)
# X = Variable("X",boolean)
# print(LogisticRegression(X,[A,B,C,D],[w0, logit(0.05)-w0, logit(0.1)-w0,
    logit(0.2)-w0, logit(0.2)-w0]).to_table(given={X:True}))
# try to predict what would happen (and then test) if we had
# w0=logit(0.01)
```

9.5 Inference Methods

Each of the inference methods implements the query method that computes the posterior probability of a variable given a dictionary of $\{variable : value\}$ observations. The methods are Displayable because they implement the *display* method which is currently text-based.

probGraphicalModels.py — (continued)

```
from display import Displayable

class InferenceMethod(Displayable):
    """The abstract class of graphical model inference methods"""
    method_name = "unnamed" # each method should have a method name

    def __init__(self,gm=None):
        self.gm = gm

    def query(self, qvar, obs={}):
        """returns a {value:prob} dictionary for the query variable"""
        raise NotImplementedError("InferenceMethod query") # abstract method
```

We use bn_4ch as the test case, in particular $P(B \mid D = true)$. This needs an error threshold, particularly for the approximate methods, where the default threshold is much too accurate.

probGraphicalModels.py — (continued)

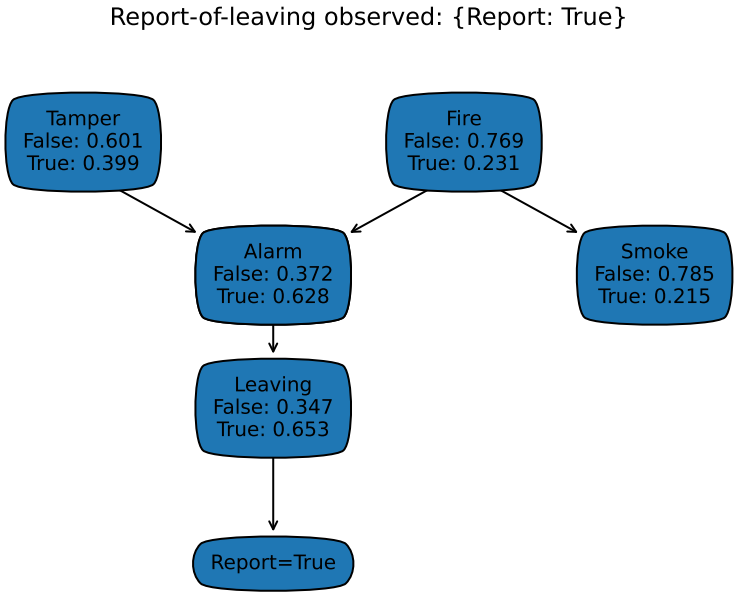


Figure 9.5: The report-of-leaving belief network with posterior distributions

```
206     def testIM(self, threshold=0.0000000001):
207         solver = self(bn_4ch)
208         res = solver.query(B,{D:True})
209         correct_answer = 0.429632380245
210         assert correct_answer-threshold < res[True] <
                correct_answer+threshold, \
211                 f"value {res[True]} not in desired range for
                        {self.method_name}"
212         print(f"Unit test passed for {self.method_name}.")
```

The following draws the posterior distribution of all variables. Figure 9.5 shows the result of `bn_reportRC.show_post({Report:True})` when run after loading `probRC.py` (see below).

probGraphicalModels.py — (continued)

```
214     def show_post(self, obs={}, format_string="{:.3f}"):
215         """draws the graphical model conditioned on observations obs
216            format_string is changed if you want more or less precision
217         """
218         plt.ion()  # interactive
219         ax = plt.figure().gca()
220         ax.set_axis_off()
221         plt.title(self.gm.title+" observed: "+str(obs))
222         bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
223         for var in reversed(self.gm.topological_sort()):
```

```
            distn = self.query(var, obs=obs)
            if var in obs:
                text = var.name + "=" + str(obs[var])
            else:
                text = var.name + "\n" + "\n".join(str(d)+":
                    "+format_string.format(v) for (d,v) in distn.items())
            if self.gm.var2parents[var]:
                for par in self.gm.var2parents[var]:
                    ax.annotate(text, par.position, xytext=var.position,
                                arrowprops={'arrowstyle':'<-'},bbox=bbox,
                                ha='center')
            else:
                x,y = var.position
                plt.text(x,y,text,bbox=bbox,ha='center')
```

9.6 Naive Search

An instance of a *ProbSearch* object takes in a graphical model. The query method uses naive search to compute the probability of a query variable given observations on other variables. See Figure 9.9 of Poole and Mackworth [2023].

probRC.py — Recursive Conditioning for Graphical Models

```
import math
from probGraphicalModels import GraphicalModel, InferenceMethod
from probFactors import Factor

class ProbSearch(InferenceMethod):
    """The class that queries graphical models using recursive conditioning

    gm is graphical model to query
    """
    method_name = "recursive conditioning"

    def __init__(self,gm=None):
        InferenceMethod.__init__(self, gm)
        ## self.max_display_level = 3

    def query(self, qvar, obs={}, split_order=None):
        """computes P(qvar | obs) where
        qvar is the query variable
        obs is a variable:value dictionary
        split_order is a list of the non-observed non-query variables in gm
        """
        if qvar in obs:
            return {val:(1 if val == obs[qvar] else 0) for val in
                qvar.domain}
        else:
           if split_order == None:
```

```
            split_order = [v for v in self.gm.variables if (v not in
                obs) and v != qvar]
        unnorm = [self.prob_search({qvar:val}|obs, self.gm.factors,
            split_order)
                  for val in qvar.domain]
        p_obs = sum(unnorm)
        return {val:pr/p_obs for val,pr in zip(qvar.domain, unnorm)}
```

The following is the naive search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm used in the subclass.

probRC.py — (continued)

```
    def prob_search(self, context, factors, split_order):
        """simple search algorithm
        context is a variable:value dictionary
        factors is a set of factors
        split_order is a list of variables in factors not assigned in
            context
        returns sum over variable assignments to variables in split order
            or product of factors """
        self.display(2,"calling prob_search,",(context,factors))
        if not factors:
            return 1
        elif to_eval := {fac for fac in factors if
            fac.can_evaluate(context)}:
            # evaluate factors when all variables are assigned
            self.display(3,"prob_search evaluating factors",to_eval)
            val = math.prod(fac.get_value(context) for fac in to_eval)
            return val * self.prob_search(context, factors-to_eval,
                split_order)
        else:
            total = 0
            var = split_order[0]
            self.display(3, "prob_search branching on", var)
            for val in var.domain:
                total += self.prob_search({var:val}|context, factors,
                    split_order[1:])
            self.display(3, "prob_search branching on", var,"returning",
                total)
            return total
```

9.7 Recursive Conditioning

The **recursive conditioning** algorithm adds forgetting and caching and recognizing disconnected components to the naive searcg. We do this by adding a cache and redefining the recursive search algorithm. In inherits the query method. See Figure 9.12 of Poole and Mackworth [2023].

probRC.py — (continued)

```
class ProbRC(ProbSearch):
    def __init__(self,gm=None):
        self.cache = {(frozenset(), frozenset()):1}
        ProbSearch.__init__(self,gm)

    def prob_search(self, context, factors, split_order):
        """ returns the number \sum_{split_order} \prod_{factors} given
            assignments in context
        context is a variable:value dictionary
        factors is a set of factors
        split_order is a list of variables in factors that are not assigned
            in context
        returns sum over variable assignments to variables in split_order
                  of the product of factors
        """
        self.display(3,"calling rc,",(context,factors))
        ce = (frozenset(context.items()), frozenset(factors)) # key for the
            cache entry
        if ce in self.cache:
            self.display(3,"rc cache lookup",(context,factors))
            return self.cache[ce]
#       if not factors: # no factors; needed if you don't have forgetting
    and caching
#           return 1
        elif vars_not_in_factors := {var for var in context
                                     if not any(var in fac.variables for
                                         fac in factors)}:
            # forget variables not in any factor
            self.display(3,"rc forgetting variables", vars_not_in_factors)
            return self.prob_search({key:val for (key,val) in
                context.items()
                                if key not in vars_not_in_factors},
                            factors, split_order)
        elif to_eval := {fac for fac in factors if
            fac.can_evaluate(context)}:
            # evaluate factors when all variables are assigned
            self.display(3,"rc evaluating factors",to_eval)
            val = math.prod(fac.get_value(context) for fac in to_eval)
            if val == 0:
                return 0
            else:
             return val * self.prob_search(context, {fac for fac in factors
                                                 if fac not in to_eval},
                                                     split_order)
        elif len(comp := connected_components(context, factors,
            split_order)) > 1:
            # there are disconnected components
            self.display(3,"splitting into connected components",comp,"in
                context",context)
```

```
104             return(math.prod(self.prob_search(context,f,eo) for (f,eo) in
                    comp))
105         else:
106             assert split_order, "split_order should not be empty to get
                    here"
107             total = 0
108             var = split_order[0]
109             self.display(3, "rc branching on", var)
110             for val in var.domain:
111                 total += self.prob_search({var:val}|context, factors,
                        split_order[1:])
112             self.cache[ce] = total
113             self.display(2, "rc branching on", var,"returning", total)
114             return total
```

`connected_components` returns a list of connected components, where a connected component is a set of factors and a set of variables, where the graph that connects variables and factors that involve them is connected. The connected components are built one at a time; with a current connected component. At all times `factors` is partitioned into 3 disjoint sets:

- `component_factors` containing factors in the current connected component where all factors that share a variable are already in the component
- `factors_to_check` containing factors in the current connected component where potentially some factors that share a variable are not in the component; these need to be checked
- `other_factors` the other factors that are not (yet) in the connected component

probRC.py — (continued)

```
116 def connected_components(context, factors, split_order):
117     """returns a list of (f,e) where f is a subset of factors and e is a
            subset of split_order
118     such that each element shares the same variables that are disjoint from
            other elements.
119     """
120     other_factors = set(factors) #copies factors
121     factors_to_check = {other_factors.pop()} # factors in connected
            component still to be checked
122     component_factors = set() # factors in first connected component
            already checked
123     component_variables = set() # variables in first connected component
124     while factors_to_check:
125         next_fac = factors_to_check.pop()
126         component_factors.add(next_fac)
127         new_vars = set(next_fac.variables) - component_variables -
                context.keys()
128         component_variables |= new_vars
```

```
        for var in new_vars:
            factors_to_check |= {f for f in other_factors if var in
                f.variables}
            other_factors -= factors_to_check # set difference
    if  other_factors:
        return ( [(component_factors,[e for e in split_order if e in
            component_variables])]
               + connected_components(context, other_factors, [e for e in
                   split_order
                                                               if e not in
                                                                   component_variables])
                                                                   )
    else:
        return [(component_factors, split_order)]
```

Testing:

probRC.py — (continued)

```
from probGraphicalModels import bn_4ch, A,B,C,D,f_a,f_b,f_c,f_d
bn_4chv = ProbRC(bn_4ch)
## bn_4chv.query(A,{})
## bn_4chv.query(D,{})
## InferenceMethod.max_display_level = 3 # show more detail in displaying
## InferenceMethod.max_display_level = 1 # show less detail in displaying
## bn_4chv.query(A,{D:True},[C,B])
## bn_4chv.query(B,{A:True,D:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportRC = ProbRC(bn_report) # answers queries using recursive
    conditioning
## bn_reportRC.query(Tamper,{})
## InferenceMethod.max_display_level = 0 # show no detail in displaying
## bn_reportRC.query(Leaving,{})
## bn_reportRC.query(Tamper,{},
    split_order=[Smoke,Fire,Alarm,Leaving,Report])
## bn_reportRC.query(Tamper,{Report:True})
## bn_reportRC.query(Tamper,{Report:True,Smoke:False})
## Note what happens to the cache when these are called in turn:
## bn_reportRC.query(Tamper,{Report:True},
    split_order=[Smoke,Fire,Alarm,Leaving])
## bn_reportRC.query(Smoke,{Report:True},
    split_order=[Tamper,Fire,Alarm,Leaving])

from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
    Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerv = ProbRC(bn_sprinkler)
## bn_sprinklerv.query(Shoes_wet,{})
## bn_sprinklerv.query(Shoes_wet,{Rained:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})
```

```
166
167 from probGraphicalModels import bn_no1, bn_lr1, Cough, Fever, Sneeze,
        Cold, Flu, Covid
168 bn_no1v = ProbRC(bn_no1)
169 bn_lr1v = ProbRC(bn_lr1)
170 ## bn_no1v.query(Flu, {Fever:1, Sneeze:1})
171 ## bn_lr1v.query(Flu, {Fever:1, Sneeze:1})
172 ## bn_lr1v.query(Cough,{})
173 ## bn_lr1v.query(Cold,{Cough:1,Sneeze:0,Fever:1})
174 ## bn_lr1v.query(Flu,{Cough:0,Sneeze:1,Fever:1})
175 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1})
176 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
177 ## bn_lr1v.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})
178
179 if __name__ == "__main__":
180     InferenceMethod.testIM(ProbRC)
```

9.8 Variable Elimination

An instance of a *VE* object takes in a graphical model. The query method uses variable elimination to compute the probability of a variable given observations on some other variables.

probVE.py — Variable Elimination for Graphical Models

```
11 from probFactors import Factor, FactorObserved, FactorSum, factor_times
12 from probGraphicalModels import GraphicalModel, InferenceMethod
13
14 class VE(InferenceMethod):
15     """The class that queries Graphical Models using variable elimination.
16
17     gm is graphical model to query
18     """
19     method_name = "variable elimination"
20
21     def __init__(self,gm=None):
22         InferenceMethod.__init__(self, gm)
23
24     def query(self,var,obs={},elim_order=None):
25         """computes P(var|obs) where
26         var is a variable
27         obs is a {variable:value} dictionary"""
28         if var in obs:
29             return {var:1 if val == obs[var] else 0 for val in var.domain}
30         else:
31             if elim_order == None:
32                 elim_order = self.gm.variables
33             projFactors = [self.project_observations(fact,obs)
34                            for fact in self.gm.factors]
35             for v in elim_order:
```

```
36              if v != var and v not in obs:
37                  projFactors = self.eliminate_var(projFactors,v)
38          unnorm = factor_times(var,projFactors)
39          p_obs=sum(unnorm)
40          self.display(1,"Unnormalized probs:",unnorm,"Prob obs:",p_obs)
41          return {val:pr/p_obs for val,pr in zip(var.domain, unnorm)}
```

A *FactorObserved* is a factor that is the result of some observations on another factor. We don't store the values in a list; we just look them up as needed. The observations can include variables that are not in the list, but should have some intersection with the variables in the factor.

probFactors.py — (continued)

```
159 class FactorObserved(Factor):
160     def __init__(self,factor,obs):
161         Factor.__init__(self, [v for v in factor.variables if v not in obs])
162         self.observed = obs
163         self.orig_factor = factor
164 
165     def get_value(self,assignment):
166         ass = assignment.copy()
167         for ob in self.observed:
168             ass[ob]=self.observed[ob]
169         return self.orig_factor.get_value(ass)
```

A *FactorSum* is a factor that is the result of summing out a variable from the product of other factors. I.e., it constructs a representation of:

$$\sum_{var} \prod_{f \in factors} f.$$

We store the values in a list in a lazy manner; if they are already computed, we used the stored values. If they are not already computed we can compute and store them.

probFactors.py — (continued)

```
171 class FactorSum(Factor):
172     def __init__(self,var,factors):
173         self.var_summed_out = var
174         self.factors = factors
175         vars = []
176         for fac in factors:
177             for v in fac.variables:
178                 if v is not var and v not in vars:
179                     vars.append(v)
180         Factor.__init__(self,vars)
181         self.values = {}
182 
183     def get_value(self,assignment):
184         """lazy implementation: if not saved, compute it. Return saved
            value"""
```

```
185         asst = frozenset(assignment.items())
186         if asst in self.values:
187             return self.values[asst]
188         else:
189             total = 0
190             new_asst = assignment.copy()
191             for val in self.var_summed_out.domain:
192                 new_asst[self.var_summed_out] = val
193                 total += math.prod(fac.get_value(new_asst) for fac in
                        self.factors)
194             self.values[asst] = total
195             return total
```

The method *factor_times* multiples a set of factors that are all factors on the same variable (or on no variables). This is the last step in variable elimination before normalizing. It returns an array giving the product for each value of *variable*.

probFactors.py — (continued)

```
197 def factor_times(variable, factors):
198     """when factors are factors just on variable (or on no variables)"""
199     prods = []
200     facs = [f for f in factors if variable in f.variables]
201     for val in variable.domain:
202         ast = {variable:val}
203         prods.append(math.prod(f.get_value(ast) for f in facs))
204     return prods
```

To project observations onto a factor, for each variable that is observed in the factor, we construct a new factor that is the factor projected onto that variable. *Factor_observed* creates a new factor that is the result is assigning a value to a single variable.

probVE.py — (continued)

```
43     def project_observations(self,factor,obs):
44         """Returns the resulting factor after observing obs
45 
46         obs is a dictionary of {variable:value} pairs.
47         """
48         if any((var in obs) for var in factor.variables):
49             # a variable in factor is observed
50             return FactorObserved(factor,obs)
51         else:
52             return factor
53 
54     def eliminate_var(self,factors,var):
55         """Eliminate a variable var from a list of factors.
56         Returns a new set of factors that has var summed out.
57         """
58         self.display(2,"eliminating ",str(var))
59         contains_var = []
60         not_contains_var = []
```

```
61          for fac in factors:
62              if var in fac.variables:
63                  contains_var.append(fac)
64              else:
65                  not_contains_var.append(fac)
66          if contains_var == []:
67              return factors
68          else:
69              newFactor = FactorSum(var,contains_var)
70              self.display(2,"Multiplying:",[str(f) for f in contains_var])
71              self.display(2,"Creating factor:", newFactor)
72              self.display(3, newFactor.to_table()) # factor in detail
73              not_contains_var.append(newFactor)
74              return not_contains_var
75
76  from probGraphicalModels import bn_4ch, A,B,C,D
77  bn_4chv = VE(bn_4ch)
78  ## bn_4chv.query(A,{})
79  ## bn_4chv.query(D,{})
80  ## InferenceMethod.max_display_level = 3 # show more detail in displaying
81  ## InferenceMethod.max_display_level = 1 # show less detail in displaying
82  ## bn_4chv.query(A,{D:True})
83  ## bn_4chv.query(B,{A:True,D:False})
84
85  from probGraphicalModels import
        bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
86  bn_reportv = VE(bn_report) # answers queries using variable elimination
87  ## bn_reportv.query(Tamper,{})
88  ## InferenceMethod.max_display_level = 0 # show no detail in displaying
89  ## bn_reportv.query(Leaving,{})
90  ## bn_reportv.query(Tamper,{},elim_order=[Smoke,Report,Leaving,Alarm,Fire])
91  ## bn_reportv.query(Tamper,{Report:True})
92  ## bn_reportv.query(Tamper,{Report:True,Smoke:False})
93
94  from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
        Grass_wet, Grass_shiny, Shoes_wet
95  bn_sprinklerv = VE(bn_sprinkler)
96  ## bn_sprinklerv.query(Shoes_wet,{})
97  ## bn_sprinklerv.query(Shoes_wet,{Rained:True})
98  ## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:True})
99  ## bn_sprinklerv.query(Shoes_wet,{Grass_shiny:False,Rained:True})
100
101 from probGraphicalModels import bn_lr1, Cough, Fever, Sneeze, Cold, Flu,
        Covid
102 vediag = VE(bn_lr1)
103 ## vediag.query(Cough,{})
104 ## vediag.query(Cold,{Cough:1,Sneeze:0,Fever:1})
105 ## vediag.query(Flu,{Cough:0,Sneeze:1,Fever:1})
106 ## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1})
107 ## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:0})
```

```
## vediag.query(Covid,{Cough:1,Sneeze:0,Fever:1,Flu:1})

if __name__ == "__main__":
    InferenceMethod.testIM(VE)
```

9.9 Stochastic Simulation

9.9.1 Sampling from a discrete distribution

The method *sample_one* generates a single sample from a (possible unnormalized) distribution. *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$. This returns a value with probability in proportion to its weight.

probStochSim.py — Probabilistic inference using stochastic simulation

```
import random
from probGraphicalModels import InferenceMethod

def sample_one(dist):
    """returns the index of a single sample from normalized distribution
        dist."""
    rand = random.random()*sum(dist.values())
    cum = 0    # cumulative weights
    for v in dist:
        cum += dist[v]
        if cum > rand:
            return v
```

If we want to generate multiple samples, repeatedly calling *sample_one* may not be efficient. If we want to generate n samples, and the distribution is over m values, *sample_one* takes time $O(mn)$. If m and n are of the same order of magnitude, we can do better.

The method *sample_multiple* generates multiple samples from a distribution defined by *dist*, where *dist* is a $\{value : weight\}$ dictionary, where $weight \geq 0$ and the weights cannot all be zero. This returns a list of values, of length *num_samples*, where each sample is selected with a probability proportional to its weight.

The method generates all of the random numbers, sorts them, and then goes through the distribution once, saving the selected samples.

probStochSim.py — (continued)

```
def sample_multiple(dist, num_samples):
    """returns a list of num_samples values selected using distribution
        dist.
    dist is a {value:weight} dictionary that does not need to be normalized
    """
    total = sum(dist.values())
    rands = sorted(random.random()*total for i in range(num_samples))
    result = []
```

```
30      dist_items = list(dist.items())
31      cum = dist_items[0][1] # cumulative sum
32      index = 0
33      for r in rands:
34          while r>cum:
35              index += 1
36              cum += dist_items[index][1]
37          result.append(dist_items[index][0])
38      return result
```

Exercise 9.1

What is the time and space complexity the following 4 methods to generate n samples, where m is the length of *dist*:

(a) n calls to *sample_one*

(b) *sample_multiple*

(c) Create the cumulative distribution (choose how this is represented) and, for each random number, do a binary search to determine the sample associated with the random number.

(d) Choose a random number in the range $[i/n, (i+1)/n)$ for each $i \in range(n)$, where n is the number of samples. Use these as the random numbers to select the particles. (Does this give random samples?)

For each method suggest when it might be the best method.

The *test_sampling* method can be used to generate the statistics from a number of samples. It is useful to see the variability as a function of the number of samples. Try it for few samples and also for many samples.

probStochSim.py — (continued)

```
40  def test_sampling(dist, num_samples):
41      """Given a distribution, dist, draw num_samples samples
42      and return the resulting counts
43      """
44      result = {v:0 for v in dist}
45      for v in sample_multiple(dist, num_samples):
46          result[v] += 1
47      return result
48
49  # try the following queries a number of times each:
50  # test_sampling({1:1,2:2,3:3,4:4}, 100)
51  # test_sampling({1:1,2:2,3:3,4:4}, 100000)
```

9.9.2 Sampling Methods for Belief Network Inference

A *SamplingInferenceMethod* is an *InferenceMethod*, but the query method also takes arguments for the number of samples and the sample-order (which is an ordering of factors). The first methods assume a belief network (and not an undirected graphical model).

probStochSim.py — (continued)

```
class SamplingInferenceMethod(InferenceMethod):
    """The abstract class of sampling-based belief network inference
        methods"""

    def __init__(self,gm=None):
        InferenceMethod.__init__(self, gm)

    def query(self,qvar,obs={},number_samples=1000,sample_order=None):
        raise NotImplementedError("SamplingInferenceMethod query") #
            abstract
```

9.9.3 Rejection Sampling

probStochSim.py — (continued)

```
class RejectionSampling(SamplingInferenceMethod):
    """The class that queries Graphical Models using Rejection Sampling.

    gm is a belief network to query
    """
    method_name = "rejection sampling"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)

    def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of variables where the parents
          come before the variable.
        """
        if sample_order is None:
            sample_order = self.gm.topological_sort()
        self.display(2,*sample_order,sep="\t")
        counts = {val:0 for val in qvar.domain}
        for i in range(number_samples):
            rejected = False
            sample = {}
            for nvar in sample_order:
                fac = self.gm.var2cpt[nvar]  #factor with nvar as child
                val = sample_one({v:fac.get_value({**sample, nvar:v}) for v
                    in nvar.domain})
                self.display(2,val,end="\t")
                if nvar in obs and obs[nvar] != val:
                    rejected = True
                    self.display(2,"Rejected")
                    break
                sample[nvar] = val
```

```
95              if not rejected:
96                  counts[sample[qvar]] += 1
97                  self.display(2,"Accepted")
98          tot = sum(counts.values())
99          # As well as the distribution we also include raw counts
100         dist = {c:v/tot if tot>0 else 1/len(qvar.domain) for (c,v) in
                counts.items()}
101         dist["raw_counts"] = counts
102         return dist
```

9.9.4 Likelihood Weighting

Likelihood weighting includes a weight for each sample. Instead of rejecting samples based on observations, likelihood weighting changes the weights of the sample in proportion with the probability of the observation. The weight then becomes the probability that the variable would have been rejected.

probStochSim.py — (continued)

```
104 class LikelihoodWeighting(SamplingInferenceMethod):
105     """The class that queries Graphical Models using Likelihood weighting.
106
107     gm is a belief network to query
108     """
109     method_name = "likelihood weighting"
110
111     def __init__(self, gm=None):
112         SamplingInferenceMethod.__init__(self, gm)
113
114     def query(self,qvar,obs={},number_samples=1000,sample_order=None):
115         """computes P(qvar | obs) where
116         qvar is a variable.
117         obs is a {variable:value} dictionary.
118         sample_order is a list of factors where factors defining the parents
119           come before the factors for the child.
120         """
121         if sample_order is None:
122             sample_order = self.gm.topological_sort()
123         self.display(2,*[v for v in sample_order
124                          if v not in obs],sep="\t")
125         counts = {val:0 for val in qvar.domain}
126         for i in range(number_samples):
127             sample = {}
128             weight = 1.0
129             for nvar in sample_order:
130                 fac = self.gm.var2cpt[nvar]
131                 if nvar in obs:
132                     sample[nvar] = obs[nvar]
133                     weight *= fac.get_value(sample)
134                 else:
135                     val = sample_one({v:fac.get_value({**sample,nvar:v}) for
                            v in nvar.domain})
```

```
                self.display(2,val,end="\t")
                sample[nvar] = val
            counts[sample[qvar]] += weight
            self.display(2,weight)
        tot = sum(counts.values())
        # as well as the distribution we also include the raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist
```

Exercise 9.2 Change this algorithm so that it does **importance sampling** using a proposal distribution. It needs *sample_one* using a different distribution and then update the weight of the current sample. For testing, use a proposal distribution that only specifies probabilities for some of the variables (and the algorithm uses the probabilities for the network in other cases).

9.9.5 Particle Filtering

In this implementation, a particle is a $\{variable : value\}$ dictionary. Because adding a new value to dictionary involves a side effect, the dictionaries need to be copied during resampling.

probStochSim.py — (continued)

```
class ParticleFiltering(SamplingInferenceMethod):
    """The class that queries Graphical Models using Particle Filtering.

    gm is a belief network to query
    """
    method_name = "particle filtering"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)

    def query(self, qvar, obs={}, number_samples=1000, sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of factors where factors defining the parents
          come before the factors for the child.
        """
        if sample_order is None:
            sample_order = self.gm.topological_sort()
        self.display(2,*[v for v in sample_order
                         if v not in obs],sep="\t")
        particles = [{} for i in range(number_samples)]
        for nvar in sample_order:
            fac = self.gm.var2cpt[nvar]
            if nvar in obs:
                weights = [fac.get_value({**part, nvar:obs[nvar]}) for part
                    in particles]
```

```
172                 particles = [{**p, nvar:obs[nvar]} for p in
                        resample(particles, weights, number_samples)]
173             else:
174                 for part in particles:
175                     part[nvar] = sample_one({v:fac.get_value({**part,
                            nvar:v}) for v in nvar.domain})
176                 self.display(2,part[nvar],end="\t")
177         counts = {val:0 for val in qvar.domain}
178         for part in particles:
179             counts[part[qvar]] += 1
180         tot = sum(counts.values())
181         # as well as the distribution we also include the raw counts
182         dist = {c:v/tot for (c,v) in counts.items()}
183         dist["raw_counts"] = counts
184         return dist
```

Resampling

Resample is based on *sample_multiple* but works with an array of particles. (Aside: Python doesn't let us use *sample_multiple* directly as it uses a dictionary, and particles, represented as dictionaries can't be the key of dictionaries).

probStochSim.py — (continued)

```
186 def resample(particles, weights, num_samples):
187     """returns num_samples copies of particles resampled according to
            weights.
188     particles is a list of particles
189     weights is a list of positive numbers, of same length as particles
190     num_samples is n integer
191     """
192     total = sum(weights)
193     rands = sorted(random.random()*total for i in range(num_samples))
194     result = []
195     cum = weights[0]   # cumulative sum
196     index = 0
197     for r in rands:
198         while r>cum:
199             index += 1
200             cum += weights[index]
201         result.append(particles[index])
202     return result
```

9.9.6 Examples

probStochSim.py — (continued)

```
204 from probGraphicalModels import bn_4ch, A,B,C,D
205 bn_4chr = RejectionSampling(bn_4ch)
206 bn_4chL = LikelihoodWeighting(bn_4ch)
```

```
## InferenceMethod.max_display_level = 2 # detailed tracing for all
    inference methods
## bn_4chr.query(A,{})
## bn_4chr.query(C,{})
## bn_4chr.query(A,{C:True})
## bn_4chr.query(B,{A:True,C:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportr = RejectionSampling(bn_report) # answers queries using
    rejection sampling
bn_reportL = LikelihoodWeighting(bn_report) # answers queries using
    likelihood weighting
bn_reportp = ParticleFiltering(bn_report) # answers queries using particle
    filtering
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{})
## bn_reportr.query(Tamper,{Report:True})
## InferenceMethod.max_display_level = 0 # no detailed tracing for all
    inference methods
## bn_reportr.query(Tamper,{Report:True},number_samples=100000)
## bn_reportr.query(Tamper,{Report:True,Smoke:False})
## bn_reportr.query(Tamper,{Report:True,Smoke:False},number_samples=100)

## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)
## bn_reportL.query(Tamper,{Report:True,Smoke:False},number_samples=100)

from probGraphicalModels import bn_sprinkler,Season, Sprinkler
from probGraphicalModels import Rained, Grass_wet, Grass_shiny, Shoes_wet
bn_sprinklerr = RejectionSampling(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerL = LikelihoodWeighting(bn_sprinkler) # answers queries using
    rejection sampling
bn_sprinklerp = ParticleFiltering(bn_sprinkler) # answers queries using
    particle filtering
#bn_sprinklerr.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerL.query(Shoes_wet,{Grass_shiny:True,Rained:True})
#bn_sprinklerp.query(Shoes_wet,{Grass_shiny:True,Rained:True})

if __name__ == "__main__":
    InferenceMethod.testIM(RejectionSampling, threshold=0.1)
    InferenceMethod.testIM(LikelihoodWeighting, threshold=0.1)
    InferenceMethod.testIM(ParticleFiltering, threshold=0.1)
```

Exercise 9.3 This code keeps regenerating the distribution of a variable given its parents. Implement one or both of the following, and compare them to the original. Make *cond_dist* return a slice that corresponds to the distribution, and then use the slice instead of the dictionary (a list slice does not generate new data structures). Make *cond_dist* remember values it has already computed, and only return these.

9.9.7 Gibbs Sampling

The following implements **Gibbs sampling**, a form of **Markov Chain Monte Carlo** MCMC.

probStochSim.py — (continued)

```
#import random
#from probGraphicalModels import InferenceMethod

#from probStochSim import sample_one, SamplingInferenceMethod

class GibbsSampling(SamplingInferenceMethod):
    """The class that queries Graphical Models using Gibbs Sampling.

    bn is a graphical model (e.g., a belief network) to query
    """
    method_name = "Gibbs sampling"

    def __init__(self, gm=None):
        SamplingInferenceMethod.__init__(self, gm)
        self.gm = gm

    def query(self, qvar, obs={}, number_samples=1000, burn_in=100,
              sample_order=None):
        """computes P(qvar | obs) where
        qvar is a variable.
        obs is a {variable:value} dictionary.
        sample_order is a list of non-observed variables in order, or
        if sample_order None, the variables are shuffled at each iteration.
        """
        counts = {val:0 for val in qvar.domain}
        if sample_order is not None:
            variables = sample_order
        else:
            variables = [v for v in self.gm.variables if v not in obs]
        var_to_factors = {v:set() for v in self.gm.variables}
        for fac in self.gm.factors:
            for var in fac.variables:
                var_to_factors[var].add(fac)
        sample = {var:random.choice(var.domain) for var in variables}
        self.display(2,"Sample:",sample)
        sample.update(obs)
        for i in range(burn_in + number_samples):
            if sample_order == None:
                random.shuffle(variables)
            for var in variables:
                # get unnormalized probability distribution of var given its
                    neighbours
                vardist = {val:1 for val in var.domain}
                for val in var.domain:
                    sample[var] = val
```

```
                    for fac in var_to_factors[var]: # Markov blanket
                        vardist[val] *= fac.get_value(sample)
                sample[var] = sample_one(vardist)
            if i >= burn_in:
                counts[sample[qvar]] +=1
        tot = sum(counts.values())
        # as well as the computed distribution, we also include raw counts
        dist = {c:v/tot for (c,v) in counts.items()}
        dist["raw_counts"] = counts
        return dist

#from probGraphicalModels import bn_4ch, A,B,C,D
bn_4chg = GibbsSampling(bn_4ch)
## InferenceMethod.max_display_level = 2 # detailed tracing for all
    inference methods
bn_4chg.query(A,{})
## bn_4chg.query(D,{})
## bn_4chg.query(B,{D:True})
## bn_4chg.query(B,{A:True,C:False})

from probGraphicalModels import
    bn_report,Alarm,Fire,Leaving,Report,Smoke,Tamper
bn_reportg = GibbsSampling(bn_report)
## bn_reportg.query(Tamper,{Report:True},number_samples=1000)

if __name__ == "__main__":
    InferenceMethod.testIM(GibbsSampling, threshold=0.1)
```

Exercise 9.4 Change the code so that it can have multiple query variables. Make the list of query variable be an input to the algorithm, so that the default value is the list of all non-observed variables.

Exercise 9.5 In this algorithm, explain where it computes the probability of a variable given its Markov blanket. Instead of returning the average of the samples for the query variable, it is possible to return the average estimate of the probability of the query variable given its Markov blanket. Does this converge to the same answer as the given code? Does it converge faster, slower, or the same?

9.9.8 Plotting Behaviour of Stochastic Simulators

The stochastic simulation runs can give different answers each time they are run. For the algorithms that give the same answer in the limit as the number of samples approaches infinity (as do all of these algorithms), the algorithms can be compared by comparing the accuracy for multiple runs. Summary statistics like the variance may provide some information, but the assumptions behind the variance being appropriate (namely that the distribution is approximately Gaussian) may not hold for cases where the predictions are bounded and often skewed.

It is more appropriate to plot the distribution of predictions over multiple runs. The *plot_stats* method plots the prediction of a particular variable (or for the partition function) for a number of runs of the same algorithm. On the x-axis, is the prediction of the algorithm. On the y-axis is the number of runs with prediction less than or equal to the x value. Thus this is like a cumulative distribution over the predictions, but with counts on the y-axis.

Note that for runs where there are no samples that are consistent with the observations (as can happen with rejection sampling), the prediction of probability is 1.0 (as a convention for $0/0$).

That variable *what* contains the query variable, or *what* is "*prob_ev*", the probability of evidence.

probStochSim.py — (continued)

```
311 import matplotlib.pyplot as plt
312
313 def plot_stats(method, qvar, qval, obs, number_runs=1000, **queryargs):
314     """Plots a cumulative distribution of the prediction of the model.
315     method is a InferenceMethod (that implements appropriate query(.))
316     plots P(qvar=qval | obs)
317     qvar is the query variable, qval is corresponding value
318     obs is the {variable:value} dictionary representing the observations
319     number_iterations is the number of runs that are plotted
320     **queryargs is the arguments to query (often number_samples for
            sampling methods)
321     """
322     plt.ion()
323     plt.xlabel("value")
324     plt.ylabel("Cumulative Number")
325     method.max_display_level, prev_mdl = 0, method.max_display_level #no
            display
326     answers = [method.query(qvar,obs,**queryargs)
327                for i in range(number_runs)]
328     values = [ans[qval] for ans in answers]
329     label = f"{method.method_name} P({qvar}={qval}|{','.join(f'{var}={val}'
            for (var,val) in obs.items())})"
330     values.sort()
331     plt.plot(values,range(number_runs),label=label)
332     plt.legend() #loc="upper left")
333     plt.draw()
334     method.max_display_level = prev_mdl # restore display level
335
336 # Try:
337 #
        plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
        number_runs=1000)
338 #
        plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
        number_runs=1000)
339 #
```

```
    plot_stats(bn_reportp,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)
#
    plot_stats(bn_reportr,Tamper,True,{Report:True,Smoke:True},number_samples=100,
    number_runs=1000)
#
    plot_stats(bn_reportL,Tamper,True,{Report:True,Smoke:True},number_samples=100,
    number_runs=1000)
#
    plot_stats(bn_reportg,Tamper,True,{Report:True,Smoke:True},number_samples=1000,
    number_runs=1000)

def plot_mult(methods, example, qvar, qval, obs, number_samples=1000,
    number_runs=1000):
    for method in methods:
        solver = method(example)
        if isinstance(method,SamplingInferenceMethod):
            plot_stats(solver, qvar, qval, obs, number_samples, number_runs)
        else:
            plot_stats(solver, qvar, qval, obs, number_runs)

from probRC import ProbRC
# Try following (but it takes a while..)
methods =
    [ProbRC,RejectionSampling,LikelihoodWeighting,ParticleFiltering,GibbsSampling]
#plot_mult(methods,bn_report,Tamper,True,{Report:True,Smoke:False},number_samples=100,
    number_runs=1000)
#
    plot_mult(methods,bn_report,Tamper,True,{Report:False,Smoke:True},number_samples=100,
    number_runs=1000)

# Sprinkler Example:
#
    plot_stats(bn_sprinklerr,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
#
    plot_stats(bn_sprinklerL,Shoes_wet,True,{Grass_shiny:True,Rained:True},number_samples=1000)
```

9.10 Hidden Markov Models

This code for hidden Markov models is independent of the graphical models code, to keep it simple. Section 9.11 gives code that models hidden Markov models, and more generally, dynamic belief networks, using the graphical models code.

This HMM code assumes there are multiple Boolean observation variables that depend on the current state and are independent of each other given the state.

probHMM.py — Hidden Markov Model

```
import random
from probStochSim import sample_one, sample_multiple

class HMM(object):
    def __init__(self, states, obsvars, pobs, trans, indist):
        """A hidden Markov model.
        states - set of states
        obsvars - set of observation variables
        pobs - probability of observations, pobs[i][s] is P(Obs_i=True |
            State=s)
        trans - transition probability - trans[i][j] gives P(State=j |
            State=i)
        indist - initial distribution - indist[s] is P(State_0 = s)
        """
        self.states = states
        self.obsvars = obsvars
        self.pobs = pobs
        self.trans = trans
        self.indist = indist
```

Consider the following example. Suppose you want to unobtrusively keep track of an animal in a triangular enclosure using sound. Suppose you have 3 microphones that provide unreliable (noisy) binary information at each time step. The animal is either close to one of the 3 points of the triangle or in the middle of the triangle.

probHMM.py — (continued)

```
# state
#       0=middle, 1,2,3 are corners
states1 = {'middle', 'c1', 'c2', 'c3'} # states
obs1 = {'m1','m2','m3'} # microphones
```

The observation model is as follows. If the animal is in a corner, it will be detected by the microphone at that corner with probability 0.6, and will be independently detected by each of the other microphones with a probability of 0.1. If the animal is in the middle, it will be detected by each microphone with a probability of 0.4.

probHMM.py — (continued)

```
# pobs gives the observation model:
#pobs[mi][state] is P(mi=on | state)
closeMic=0.6; farMic=0.1; midMic=0.4
pobs1 = {'m1':{'middle':midMic, 'c1':closeMic, 'c2':farMic, 'c3':farMic},
    # mic 1
        'm2':{'middle':midMic, 'c1':farMic, 'c2':closeMic, 'c3':farMic}, #
            mic 2
        'm3':{'middle':midMic, 'c1':farMic, 'c2':farMic, 'c3':closeMic}} #
            mic 3
```

The transition model is as follows: If the animal is in a corner it stays in the same corner with probability 0.80, goes to the middle with probability 0.1

or goes to one of the other corners with probability 0.05 each. If it is in the middle, it stays in the middle with probability 0.7, otherwise it moves to one the corners, each with probability 0.1.

probHMM.py — (continued)

```
# trans specifies the dynamics
# trans[i] is the distribution over states resulting from state i
# trans[i][j] gives P(S=j | S=i)
sm=0.7; mmc=0.1             # transition probabilities when in middle
sc=0.8; mcm=0.1; mcc=0.05   # transition probabilities when in a corner
trans1 = {'middle':{'middle':sm, 'c1':mmc, 'c2':mmc, 'c3':mmc}, # was in middle
          'c1':{'middle':mcm, 'c1':sc, 'c2':mcc, 'c3':mcc}, # was in corner 1
          'c2':{'middle':mcm, 'c1':mcc, 'c2':sc, 'c3':mcc}, # was in corner 2
          'c3':{'middle':mcm, 'c1':mcc, 'c2':mcc, 'c3':sc}} # was in corner 3
```

Initially the animal is in one of the four states, with equal probability.

probHMM.py — (continued)

```
# initially we have a uniform distribution over the animal's state
indist1 = {st:1.0/len(states1) for st in states1}

hmm1 = HMM(states1, obs1, pobs1, trans1, indist1)
```

9.10.1 Exact Filtering for HMMs

A *HMMVEfilter* has a current state distribution which can be updated by observing or by advancing to the next time.

probHMM.py — (continued)

```
from display import Displayable

class HMMVEfilter(Displayable):
    def __init__(self,hmm):
        self.hmm = hmm
        self.state_dist = hmm.indist

    def filter(self, obsseq):
        """updates and returns the state distribution following the sequence of
        observations in obsseq using variable elimination.

        Note that it first advances time.
        This is what is required if it is called sequentially.
        If that is not what is wanted initially, do an observe first.
        """
        for obs in obsseq:
```

```
72              self.advance()    # advance time
73              self.observe(obs) # observe
74          return self.state_dist
75
76      def observe(self, obs):
77          """updates state conditioned on observations.
78          obs is a list of values for each observation variable"""
79          for i in self.hmm.obsvars:
80              self.state_dist = {st:self.state_dist[st]*(self.hmm.pobs[i][st]
81                                                  if obs[i] else
                                                      (1-self.hmm.pobs[i][st]))
82                                 for st in self.hmm.states}
83          norm = sum(self.state_dist.values()) # normalizing constant
84          self.state_dist = {st:self.state_dist[st]/norm for st in
                self.hmm.states}
85          self.display(2,"After observing",obs,"state
                distribution:",self.state_dist)
86
87      def advance(self):
88          """advance to the next time"""
89          nextstate = {st:0.0 for st in self.hmm.states}  # distribution over
                next states
90          for j in self.hmm.states:     # j ranges over next states
91              for i in self.hmm.states: # i ranges over previous states
92                  nextstate[j] += self.hmm.trans[i][j]*self.state_dist[i]
93          self.state_dist = nextstate
94          self.display(2,"After advancing state
                distribution:",self.state_dist)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
96  hmm1f1 = HMMVEfilter(hmm1)
97  # hmm1f1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
98  ## HMMVEfilter.max_display_level = 2 # show more detail in displaying
99  # hmm1f2 = HMMVEfilter(hmm1)
100 # hmm1f2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
        {'m1':1, 'm2':0, 'm3':0},
101 #              {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
        {'m1':0, 'm2':0, 'm3':0},
102 #              {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
        {'m1':0, 'm2':0, 'm3':1},
103 #              {'m1':0, 'm2':0, 'm3':1}])
104 # hmm1f3 = HMMVEfilter(hmm1)
105 # hmm1f3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
        {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])
106
107 # How do the following differ in the resulting state distribution?
108 # Note they start the same, but have different initial observations.
109 ## HMMVEfilter.max_display_level = 1 # show less detail in displaying
110 # for i in range(100): hmm1f1.advance()
```

```
# hmm1f1.state_dist
# for i in range(100): hmm1f3.advance()
# hmm1f3.state_dist
```

Exercise 9.6 The representation assumes that there are a list of Boolean observations. Extend the representation so that the each observation variable can have multiple discrete values. You need to choose a representation for the model, and change the algorithm.

9.10.2 Localization

The localization example in the book is a controlled HMM, where there is a given action at each time and the transition depends on the action. In this class, the transition is set to None initially, and needs to be provided with an action to determine the transition probability.

probLocalization.py — Controlled HMM and Localization example

```
from probHMM import HMMVEfilter, HMM
from display import Displayable
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons

class HMM_Controlled(HMM):
    """A controlled HMM, where the transition probability depends on the
       action.
       Instead of the transition probability, it has a function act2trans
       from action to transition probability.
       Any algorithms need to select the transition probability according
          to the action.
    """
    def __init__(self, states, obsvars, pobs, act2trans, indist):
        self.act2trans = act2trans
        HMM.__init__(self, states, obsvars, pobs, None, indist)


local_states = list(range(16))
door_positions = {2,4,7,11}
def prob_door(loc): return 0.8 if loc in door_positions else 0.1
local_obs = {'door':[prob_door(i) for i in range(16)]}
act2trans = {'right': [[0.1 if next == current
                            else 0.8 if next == (current+1)%16
                            else 0.074 if next == (current+2)%16
                            else 0.002 for next in range(16)] for
                                current in range(16)],
                     'left': [[0.1 if next == current
                            else 0.8 if next == (current-1)%16
                            else 0.074 if next == (current-2)%16
                            else 0.002 for next in range(16)] for
                                current in range(16)]}
```

```
hmm_16pos = HMM_Controlled(local_states, {'door'}, local_obs, act2trans,
    [1/16 for i in range(16)])
```

To change the VE localization code to allow for controlled HMMs, notice that the action selects which transition probability to us.

probLocalization.py — (continued)

```
class HMM_Local(HMMVEfilter):
    """VE filter for controlled HMMs
    """
    def __init__(self, hmm):
        HMMVEfilter.__init__(self, hmm)

    def go(self, action):
        self.hmm.trans = self.hmm.act2trans[action]
        self.advance()

loc_filt = HMM_Local(hmm_16pos)
# loc_filt.observe({'door':True}); loc_filt.go("right");
    loc_filt.observe({'door':False}); loc_filt.go("right");
    loc_filt.observe({'door':True})
# loc_filt.state_dist
```

The following lets us interactively move the agent and provide observations. It shows the distribution over locations.

probLocalization.py — (continued)

```
class Show_Localization(Displayable):
    def __init__(self,hmm):
        self.hmm = hmm
        self.loc_filt = HMM_Local(hmm)
        fig,(self.ax) = plt.subplots()
        plt.subplots_adjust(bottom=0.2)
        left_butt = Button(plt.axes([0.05,0.02,0.1,0.05]), "left")
        left_butt.on_clicked(self.left)
        right_butt = Button(plt.axes([0.25,0.02,0.1,0.05]), "right")
        right_butt.on_clicked(self.right)
        door_butt = Button(plt.axes([0.45,0.02,0.1,0.05]), "door")
        door_butt.on_clicked(self.door)
        nodoor_butt = Button(plt.axes([0.65,0.02,0.1,0.05]), "no door")
        nodoor_butt.on_clicked(self.nodoor)
        reset_butt = Button(plt.axes([0.85,0.02,0.1,0.05]), "reset")
        reset_butt.on_clicked(self.reset)
               #this makes sure y-axis goes to 1, graph overwritten in
                   draw_dist
        self.draw_dist()
        plt.show()

    def draw_dist(self):
        self.ax.clear()
        plt.ylim(0,1)
```

```
77          self.ax.set_ylabel("Probability")
78          self.ax.set_xlabel("Location")
79          self.ax.set_title("Location Probability Distribution")
80          self.ax.set_xticks(self.hmm.states)
81          vals = [self.loc_filt.state_dist[i] for i in self.hmm.states]
82          self.bars = self.ax.bar(self.hmm.states, vals, color='black')
83          self.ax.bar_label(self.bars,["{v:.2f}".format(v=v) for v in vals],
                padding = 1)
84          plt.draw()
85
86      def left(self,event):
87          self.loc_filt.go("left")
88          self.draw_dist()
89      def right(self,event):
90          self.loc_filt.go("right")
91          self.draw_dist()
92      def door(self,event):
93          self.loc_filt.observe({'door':True})
94          self.draw_dist()
95      def nodoor(self,event):
96          self.loc_filt.observe({'door':False})
97          self.draw_dist()
98      def reset(self,event):
99          self.loc_filt.state_dist = {i:1/16 for i in range(16)}
100         self.draw_dist()
101
102 # sl = Show_Localization(hmm_16pos)
```

9.10.3 Particle Filtering for HMMs

In this implementation a particle is just a state. If you want to do some form of smoothing, a particle should probably be a history of states. This maintains, *particles*, an array of states, *weights* an array of (non-negative) real numbers, such that $weights[i]$ is the weight of $particles[i]$.

probHMM.py — (continued)

```
114 from display import Displayable
115 from probStochSim import resample
116
117 class HMMparticleFilter(Displayable):
118     def __init__(self,hmm,number_particles=1000):
119         self.hmm = hmm
120         self.particles = [sample_one(hmm.indist)
121                           for i in range(number_particles)]
122         self.weights = [1 for i in range(number_particles)]
123
124     def filter(self, obsseq):
125         """returns the state distribution following the sequence of
126         observations in obsseq using particle filtering.
```

```
127
128         Note that it first advances time.
129         This is what is required if it is called after previous filtering.
130         If that is not what is wanted initially, do an observe first.
131         """
132         for obs in obsseq:
133             self.advance()   # advance time
134             self.observe(obs) # observe
135             self.resample_particles()
136             self.display(2,"After observing", str(obs),
137                          "state distribution:",
                                 self.histogram(self.particles))
138         self.display(1,"Final state distribution:",
                self.histogram(self.particles))
139         return self.histogram(self.particles)
140
141     def advance(self):
142         """advance to the next time.
143         This assumes that all of the weights are 1."""
144         self.particles = [sample_one(self.hmm.trans[st])
145                           for st in self.particles]
146
147     def observe(self, obs):
148         """reweighs the particles to incorporate observations obs"""
149         for i in range(len(self.particles)):
150             for obv in obs:
151                 if obs[obv]:
152                     self.weights[i] *= self.hmm.pobs[obv][self.particles[i]]
153                 else:
154                     self.weights[i] *=
                            1-self.hmm.pobs[obv][self.particles[i]]
155
156     def histogram(self, particles):
157         """returns list of the probability of each state as represented by
158         the particles"""
159         tot=0
160         hist = {st: 0.0 for st in self.hmm.states}
161         for (st,wt) in zip(self.particles,self.weights):
162             hist[st]+=wt
163             tot += wt
164         return {st:hist[st]/tot for st in hist}
165
166     def resample_particles(self):
167         """resamples to give a new set of particles."""
168         self.particles = resample(self.particles, self.weights,
                len(self.particles))
169         self.weights = [1] * len(self.particles)
```

The following are some queries for *hmm*1.

probHMM.py — (continued)

```
hmm1pf1 = HMMparticleFilter(hmm1)
# HMMparticleFilter.max_display_level = 2 # show each step
# hmm1pf1.filter([{'m1':0, 'm2':1, 'm3':1}, {'m1':1, 'm2':0, 'm3':1}])
# hmm1pf2 = HMMparticleFilter(hmm1)
# hmm1pf2.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':1, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':0, 'm2':0, 'm3':0},
#               {'m1':0, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':1},
    {'m1':0, 'm2':0, 'm3':1},
#               {'m1':0, 'm2':0, 'm3':1}])
# hmm1pf3 = HMMparticleFilter(hmm1)
# hmm1pf3.filter([{'m1':1, 'm2':0, 'm3':0}, {'m1':0, 'm2':0, 'm3':0},
    {'m1':1, 'm2':0, 'm3':0}, {'m1':1, 'm2':0, 'm3':1}])
```

Exercise 9.7 A form of importance sampling can be obtained by not resampling. Is it better or worse than particle filtering? Hint: you need to think about how they can be compared. Is the comparison different if there are more states than particles?

Exercise 9.8 Extend the particle filtering code to continuous variables and observations. In particular, suppose the state transition is a linear function with Gaussian noise of the previous state, and the observations are linear functions with Gaussian noise of the state. You may need to research how to sample from a Gaussian distribution.

9.10.4 Generating Examples

The following code is useful for generating examples.

probHMM.py — (continued)

```
def simulate(hmm,horizon):
    """returns a pair of (state sequence, observation sequence) of length
        horizon.
    for each time t, the agent is in state_sequence[t] and
    observes observation_sequence[t]
    """
    state = sample_one(hmm.indist)
    obsseq=[]
    stateseq=[]
    for time in range(horizon):
        stateseq.append(state)
        newobs =
            {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
                 for obs in hmm.obsvars}
        obsseq.append(newobs)
        state = sample_one(hmm.trans[state])
    return stateseq,obsseq

def simobs(hmm,stateseq):
```

```
    """returns observation sequence for the state sequence"""
    obsseq=[]
    for state in stateseq:
        newobs =
            {obs:sample_one({0:1-hmm.pobs[obs][state],1:hmm.pobs[obs][state]})
                 for obs in hmm.obsvars}
        obsseq.append(newobs)
    return obsseq

def create_eg(hmm,n):
    """Create an annotated example for horizon n"""
    seq,obs = simulate(hmm,n)
    print("True state sequence:",seq)
    print("Sequence of observations:\n",obs)
    hmmfilter = HMMVEfilter(hmm)
    dist = hmmfilter.filter(obs)
    print("Resulting distribution over states:\n",dist)
```

9.11 Dynamic Belief Networks

A **dynamic belief network (DBN)** is a belief network that extends in time.

There are a number of ways that reasoning can be carried out in a DBN, including:

- Rolling out the DBN for some time period, and using standard belief network inference. The latest time that needs to be in the rolled out network is the time of the latest observation or the time of a query (whichever is later). This allows us to observe any variables at any time and query any variables at any time. This is covered in Section 9.11.2.

- An unrolled belief network may be very large, and we might only be interested in asking about "now". In this case we can just representing the variables "now". In this approach we can observe and query the current variables. We can them move to the next time. This does not allow for arbitrary historical queries (about the past or the future), but can be much simpler. This is covered in Section 9.11.3.

9.11.1 Representing Dynamic Belief Networks

To specify a DBN, think about the distribution *now*. *Now* will be represented as time 1. Each variable will have a corresponding previous variable; these will be created together.

A dynamic belief network consists of:

- A set of features. A variable is a feature-time pair.

- An initial distribution over the features "now" (time 1). This is a belief network with all variables being time 1 variables.

- A specification of the dynamics. We define the how the variables *now* (time 1) depend on variables *now* and the previous time (time 0), in such a way that the graph is acyclic.

probDBN.py — Dynamic belief networks

```
from variable import Variable
from probGraphicalModels import GraphicalModel, BeliefNetwork
from probFactors import Prob, Factor, CPD
from probVE import VE
from display import Displayable

class DBNvariable(Variable):
    """A random variable that incorporates the stage (time)

    A variable can have both a name and an index. The index defaults to 1.
    """
    def __init__(self,name,domain=[False,True],index=1):
        Variable.__init__(self,f"{name}_{index}",domain)
        self.basename = name
        self.domain = domain
        self.index = index
        self.previous = None

    def __lt__(self,other):
        if self.name != other.name:
            return self.name<other.name
        else:
            return self.index<other.index

    def __gt__(self,other):
        return other<self

def variable_pair(name,domain=[False,True]):
    """returns a variable and its predecessor. This is used to define
        2-stage DBNs

    If the name is X, it returns the pair of variables X_prev,X_now"""
    var_now = DBNvariable(name,domain,index='now')
    var_prev = DBNvariable(name,domain,index='prev')
    var_now.previous = var_prev
    return var_prev, var_now
```

A *FactorRename* is a factor that is the result renaming the variables in the factor. It takes a factor, *fac*, and a $\{new : old\}$ dictionary, where *new* is the name of a variable in the resulting factor and *old* is the corresponding name in *fac*. This assumes that the all variables are renamed.

probDBN.py — (continued)

```
class FactorRename(Factor):
    def __init__(self,fac,renaming):
        """A renamed factor.
        fac is a factor
        renaming is a dictionary of the form {new:old} where old and new
            var variables,
           where the variables in fac appear exactly once in the renaming
        """
        Factor.__init__(self,[n for (n,o) in renaming.items() if o in
            fac.variables])
        self.orig_fac = fac
        self.renaming = renaming

    def get_value(self,assignment):
        return self.orig_fac.get_value({self.renaming[var]:val
                                        for (var,val) in assignment.items()
                                        if var in self.variables})
```

The following class renames the variables of a conditional probability distribution. It is used for template models (e.g., dynamic decision networks or relational models)

probDBN.py — (continued)

```
class CPDrename(FactorRename, CPD):
    def __init__(self, cpd, renaming):
        renaming_inverse = {old:new for (new,old) in renaming.items()}
        CPD.__init__(self,renaming_inverse[cpd.child],[renaming_inverse[p]
            for p in cpd.parents])
        self.orig_fac = cpd
        self.renaming = renaming
```

probDBN.py — (continued)

```
class DBN(Displayable):
    """The class of stationary Dynamic Belief networks.
    * name is the DBN name
    * vars_now is a list of current variables (each must have
    previous variable).
    * transition_factors is a list of factors for P(X|parents) where X
    is a current variable and parents is a list of current or previous
        variables.
    * init_factors is a list of factors for P(X|parents) where X is a
    current variable and parents can only include current variables
    The graph of transition factors + init factors must be acyclic.

    """
    def __init__(self, title, vars_now, transition_factors=None,
        init_factors=None):
        self.title = title
        self.vars_now = vars_now
```

```
85          self.vars_prev = [v.previous for v in vars_now]
86          self.transition_factors = transition_factors
87          self.init_factors = init_factors
88          self.var_index = {}    # var_index[v] is the index of variable v
89          for i,v in enumerate(vars_now):
90              self.var_index[v]=i
```

Here is a 3 variable DBN:

probDBN.py — (continued)

```
92  A0,A1 = variable_pair("A", domain=[False,True])
93  B0,B1 = variable_pair("B", domain=[False,True])
94  C0,C1 = variable_pair("C", domain=[False,True])
95
96  # dynamics
97  pc = Prob(C1,[B1,C0],[[[0.03,0.97],[0.38,0.62]],[[0.23,0.77],[0.78,0.22]]])
98  pb = Prob(B1,[A0,A1],[[[0.5,0.5],[0.77,0.23]],[[0.4,0.6],[0.83,0.17]]])
99  pa = Prob(A1,[A0,B0],[[[0.1,0.9],[0.65,0.35]],[[0.3,0.7],[0.8,0.2]]])
100
101 # initial distribution
102 pa0 = Prob(A1,[],[0.9,0.1])
103 pb0 = Prob(B1,[A1],[[0.3,0.7],[0.8,0.2]])
104 pc0 = Prob(C1,[],[0.2,0.8])
105
106 dbn1 = DBN("Simple DBN",[A1,B1,C1],[pa,pb,pc],[pa0,pb0,pc0])
```

Here is the animal example

probDBN.py — (continued)

```
108 from probHMM import closeMic, farMic, midMic, sm, mmc, sc, mcm, mcc
109
110 Pos_0,Pos_1 = variable_pair("Position",domain=[0,1,2,3])
111 Mic1_0,Mic1_1 = variable_pair("Mic1")
112 Mic2_0,Mic2_1 = variable_pair("Mic2")
113 Mic3_0,Mic3_1 = variable_pair("Mic3")
114
115 # conditional probabilities - see hmm for the values of sm,mmc, etc
116 ppos = Prob(Pos_1, [Pos_0],
117             [[sm, mmc, mmc, mmc], #was in middle
118              [mcm, sc, mcc, mcc], #was in corner 1
119              [mcm, mcc, sc, mcc], #was in corner 2
120              [mcm, mcc, mcc, sc]]) #was in corner 3
121 pm1 = Prob(Mic1_1, [Pos_1], [[1-midMic, midMic], [1-closeMic, closeMic],
122                              [1-farMic, farMic], [1-farMic, farMic]])
123 pm2 = Prob(Mic2_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
124                              [1-closeMic, closeMic], [1-farMic, farMic]])
125 pm3 = Prob(Mic3_1, [Pos_1], [[1-midMic, midMic], [1-farMic, farMic],
126                              [1-farMic, farMic], [1-closeMic, closeMic]])
127 ipos = Prob(Pos_1,[], [0.25, 0.25, 0.25, 0.25])
128 dbn_an =DBN("Animal DBN",[Pos_1,Mic1_1,Mic2_1,Mic3_1],
129             [ppos, pm1, pm2, pm3],
130             [ipos, pm1, pm2, pm3])
```

9.11.2 Unrolling DBNs

probDBN.py — (continued)

```
class BNfromDBN(BeliefNetwork):
    """Belief Network unrolled from a dynamic belief network
    """

    def __init__(self,dbn,horizon):
        """dbn is the dynamic belief network being unrolled
        horizon>0 is the number of steps (so there will be horizon+1
            variables for each DBN variable.
        """
        self.name2var = {var.basename:
            [DBNvariable(var.basename,var.domain,index) for index in
            range(horizon+1)]
                         for var in dbn.vars_now}
        self.display(1,f"name2var={self.name2var}")
        variables = {v for vs in self.name2var.values() for v in vs}
        self.display(1,f"variables={variables}")
        bnfactors = {CPDrename(fac,{self.name2var[var.basename][0]:var
                                    for var in fac.variables})
                     for fac in dbn.init_factors}
        bnfactors |= {CPDrename(fac,{self.name2var[var.basename][i]:var
                                    for var in fac.variables if
                                        var.index=='prev'}
                                | {self.name2var[var.basename][i+1]:var
                                    for var in fac.variables if
                                        var.index=='now'})
                     for fac in dbn.transition_factors
                         for i in range(horizon)}
        self.display(1,f"bnfactors={bnfactors}")
        BeliefNetwork.__init__(self, dbn.title, variables, bnfactors)
```

Here are two examples. Note that we need to use `bn.name2var['B'][2]` to get the variable B2 (B at time 2).

probDBN.py — (continued)

```
# Try
#from probRC import ProbRC
#bn = BNfromDBN(dbn1,2) # construct belief network
#drc = ProbRC(bn)            # initialize recursive conditioning
#B2 = bn.name2var['B'][2]
#drc.query(B2) #P(B2)
#drc.query(bn.name2var['B'][1],{bn.name2var['B'][0]:True,bn.name2var['C'][1]:False})
    #P(B1|B0,C1)
```

9.11.3 DBN Filtering

If we only wanted to ask questions about the current state, we can save space by forgetting the history variables.

probDBN.py — (continued)

```
164 class DBNVEfilter(VE):
165     def __init__(self,dbn):
166         self.dbn = dbn
167         self.current_factors = dbn.init_factors
168         self.current_obs = {}
169 
170     def observe(self, obs):
171         """updates the current observations with obs.
172         obs is a variable:value dictionary where variable is a current
173         variable.
174         """
175         assert all(self.current_obs[var]==obs[var] for var in obs
176                    if var in self.current_obs),"inconsistent current
                           observations"
177         self.current_obs.update(obs) # note 'update' is a dict method
178 
179     def query(self,var):
180         """returns the posterior probability of current variable var"""
181         return
                VE(GraphicalModel(self.dbn.title,self.dbn.vars_now,self.current_factors)).query(var,self.c
182 
183     def advance(self):
184         """advance to the next time"""
185         prev_factors = [self.make_previous(fac) for fac in
                self.current_factors]
186         prev_obs = {var.previous:val for var,val in
                self.current_obs.items()}
187         two_stage_factors = prev_factors + self.dbn.transition_factors
188         self.current_factors =
                self.elim_vars(two_stage_factors,self.dbn.vars_prev,prev_obs)
189         self.current_obs = {}
190 
191     def make_previous(self,fac):
192          """Creates new factor from fac where the current variables in fac
193          are renamed to previous variables.
194          """
195          return FactorRename(fac, {var.previous:var for var in
                 fac.variables})
196 
197     def elim_vars(self,factors, vars, obs):
198         for var in vars:
199            if var in obs:
200                factors = [self.project_observations(fac,obs) for fac in
                       factors]
201            else:
202                factors = self.eliminate_var(factors, var)
203         return factors
```

Example queries:

probDBN.py — (continued)

```
#df = DBNVEfilter(dbn1)
#df.observe({B1:True}); df.advance(); df.observe({C1:False})
#df.query(B1)  #P(B1|B0,C1)
#df.advance(); df.query(B1)
#dfa = DBNVEfilter(dbn_an)
# dfa.observe({Mic1_1:0, Mic2_1:1, Mic3_1:1})
# dfa.advance()
# dfa.observe({Mic1_1:1, Mic2_1:0, Mic3_1:1})
# dfa.query(Pos_1)
```

Chapter 10

Learning with Uncertainty

10.1 K-means

The k-means learner maintains two lists that suffice as sufficient statistics to classify examples, and to learn the classification:

- *class_counts* is a list such that $class\_counts[c]$ is the number of examples in the training set with $class = c$.
- *feature_sum* is a list such that $feature\_sum[i][c]$ is sum of the values for the i'th feature i for members of class c. The average value of the ith feature in class i is

$$\frac{feature\_sum[i][c]}{class\_counts[c]}$$

The class is initialized by randomly assigning examples to classes, and updating the statistics for *class_counts* and *feature_sum*.

learnKMeans.py — k-means learning

```
11 from learnProblem import Data_set, Learner, Data_from_file
12 import random
13 import matplotlib.pyplot as plt
14
15 class K_means_learner(Learner):
16     def __init__(self,dataset, num_classes):
17         self.dataset = dataset
18         self.num_classes = num_classes
19         self.random_initialize()
20
21     def random_initialize(self):
```

```
22          # class_counts[c] is the number of examples with class=c
23          self.class_counts = [0]*self.num_classes
24          # feature_sum[i][c] is the sum of the values of feature i for class
                c
25          self.feature_sum = [[0]*self.num_classes
26                              for feat in self.dataset.input_features]
27          for eg in self.dataset.train:
28              cl = random.randrange(self.num_classes) # assign eg to random
                    class
29              self.class_counts[cl] += 1
30              for (ind,feat) in enumerate(self.dataset.input_features):
31                  self.feature_sum[ind][cl] += feat(eg)
32          self.num_iterations = 0
33          self.display(1,"Initial class counts: ",self.class_counts)
```

The distance from (the mean of) a class to an example is the sum, over all fratures, of the sum-of-squares differences of the class mean and the example value.

learnKMeans.py — (continued)

```
35      def distance(self,cl,eg):
36          """distance of the eg from the mean of the class"""
37          return sum( (self.class_prediction(ind,cl)-feat(eg))**2
38                          for (ind,feat) in
39                              enumerate(self.dataset.input_features))
40
41      def class_prediction(self,feat_ind,cl):
42          """prediction of the class cl on the feature with index feat_ind"""
43          if self.class_counts[cl] == 0:
44              return 0 # there are no examples so we can choose any value
45          else:
46              return self.feature_sum[feat_ind][cl]/self.class_counts[cl]
47
48      def class_of_eg(self,eg):
49          """class to which eg is assigned"""
50          return (min((self.distance(cl,eg),cl)
51                          for cl in range(self.num_classes)))[1]
52                # second element of tuple, which is a class with minimum
                      distance
```

One step of k-means updates the *class_counts* and *feature_sum*. It uses the old values to determine the classes, and so the new values for *class_counts* and *feature_sum*. At the end it determines whether the values of these have changes, and then replaces the old ones with the new ones. It returns an indicator of whether the values are stable (have not changed).

learnKMeans.py — (continued)

```
53      def k_means_step(self):
54          """Updates the model with one step of k-means.
55          Returns whether the assignment is stable.
56          """
```

```
57          new_class_counts = [0]*self.num_classes
58          # feature_sum[i][c] is the sum of the values of feature i for class
                c
59          new_feature_sum = [[0]*self.num_classes
60                             for feat in self.dataset.input_features]
61          for eg in self.dataset.train:
62              cl = self.class_of_eg(eg)
63              new_class_counts[cl] += 1
64              for (ind,feat) in enumerate(self.dataset.input_features):
65                  new_feature_sum[ind][cl] += feat(eg)
66          stable = (new_class_counts == self.class_counts) and
                (self.feature_sum == new_feature_sum)
67          self.class_counts = new_class_counts
68          self.feature_sum = new_feature_sum
69          self.num_iterations += 1
70          return stable
71
72
73      def learn(self,n=100):
74          """do n steps of k-means, or until convergence"""
75          i=0
76          stable = False
77          while i<n and not stable:
78              stable = self.k_means_step()
79              i += 1
80              self.display(1,"Iteration",self.num_iterations,
81                           "class counts: ",self.class_counts,"
                                Stable=",stable)
82          return stable
83
84      def show_classes(self):
85          """sorts the data by the class and prints in order.
86          For visualizing small data sets
87          """
88          class_examples = [[] for i in range(self.num_classes)]
89          for eg in self.dataset.train:
90              class_examples[self.class_of_eg(eg)].append(eg)
91          print("Class","Example",sep='\t')
92          for cl in range(self.num_classes):
93              for eg in class_examples[cl]:
94                  print(cl,*eg,sep='\t')
95
96      def plot_error(self, maxstep=20):
97          """Plots the sum-of-suares error as a function of the number of
                steps"""
98          plt.ion()
99          plt.xlabel("step")
100         plt.ylabel("Ave sum-of-squares error")
101         train_errors = []
102         if self.dataset.test:
```

```
103             test_errors = []
104         for i in range(maxstep):
105             self.learn(1)
106             train_errors.append( sum(self.distance(self.class_of_eg(eg),eg)
107                                      for eg in self.dataset.train)
108                                  /len(self.dataset.train))
109             if self.dataset.test:
110                 test_errors.append(
                        sum(self.distance(self.class_of_eg(eg),eg)
111                                       for eg in self.dataset.test)
112                                   /len(self.dataset.test))
113         plt.plot(range(1,maxstep+1),train_errors,
114                  label=str(self.num_classes)+" classes. Training set")
115         if self.dataset.test:
116             plt.plot(range(1,maxstep+1),test_errors,
117                      label=str(self.num_classes)+" classes. Test set")
118         plt.legend()
119         plt.draw()
120
121 %data = Data_from_file('data/emdata1.csv', num_train=10,
        target_index=2000) % trivial example
122 data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
123 %data = Data_from_file('data/emdata0.csv', num_train=14,
        target_index=2000) % example from textbook
124 kml = K_means_learner(data,2)
125 num_iter=4
126 print("Class assignment after",num_iter,"iterations:")
127 kml.learn(num_iter); kml.show_classes()
128
129 # Plot the error
130 # km2=K_means_learner(data,2); km2.plot_error(20) # 2 classes
131 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
132 # km13=K_means_learner(data,13); km13.plot_error(20) # 13 classes
133
134 # data = Data_from_file('data/carbool.csv',
        target_index=2000,boolean_features=True)
135 # kml = K_means_learner(data,3)
136 # kml.learn(20); kml.show_classes()
137 # km3=K_means_learner(data,3); km3.plot_error(20) # 3 classes
138 # km3=K_means_learner(data,30); km3.plot_error(20) # 30 classes
```

Exercise 10.1 Change *boolean_features = True* flag to allow for numerical features. K-means assumes the features are numerical, so we want to make non-numerical features into numerical features (using characteristic functions) but we probably don't want to change numerical features into Boolean.

Exercise 10.2 If there are many classes, some of the classes can become empty (e.g., try 100 classes with carbool.csv). Implement a way to put some examples into a class, if possible. Two ideas are:

(a) Initialize the classes with actual examples, so that the classes will not start empty. (Do the classes become empty?)

(b) In *class_prediction*, we test whether the code is empty, and make a prediction of 0 for an empty class. It is possible to make a different prediction to "steal" an example (but you should make sure that a class has a consistent value for each feature in a loop).

Make your own suggestions, and compare it with the original, and whichever of these you think may work better.

10.2 EM

In the following definition, a class, c, is a integer in range $[0, num\_classes)$. i is an index of a feature, so $feat[i]$ is the ith feature, and a feature is a function from tuples to values. val is a value of a feature.

A model consists of 2 lists, which form the sufficient statistics:

- $class\_counts$ is a list such that $class\_counts[c]$ is the number of tuples with $class = c$, where each tuple is weighted by its probability, i.e.,

$$class\_counts[c] = \sum_{t:class(t)=c} P(t)$$

- $feature\_counts$ is a list such that $feature\_counts[i][val][c]$ is the weighted count of the number of tuples t with $feat[i](t) = val$ and $class(t) = c$, each tuple is weighted by its probability, i.e.,

$$feature\_counts[i][val][c] = \sum_{t:feat[i](t)=val \text{ and } class(t)=c} P(t)$$

learnEM.py — EM Learning

```
from learnProblem import Data_set, Learner, Data_from_file
import random
import math
import matplotlib.pyplot as plt

class EM_learner(Learner):
    def __init__(self,dataset, num_classes):
        self.dataset = dataset
        self.num_classes = num_classes
        self.class_counts = None
        self.feature_counts = None
```

The function *em_step* goes though the training examples, and updates these counts. The first time it is run, when there is no model, it uses random distributions.

learnEM.py — (continued)

```
23      def em_step(self, orig_class_counts, orig_feature_counts):
24          """updates the model."""
25          class_counts = [0]*self.num_classes
26          feature_counts = [{val:[0]*self.num_classes
27                             for val in feat.frange}
28                             for feat in self.dataset.input_features]
29          for tple in self.dataset.train:
30              if orig_class_counts: # a model exists
31                  tpl_class_dist = self.prob(tple, orig_class_counts,
                        orig_feature_counts)
32              else:               # initially, with no model, return a random
                    distribution
33                  tpl_class_dist = random_dist(self.num_classes)
34              for cl in range(self.num_classes):
35                  class_counts[cl] += tpl_class_dist[cl]
36                  for (ind,feat) in enumerate(self.dataset.input_features):
37                      feature_counts[ind][feat(tple)][cl] += tpl_class_dist[cl]
38          return class_counts, feature_counts
```

prob computes the probability of a class c for a tuple *tpl*, given the current statistics.

$$\begin{aligned} P(c \mid tple) &\propto P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \frac{class\_counts[c]}{len(self.dataset)} * \prod_i \frac{feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]} \\ &\propto \frac{\prod_i feature\_counts[i][feat_i(tple)][c]}{class\_counts[c]^{|feats|-1}} \end{aligned}$$

The last step is because $len(self.dataset)$ is a constant (independent of c). $class\_counts[c]$ can be taken out of the product, but needs to be raised to the power of the number of features, and one of them cancels.

learnEM.py — (continued)

```
40      def prob(self, tple, class_counts, feature_counts):
41          """returns a distribution over the classes for tuple tple in the
                model defined by the counts
42          """
43          feats = self.dataset.input_features
44          unnorm = [prod(feature_counts[i][feat(tple)][c]
45                         for (i,feat) in enumerate(feats))
46                        /(class_counts[c]**(len(feats)-1))
47                    for c in range(self.num_classes)]
48          thesum = sum(unnorm)
49          return [un/thesum for un in unnorm]
```

learn does n steps of EM:

learnEM.py — (continued)

```
51      def learn(self,n):
52          """do n steps of em"""
53          for i in range(n):
54              self.class_counts,self.feature_counts =
                    self.em_step(self.class_counts,
55                                                          self.feature_counts)
```

The following is for visualizing the classes. It prints the dataset ordered by the probability of class c.

learnEM.py — (continued)

```
57      def show_class(self,c):
58          """sorts the data by the class and prints in order.
59          For visualizing small data sets
60          """
61          sorted_data =
                sorted((self.prob(tpl,self.class_counts,self.feature_counts)[c],
62                                ind,  # preserve ordering for equal
                                      probabilities
63                                tpl)
64                               for (ind,tpl) in enumerate(self.dataset.train))
65          for cc,r,tpl in sorted_data:
66              print(cc,*tpl,sep='\t')
```

The following are for evaluating the classes.

The probability of a tuple can be evaluated by marginalizing over the classes:

$$\begin{aligned} P(tple) &= \sum_c P(c) * \prod_i P(X_i{=}tple(i) \mid c) \\ &= \sum_c \frac{cc[c]}{len(self.dataset)} * \prod_i \frac{fc[i][feat_i(tple)][c]}{cc[c]} \end{aligned}$$

where cc is the class count and fc is feature count. $len(self.dataset)$ can be distributed out of the sum, and $cc[c]$ can be taken out of the product:

$$= \frac{1}{len(self.dataset)} \sum_c \frac{1}{cc[c]^{\#feats-1}} * \prod_i fc[i][feat_i(tple)][c]$$

Given the probability of each tuple, we can evaluate the logloss, as the negative of the log probability:

learnEM.py — (continued)

```
68      def logloss(self,tple):
69          """returns the logloss of the prediction on tple, which is
                -log(P(tple))
70          based on the current class counts and feature counts
71          """
72          feats = self.dataset.input_features
73          res = 0
74          cc = self.class_counts
75          fc = self.feature_counts
```

```
        for c in range(self.num_classes):
            res += prod(fc[i][feat(tple)][c]
                        for (i,feat) in
                            enumerate(feats))/(cc[c]**(len(feats)-1))
        if res>0:
            return -math.log2(res/len(self.dataset.train))
        else:
            return float("inf") #infinity

    def plot_error(self, maxstep=20):
        """Plots the logloss error as a function of the number of steps"""
        plt.ion()
        plt.xlabel("step")
        plt.ylabel("Ave Logloss (bits)")
        train_errors = []
        if self.dataset.test:
            test_errors = []
        for i in range(maxstep):
            self.learn(1)
            train_errors.append( sum(self.logloss(tple) for tple in
                self.dataset.train)
                                 /len(self.dataset.train))
            if self.dataset.test:
                test_errors.append( sum(self.logloss(tple) for tple in
                    self.dataset.test)
                                    /len(self.dataset.test))
        plt.plot(range(1,maxstep+1),train_errors,
                 label=str(self.num_classes)+" classes. Training set")
        if self.dataset.test:
            plt.plot(range(1,maxstep+1),test_errors,
                     label=str(self.num_classes)+" classes. Test set")
        plt.legend()
        plt.draw()

def prod(L):
    """returns the product of the elements of L"""
    res = 1
    for e in L:
        res *= e
    return res

def random_dist(k):
    """generate k random numbers that sum to 1"""
    res = [random.random() for i in range(k)]
    s = sum(res)
    return [v/s for v in res]

data = Data_from_file('data/emdata2.csv', num_train=10, target_index=2000)
eml = EM_learner(data,2)
num_iter=2
```

```
print("Class assignment after",num_iter,"iterations:")
eml.learn(num_iter); eml.show_class(0)

# Plot the error
# em2=EM_learner(data,2); em2.plot_error(40) # 2 classes
# em3=EM_learner(data,3); em3.plot_error(40) # 3 classes
# em13=EM_learner(data,13); em13.plot_error(40) # 13 classes

# data = Data_from_file('data/carbool.csv',
    target_index=2000,boolean_features=False)
# [f.frange for f in data.input_features]
# eml = EM_learner(data,3)
# eml.learn(20); eml.show_class(0)
# em3=EM_learner(data,3); em3.plot_error(60) # 3 classes
# em3=EM_learner(data,30); em3.plot_error(60) # 30 classes
```

Exercise 10.3 For the EM data, where there are naturally 2 classes, 3 classes does better on the training set after a while than 2 classes, but worse on the test set. Explain why. Hint: look what the 3 classes are. Use "em3.show_class(i)" for each of the classes $i \in [0, 3)$.

Exercise 10.4 Write code to plot the logloss as a function of the number of classes (from 1 to say 15) for a fixed number of iterations. (From the experience with the existing code, think about how many iterations is appropriate.)

Chapter 11

Causality

11.1 Do Questions

A causal model can answer "do" questions.

The following adds the queryDo method to the InferenceMethod class, so it can be used with any inference method.

probDo.py — Probabilistic inference with the do operator

```
from probGraphicalModels import InferenceMethod, BeliefNetwork
from probFactors import CPD, ConstantCPD

def queryDo(self, qvar, obs={}, do={}):
    assert isinstance(self.gm, BeliefNetwork), "Do only applies to belief
        networks"
    if do=={}:
        return self.query(qvar, obs)
    else:
        newfacs = ({f for (ch,f) in self.gm.var2cpt.items() if ch not in
            do} |
                   {ConstantCPD(v,c) for (v,c) in do.items()})
        self.modBN = BeliefNetwork(self.gm.title+"(mod)",
            self.gm.variables, newfacs)
        oldBN, self.gm = self.gm, self.modBN
        result = self.query(qvar, obs)
        self.gm = oldBN # restore original
        return result

InferenceMethod.queryDo = queryDo
```

probDo.py — (continued)

```
from probRC import ProbRC
```

```
30
31 from probGraphicalModels import bn_sprinkler, Season, Sprinkler, Rained,
       Grass_wet, Grass_shiny, Shoes_wet, bn_sprinkler_soff
32 bn_sprinklerv = ProbRC(bn_sprinkler)
33 ## bn_sprinklerv.queryDo(Shoes_wet)
34 ## bn_sprinklerv.queryDo(Shoes_wet,obs={Sprinkler:"off"})
35 ## bn_sprinklerv.queryDo(Shoes_wet,do={Sprinkler:"off"})
36 ## ProbRC(bn_sprinkler_soff).query(Shoes_wet) # should be same as previous
       case
37 ## bn_sprinklerv.queryDo(Season, obs={Sprinkler:"off"})
38 ## bn_sprinklerv.queryDo(Season, do={Sprinkler:"off"})
```

The following is a representation of a possible model where marijuana is a gateway drug to harder drugs (or not). Try the queries at the end.

probDo.py — (continued)

```
40 from probVariables import Variable
41 from probFactors import Prob
42 from probGraphicalModels import boolean, BeliefNetwork
43
44 Takes_Marijuana = Variable("Takes_Marijuana", boolean, position=(0.1,0.5))
45 Drug_Prone = Variable("Drug_Prone", boolean, position=(0.1,0.5)) #
       (0.5,0.9))
46 Side_Effects = Variable("Side_Effects", boolean, position=(0.1,0.5)) #
       (0.5,0.1))
47 Takes_Hard_Drugs = Variable("Takes_Hard_Drugs", boolean,
       position=(0.9,0.5))
48
49 p_tm = Prob(Takes_Marijuana, [Drug_Prone], [[0.98, 0.02], [0.2, 0.8]])
50 p_dp = Prob(Drug_Prone, [], [0.8, 0.2])
51 p_be = Prob(Side_Effects, [Takes_Marijuana], [[1, 0], [0.4, 0.6]])
52 p_thd = Prob(Takes_Hard_Drugs, [Side_Effects, Drug_Prone],
53                # Drug_Prone=False  Drug_Prone=True
54                [[[0.999, 0.001],   [0.6, 0.4]], # Side_Effects=False
55                 [[0.99999, 0.00001], [0.995, 0.005]]]) # Side_Effects=True
56
57 drugs = BeliefNetwork("Gateway Drug?",
58                   [Takes_Marijuana,Drug_Prone,Side_Effects,Takes_Hard_Drugs],
59                   [p_tm, p_dp, p_be, p_thd])
60
61 drugsq = ProbRC(drugs)
62 # drugsq.queryDo(Takes_Hard_Drugs)
63 # drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: True})
64 # drugsq.queryDo(Takes_Hard_Drugs, obs = {Takes_Marijuana: False})
65 # drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: True})
66 # drugsq.queryDo(Takes_Hard_Drugs, do = {Takes_Marijuana: False})
```

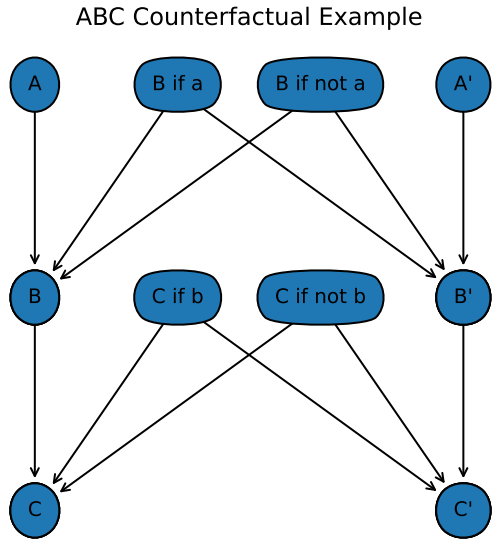


Figure 11.1: $A \rightarrow B \rightarrow C$ belief network for "what if A"

11.2 Counterfactual Example

This is for a chain $A \rightarrow B \rightarrow C$ where the query is A=*true*, C=*true* is observed; what is the probability of C is A were false. See Figure 11.1.

probCounterfactual.py — Counterfactual Query Example

```
from probVariables import Variable
from probFactors import Prob
from probGraphicalModels import boolean, BeliefNetwork
from probRC import ProbRC
from probDo import queryDo

# without a deterministic system
Ap = Variable("Ap", boolean, position=(0.2,0.8))
Bp = Variable("Bp", boolean, position=(0.2,0.4))
Cp = Variable("Cp", boolean, position=(0.2,0.0))
p_Ap = Prob(Ap, [], [0.5,0.5])
p_Bp = Prob(Bp, [Ap], [[0.6,0.4], [0.6,0.4]]) # does not depend on A!
p_Cp = Prob(Cp, [Bp], [[0.2,0.8], [0.9,0.1]])
abcSimple = BeliefNetwork("ABC Simple",
                      [Ap,Bp,Cp],
                      [p_Ap, p_Bp, p_Cp])
ABCsimpq = ProbRC(abcSimple)
# ABCsimpq.show_post(obs = {Ap:True, Cp:True})
```

```
29
30 # as a deterministic system with independent noise
31 A = Variable("A", boolean, position=(0.2,0.8))
32 B = Variable("B", boolean, position=(0.2,0.4))
33 C = Variable("C", boolean, position=(0.2,0.0))
34 Aprime = Variable("A'", boolean, position=(0.8,0.8))
35 Bprime = Variable("B'", boolean, position=(0.8,0.4))
36 Cprime = Variable("C'", boolean, position=(0.8,0.0))
37 BifA = Variable("B if a", boolean, position=(0.4,0.8))
38 BifnA = Variable("B if not a", boolean, position=(0.6,0.8))
39 CifB = Variable("C if b", boolean, position=(0.4,0.4))
40 CifnB = Variable("C if not b", boolean, position=(0.6,0.4))
41
42 p_A = Prob(A, [], [0.5,0.5])
43 p_B = Prob(B, [A, BifA, BifnA], [[[[1,0],[0,1]],[[1,0],[0,1]]], # A=0
44                                  [[[1,0],[1,0]],[[0,1],[0,1]]]]) # A=1
45 p_C = Prob(C, [B, CifB, CifnB], [[[[1,0],[0,1]],[[1,0],[0,1]]], # B=0
46                                  [[[1,0],[1,0]],[[0,1],[0,1]]]]) # B=1
47 p_Aprime = Prob(Aprime,[], [0.6,0.4])
48 p_Bprime = Prob(Bprime, [Aprime, BifA, BifnA],
       [[[[1,0],[0,1]],[[1,0],[0,1]]], # A=0
49                                  [[[1,0],[1,0]],[[0,1],[0,1]]]]) # A=1
50 p_Cprime = Prob(Cprime, [Bprime, CifB, CifnB],
       [[[[1,0],[0,1]],[[1,0],[0,1]]], # B=0
51                                  [[[1,0],[1,0]],[[0,1],[0,1]]]]) # B=1
52 p_bifa = Prob(BifA, [], [0.6,0.4]) # Does not actually depend on A!!!
53 p_bifna = Prob(BifnA, [], [0.6,0.4])
54 p_cifb = Prob(CifB, [], [0.9,0.1])
55 p_cifnb = Prob(CifnB, [], [0.2,0.8])
56
57 abcCounter = BeliefNetwork("ABC Counterfactual Example",
58                         [A,B,C,Aprime,Bprime,Cprime,BifA, BifnA, CifB,
                               CifnB],
59                         [p_A,p_B,p_C,p_Aprime,p_Bprime, p_Cprime, p_bifa,
                               p_bifna, p_cifb, p_cifnb])
60
61 abcq = ProbRC(abcCounter)
62 # abcq.queryDo(Cprime, obs = {Aprime:False, A:True})
63 # abcq.queryDo(Cprime, obs = {C:True, Aprime:False})
64 # abcq.queryDo(Cprime, obs = {A:True, C:True, Aprime:False})
65 # abcq.queryDo(Cprime, obs = {A:True, C:True, Aprime:False})
66 # abcq.queryDo(Cprime, obs = {A:False, C:True, Aprime:False})
67 # abcq.queryDo(CifB, obs = {C:True,Aprime:False})
68 # abcq.queryDo(CifnB, obs = {C:True,Aprime:False})
69
70 # abcq.show_post(obs = {})
71 # abcq.show_post(obs = {Aprime:False, A:True})
72 # abcq.show_post(obs = {A:True, C:True, Aprime:False})
73 # abcq.show_post(obs = {A:True, C:True, Aprime:True})
```

The following is the firing squad example of Pearl. See Figure 11.2.

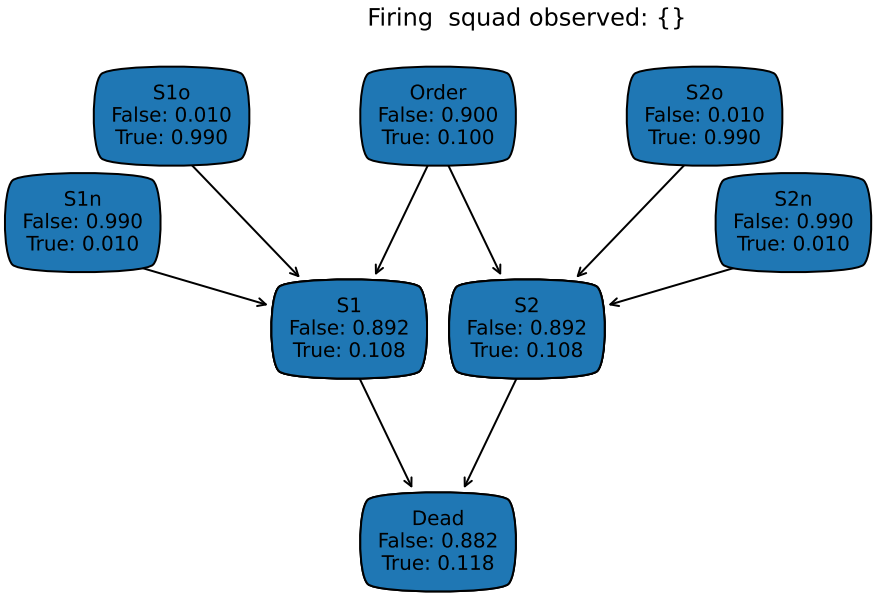


Figure 11.2: Firing squad belief network

probCounterfactual.py — (continued)

```
75 Order = Variable("Order", boolean, position=(0.4,0.8))
76 S1 = Variable("S1", boolean, position=(0.3,0.4))
77 S1o = Variable("S1o", boolean, position=(0.1,0.8))
78 S1n = Variable("S1n", boolean, position=(0.0,0.6))
79 S2 = Variable("S2", boolean, position=(0.5,0.4))
80 S2o = Variable("S2o", boolean, position=(0.7,0.8))
81 S2n = Variable("S2n", boolean, position=(0.8,0.6))
82 Dead = Variable("Dead", boolean, position=(0.4,0.0))
83
84 p_S1 = Prob(S1, [Order, S1o, S1n], [[[[1,0],[0,1]],[[1,0],[0,1]]], #
       Order=0
85                                    [[[1,0],[1,0]],[[0,1],[0,1]]]]) # Order=1
86 p_S2 = Prob(S2, [Order, S2o, S2n], [[[[1,0],[0,1]],[[1,0],[0,1]]], #
       Order=0
87                                    [[[1,0],[1,0]],[[0,1],[0,1]]]]) # Order=1
88 p_dead = Prob(Dead, [S1,S2], [[[1,0],[0,1]],[[0,1],[0,1]]])
89 p_order = Prob(Order, [], [0.9, 0.1])
90 p_s1o = Prob(S1o, [], [0.01, 0.99])
91 p_s1n = Prob(S1n, [], [0.99, 0.01])
92 p_s2o = Prob(S2o, [], [0.01, 0.99])
93 p_s2n = Prob(S2n, [], [0.99, 0.01])
94
95 firing_squad = BeliefNetwork("Firing squad",
```

```
                          [Order, S1, S1o, S1n, S2, S2o, S2n, Dead],
                          [p_order, p_dead, p_S1, p_s1o, p_s1n, p_S2, p_s2o,
                              p_s2n])
fsq = ProbRC(firing_squad)
# fsq.queryDo(Dead)
# fsq.queryDo(Order, obs={Dead:True})
# fsq.queryDo(Dead, obs={Order:True})
```

Chapter 12

Planning with Uncertainty

12.1 Decision Networks

The decision network code builds on the representation for belief networks of Chapter 9.

We first allow for factors that define the utility. Here the **utility** is a function of the variables in *vars*. In a **utility table** the utility is defined in terms of a tabular factor – a list that enumerates the values – as in Section 9.3.3.

decnNetworks.py — Representations for Decision Networks

```
from probGraphicalModels import GraphicalModel, BeliefNetwork
from probFactors import Factor, CPD, TabFactor, factor_times, Prob
from probVariables import Variable
import matplotlib.pyplot as plt

class Utility(Factor):
    """A factor defining a utility"""
    pass

class UtilityTable(TabFactor, Utility):
    """A factor defining a utility using a table"""
    def __init__(self, vars, table, position=None):
        """Creates a factor on vars from the table.
        The table is ordered according to vars.
        """
        TabFactor.__init__(self,vars,table)
        self.position = position
```

A **decision variable** is a like a random variable with a string name, and a domain, which is a list of possible values. The decision variable also includes the parents, a list of the variables whose value will be known when the decision is made. It also includes a potion, which is only used for plotting.

decnNetworks.py — (continued)

```
class DecisionVariable(Variable):
    def __init__(self, name, domain, parents, position=None):
        Variable.__init__(self, name, domain, position)
        self.parents = parents
        self.all_vars = set(parents) | {self}
```

A decision network is a graphical model where the variables can be random variables or decision variables. Among the factors we assume there is one utility factor.

decnNetworks.py — (continued)

```
class DecisionNetwork(BeliefNetwork):
    def __init__(self, title, vars, factors):
        """vars is a list of variables
        factors is a list of factors (instances of CPD and Utility)
        """
        GraphicalModel.__init__(self, title, vars, factors) # don't call
            init for BeliefNetwork
        self.var2parents = ({v : v.parents for v in vars if
            isinstance(v,DecisionVariable)}
                    | {f.child:f.parents for f in factors if
                        isinstance(f,CPD)})
        self.children = {n:[] for n in self.variables}
        for v in self.var2parents:
            for par in self.var2parents[v]:
                self.children[par].append(v)
        self.utility_factor = [f for f in factors if
            isinstance(f,Utility)][0]
        self.topological_sort_saved = None
```

The split order ensures that the parents of a decision node are split before the decision node, and no other variables (if that is possible).

decnNetworks.py — (continued)

```
    def split_order(self):
        so = []
        tops = self.topological_sort()
        for v in tops:
            if isinstance(v,DecisionVariable):
                so += [p for p in v.parents if p not in so]
                so.append(v)
        so += [v for v in tops if v not in so]
        return so
```

decnNetworks.py — (continued)

```
    def show(self):
        plt.ion()  # interactive
        ax = plt.figure().gca()
        ax.set_axis_off()
        plt.title(self.title)
```

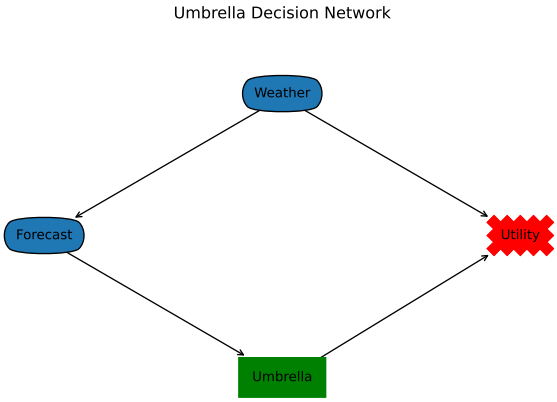


Figure 12.1: The umbrella decision network

```
65          for par in self.utility_factor.variables:
66              ax.annotate("Utility", par.position,
                    xytext=self.utility_factor.position,
67                                  arrowprops={'arrowstyle':'<-'},bbox=dict(boxstyle="sawtooth,pad=1
68                                  ha='center')
69          for var in reversed(self.topological_sort()):
70              if isinstance(var,DecisionVariable):
71                  bbox = dict(boxstyle="square,pad=1.0",color="green")
72              else:
73                 bbox = dict(boxstyle="round4,pad=1.0,rounding_size=0.5")
74              if self.var2parents[var]:
75                  for par in self.var2parents[var]:
76                      ax.annotate(var.name, par.position, xytext=var.position,
77                                  arrowprops={'arrowstyle':'<-'},bbox=bbox,
78                                  ha='center')
79              else:
80                  x,y = var.position
81                  plt.text(x,y,var.name,bbox=bbox,ha='center')
```

12.1.1 Example Decision Networks

Umbrella Decision Network

Here is a simple "umbrella" decision network. The output of `umbrella_dn.show()` is shown in Figure 12.1.

decnNetworks.py — (continued)

```
83 Weather = Variable("Weather", ["NoRain", "Rain"], position=(0.5,0.8))
84 Forecast = Variable("Forecast", ["Sunny", "Cloudy", "Rainy"],
       position=(0,0.4))
85 # Each variant uses one of the following:
```

```
Umbrella = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast},
    position=(0.5,0))

p_weather = Prob(Weather, [], [0.7, 0.3])
p_forecast = Prob(Forecast, [Weather], [[0.7, 0.2, 0.1], [0.15, 0.25,
    0.6]])
umb_utility = UtilityTable([Weather, Umbrella], [[20, 100], [70, 0]],
    position=(1,0.4))

umbrella_dn = DecisionNetwork("Umbrella Decision Network",
                              {Weather, Forecast, Umbrella},
                              {p_weather, p_forecast, umb_utility})
```

The following is a variant with the umbrella decision having 2 parents; nothing else has changed. This is interesting because one of the parents is not needed; if the agent knows the weather, it can ignore the forecast.

decnNetworks.py — (continued)

```
Umbrella2p = DecisionVariable("Umbrella", ["Take", "Leave"], {Forecast,
    Weather}, position=(0.5,0))
umb_utility2p = UtilityTable([Weather, Umbrella2p], [[20, 100], [70, 0]],
    position=(1,0.4))
umbrella_dn2p = DecisionNetwork("Umbrella Decision Network (extra arc)",
                              {Weather, Forecast, Umbrella2p},
                              {p_weather, p_forecast, umb_utility2p})
```

Fire Decision Network

The fire decision network of Figure 12.2 (showing the result of `fire_dn.show()`) is represented as:

decnNetworks.py — (continued)

```
boolean = [False, True]
Alarm = Variable("Alarm", boolean, position=(0.25,0.633))
Fire = Variable("Fire", boolean, position=(0.5,0.9))
Leaving = Variable("Leaving", boolean, position=(0.25,0.366))
Report = Variable("Report", boolean, position=(0.25,0.1))
Smoke = Variable("Smoke", boolean, position=(0.75,0.633))
Tamper = Variable("Tamper", boolean, position=(0,0.9))

See_Sm = Variable("See_Sm", boolean, position=(0.75,0.366) )
Chk_Sm = DecisionVariable("Chk_Sm", boolean, {Report}, position=(0.5,
    0.366))
Call = DecisionVariable("Call", boolean,{See_Sm,Chk_Sm,Report},
    position=(0.75,0.1))

f_ta = Prob(Tamper,[],[0.98,0.02])
f_fi = Prob(Fire,[],[0.99,0.01])
f_sm = Prob(Smoke,[Fire],[[0.99,0.01],[0.1,0.9]])
f_al = Prob(Alarm,[Fire,Tamper],[[[0.9999, 0.0001], [0.15, 0.85]], [[0.01,
    0.99], [0.5, 0.5]]])
```

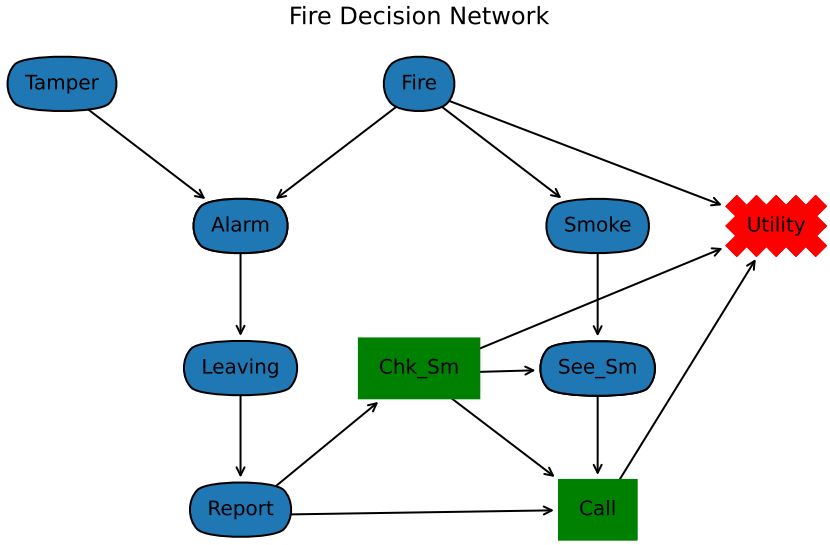


Figure 12.2: Fire Decision Network

```
f_lv = Prob(Leaving,[Alarm],[[0.999, 0.001], [0.12, 0.88]])
f_re = Prob(Report,[Leaving],[[0.99, 0.01], [0.25, 0.75]])
f_ss = Prob(See_Sm,[Chk_Sm,Smoke],[[[1,0],[1,0]],[[1,0],[0,1]]])

ut =
    UtilityTable([Chk_Sm,Fire,Call],[[[0,-200],[-5000,-200]],[[-20,-220],[-5020,-220]]],
    position=(1,0.633))

fire_dn = DecisionNetwork("Fire Decision Network",
                          {Tamper,Fire,Alarm,Leaving,Smoke,Call,See_Sm,Chk_Sm,Report},
                          {f_ta,f_fi,f_sm,f_al,f_lv,f_re,f_ss,ut})
```

Cheating Decision Network

The following is the representation of the cheating decision of Figure 12.3. Note that we keep the names of the variables short (less than 8 characters) so that the tables look good when printed.

decnNetworks.py — (continued)

```
grades = ['A','B','C','F']
Watched = Variable("Watched", boolean, position=(0,0.9))
Caught1 = Variable("Caught1", boolean, position=(0.2,0.7))
Caught2 = Variable("Caught2", boolean, position=(0.6,0.7))
```

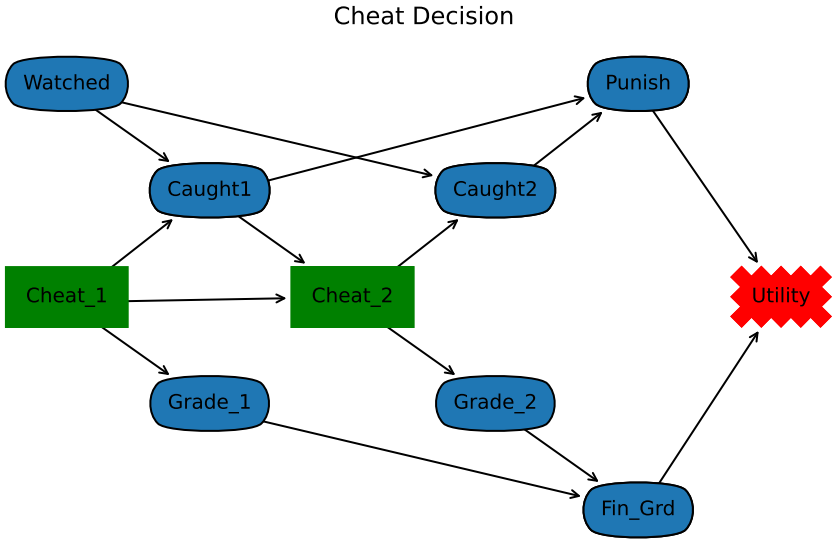


Figure 12.3: Cheating Decision Network

```
Punish = Variable("Punish", ["None","Suspension","Recorded"],
    position=(0.8,0.9))
Grade_1 = Variable("Grade_1", grades, position=(0.2,0.3))
Grade_2 = Variable("Grade_2", grades, position=(0.6,0.3))
Fin_Grd = Variable("Fin_Grd", grades, position=(0.8,0.1))
Cheat_1 = DecisionVariable("Cheat_1", boolean, set(), position=(0,0.5))
    #no parents
Cheat_2 = DecisionVariable("Cheat_2", boolean, {Cheat_1,Caught1},
    position=(0.4,0.5))

p_wa = Prob(Watched,[],[0.7, 0.3])
p_cc1 = Prob(Caught1,[Watched,Cheat_1],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_cc2 = Prob(Caught2,[Watched,Cheat_2],[[[1.0, 0.0], [0.9, 0.1]], [[1.0,
    0.0], [0.5, 0.5]]])
p_pun = Prob(Punish,[Caught1,Caught2],[[[1.0, 0.0, 0.0], [0.5, 0.4, 0.1]],
    [[0.6, 0.2, 0.2], [0.2, 0.5, 0.3]]])
p_gr1 = Prob(Grade_1,[Cheat_1], [{'A':0.2, 'B':0.3, 'C':0.3, 'D': 0.2},
    {'A':0.5, 'B':0.3, 'C':0.2, 'D':0.0}])
p_gr2 = Prob(Grade_2,[Cheat_2], [{'A':0.2, 'B':0.3, 'C':0.3, 'D': 0.2},
    {'A':0.5, 'B':0.3, 'C':0.2, 'D':0.0}])
p_fg = Prob(Fin_Grd,[Grade_1,Grade_2],
       {'A':{'A':{'A':1.0, 'B':0.0, 'C': 0.0, 'D':0.0},
             'B': {'A':0.5, 'B':0.5, 'C': 0.0, 'D':0.0},
```

```
148             'C':{'A':0.25, 'B':0.5, 'C': 0.25, 'D':0.0},
149             'D':{'A':0.25, 'B':0.25, 'C': 0.25, 'D':0.25}},
150         'B':{'A':{'A':0.5, 'B':0.5, 'C': 0.0, 'D':0.0},
151             'B': {'A':0.0, 'B':1, 'C': 0.0, 'D':0.0},
152             'C':{'A':0.0, 'B':0.5, 'C': 0.5, 'D':0.0},
153             'D':{'A':0.0, 'B':0.25, 'C': 0.5, 'D':0.25}},
154         'C':{'A':{'A':0.25, 'B':0.5, 'C': 0.25, 'D':0.0},
155             'B': {'A':0.0, 'B':0.5, 'C': 0.5, 'D':0.0},
156             'C':{'A':0.0, 'B':0.0, 'C': 1, 'D':0.0},
157             'D':{'A':0.0, 'B':0.0, 'C': 0.5, 'D':0.5}},
158         'D':{'A':{'A':0.25, 'B':0.25, 'C': 0.25, 'D':0.25},
159             'B': {'A':0.0, 'B':0.25, 'C': 0.5, 'D':0.25},
160             'C':{'A':0.0, 'B':0.0, 'C': 0.5, 'D':0.5},
161             'D':{'A':0.0, 'B':0.0, 'C': 0, 'D':1.0}}})
162
163 utc = UtilityTable([Punish,Fin_Grd],{'None':{'A':100, 'B':90, 'C': 70,
        'D':50},
164                                      'Suspension':{'A':40, 'B':20, 'C': 10,
                                             'D':0},
165                                      'Recorded':{'A':70, 'B':60, 'C': 40,
                                             'D':20}}, position=(1,0.5))
166
167 cheating_dn = DecisionNetwork("Cheating Decision Network",
168                               {Punish,Caught2,Watched,Fin_Grd,Grade_2,Grade_1,Cheat_2,Caught1,Cheat_1},
169                               {p_wa, p_cc1, p_cc2, p_pun, p_gr1,
                                      p_gr2,p_fg,utc})
```

Chain of 3 decisions

The following example is a finite-stage fully-observable Markov decision process with a single reward (utility) at the end. It is interesting because the parents do not include all predecessors. The methods we use will work without change on this, even though the agent does not condition on all of its previous observations and actions. The output of `ch3.show()` is shown in Figure 12.4.

decnNetworks.py — (continued)

```
171 S0 = Variable('S0', boolean, position=(0,0.5))
172 D0 = DecisionVariable('D0', boolean, {S0}, position=(1/7,0.1))
173 S1 = Variable('S1', boolean, position=(2/7,0.5))
174 D1 = DecisionVariable('D1', boolean, {S1}, position=(3/7,0.1))
175 S2 = Variable('S2', boolean, position=(4/7,0.5))
176 D2 = DecisionVariable('D2', boolean, {S2}, position=(5/7,0.1))
177 S3 = Variable('S3', boolean, position=(6/7,0.5))
178
179 p_s0 = Prob(S0, [], [0.5,0.5])
180 tr = [[[0.1, 0.9], [0.9, 0.1]], [[0.2, 0.8], [0.8, 0.2]]] # 0 is flip, 1
        is keep value
181 p_s1 = Prob(S1, [D0,S0], tr)
182 p_s2 = Prob(S2, [D1,S1], tr)
183 p_s3 = Prob(S3, [D2,S2], tr)
```

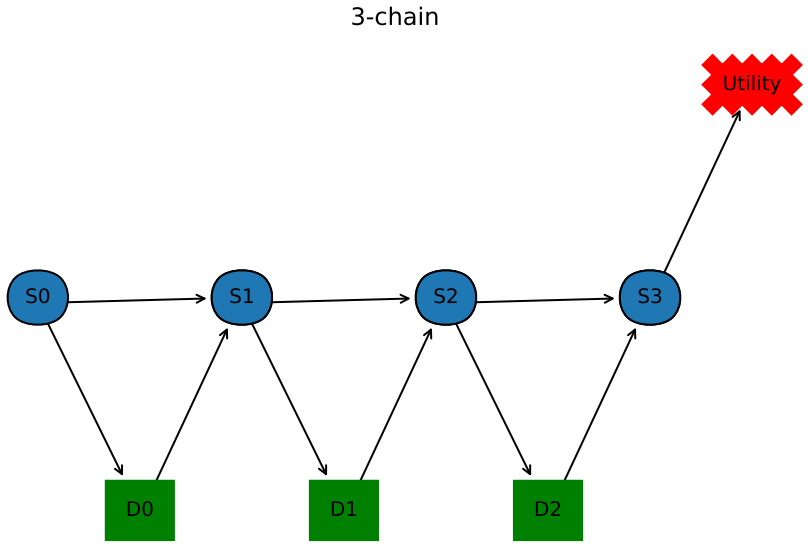


Figure 12.4: A decision network that is a chain of 3 decisions

```
184
185 ch3U = UtilityTable([S3],[0,1], position=(7/7,0.9))
186
187 ch3 = DecisionNetwork("3-chain",
        {S0,D0,S1,D1,S2,D2,S3},{p_s0,p_s1,p_s2,p_s3,ch3U})
188 #rc3 = RC_DN(ch3)
189 #rc3.optimize()
190 #rc3.opt_policy
```

12.1.2 Recursive Conditioning for decision networks

An instance of a `RC_DN` object takes in a decision network. The query method uses recursive conditioning to compute the expected utility of the optimal policy. `self.opt_policy` becomes the optimal policy.

decnNetworks.py — (continued)

```
192 import math
193 from probGraphicalModels import GraphicalModel, InferenceMethod
194 from probFactors import Factor
195 from probRC import connected_components
196
197 class RC_DN(InferenceMethod):
198     """The class that queries graphical models using recursive conditioning
199
```

```
    gm is graphical model to query
    """

    def __init__(self,gm=None):
        self.gm = gm
        self.cache = {(frozenset(), frozenset()):1}
        ## self.max_display_level = 3

    def optimize(self, split_order=None):
        """computes expected utility, and creates optimal decision
            functions, where
        elim_order is a list of the non-observed non-query variables in gm
        """
        if split_order == None:
            split_order = self.gm.split_order()
        self.opt_policy = {}
        return self.rc({}, self.gm.factors, split_order)
```

The following us the simplest search-based algorithm. It is exponential in the number of variables, so is not very useful. However, it is simple, and useful to understand before looking at the more complicated algorithm. Note that the above code does not call `rc0`; you will need to change the `self.rc` to `self.rc0` in above code to use it.

decnNetworks.py — (continued)

```
    def rc0(self, context, factors, split_order):
        """simplest search algorithm"""
        self.display(2,"calling rc0,",(context,factors),"with
            SO",split_order)
        if not factors:
            return 1
        elif to_eval := {fac for fac in factors if
            fac.can_evaluate(context)}:
            self.display(3,"rc0 evaluating factors",to_eval)
            val = math.prod(fac.get_value(context) for fac in to_eval)
            return val * self.rc0(context, factors-to_eval, split_order)
        else:
            var = split_order[0]
            self.display(3, "rc0 branching on", var)
            if isinstance(var,DecisionVariable):
                assert set(context) <= set(var.parents), f"cannot optimize
                    {var} in context {context}"
                maxres = -math.inf
                for val in var.domain:
                    self.display(3,"In rc0, branching on",var,"=",val)
                    newres = self.rc0({var:val}|context, factors,
                        split_order[1:])
                    if newres > maxres:
                        maxres = newres
                        theval = val
                self.opt_policy[frozenset(context.items())] = (var,theval)
```

```
239             return maxres
240         else:
241             total = 0
242             for val in var.domain:
243                 total += self.rc0({var:val}|context, factors,
                        split_order[1:])
244             self.display(3, "rc0 branching on", var,"returning", total)
245             return total
```

We can combine the optimization for decision networks above, with the improvements of recursive conditioning used for graphical models (Section 9.7, page 198).

decnNetworks.py — (continued)

```
247     def rc(self, context, factors, split_order):
248         """ returns the number \sum_{split_order} \prod_{factors} given
                assignments in context
249         context is a variable:value dictionary
250         factors is a set of factors
251         split_order is a list of variables in factors that are not in
                context
252         """
253         self.display(3,"calling rc,",(context,factors))
254         ce = (frozenset(context.items()), frozenset(factors)) # key for the
                cache entry
255         if ce in self.cache:
256             self.display(2,"rc cache lookup",(context,factors))
257             return self.cache[ce]
258 #       if not factors: # no factors; needed if you don't have forgetting
        and caching
259 #           return 1
260         elif vars_not_in_factors := {var for var in context
261                                      if not any(var in fac.variables for
                                            fac in factors)}:
262             # forget variables not in any factor
263             self.display(3,"rc forgetting variables", vars_not_in_factors)
264             return self.rc({key:val for (key,val) in context.items()
265                             if key not in vars_not_in_factors},
266                            factors, split_order)
267         elif to_eval := {fac for fac in factors if
                fac.can_evaluate(context)}:
268             # evaluate factors when all variables are assigned
269             self.display(3,"rc evaluating factors",to_eval)
270             val = math.prod(fac.get_value(context) for fac in to_eval)
271             if val == 0:
272                 return 0
273             else:
274                 return val * self.rc(context, {fac for fac in factors if fac
                        not in to_eval}, split_order)
275         elif len(comp := connected_components(context, factors,
                split_order)) > 1:
```

```
            # there are disconnected components
            self.display(2,"splitting into connected components",comp)
            return(math.prod(self.rc(context,f,eo) for (f,eo) in comp))
        else:
            assert split_order, f"split_order empty rc({context},{factors})"
            var = split_order[0]
            self.display(3, "rc branching on", var)
            if isinstance(var,DecisionVariable):
                assert set(context) <= set(var.parents), f"cannot optimize
                    {var} in context {context}"
                maxres = -math.inf
                for val in var.domain:
                    self.display(3,"In rc, branching on",var,"=",val)
                    newres = self.rc({var:val}|context, factors,
                        split_order[1:])
                    if newres > maxres:
                        maxres = newres
                        theval = val
                self.opt_policy[frozenset(context.items())] = (var,theval)
                self.cache[ce] = maxres
                return maxres
            else:
                total = 0
                for val in var.domain:
                    total += self.rc({var:val}|context, factors,
                        split_order[1:])
                self.display(3, "rc branching on", var,"returning", total)
                self.cache[ce] = total
                return total
```

Here is how to run the optimize the example decision networks:

decnNetworks.py — (continued)

```
# Umbrella decision network
#urc = RC_DN(umberella_dn)
#urc.optimize()
#urc.opt_policy

#rc_fire = RC_DN(fire_dn)
#rc_fire.optimize()
#rc_fire.opt_policy

#rc_cheat = RC_DN(cheating_dn)
#rc_cheat.optimize()
#rc_cheat.opt_policy

#rc_ch3 = RC_DN(ch3)
#rc_ch3.optimize()
#rc_ch3.opt_policy
```

12.1.3 Variable elimination for decision networks

VE_DN is variable elimination for decision networks. The method *optimize* is used to optimize all the decisions. Note that *optimize* requires a legal elimination ordering of the random and decision variables, otherwise it will give an exception. (A decision node can only be maximized if the variables that are not its parents have already been eliminated.)

decnNetworks.py — (continued)

```
320 from probVE import VE
321
322 class VE_DN(VE):
323     """Variable Elimination for Decision Networks"""
324     def __init__(self,dn=None):
325         """dn is a decision network"""
326         VE.__init__(self,dn)
327         self.dn = dn
328
329     def optimize(self,elim_order=None,obs={}):
330         if elim_order == None:
331                 elim_order = reversed(self.gm.split_order())
332         policy = []
333         proj_factors = [self.project_observations(fact,obs)
334                         for fact in self.dn.factors]
335         for v in elim_order:
336             if isinstance(v,DecisionVariable):
337                 to_max = [fac for fac in proj_factors
338                           if v in fac.variables and set(fac.variables) <=
                              v.all_vars]
339                 assert len(to_max)==1, "illegal variable order
                      "+str(elim_order)+" at "+str(v)
340                 newFac = FactorMax(v, to_max[0])
341                 policy.append(newFac.decision_fun)
342                 proj_factors = [fac for fac in proj_factors if fac is not
                      to_max[0]]+[newFac]
343                 self.display(2,"maximizing",v,"resulting
                      factor",newFac.brief() )
344                 self.display(3,newFac)
345             else:
346                 proj_factors = self.eliminate_var(proj_factors, v)
347         assert len(proj_factors)==1,"Should there be only one element of
                proj_factors?"
348         value = proj_factors[0].get_value({})
349         return value,policy
```

decnNetworks.py — (continued)

```
351 class FactorMax(Factor):
352     """A factor obtained by maximizing a variable in a factor.
353     Also builds a decision_function. This is based on FactorSum.
354     """
```

```
355
356     def __init__(self, dvar, factor):
357         """dvar is a decision variable.
358         factor is a factor that contains dvar and only parents of dvar
359         """
360         self.dvar = dvar
361         self.factor = factor
362         vars = [v for v in factor.variables if v is not dvar]
363         Factor.__init__(self,vars)
364         self.values = [None]*self.size
365         self.decision_fun = FactorDF(dvar,vars,[None]*self.size)
366
367     def get_value(self,assignment):
368         """lazy implementation: if saved, return saved value, else compute
                it"""
369         index = self.assignment_to_index(assignment)
370         if self.values[index]:
371             return self.values[index]
372         else:
373             max_val = float("-inf") # -infinity
374             new_asst = assignment.copy()
375             for elt in self.dvar.domain:
376                 new_asst[self.dvar] = elt
377                 fac_val = self.factor.get_value(new_asst)
378                 if fac_val>max_val:
379                     max_val = fac_val
380                     best_elt = elt
381             self.values[index] = max_val
382             self.decision_fun.values[index] = best_elt
383             return max_val
```

A decision function is a stored factor.

decnNetworks.py — (continued)

```
385 class FactorDF(TabFactor):
386     """A decision function"""
387     def __init__(self,dvar, vars, values):
388         TabStored.__init__(self,vars,values)
389         self.dvar = dvar
390         self.name = str(dvar) # Used in printing
```

Here are some example queries:

decnNetworks.py — (continued)

```
392 # Example queries:
393 # v,p = VE_DN(fire_dn).optimize(); print(v)
394 # for df in p: print(df,"\n")
395
396 # VE_DN.max_display_level = 3 # if you want to show lots of detail
397 # v,p = VE_DN(cheating_dn).optimize(); print(v)
398 # for df in p: print(df,"\n") # print decision functions
```

12.2 Markov Decision Processes

We will represent a **Markov decision process** (**MDP**) directly, rather than using the recursive conditioning or variable elimination code, as we did for decision networks.

mdpProblem.py — Representations for Markov Decision Processes

```
import random
import matplotlib.pyplot as plt
from matplotlib.widgets import Button, CheckButtons, TextBox
from display import Displayable
from utilities import argmaxd

class MDP(Displayable):
    """A Markov Decision Process. Must define:
    self.states the set (or list) of states
    self.actions the set (or list) of actions
    self.discount a real-valued discount
    """

    def __init__(self, states, actions, discount, init=0):
        self.states = states
        self.actions = actions
        self.discount = discount
        self.initv = self.v = {s:init for s in self.states}
        self.initq = self.q = {s: {a: init for a in self.actions} for s in
            self.states}

    def P(self,s,a):
        """Transition probability function
        returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
            probabilities are zero.
        """
        raise NotImplementedError("P") # abstract method

    def R(self,s,a):
        """Reward function R(s,a)
        returns the expected reward for doing a in state s.
        """
        raise NotImplementedError("R") # abstract method
```

Two state partying example (Example 9.27 in Poole and Mackworth [2017]):

mdpExamples.py — MDP Examples

```
from mdpProblem import MDP, GridMDP
import matplotlib.pyplot as plt

class party(MDP):
    """Simple 2-state, 2-Action Partying MDP Example"""
    def __init__(self, discount=0.9):
        states = {'healthy','sick'}
```

```
18          actions = {'relax', 'party'}
19          MDP.__init__(self, states, actions, discount)
20
21      def R(self,s,a):
22          "R(s,a)"
23          return { 'healthy': {'relax': 7, 'party': 10},
24                   'sick':   {'relax': 0, 'party': 2 }}[s][a]
25
26      def P(self,s,a):
27          "returns a dictionary of {s1:p1} such that P(s1 | s,a)=p1. Other
                probabilities are zero."
28          phealthy = { # P('healthy' | s, a)
29                       'healthy': {'relax': 0.95, 'party': 0.7},
30                       'sick': {'relax': 0.5, 'party': 0.1 }}[s][a]
31          return {'healthy':phealthy, 'sick':1-phealthy}
```

The next example is the tiny game from Example 12.1 and Figure 12.1 of Poole and Mackworth [2017]. The state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left state. The actions are *upC* for up-careful, and *upR* for up-risky. (Note that `GridMDP` is just a type of MDP for which we have methods to show; you can assume it is just `MDP` here).

mdpExamples.py — (continued)

```
34  class MDPtiny(GridMDP):
35      def __init__(self, discount=0.9):
36          actions = ['right', 'upC', 'left', 'upR']
37          self.x_dim = 2 # x-dimension
38          self.y_dim = 3
39          states = [(x,y) for x in range(self.x_dim) for y in
                range(self.y_dim)]
40          # for GridMDP
41          self.xoff = {'right':0.25, 'upC':0, 'left':-0.25, 'upR':0}
42          self.yoff = {'right':0, 'upC':-0.25, 'left':0, 'upR':0.25}
43          GridMDP.__init__(self, states, actions, discount)
44
45      def P(self,s,a):
46          """return a dictionary of {s1:p1} if P(s1 | s,a)=p1. Other
                probabilities are zero.
47          """
48          (x,y) = s
49          if a == 'right':
50              return {(1,y):1}
51          elif a == 'upC':
52              return {(x,min(y+1,2)):1}
53          elif a == 'left':
54              if (x,y) == (0,2): return {(0,0):1}
55              else: return {(0,y): 1}
56          elif a == 'upR':
57              if x==0:
58                  if y<2: return {(x,y):0.1, (x+1,y):0.1, (x,y+1):0.8}
```

```
59              else: # at (0,2)
60                  return {(0,0):0.1, (1,2): 0.1, (0,2): 0.8}
61          elif y < 2: # x==1
62              return {(0,y):0.1, (1,y):0.1, (1,y+1):0.8}
63          else: # at (1,2)
64             return {(0,2):0.1, (1,2): 0.9}
65
66      def R(self,s,a):
67          (x,y) = s
68          if a == 'right':
69              return [0,-1][x]
70          elif a == 'upC':
71              return [-1,-1,-2][y]
72          elif a == 'left':
73              if x==0:
74                  return [-1, -100, 10][y]
75              else: return 0
76          elif a == 'upR':
77              return [[-0.1, -10, 0.2],[-0.1, -0.1, -0.9]][x][y]
78                  # at (0,2) reward is 0.1*10+0.8*-1=0.2
```

Here is the domain of Example 9.28 of Poole and Mackworth [2017]. Here the state is represented as (x, y) where x counts from zero from the left, and y counts from zero upwards, so the state $(0, 0)$ is on the bottom-left state.

mdpExamples.py — (continued)

```
80  class grid(GridMDP):
81      """ x_dim * y_dim grid with rewarding states"""
82      def __init__(self, discount=0.9, x_dim=10, y_dim=10):
83          self.x_dim = x_dim # size in x-direction
84          self.y_dim = y_dim # size in y-direction
85          actions = ['up', 'down', 'right', 'left']
86          states = [(x,y) for x in range(y_dim) for y in range(y_dim)]
87          self.rewarding_states = {(3,2):-10, (3,5):-5, (8,2):10, (7,7):3}
88          self.fling_states = {(8,2), (7,7)}
89          self.xoff = {'right':0.25, 'up':0, 'left':-0.25, 'down':0}
90          self.yoff = {'right':0, 'up':0.25, 'left':0, 'down':-0.25}
91          GridMDP.__init__(self, states, actions, discount)
92
93      def intended_next(self,s,a):
94          """returns the next state in the direction a.
95          This is where the agent will end up if to goes in its
                intended_direction
96               (which it does with probability 0.7).
97          """
98          (x,y) = s
99          if a=='up':
100             return (x, y+1 if y+1 < self.y_dim else y)
101         if a=='down':
102             return (x, y-1 if y > 0 else y)
103         if a=='right':
```

```
104             return (x+1 if x+1 < self.x_dim else x,y)
105         if a=='left':
106             return (x-1 if x > 0 else x,y)
107 
108     def P(self,s,a):
109         """return a dictionary of {s1:p1} if P(s1 | s,a)=p1. Other
                probabilities are zero.
110         Corners are tricky because different actions result in same state.
111         """
112         if s in self.fling_states:
113             return {(0,0): 0.25, (self.x_dim-1,0):0.25,
                    (0,self.y_dim-1):0.25, (self.x_dim-1,self.y_dim-1):0.25}
114         res = dict()
115         for ai in self.actions:
116             s1 = self.intended_next(s,ai)
117             ps1 = 0.7 if ai==a else 0.1
118             if s1 in res: # occurs in corners
119                 res[s1] += ps1
120             else:
121                 res[s1] = ps1
122         return res
123 
124     def R(self,s,a):
125          if s in self.rewarding_states:
126              return self.rewarding_states[s]
127          else:
128              (x,y) = s
129              rew = 0
130              # rewards from crashing:
131              if y==0: ## on bottom.
132                  rew += -0.7 if a == 'down' else -0.1
133              if y==self.y_dim-1: ## on top.
134                  rew += -0.7 if a == 'up' else -0.1
135              if x==0: ## on left
136                  rew += -0.7 if a == 'left' else -0.1
137              if x==self.x_dim-1: ## on right.
138                  rew += -0.7 if a == 'right' else -0.1
139              return rew
```

12.2.1 Value Iteration

This implements value iteration.

This uses indexes of the states and actions (not the names). The value function is represented so $v[s]$ is the value of state with index s. A Q function is represented so $q[s][a]$ is the value for doing action with index a state with index s. Similarly a policy π is represented as a list where $pi[s]$, where s is the index of a state, returns the index of the action.

mdpProblem.py — (continued)

```
43      def vi(self, n):
44          """carries out n iterations of value iteration, updating value
                function self.v
45          Returns a Q-function, value function, policy
46          """
47          self.display(3,"calling vi")
48          assert n>0,"You must carry out at least one iteration of vi.
                n="+str(n)
49          #v = v0 if v0 is not None else {s:0 for s in self.states}
50          for i in range(n):
51              self.q = {s: {a: self.R(s,a)+self.discount*sum(p1*self.v[s1]
52                                                             for (s1,p1) in
                                                                 self.P(s,a).items())
53                        for a in self.actions}
54                    for s in self.states}
55              self.v = {s: max(self.q[s][a] for a in self.actions)
56                        for s in self.states}
57          self.pi = {s: argmaxd(self.q[s])
58                     for s in self.states}
59          return self.q, self.v, self.pi
```

The following shows how this can be used.

mdpExamples.py — (continued)

```
141 ## Testing value iteration
142 # Try the following:
143 # pt = party(discount=0.9)
144 # pt.vi(1)
145 # pt.vi(100)
146 # party(discount=0.99).vi(100)
147 # party(discount=0.4).vi(100)
148
149 # gr = grid(discount=0.9)
150 # gr.show()
151 # q,v,pi = gr.vi(100)
152 # q[(7,2)]
```

12.2.2 Showing Grid MDPs

A `GridMDP` is a type of MDP where we the states are (x,y) positions. It is a special sort of MDP only because we have methods to show it.

mdpProblem.py — (continued)

```
61  class GridMDP(MDP):
62      def __init__(self, states, actions, discount):
63          MDP.__init__(self, states, actions, discount)
64
65      def show(self):
66          #plt.ion()  # interactive
67          fig,(self.ax) = plt.subplots()
```

```
68          plt.subplots_adjust(bottom=0.2)
69          stepB = Button(plt.axes([0.8,0.05,0.1,0.075]), "step")
70          stepB.on_clicked(self.on_step)
71          resetB = Button(plt.axes([0.65,0.05,0.1,0.075]), "reset")
72          resetB.on_clicked(self.on_reset)
73          self.qcheck = CheckButtons(plt.axes([0.2,0.05,0.35,0.075]),
74                                     ["show q-values","show policy"])
75          self.qcheck.on_clicked(self.show_vals)
76          self.font_box = TextBox(plt.axes([0.1,0.05,0.05,0.075]),"Font:",
                textalignment="center")
77          self.font_box.on_submit(self.set_font_size)
78          self.font_box.set_val(str(plt.rcParams['font.size']))
79          self.show_vals(None)
80          plt.show()
81
82      def set_font_size(self, s):
83          plt.rcParams.update({'font.size': eval(s)})
84          plt.draw()
85
86      def show_vals(self,event):
87          self.ax.cla()
88          array = [[self.v[(x,y)] for x in range(self.x_dim)]
89                                          for y in range(self.y_dim)]
90          self.ax.pcolormesh([x-0.5 for x in range(self.x_dim+1)],
91                             [x-0.5 for x in range(self.y_dim+1)],
92                             array, edgecolors='black',cmap='summer')
93              # for cmap see
                    https://matplotlib.org/stable/tutorials/colors/colormaps.html
94          if self.qcheck.get_status()[1]: # "show policy"
95                 for (x,y) in self.q:
96                    maxv = max(self.q[(x,y)][a] for a in self.actions)
97                    for a in self.actions:
98                        if self.q[(x,y)][a] == maxv:
99                           # draw arrow in appropriate direction
100                          self.ax.arrow(x,y,self.xoff[a]*2,self.yoff[a]*2,
101                                   color='red',width=0.05, head_width=0.2,
                                        length_includes_head=True)
102         if self.qcheck.get_status()[0]: # "show q-values"
103            self.show_q(event)
104         else:
105            self.show_v(event)
106         self.ax.set_xticks(range(self.x_dim))
107         self.ax.set_xticklabels(range(self.x_dim))
108         self.ax.set_yticks(range(self.y_dim))
109         self.ax.set_yticklabels(range(self.y_dim))
110         plt.draw()
111
112     def on_step(self,event):
113         self.vi(1)
114         self.show_vals(event)
```

```
115
116     def show_v(self,event):
117         """show values"""
118         for (x,y) in self.v:
119             self.ax.text(x,y,"{val:.2f}".format(val=self.v[(x,y)]),ha='center')
120
121     def show_q(self,event):
122         """show q-values"""
123         for (x,y) in self.q:
124             for a in self.actions:
125                 self.ax.text(x+self.xoff[a],y+self.yoff[a],
126                              "{val:.2f}".format(val=self.q[(x,y)][a]),ha='center')
127
128     def on_reset(self,event):
129        self.v = self.initv
130        self.q = self.initq
131        self.show_vals(event)
```

Figure 12.5 shows the user interface for the tiny domain, which can be obtained using
`MDPtiny(discount=0.9).show()`
resizing it, checking "show q-values" and "show policy", and clicking "step" a few times.

To run the demo in class do:
```
% python -i mdpExamples.py
MDPtiny(discount=0.9).show()
```

Figure 12.6 shows the user interface for the grid domain, which can be obtained using
`grid(discount=0.9).show()`
resizing it, checking "show q-values" and "show policy", and clicking "step" a few times.

Exercise 12.1 Computing q before v may seem like a waste of space because we don't need to store q in order to compute value function or the policy. Change the algorithm so that it loops through the states and actions once per iteration, and only stores the value function and the policy. Note that to get the same results as before, you would need to make sure that you use the previous value of v in the computation not the current value of v. Does using the current value of v hurt the algorithm or make it better (in approaching the actual value function)?

12.2.3 Asynchronous Value Iteration

This implements asynchronous value iteration, storing Q.

A Q function is represented so $q[s][a]$ is the value for doing action with index a state with index s.

mdpProblem.py — (continued)

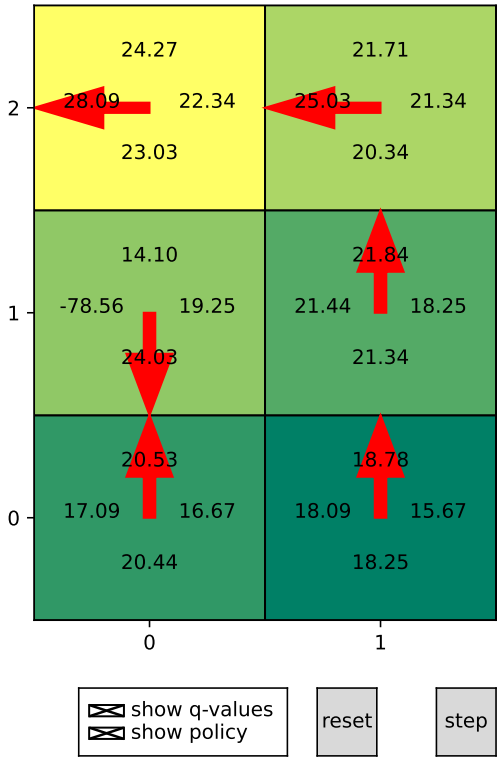


Figure 12.5: Interface for tiny example, after a number of steps. Each rectangle represents a state. In each rectangle are the 4 Q-values for the state. The leftmost number is the for the left action; the rightmost number is for the right action; the upper most is for the upR (up-risky) action and the lowest number is for the upC action. The arrow points to the action(s) with the maximum Q-value. Use `MDPtiny().show()` after loading `mdpExamples.py`

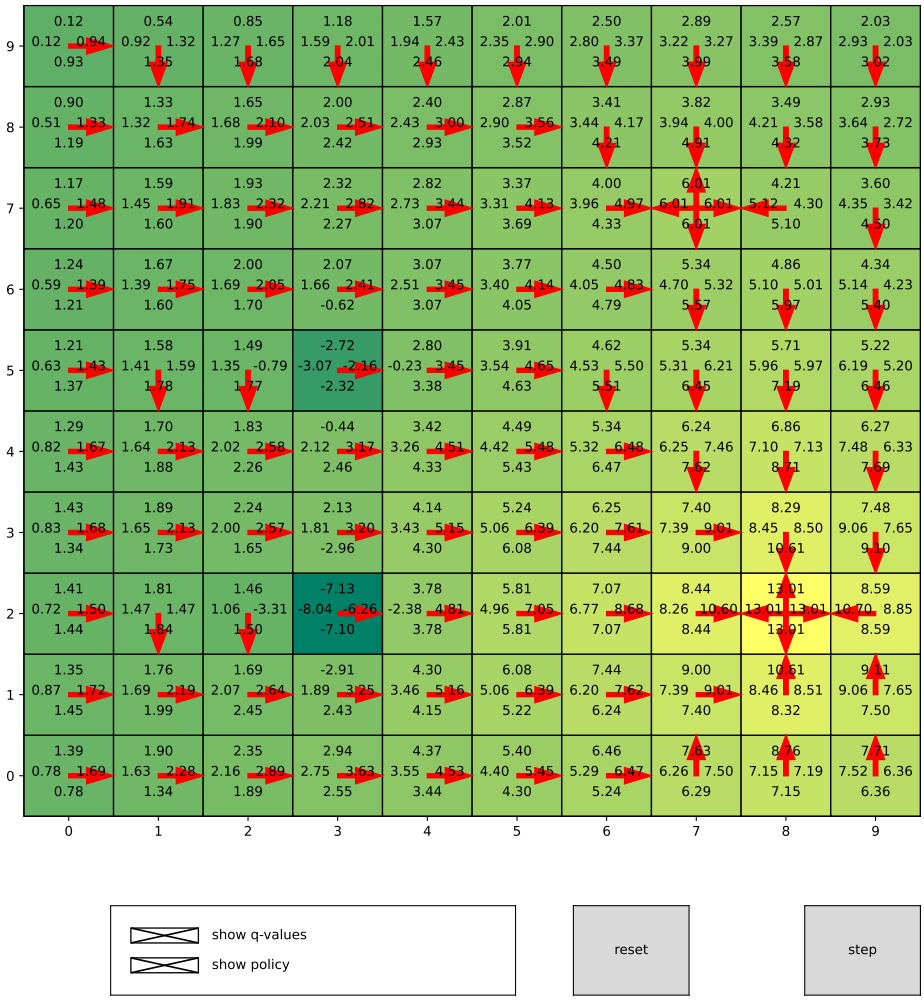


Figure 12.6: Interface for grid example, after a number of steps. Each rectangle represents a state. In each rectangle are the 4 Q-values for the state. The leftmost number is the for the left action; the rightmost number is for the right action; the upper most is for the up action and the lowest number is for the down action. The arrow points to the action(s) with the maximum Q-value. From `grid(discount=0.9).show()`

```
133     def avi(self,n):
134         states = list(self.states)
135         actions = list(self.actions)
136         for i in range(n):
137             s = random.choice(states)
138             a = random.choice(actions)
139             self.q[s][a] = (self.R(s,a) + self.discount *
140                             sum(p1 * max(self.q[s1][a1]
141                                          for a1 in self.actions)
142                                 for (s1,p1) in self.P(s,a).items()))
143         return Q
```

The following shows how `avi` can be used.

mdpExamples.py — (continued)

```
155 ## Testing asynchronous value iteration
156 # Try the following:
157 # pt = party(discount=0.9)
158 # pt.avi(10)
159 # pt.vi(1000)
160 
161 # gr = grid(discount=0.9)
162 # q = gr.avi(100000)
163 # q[(7,2)]
```

Exercise 12.2 Implement value iteration that stores the V-values rather than the Q-values. Does it work better than storing Q? (What might better mean?)

Exercise 12.3 In asynchronous value iteration, try a number of different ways to choose the states and actions to update (e.g., sweeping through the state-action pairs, choosing them at random). Note that the best way may be to determine which states have had their Q-values change the most, and then update the previous ones, but that is not so straightforward to implement, because you need to find those previous states.

Chapter 13

Reinforcement Learning

13.1 Representing Agents and Environments

The reinforcement learning agents and environments are instances of the general agent architecture of Section 2.1, where the percepts are reward–state pairs. The *state* is the world state; this is the fully observable assumption. In particular:

- An agent implements the method `select_action` that takes the reward and environment state and returns the next action (and updates the state of the agent).
- An environment implements the method `do` that takes the action and returns a pair of the reward and the resulting environment state.

These are chained together to simulate the system.

The only difference between the architecture of Section 2.1 is that the simulation starts by calling the agent method `initial_action(state)`, which is generally to save the state and return a random action.

The environments have names for the roles of agents participating. In this chapter, where we assume a single agent, this is used as the name of the environment.

rlProblem.py — Representations for Reinforcement Learning

```
11 import random
12 import math
13 from display import Displayable
14 from agents import Agent, Environment
15 from utilities import pick_from_dist, argmaxe, argmaxd, flip
16
17 class RL_env(Environment):
```

```
18      def __init__(self, name, actions, state):
19          """creates an environment given name, list of actions, and initial
                state"""
20          self.name = name       # the role for an agent
21          self.actions = actions # list of all actions
22          self.state = state     # initial state
23
24      # must implement do(action)->(reward,state)
```

rlProblem.py — (continued)

```
26
27  class RL_agent(Agent):
28      """An RL_Agent
29      has percepts (s, r) for some state s and real reward r
30      """
31      def __init__(self, actions):
32         self.actions = actions
33
34      def initial_action(self, env_state):
35          """return the initial action, and remember the state and action
36          Act randomly initially
37          Could be overridden to initialize data structures (as the agent now
                knows about one state)
38          """
39          self.state = env_state
40          self.action = random.choice(self.actions)
41          return self.action
42
43      def select_action(self, reward, state):
44          """
45          Select the action given the reward and next state
46          Remember the action in self.action
47          This implements "Act randomly" and should be overridden!
48          """
49          self.action = random.choice(self.actions)
50          return self.action
```

This is similar to `Simulate` of Section 2.1, except it is initialized by `agent.initial_action(state)`.

rlProblem.py — (continued)

```
52  import matplotlib.pyplot as plt
53
54  class Simulate(Displayable):
55      """simulate the interaction between the agent and the environment
56      for n time steps.
57      Returns a pair of the agent state and the environment state.
58      """
59      def __init__(self, agent, environment):
60          self.agent = agent
61          self.env = environment
62          self.action = agent.initial_action(self.env.state)
```

```
63          self.reward_history = [] # for plotting
64
65      def go(self, n):
66          for i in range(n):
67              (reward,state) = self.env.do(self.action)
68              self.display(2,f"i={i} reward={reward}, state={state}")
69              self.reward_history.append(reward)
70              self.action = self.agent.select_action(reward,state)
71              self.display(2,f"    action={self.action}")
72          return self
```

The following plots the sum of rewards as a function of the step in a simulation.

rlProblem.py — (continued)

```
74      def plot(self, label=None, step_size=None, xscale='linear'):
75          """
76          plots the rewards history in the simulation
77          label is the label for the plot
78          step_size is the number of steps between each point plotted
79          xscale is 'log' or 'linear'
80
81          returns sum of rewards
82          """
83          if step_size is None: #for long simulations (> 999), only plot some
                points
84              step_size = max(1,len(self.reward_history)//500)
85          if label is None:
86              label = self.agent.method
87          plt.ion()
88          plt.xscale(xscale)
89          plt.xlabel("step")
90          plt.ylabel("Sum of rewards")
91          sum_history, sum_rewards = acc_rews(self.reward_history, step_size)
92          plt.plot(range(0,len(self.reward_history),step_size), sum_history,
                label=label)
93          plt.legend()
94          plt.draw()
95          return sum_rewards
96
97  def acc_rews(rews,step_size):
98      """returns the rolling sum of the values, sampled each step_size, and
            the sum
99      """
100     acc = []
101     sumr = 0; i=0
102     for e in rews:
103        sumr += e
104        i += 1
105        if (i%step_size == 0): acc.append(sumr)
106     return acc, sumr
```

Here is the definition of the simple 2-state, 2-action decision about whether to party or relax (Example 12.29 in Poole and Mackworth [2023]).

rlExamples.py — Some example reinforcement learning environments

```
from rlProblem import RL_env
class Healthy_env(RL_env):
    def __init__(self):
        RL_env.__init__(self, "Party Decision", ["party", "relax"],
            "healthy")

    def do(self, action):
        """updates the state based on the agent doing action.
        returns reward,state
        """
        if self.state=="healthy":
            if action=="party":
                self.state = "healthy" if flip(0.7) else "sick"
                reward = 10
            else: # action=="relax"
                self.state = "healthy" if flip(0.95) else "sick"
                reward = 7
        else: # self.state=="sick"
            if action=="party":
                self.state = "healthy" if flip(0.1) else "sick"
                reward = 2
            else:
                self.state = "healthy" if flip(0.5) else "sick"
                reward = 0
        return reward, self.state
```

13.1.1 Simulating an environment from an MDP

Given the definition for an MDP (page 262), *Env_from_MDP* takes in an MDP and simulates the environment with those dynamics.

Note that the representation of an MDP does not contain enough information to simulate a system, because does not specify the distribution over initial states, and it loses any dependency between the rewards and the resulting state (e.g., hitting the wall and having a negative reward may be correlated). The following code assumes the agent always received the average reward for the state and action.

rlProblem.py — (continued)

```
class Env_from_MDP(RL_env):
    def __init__(self, mdp):
        initial_state = random.choice(mdp.states)
        RL_env.__init__(self, "From MDP", mdp.actions, initial_state)
        self.mdp = mdp

    def do(self, action):
```

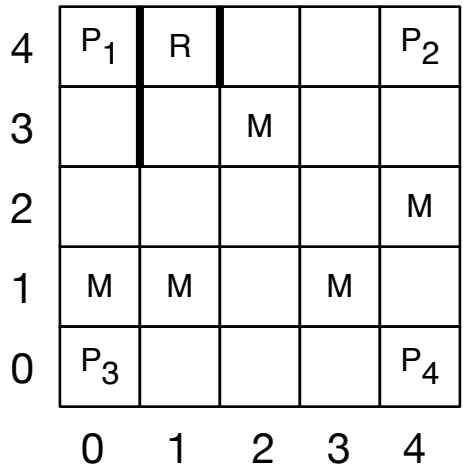


Figure 13.1: Monster game

```
116         """updates the state based on the agent doing action.
117         returns state,reward
118         """
119         reward = self.mdp.R(self.state,action)
120         self.state = pick_from_dist(self.mdp.P(self.state,action))
121         return reward,self.state
```

13.1.2 Monster Game

This is for the game depicted in Figure 13.1 (Example 13.2 of Poole and Mackworth [2023]).

rlExamples.py — (continued)

```
36  import random
37  from utilities import flip
38  from rlProblem import RL_env
39
40  class Monster_game_env(RL_env):
41      xdim = 5
42      ydim = 5
43
44      vwalls = [(0,3), (0,4), (1,4)] # vertical walls right of these locations
45      hwalls = [] # not implemented
46      crashed_reward = -1
47
48      prize_locs = [(0,0), (0,4), (4,0), (4,4)]
49      prize_apears_prob = 0.3
50      prize_reward = 10
51
52      monster_locs = [(0,1), (1,1), (2,3), (3,1), (4,2)]
```

```
    monster_appears_prob = 0.4
    monster_reward_when_damaged = -10
    repair_stations = [(1,4)]

    actions = ["up","down","left","right"]

    def __init__(self):
        # State:
        self.x = 2
        self.y = 2
        self.damaged = False
        self.prize = None
        # Statistics
        self.number_steps = 0
        self.accumulated_rewards = 0 # sum of rewards received
        self.min_accumulated_rewards = 0
        self.min_step = 0
        self.zero_crossing = 0
        RL_env.__init__(self, "Monster Game", self.actions, (self.x,
            self.y, self.damaged, self.prize))
        self.display(2,"","Step","Tot Rew","Ave Rew",sep="\t")

    def do(self,action):
        """updates the state based on the agent doing action.
        returns reward,state
        """
        assert action in self.actions, f"Monster game, unknown action:
            {action}"
        reward = 0.0
        # A prize can appear:
        if self.prize is None and flip(self.prize_apears_prob):
               self.prize = random.choice(self.prize_locs)
        # Actions can be noisy
        if flip(0.4):
            actual_direction = random.choice(self.actions)
        else:
            actual_direction = action
        # Modeling the actions given the actual direction
        if actual_direction == "right":
            if self.x==self.xdim-1 or (self.x,self.y) in self.vwalls:
                reward += self.crashed_reward
            else:
                self.x += 1
        elif actual_direction == "left":
            if self.x==0 or (self.x-1,self.y) in self.vwalls:
                reward += self.crashed_reward
            else:
                self.x += -1
        elif actual_direction == "up":
            if self.y==self.ydim-1:
```

```
101             reward += self.crashed_reward
102         else:
103             self.y += 1
104     elif actual_direction == "down":
105         if self.y==0:
106             reward += self.crashed_reward
107         else:
108             self.y += -1
109     else:
110         raise RuntimeError(f"unknown_direction: {actual_direction}")
111 
112     # Monsters
113     if (self.x,self.y) in self.monster_locs and
            flip(self.monster_appears_prob):
114         if self.damaged:
115             reward += self.monster_reward_when_damaged
116         else:
117             self.damaged = True
118     if (self.x,self.y) in self.repair_stations:
119         self.damaged = False
120 
121     # Prizes
122     if (self.x,self.y) == self.prize:
123         reward += self.prize_reward
124         self.prize = None
125 
126     # Statistics
127     self.number_steps += 1
128     self.accumulated_rewards += reward
129     if self.accumulated_rewards < self.min_accumulated_rewards:
130         self.min_accumulated_rewards = self.accumulated_rewards
131         self.min_step = self.number_steps
132     if self.accumulated_rewards>0 and reward>self.accumulated_rewards:
133         self.zero_crossing = self.number_steps
134     self.display(2,"",self.number_steps,self.accumulated_rewards,
135                  self.accumulated_rewards/self.number_steps,sep="\t")
136 
137     return reward, (self.x, self.y, self.damaged, self.prize)
```

13.2 Q Learning

To run the Q-learning demo, in folder "aipython", load "rlQLearner.py", and copy and paste the example queries at the bottom of that file.

rlQLearner.py — Q Learning

```
11 import random
12 import math
```

```
from display import Displayable
from utilities import argmaxe, argmaxd, flip
from rlProblem import RL_agent, epsilon_greedy, ucb

class Q_learner(RL_agent):
    """A Q-learning agent has
    belief-state consisting of
        state is the previous state (initialized by RL_agent
        q is a {(state,action):value} dict
        visits is a {(state,action):n} dict. n is how many times action was
            done in state
        acc_rewards is the accumulated reward

    """
```

rlQLearner.py — (continued)

```
    def __init__(self, role, actions, discount,
                 exploration_strategy=epsilon_greedy, es_kwargs={},
                 alpha_fun=lambda _:0.2,
                 Qinit=0, method="Q_learner"):
        """
        role is the role of the agent (e.g., in a game)
        actions is the set of actions the agent can do
        discount is the discount factor
        exploration_strategy is the exploration function, default
            "epsilon_greedy"
        es_kwargs is extra arguments of exploration_strategy
        alpha_fun is a function that computes alpha from the number of
            visits
        Qinit is the initial q-value
        method gives the method used to implement the role (for plotting)
        """
        RL_agent.__init__(self, actions)
        self.role = role
        self.discount = discount
        self.exploration_strategy = exploration_strategy
        self.es_kwargs = es_kwargs
        self.alpha_fun = alpha_fun
        self.Qinit = Qinit
        self.method = method
        self.acc_rewards = 0
        self.Q = {}
        self.visits = {}
```

The initial action is a random action. It remembers the state, and initializes the data structures.

rlQLearner.py — (continued)

```
    def initial_action(self, state):
        """ Returns the initial action; selected at random
        Initialize Data Structures
```

```
56          """
57          self.Q[state] = {act:self.Qinit for act in self.actions}
58          self.action = random.choice(self.actions)
59          self.visits[state] = {act:0 for act in self.actions}
60          self.state = state
61          self.display(2, f"Initial State: {state} Action {self.action}")
62          self.display(2,"s\ta\tr\ts'\tQ")
63          return self.action
64
65      def select_action(self, reward, next_state):
66          """give reward and next state, select next action to be carried
                out"""
67          if next_state not in self.visits: # next state not seen before
68              self.Q[next_state] = {act:self.Qinit for act in self.actions}
69              self.visits[next_state] = {act:0 for act in self.actions}
70          self.visits[self.state][self.action] +=1
71          alpha = self.alpha_fun(self.visits[self.state][self.action])
72          self.Q[self.state][self.action] += alpha*(
73                              reward
74                              + self.discount * max(self.Q[next_state].values())
75                              - self.Q[self.state][self.action])
76          self.display(2,self.state, self.action, reward, next_state,
77                       self.Q[self.state][self.action], sep='\t')
78          self.state = next_state
79          self.action = self.exploration_strategy(self.Q[next_state],
80                                      self.visits[next_state],**self.es_kwargs)
81          self.display(3,f"Agent {self.role} doing {self.action} in state
                {self.state}")
82          return self.action
```

Exercise 13.1 Implement SARSA. Hint: it does not do a *max* in *do*. Instead it needs to choose *next_act* before it does the update.

13.2.1 Exploration Strategies

Two explorations strategies are defined: epsilon-greedy and UCB.

In general an exploration strategy takes two arguments

- *Qs* is a $\{action : q\_value\}$ dictionary for the current state
- *Vs* is a $\{action : visits\}$ dictionary for the current state; where *visits* is the number of times that the action has been carried out in the current state.

rlProblem.py — (continued)

```
123 def epsilon_greedy(Qs, Vs={}, epsilon=0.1):
124         """select action given epsilon greedy
125         Qs is the {action:Q-value} dictionary for current state
126         Vs is ignored
127         """
```

```
128         if flip(epsilon):
129             return random.choice(list(Qs.keys())) # act randomly
130         else:
131             return argmaxd(Qs)
132 
133 def ucb(Qs, Vs, c=1.4):
134         """select action given upper-confidence bound
135         Qs is the {action:Q-value} dictionary for current state
136         Vs is the {action:visits} dictionary for current state
137 
138         0.01 is to prevent divide-by zero (could just be infinity)
139         """
140         Ns = sum(Vs.values())
141         ucb1 = {a:Qs[a]+c*math.sqrt(Ns/(0.01+Vs[a]))
142                     for a in Qs.keys()}
143         action = argmaxd(ucb1)
144         return action
```

Exercise 13.2 Implement a soft-max action selection. Choose a temperature that works well for the domain. Explain how you picked this temperature. Compare the epsilon-greedy, soft-max and optimism in the face of uncertainty.

13.2.2 Testing Q-learning

The first tests are for the 2-action 2-state decision about whether to relax or party (Example 12.29 of Poole and Mackworth [2023].

rlQLearner.py — (continued)

```
84  ####### TEST CASES ########
85  from rlProblem import Simulate,epsilon_greedy, ucb
86  from rlExamples import Healthy_env, Monster_game_env
87  from rlQLearner import Q_learner
88  
89  env = Healthy_env()
90  # Some RL agents with different parameters:
91  ag = Q_learner(env.name, env.actions, 0.7)
92  ag_ucb = Q_learner(env.name, env.actions, 0.7, exploration_strategy = ucb,
        es_kwargs={'c':0.1}, method="ucb")
93  ag_opt = Q_learner(env.name, env.actions, 0.7, Qinit=100,
        method="optimistic" )
94  ag_exp_m = Q_learner(env.name, env.actions, 0.7,
        es_kwargs={'epsilon':0.5}, method="more explore")
95  ag_greedy = Q_learner(env.name, env.actions, 0.1, Qinit=100, method="disc
        0.1")
96  
97  sim_ag = Simulate(ag,env)
98  
99  # sim_ag.go(100)
100 # ag.Q   # get the learned Q-values
101 # sim_ag.plot()
```

```
# Simulate(ag_ucb,env).go(100).plot()
# Simulate(ag_opt,env).go(100).plot()
# Simulate(ag_exp_m,env).go(100).plot()
# Simulate(ag_greedy,env).go(100).plot()


from mdpExamples import MDPtiny
from rlProblem import Env_from_MDP
envt = Env_from_MDP(MDPtiny())
agt = Q_learner(envt.name, envt.actions, 0.8)
#Simulate(agt, envt).go(1000).plot()

mon_env = Monster_game_env()
mag1 = Q_learner(mon_env.name, mon_env.actions,0.9)
#Simulate(mag1,mon_env).go(100000).plot()
mag_ucb = Q_learner(mon_env.name, mon_env.actions,0.9,exploration_strategy
    = ucb,es_kwargs={'c':0.1},method="UCB(0.1)")
#Simulate(mag_ucb,mon_env).go(100000).plot()

mag2 = Q_learner(mon_env.name, mon_env.actions,
    0.9,es_kwargs={'epsilon':0.2},alpha_fun=lambda
    k:1/k,method="alpha=1/k")
#Simulate(mag2,mon_env).go(100000).plot()
mag3 = Q_learner(mon_env.name, mon_env.actions, 0.9,alpha_fun=lambda
    k:10/(9+k),method="alpha=10/(9+k)")
#Simulate(mag3,mon_env).go(100000).plot()
```

13.3 Q-leaning with Experience Replay

A bounded buffer remembers values up to size `buffer_size`. Once it is full, all old experiences have the same chance of being in the buffer.

rlQExperienceReplay.py — Q-Learner with Experience Replay

```
from rlQLearner import Q_learner
from utilities import flip
import random

class BoundedBuffer(object):
    def __init__(self, buffer_size=1000):
        self.buffer_size = buffer_size
        self.buffer = [0]*buffer_size
        self.number_added = 0

    def add(self,experience):
        if self.number_added < self.buffer_size:
            self.buffer[self.number_added] = experience
        else:
            if flip(self.buffer_size/self.number_added):
                position = random.randrange(self.buffer_size)
```

```
27              self.buffer[position] = experience
28          self.number_added += 1
29
30      def get(self):
31          return self.buffer[random.randrange(min(self.number_added,
                self.buffer_size))]
```

A `Q_ER_Learner` does Q-leaning with experience replay. It only uses action replay after `burn_in` number of steps.

rlQExperienceReplay.py — (continued)

```
33  class Q_ER_learner(Q_learner):
34      def __init__(self, role, actions, discount,
35                   max_buffer_size=10000,
36                   num_updates_per_action=5, burn_in=1000,
37                  method="Q_ER_learner", **q_kwargs):
38          """Q-learner with experience replay
39          role is the role of the agent (e.g., in a game)
40          actions is the set of actions the agent can do
41          discount is the discount factor
42          max_buffer_size is the maximum number of past experiences that is
                remembered
43          burn_in is the number of steps before using old experiences
44          num_updates_per_action is the number of q-updates for past
                experiences per action
45          q_kwargs are any extra parameters for Q_learner
46          """
47          Q_learner.__init__(self, role, actions, discount, method=method,
                **q_kwargs)
48          self.experience_buffer = BoundedBuffer(max_buffer_size)
49          self.num_updates_per_action = num_updates_per_action
50          self.burn_in = burn_in
51
52      def select_action(self, reward, next_state):
53          """give reward and new state, select next action to be carried
                out"""
54          self.experience_buffer.add((self.state,self.action,reward,next_state))
                #remember experience
55          if next_state not in self.Q: # Q and visits are defined on the same
                states
56              self.Q[next_state] = {act:self.Qinit for act in self.actions}
57              self.visits[next_state] = {act:0 for act in self.actions}
58          self.visits[self.state][self.action] +=1
59          alpha = self.alpha_fun(self.visits[self.state][self.action])
60          self.Q[self.state][self.action] += alpha*(
61                           reward
62                           + self.discount * max(self.Q[next_state].values())
63                           - self.Q[self.state][self.action])
64          self.display(2,self.state, self.action, reward, next_state,
65                       self.Q[self.state][self.action], sep='\t')
66          self.state = next_state
```

```
        # do some updates from experience buffer
        if self.experience_buffer.number_added > self.burn_in:
            for i in range(self.num_updates_per_action):
                (s,a,r,ns) = self.experience_buffer.get()
                self.visits[s][a] +=1 # is this correct?
                alpha = self.alpha_fun(self.visits[s][a])
                self.Q[s][a] += alpha * (r +
                                  self.discount* max(self.Q[ns][na]
                                         for na in self.actions)
                                  -self.Q[s][a] )
        ### CHOOSE NEXT ACTION ###
        self.action = self.exploration_strategy(self.Q[next_state],
                                     self.visits[next_state],**self.es_kwargs)
        self.display(3,f"Agent {self.role} doing {self.action} in state
            {self.state}")
        return self.action
```

rlQExperienceReplay.py — (continued)

```
from rlProblem import Simulate
from rlExamples import Monster_game_env
from rlQLearner import mag1, mag2, mag3

mon_env = Monster_game_env()
mag1ar = Q_ER_learner(mon_env.name, mon_env.actions,0.9,method="Q_ER")
# Simulate(mag1ar,mon_env).go(100000).plot()

mag3ar = Q_ER_learner(mon_env.name, mon_env.actions, 0.9, alpha_fun=lambda
    k:10/(9+k),method="Q_ER alpha=10/(9+k)")
# Simulate(mag3ar,mon_env).go(100000).plot()
```

13.4 Model-based Reinforcement Learner

To run the demo, in folder "aipython", load "rlModelLearner.py", and copy and paste the example queries at the bottom of that file. This assumes Python 3.

A model-based reinforcement learner builds a Markov decision process model of the domain, simultaneously learns the model and plans with that model.

The model-based reinforcement learner used the following data structures:

- $Q[s][a]$ is dictionary that, given state s and action a returns the Q-value, the estimate of the future (discounted) value of being in state s and doing action a.
- $R[s][a]$ is dictionary that, given a (s,a) state s and action a is the average reward received from doing a in state s.

- $T[s][a][s']$ is dictionary that, given states s and s' and action a returns the number of times a was done in state s and the result was state s'. Note that s' is only a key if it has been the result of doing a in s; there are no 0 counts recorded.

- $visits[s][a]$ is dictionary that, given state s and action a returns the number of times action a was carried out in state s. This is the C of Figure 13.6 of Poole and Mackworth [2023].

  Note that $visits[s][a] = \sum_{s'} T[s][a][s']$ but is stored separately to keep the code more readable.

The main difference to Figure 13.6 of Poole and Mackworth [2023] is the code below does a fixed number of asynchronous value iteration updates per step.

rlModelLearner.py — Model-based Reinforcement Learner

```
import random
from rlProblem import RL_agent, Simulate, epsilon_greedy, ucb
from display import Displayable
from utilities import argmaxe, flip

class Model_based_reinforcement_learner(RL_agent):
    """A Model-based reinforcement learner
    """

    def __init__(self, role, actions, discount,
                 exploration_strategy=epsilon_greedy, es_kwargs={},
                 Qinit=0,
                 updates_per_step=10, method="MBR_learner"):
        """role is the role of the agent (e.g., in a game)
        actions is the list of actions the agent can do
        discount is the discount factor
        explore is the proportion of time the agent will explore
        Qinit is the initial value of the Q's
        updates_per_step is the number of AVI updates per action
        label is the label for plotting
        """
        RL_agent.__init__(self, actions)
        self.role = role
        self.actions = actions
        self.discount = discount
        self.exploration_strategy = exploration_strategy
        self.es_kwargs = es_kwargs
        self.Qinit = Qinit
        self.updates_per_step = updates_per_step
        self.method = method
```

rlModelLearner.py — (continued)

```
    def initial_action(self, state):
        """ Returns the initial action; selected at random
```

```
        Initialize Data Structures

        """
        self.state = state
        self.T = {self.state: {a: {} for a in self.actions}}
        self.visits = {self.state: {a: 0 for a in self.actions}}
        self.Q = {self.state: {a: self.Qinit for a in self.actions}}
        self.R = {self.state: {a: 0 for a in self.actions}}
        self.states_list = [self.state] # list of states encountered
        self.action = random.choice(self.actions)
        self.display(2, f"Initial State: {state} Action {self.action}")
        self.display(2,"s\ta\tr\ts'\tQ")
        return self.action
```

rlModelLearner.py — (continued)

```
    def select_action(self, reward, next_state):
        """do num_steps of interaction with the environment
        for each action, do updates_per_step iterations of asynchronous
            value iteration
        """
        if next_state not in self.visits: # has not been encountered before
            self.states_list.append(next_state)
            self.visits[next_state] = {a:0 for a in self.actions}
            self.T[next_state] = {a:{} for a in self.actions}
            self.Q[next_state] = {a:self.Qinit for a in self.actions}
            self.R[next_state] = {a:0 for a in self.actions}
        if next_state in self.T[self.state][self.action]:
            self.T[self.state][self.action][next_state] += 1
        else:
            self.T[self.state][self.action][next_state] = 1
        self.visits[self.state][self.action] += 1
        self.R[self.state][self.action] +=
            (reward-self.R[self.state][self.action])/self.visits[self.state][self.action]
        st,act = self.state,self.action #initial state-action pair for AVI
        for update in range(self.updates_per_step):
            self.Q[st][act] = self.R[st][act]+self.discount*(
                sum(self.T[st][act][nst]/self.visits[st][act]*
                    max(self.Q[nst][nact] for nact in self.actions)
                    for nst in self.T[st][act].keys()))
            st = random.choice(self.states_list)
            act = random.choice(self.actions)
        self.state = next_state
        self.action = self.exploration_strategy(self.Q[next_state],
                                self.visits[next_state],**self.es_kwargs)
        return self.action
```

rlModelLearner.py — (continued)

```
from rlExamples import Monster_game_env
mon_env = Monster_game_env()
```

```
mbl1 = Model_based_reinforcement_learner(mon_env.name, mon_env.actions,
    0.9, updates_per_step=10,method="model-based(10)")
# Simulate(mbl1,mon_env).go(100000).plot()
mbl2 = Model_based_reinforcement_learner(mon_env.name, mon_env.actions,
    0.9, updates_per_step=1, method="model-based(1)")
# Simulate(mbl2,mon_env).go(100000).plot()
```

Exercise 13.3 If there was only one update per step, the algorithm can be made simpler and use less space. Explain how. Does it make it more efficient? Is it worthwhile having more than one update per step for the games implemented here?

Exercise 13.4 It is possible to implement the model-based reinforcement learner by replacing Q, R, T, *visits*, *res_states* with a single dictionary that, given a state and action returns a tuple corresponding to these data structures. Does this make the algorithm easier to understand? Does this make the algorithm more efficient?

Exercise 13.5 If the states and the actions were mapped into integers, the dictionaries could be implemented perhaps more efficiently as arrays. How does the code need to change?. Implement this for the monster game. Is it more efficient?

Exercise 13.6 In `random_choice` in the updates of `select_action`, all state-action pairs have the same chance of being chosen. Does selecting state-action pairs proportionally to the number of times visited work better than what is implemented? Provide evidence for your answer.

13.5 Reinforcement Learning with Features

To run the demo, in folder "`aipython`", load "`rlFeatures.py`", and copy and paste the example queries at the bottom of that file. This assumes Python 3.

13.5.1 Representing Features

A feature is a function from state and action. To construct the features for a domain, we construct a function that takes a state and an action and returns the list of all feature values for that state and action. This feature set is redesigned for each problem.

get_features$(state, action)$ returns the feature values appropriate for the monster game.

rlMonsterGameFeatures.py — Feature-based Reinforcement Learner

```
from rlExamples import Monster_game_env
from rlProblem import RL_env

def get_features(state,action):
    """returns the list of feature values for the state-action pair
    """
```

```
    assert action in Monster_game_env.actions, f"Monster game, unknown
        action: {action}"
    (x,y,d,p) = state
    # f1: would go to a monster
    f1 = monster_ahead(x,y,action)
    # f2: would crash into wall
    f2 = wall_ahead(x,y,action)
    # f3: action is towards a prize
    f3 = towards_prize(x,y,action,p)
    # f4: damaged and action is toward repair station
    f4 = towards_repair(x,y,action) if d else 0
    # f5: damaged and towards monster
    f5 = 1 if d and f1 else 0
    # f6: damaged
    f6 = 1 if d else 0
    # f7: not damaged
    f7 = 1-f6
    # f8: damaged and prize ahead
    f8 = 1 if d and f3 else 0
    # f9: not damaged and prize ahead
    f9 = 1 if not d and f3 else 0
    features = [1,f1,f2,f3,f4,f5,f6,f7,f8,f9]
    # the next 20 features are for 5 prize locations
    # and 4 distances from outside in all directions
    for pr in Monster_game_env.prize_locs+[None]:
        if p==pr:
            features += [x, 4-x, y, 4-y]
        else:
            features += [0, 0, 0, 0]
    # fp04 feature for y when prize is at 0,4
    # this knows about the wall to the right of the prize
    if p==(0,4):
        if x==0:
            fp04 = y
        elif y<3:
            fp04 = y
        else:
            fp04 = 4-y
    else:
        fp04 = 0
    features.append(fp04)
    return features

def monster_ahead(x,y,action):
    """returns 1 if the location expected to get to by doing
    action from (x,y) can contain a monster.
    """
    if action == "right" and (x+1,y) in Monster_game_env.monster_locs:
        return 1
    elif action == "left" and (x-1,y) in Monster_game_env.monster_locs:
```

```
66          return 1
67      elif action == "up" and (x,y+1) in Monster_game_env.monster_locs:
68          return 1
69      elif action == "down" and (x,y-1) in Monster_game_env.monster_locs:
70          return 1
71      else:
72          return 0
73
74  def wall_ahead(x,y,action):
75      """returns 1 if there is a wall in the direction of action from (x,y).
76      This is complicated by the internal walls.
77      """
78      if action == "right" and (x==Monster_game_env.xdim-1 or (x,y) in
            Monster_game_env.vwalls):
79          return 1
80      elif action == "left" and (x==0 or (x-1,y) in Monster_game_env.vwalls):
81          return 1
82      elif action == "up" and y==Monster_game_env.ydim-1:
83          return 1
84      elif action == "down" and y==0:
85          return 1
86      else:
87          return 0
88
89  def towards_prize(x,y,action,p):
90      """action goes in the direction of the prize from (x,y)"""
91      if p is None:
92          return 0
93      elif p==(0,4): # take into account the wall near the top-left prize
94          if action == "left" and (x>1 or x==1 and y<3):
95              return 1
96          elif action == "down" and (x>0 and y>2):
97              return 1
98          elif action == "up" and (x==0 or y<2):
99              return 1
100         else:
101             return 0
102     else:
103         px,py = p
104         if p==(4,4) and x==0:
105             if (action=="right" and y<3) or (action=="down" and y>2) or
                    (action=="up" and y<2):
106                 return 1
107             else:
108                 return 0
109         if (action == "up" and y<py) or (action == "down" and py<y):
110             return 1
111         elif (action == "left" and px<x) or (action == "right" and x<px):
112             return 1
113         else:
```

```
            return 0

def towards_repair(x,y,action):
    """returns 1 if action is towards the repair station.
    """
    if action == "up" and (x>0 and y<4 or x==0 and y<2):
        return 1
    elif action == "left" and x>1:
        return 1
    elif action == "right" and x==0 and y<3:
        return 1
    elif action == "down" and x==0 and y>2:
        return 1
    else:
        return 0
```

The following uses a simpler set of features. In particular, it only considers whether the action will most likely result in a monster position or a wall, and whether the action moves towards the current prize.

rlMonsterGameFeatures.py — (continued)

```
def simp_features(state,action):
    """returns a list of feature values for the state-action pair
    """
    assert action in Monster_game_env.actions
    (x,y,d,p) = state
    # f1: would go to a monster
    f1 = monster_ahead(x,y,action)
    # f2: would crash into wall
    f2 = wall_ahead(x,y,action)
    # f3: action is towards a prize
    f3 = towards_prize(x,y,action,p)
    return [1,f1,f2,f3]
```

13.5.2 Feature-based RL learner

This learns a linear function approximation of the Q-values. It requires the function *get_features* that given a state and an action returns a list of values for all of the features. Each environment requires this function to be provided.

rlFeatures.py — Feature-based Reinforcement Learner

```
import random
from rlProblem import RL_agent, epsilon_greedy, ucb
from display import Displayable
from utilities import argmaxe, flip

class SARSA_LFA_learner(RL_agent):
    """A SARSA with linear function approximation (LFA) learning agent has
    """
    def __init__(self, role, actions, discount, get_features,
```

```
                exploration_strategy=epsilon_greedy, es_kwargs={},
                step_size=0.01, winit=0, method="SARSA_LFA"):
        """role is the role of the agent (e.g., in a game)
        actions is the set of actions the agent can do
        discount is the discount factor
        get_features is a function get_features(state,action) -> list of
            feature values
        exploration_strategy is the exploration function, default
            "epsilon_greedy"
        es_kwargs is extra keyword arguments of the exploration_strategy
        step_size is gradient descent step size
        winit is the initial value of the weights
        method gives the method used to implement the role (for plotting)
        """
        RL_agent.__init__(self, actions)
        self.role = role
        self.discount = discount
        self.exploration_strategy = exploration_strategy
        self.es_kwargs = es_kwargs
        self.get_features = get_features
        self.step_size = step_size
        self.winit = winit
        self.method = method
```

The initial action is a random action. It remembers the state, and initializes the data structures.

rlFeatures.py — (continued)

```
    def initial_action(self, state):
        """ Returns the initial action; selected at random
        Initialize Data Structures
        """
        self.action = random.choice(self.actions)
        self.features = self.get_features(state, self.action)
        self.weights = [self.winit for f in self.features]
        self.state = state
        self.display(2, f"Initial State: {state} Action {self.action}")
        self.display(2,"s\ta\tr\ts'\tQ")
        return self.action
```

do takes in the number of steps.

rlFeatures.py — (continued)

```

    def Q(self, state,action):
        """returns Q-value of the state and action for current weights
        """
        return dot_product(self.weights, self.get_features(state,action))

    def select_action(self, reward, next_state):
        """do num_steps of interaction with the environment"""
        feature_values = self.get_features(self.state,self.action)
```

```
        oldQ = self.Q(self.state,self.action)
        next_action = self.exploration_strategy({a:self.Q(next_state,a)
                                                 for a in self.actions}, {})
        nextQ = self.Q(next_state,next_action)
        delta = reward + self.discount * nextQ - oldQ
        for i in range(len(self.weights)):
            self.weights[i] += self.step_size * delta * feature_values[i]
        self.display(2,self.state, self.action, reward, next_state,
                     self.Q(self.state,self.action), delta, sep='\t')
        self.state = next_state
        self.action = next_action
        return self.action

    def show_actions(self,state=None):
        """prints the value for each action in a state.
        This may be useful for debugging.
        """
        if state is None:
            state = self.state
        for next_act in self.actions:
            print(next_act,dot_product(self.weights,
                self.get_features(state,next_act)))

def dot_product(l1,l2):
    return sum(e1*e2 for (e1,e2) in zip(l1,l2))
```

Test code:

rlFeatures.py — (continued)

```
from rlProblem import Simulate
from rlExamples import Monster_game_env # monster game environment
import rlMonsterGameFeatures

mon_env = Monster_game_env()
fa1 = SARSA_LFA_learner(mon_env.name, mon_env.actions, 0.9,
    rlMonsterGameFeatures.get_features)
# Simulate(fa1,mon_env).go(100000).plot()
fas1 = SARSA_LFA_learner(mon_env.name, mon_env.actions, 0.9,
    rlMonsterGameFeatures.simp_features, method="LFA (simp features)")
#Simulate(fas1,mon_env).go(100000).plot()
```

Exercise 13.7 How does the step-size affect performance? Try different step sizes (e.g., 0.1, 0.001, other sizes in between). Explain the behavior you observe. Which step size works best for this example. Explain what evidence you are basing your prediction on.

Exercise 13.8 Does having extra features always help? Does it sometime help? Does whether it helps depend on the step size? Give evidence for your claims.

Exercise 13.9 For each of the following first predict, then plot, then explain the behavior you observed:

(a) SARSA_LFA, Model-based learning (with 1 update per step) and Q-learning for 10,000 steps 20% exploring followed by 10,000 steps 100% exploiting

(b) SARSA_LFA, model-based learning and Q-learning for

   i) 100,000 steps 20% exploring followed by 100,000 steps 100% exploit

   ii) 10,000 steps 20% exploring followed by 190,000 steps 100% exploit

(c) Suppose your goal was to have the best accumulated reward after 200,000 steps. You are allowed to change the exploration rate at a fixed number of steps. For each of the methods, which is the best position to start exploiting more? Which method is better? What if you wanted to have the best reward after 10,000 or 1,000 steps?

Based on this evidence, explain when it is preferable to use SARSA_LFA, Model-based learner, or Q-learning.

Important: you need to run each algorithm more than once. Your explanation should include the variability as well as the typical behavior.

Exercise 13.10 In the call to `self.exploration_strategy`, what should the counts be? (The code above will fail for ucb, for example.) Think about the case where there are too many states. Suppose we are just learning for a neighborhood of a current state (e.g., a fixed number of steps away the from the current state); how could the algorithm be modifies to make sure it has at least explored the close neighborhood of the current state?

Chapter 14

Multiagent Systems

14.1 Minimax

Here we consider two-player zero-sum games. Here a player only wins when another player loses. This can be modeled as where there is a single utility which one agent (the maximizing agent) is trying minimize and the other agent (the minimizing agent) is trying to minimize.

14.1.1 Creating a two-player game

masProblem.py — A Multiagent Problem

```
11 from display import Displayable
12
13 class Node(Displayable):
14     """A node in a search tree. It has a
15     name a string
16     isMax is True if it is a maximizing node, otherwise it is minimizing
           node
17     children is the list of children
18     value is what it evaluates to if it is a leaf.
19     """
20     def __init__(self, name, isMax, value, children):
21         self.name = name
22         self.isMax = isMax
23         self.value = value
24         self.allchildren = children
25
26     def isLeaf(self):
27         """returns true of this is a leaf node"""
28         return self.allchildren is None
```

```
29
30      def children(self):
31          """returns the list of all children."""
32          return self.allchildren
33
34      def evaluate(self):
35          """returns the evaluation for this node if it is a leaf"""
36          return self.value
```

The following gives the tree from Figure 11.5 of the book. Note how 888 is used as a value here, but never appears in the trace.

masProblem.py — (continued)

```
38  fig10_5 = Node("a",True,None, [
39              Node("b",False,None, [
40                  Node("d",True,None, [
41                      Node("h",False,None, [
42                          Node("h1",True,7,None),
43                          Node("h2",True,9,None)]),
44                      Node("i",False,None, [
45                          Node("i1",True,6,None),
46                          Node("i2",True,888,None)])]),
47                  Node("e",True,None, [
48                      Node("j",False,None, [
49                          Node("j1",True,11,None),
50                          Node("j2",True,12,None)]),
51                      Node("k",False,None, [
52                          Node("k1",True,888,None),
53                          Node("k2",True,888,None)])])]),
54              Node("c",False,None, [
55                  Node("f",True,None, [
56                      Node("l",False,None, [
57                          Node("l1",True,5,None),
58                          Node("l2",True,888,None)]),
59                      Node("m",False,None, [
60                          Node("m1",True,4,None),
61                          Node("m2",True,888,None)])]),
62                  Node("g",True,None, [
63                      Node("n",False,None, [
64                          Node("n1",True,888,None),
65                          Node("n2",True,888,None)]),
66                      Node("o",False,None, [
67                          Node("o1",True,888,None),
68                          Node("o2",True,888,None)])])])])
```

The following is a representation of a **magic-sum game**, where players take turns picking a number in the range $[1, 9]$, and the first player to have 3 numbers that sum to 15 wins. Note that this is a syntactic variant of **tic-tac-toe** or **naughts and crosses**. To see this, consider the numbers on a **magic square** (Figure 14.1); 3 numbers that add to 15 correspond exactly to the winning positions

6	1	8
7	5	3
2	9	4

Figure 14.1: Magic Square

of tic-tac-toe played on the magic square.

Note that we do not remove symmetries. (What are the symmetries? How do the symmetries of tic-tac-toe translate here?)

masProblem.py — (continued)

```
70
71  class Magic_sum(Node):
72      def __init__(self, xmove=True, last_move=None,
73                   available=[1,2,3,4,5,6,7,8,9], x=[], o=[]):
74          """This is a node in the search for the magic-sum game.
75          xmove is True if the next move belongs to X.
76          last_move is the number selected in the last move
77          available is the list of numbers that are available to be chosen
78          x is the list of numbers already chosen by x
79          o is the list of numbers already chosen by o
80          """
81          self.isMax = self.xmove = xmove
82          self.last_move = last_move
83          self.available = available
84          self.x = x
85          self.o = o
86          self.allchildren = None #computed on demand
87          lm = str(last_move)
88          self.name = "start" if not last_move else "o="+lm if xmove else
                "x="+lm
89
90      def children(self):
91          if self.allchildren is None:
92              if self.xmove:
93                  self.allchildren = [
94                      Magic_sum(xmove = not self.xmove,
95                                last_move = sel,
96                                available = [e for e in self.available if e is
                                    not sel],
97                                x = self.x+[sel],
98                                o = self.o)
99                               for sel in self.available]
100             else:
101                 self.allchildren = [
102                     Magic_sum(xmove = not self.xmove,
103                               last_move = sel,
104                               available = [e for e in self.available if e is
                                    not sel],
```

```
                            x = self.x,
                            o = self.o+[sel])
                           for sel in self.available]
        return self.allchildren

    def isLeaf(self):
        """A leaf has no numbers available or is a win for one of the
            players.
        We only need to check for a win for o if it is currently x's turn,
        and only check for a win for x if it is o's turn (otherwise it would
        have been a win earlier).
        """
        return (self.available == [] or
                (sum_to_15(self.last_move,self.o)
                 if self.xmove
                 else sum_to_15(self.last_move,self.x)))

    def evaluate(self):
        if self.xmove and sum_to_15(self.last_move,self.o):
            return -1
        elif not self.xmove and sum_to_15(self.last_move,self.x):
            return 1
        else:
            return 0

def sum_to_15(last,selected):
    """is true if last, together with two other elements of selected sum to
        15.
    """
    return any(last+a+b == 15
               for a in selected if a != last
               for b in selected if b != last and b != a)
```

14.1.2 Minimax and α-β Pruning

This is a naive depth-first **minimax algorithm**:

masMiniMax.py — Minimax search with alpha-beta pruning

```
def minimax(node,depth):
    """returns the value of node, and a best path for the agents
    """
    if node.isLeaf():
        return node.evaluate(),None
    elif node.isMax:
        max_score = float("-inf")
        max_path = None
        for C in node.children():
            score,path = minimax(C,depth+1)
            if score > max_score:
                max_score = score
                max_path = C.name,path
```

```
24          return max_score,max_path
25      else:
26          min_score = float("inf")
27          min_path = None
28          for C in node.children():
29              score,path = minimax(C,depth+1)
30              if score < min_score:
31                  min_score = score
32                  min_path = C.name,path
33          return min_score,min_path
```

The following is a depth-first minimax with α-β **pruning**. It returns the value for a node as well as a best path for the agents.

masMiniMax.py — (continued)

```
35  def minimax_alpha_beta(node,alpha,beta,depth=0):
36      """node is a Node, alpha and beta are cutoffs, depth is the depth
37      returns value, path
38      where path is a sequence of nodes that results in the value
39      """
40      node.display(2," "*depth,"minimax_alpha_beta(",node.name,", ",alpha, ",
            ", beta,")")
41      best=None     # only used if it will be pruned
42      if node.isLeaf():
43          node.display(2," "*depth,"returning leaf value",node.evaluate())
44          return node.evaluate(),None
45      elif node.isMax:
46          for C in node.children():
47              score,path = minimax_alpha_beta(C,alpha,beta,depth+1)
48              if score >= beta:  # beta pruning
49                  node.display(2," "*depth,"pruned due to
                        beta=",beta,"C=",C.name)
50                  return score, None
51              if score > alpha:
52                  alpha = score
53                  best = C.name, path
54          node.display(2," "*depth,"returning max alpha",alpha,"best",best)
55          return alpha,best
56      else:
57          for C in node.children():
58              score,path = minimax_alpha_beta(C,alpha,beta,depth+1)
59              if score <= alpha:   # alpha pruning
60                  node.display(2," "*depth,"pruned due to
                        alpha=",alpha,"C=",C.name)
61                  return score, None
62              if score < beta:
63                  beta=score
64                  best = C.name,path
65          node.display(2," "*depth,"returning min beta",beta,"best=",best)
66          return beta,best
```

Testing:

masMiniMax.py — (continued)

```
from masProblem import fig10_5, Magic_sum, Node

# Node.max_display_level=2 # print detailed trace
# minimax_alpha_beta(fig10_5, -9999, 9999,0)
# minimax_alpha_beta(Magic_sum(), -9999, 9999,0)

#To see how much time alpha-beta pruning can save over minimax, uncomment the following:
## import timeit
## timeit.Timer("minimax(Magic_sum(),0)",setup="from __main__ import minimax, Magic_sum"
##             ).timeit(number=1)
## trace=False
## timeit.Timer("minimax_alpha_beta(Magic_sum(), -9999, 9999,0)",
##             setup="from __main__ import minimax_alpha_beta, Magic_sum"
##             ).timeit(number=1)
```

14.2 Multiagent Learning

The next code of for multiple agents that learn when interacting with other agents. The main difference with the learners from the last chapter is that the games take actions from all agents and provide a separate reward to each agent.

The following agent maintains a stochastic policy; it learns a distribution over actions for each state.

masLearn.py — Simulations of agents learning

```
from display import Displayable
import utilities # argmaxall for (element,value) pairs
import matplotlib.pyplot as plt
import random
from rlProblem import RL_agent


class StochasticPIAgent(RL_agent):
    """This agent maintains the Q-function for each state.
    Chooses the best action using empirical distribution over actions
    """
    def __init__(self, role, actions, discount=0,
                 alpha_fun=lambda k:10/(9+k), Qinit=1, pi_init=1,
                     method="Stochastic Q_learner"):
        """
        role is the role of the agent (e.g., in a game)
        actions is the set of actions the agent can do.
        discount is the discount factor (0 is appropriate if there is a single state)
        alpha_fun is a function that computes alpha from the number of visits
```

```
        Qinit is the initial q-values
        pi_init gives the prior counts (Dirichlet prior) for the policy
            (must be >0)
        method gives the method used to implement the role
        """
        #self.max_display_level = 3
        RL_agent.__init__(self, actions)
        self.role = role
        self.discount = discount
        self.alpha_fun = alpha_fun
        self.Qinit = Qinit
        self.pi_init = pi_init
        self.method = method
        self.Q = {}
        self.pi = {}
        self.visits = {}

    def initial_action(self, state):
        """ Returns the initial action; selected at random
        Initialize Data Structures
        """
        self.Q[state] = {act:self.Qinit for act in self.actions}
        self.pi[state] = {act:self.pi_init for act in self.actions}
        self.action = random.choice(self.actions)
        self.visits[state] = {act:0 for act in self.actions}
        self.state = state
        self.display(2, f"Initial State: {state} Action {self.action}")
        self.display(2,"s\ta\tr\ts'\tQ")
        return self.action

    def select_action(self, reward, next_state):
        """give reward and next state, select next action to be carried
            out"""
        if next_state not in self.visits: # next state not seen before
            self.Q[next_state] = {act:self.Qinit for act in self.actions}
            self.pi[next_state] = {act:self.pi_init for act in
              self.actions}
            self.visits[next_state] = {act:0 for act in self.actions}
        self.visits[self.state][self.action] +=1
        alpha = self.alpha_fun(self.visits[self.state][self.action])
        self.Q[self.state][self.action] += alpha*(
                          reward
                          + self.discount * max(self.Q[next_state].values())
                          - self.Q[self.state][self.action])
        a_best = utilities.argmaxd(self.Q[self.state])
        self.pi[self.state][a_best] +=1
        self.display(2,self.state, self.action, reward, next_state,
                    self.Q[self.state][self.action], sep='\t')
        self.state = next_state
        self.action = select_from_dist(self.pi[next_state])
```

```
77          self.display(3,f"Agent {self.role} doing {self.action} in state
                {self.state}")
78          return self.action
79
80
81  def normalize(dist):
82      """dict is a {value:number} dictionary, where the numbers are all
            non-negative
83      returns dict where the numbers sum to one
84      """
85      tot = sum(dist.values())
86      return {var:val/tot for (var,val) in dist.items()}
87
88  def select_from_dist(dist):
89      rand = random.random()
90      for (act,prob) in normalize(dist).items():
91          rand -= prob
92          if rand < 0:
93              return act
```

The agent can be tested on the reinforcement learning benchmarks from the previous chapter:

masLearn.py — (continued)

```
95  #### Testing on RL benchmarks #####
96  from rlProblem import Simulate
97  from rlExamples import Healthy_env, Monster_game_env
98  mon_env = Monster_game_env()
99  magspi =StochasticPIAgent(mon_env.name, mon_env.actions,0.9)
100 #Simulate(magspi,mon_env).go(100000).plot()
```

The simulation for a game passes the joint action from all agents to the environment, which returns a tuple of rewards – one for each agent – and the next state.

masLearn.py — (continued)

```
102 class SimulateGame(Displayable):
103     def __init__(self, game, agent_types):
104         #self.max_display_level = 3
105         self.game = game
106         self.agents = [agent_types[i](game.players[i], game.actions[i], 0)
                for i in range(game.num_agents)] # list of agents
107         self.action_dists = [{act:0 for act in game.actions[i]} for i in
                range(game.num_agents)]
108         self.action_history = []
109         self.state_history = []
110         self.reward_history = []
111         self.dist = {}
112         self.dist_history = []
113         self.actions = tuple(ag.initial_action(game.initial_state) for ag
                in self.agents)
```

```
        self.num_steps = 0

    def go(self, steps):
        for i in range(steps):
            self.num_steps += 1
            (self.rewards, state) = self.game.play(self.actions)
            self.display(3, f"In go rewards={self.rewards}, state={state}")
            self.reward_history.append(self.rewards)
            self.state_history.append(state)
            self.actions = tuple(agent.select_action(reward, state)
                                 for (agent,reward) in
                                     zip(self.agents,self.rewards))
            self.action_history.append(self.actions)
            for i in range(self.game.num_agents):
                self.action_dists[i][self.actions[i]] += 1
            self.dist_history.append([{a:i for (a,i) in elt.items()} for
                elt in self.action_dists]) # deep copy
        #print("Scores:", ' '.join(f"{self.agents[i].role} average
            reward={ag.total_score/self.num_steps}" for ag in self.agents))
        print("Distributions:", '
            '.join(str({a:self.dist_history[-1][i][a]/sum(self.dist_history[-1][i].values())
            for a in self.game.actions[i]})
                                          for  i in
                                              range(self.game.num_agents)))
        #return self.reward_history, self.action_history

    def action_dist(self,which_actions=[1,1]):
        """ which actions is [a0,a1]
        returns the empirical distribution of actions for agents,
           where ai specifies the index of the actions for agent i
        remove this???
        """
        return [sum(1 for a in sim.action_history
                       if
                           a[i]==gm.actions[i][which_actions[i]])/len(sim.action_history)
                    for i in range(2)]
```

The plotting shows how the empirical distributions of the first two agents changes as the learning continues.

masLearn.py — (continued)

```
    def plot_dynamics(self, x_action=0, y_action=0):
        plt.ion() # make it interactive
        agents = self.agents
        x_act = self.game.actions[0][x_action]
        y_act = self.game.actions[1][y_action]
        plt.xlabel(f"Probability {self.game.players[0]}
            {self.agents[0].actions[x_action]}")
        plt.ylabel(f"Probability {self.game.players[1]}
            {self.agents[1].actions[y_action]}")
```

```
151         plt.plot([self.dist_history[i][0][x_act]/sum(self.dist_history[i][0].values())
                for i in range(len(self.dist_history))],
152                 [self.dist_history[i][1][y_act]/sum(self.dist_history[i][1].values())
                        for i in range(len(self.dist_history))])
153         #plt.legend()
154         plt.savefig('soccerplot.pdf')
155         plt.show()
```

masLearn.py — (continued)

```
157
158 class ShoppingGame(Displayable):
159     def __init__(self):
160         self.num_agents = 2
161         self.states = ['s']
162         self.initial_state = 's'
163         self.actions = [['shopping', 'football']]*2
164         self.players = ['football-preferrer goes to', 'shopping-preferrer
                goes to']
165
166     def play(self, actions):
167         """Given (action1,action2) returns (resulting_state, (rewward1,
                reward2))
168         """
169         return ({('football', 'football'): (2, 1),
170                 ('football', 'shopping'): (0, 0),
171                 ('shopping', 'football'): (0, 0),
172                 ('shopping', 'shopping'): (1, 2)
173                     }[actions], 's')
174
175
176 class SoccerGame(Displayable):
177     def __init__(self):
178         self.num_agents = 2
179         self.states = ['s']
180         self.initial_state = 's'
181         self.initial_state = 's'
182         self.actions = [['right', 'left']]*2
183         self.players = ['goalkeeper', 'kicker']
184
185     def play(self, actions):
186         """Given (action1,action2) returns (resulting_state, (rewward1,
                reward2))
187         resulting state is 's'
188         """
189         return ({('left', 'left'): (0.6, 0.4),
190                 ('left', 'right'): (0.3, 0.7),
191                 ('right', 'left'): (0.2, 0.8),
192                 ('right', 'right'): (0.9,0.1)
193                }[actions], 's')
194
```

```
195 class GameShow(Displayable):
196     def __init__(self):
197         self.num_agents = 2
198         self.states = ['s']
199         self.initial_state = 's'
200         self.actions = [['takes', 'gives']]*2
201         self.players = ['Agent 1', 'Agent 2']
202 
203     def play(self, actions):
204         return ({('takes', 'takes'): (1, 1),
205                  ('takes', 'gives'): (11, 0),
206                  ('gives', 'takes'): (0, 11),
207                  ('gives', 'gives'): (10, 10)
208                 }[actions], 's')
209 
210 
211 class UniqueNEGameExample(Displayable):
212     def __init__(self):
213         self.num_agents = 2
214         self.states = ['s']
215         self.initial_state = 's'
216         self.actions = [['a1', 'b1', 'c1'],['d2', 'e2', 'f2']]
217         self.players = ['agent 1 does', 'agent 2 does']
218 
219     def play(self, actions):
220         return ({('a1', 'd2'): (3, 5),
221                  ('a1', 'e2'): (5, 1),
222                  ('a1', 'f2'): (1, 2),
223                  ('b1', 'd2'): (1, 1),
224                  ('b1', 'e2'): (2, 9),
225                  ('b1', 'f2'): (6, 4),
226                  ('c1', 'd2'): (2, 6),
227                  ('c1', 'e2'): (4, 7),
228                  ('c1', 'f2'): (0, 8)
229                     }[actions], 's')
```

masLearn.py — (continued)

```
232 # Choose one:
233 # gm = ShoppingGame()
234 # gm = SoccerGame()
235 # gm = GameShow()
236 # gm = UniqueNEGameExample()
237 
238 from rlQLearner import Q_learner
239 from rlProblem import RL_agent
240 # Choose one of the combinations of learners:
241 # sim=SimulateGame(gm,[StochasticPIAgent, StochasticPIAgent]);
        sim.go(10000)
242 # sim= SimulateGame(gm,[Q_learner, Q_learner]); sim.go(10000)
243 # sim=SimulateGame(gm,[Q_learner, StochasticPIAgent]); sim.go(10000)
```

```
244
245
246 # sim.plot_dynamics()
247
248 # empirical proportion that agents did their action at index 1:
249 # sim.action_dist([1,1])
250
251 # (unnormalized) empirical distribution for agent 0
252 # sim.agents[0].dist
```

Exercise 14.1 Try the game show game (prisoner's dilemma) with two `StochasticPIAgent` agents and `alpha_fun=lambda k:0.1`. Try also 0.01. Why does this work qualitatively different? Is this better?

Chapter 15

Relational Learning

15.1 Collaborative Filtering

Based on gradient descent algorithm of Koren, Y., Bell, R. and Volinsky, C., Matrix Factorization Techniques for Recommender Systems, IEEE Computer 2009.

This assumes the form of the dataset from movielens (`http://grouplens.org/datasets/movielens/`). The rating are a set of $(user, item, rating, timestamp)$ tuples.

relnCollFilt.py — Latent Property-based Collaborative Filtering

```
11 import random
12 import matplotlib.pyplot as plt
13 import urllib.request
14 from learnProblem import Learner
15 from display import Displayable
16
17 class CF_learner(Learner):
18     def __init__(self,
19                  rating_set,         # a Rating_set object
20                  rating_subset = None, # subset of ratings to be used as
                         training ratings
21                  test_subset = None, # subset of ratings to be used as test
                         ratings
22                  step_size = 0.01,   # gradient descent step size
23                  reglz = 1.0,        # the weight for the regularization
                         terms
24                  num_properties = 10,  # number of hidden properties
25                  property_range = 0.02 # properties are initialized to be
                         between
26                                      # -property_range and property_range
```

```
27              ):
28          self.rating_set = rating_set
29          self.ratings = rating_subset or rating_set.training_ratings #
                whichever is not empty
30          if test_subset is None:
31              self.test_ratings = self.rating_set.test_ratings
32          else:
33              self.test_ratings = test_subset
34          self.step_size = step_size
35          self.reglz = reglz
36          self.num_properties = num_properties
37          self.num_ratings = len(self.ratings)
38          self.ave_rating = (sum(r for (u,i,r,t) in self.ratings)
39                             /self.num_ratings)
40          self.users = {u for (u,i,r,t) in self.ratings}
41          self.items = {i for (u,i,r,t) in self.ratings}
42          self.user_bias = {u:0 for u in self.users}
43          self.item_bias = {i:0 for i in self.items}
44          self.user_prop = {u:[random.uniform(-property_range,property_range)
45                               for p in range(num_properties)]
46                             for u in self.users}
47          self.item_prop = {i:[random.uniform(-property_range,property_range)
48                               for p in range(num_properties)]
49                             for i in self.items}
50          self.zeros = [0 for p in range(num_properties)]
51          self.iter=0
52
53      def stats(self):
54          self.display(1,"ave sumsq error of mean for training=",
55                       sum((self.ave_rating-rating)**2 for
56                           (user,item,rating,timestamp)
57                           in self.ratings)/len(self.ratings))
58          self.display(1,"ave sumsq error of mean for test=",
59                       sum((self.ave_rating-rating)**2 for
60                           (user,item,rating,timestamp)
61                           in self.test_ratings)/len(self.test_ratings))
62          self.display(1,"error on training set",
63                       self.evaluate(self.ratings))
64          self.display(1,"error on test set",
65                       self.evaluate(self.test_ratings))
```

learn carries out *num_iter* steps of gradient descent.

relnCollFilt.py — (continued)

```
66      def prediction(self,user,item):
67          """Returns prediction for this user on this item.
68          The use of .get() is to handle users or items not in the training
                set.
69          """
70          return (self.ave_rating
71                  +  self.user_bias.get(user,0) #self.user_bias[user]
```

```
71              + self.item_bias.get(item,0) #self.item_bias[item]
72              +
                    sum([self.user_prop.get(user,self.zeros)[p]*self.item_prop.get(item,self.zeros)
73                     for p in range(self.num_properties)]))
74
75      def learn(self, num_iter = 50):
76          """ do num_iter iterations of gradient descent."""
77          for i in range(num_iter):
78              self.iter += 1
79              abs_error=0
80              sumsq_error=0
81              for (user,item,rating,timestamp) in
                    random.sample(self.ratings,len(self.ratings)):
82                  error = self.prediction(user,item) - rating
83                  abs_error += abs(error)
84                  sumsq_error += error * error
85                  self.user_bias[user] -= self.step_size*error
86                  self.item_bias[item] -= self.step_size*error
87                  for p in range(self.num_properties):
88                      self.user_prop[user][p] -=
                            self.step_size*error*self.item_prop[item][p]
89                      self.item_prop[item][p] -=
                            self.step_size*error*self.user_prop[user][p]
90              for user in self.users:
91                  self.user_bias[user] -= self.step_size*self.reglz*
                        self.user_bias[user]
92                  for p in range(self.num_properties):
93                      self.user_prop[user][p] -=
                            self.step_size*self.reglz*self.user_prop[user][p]
94              for item in self.items:
95                  self.item_bias[item] -=
                        self.step_size*self.reglz*self.item_bias[item]
96                  for p in range(self.num_properties):
97                      self.item_prop[item][p] -=
                            self.step_size*self.reglz*self.item_prop[item][p]
98              self.display(1,"Iteration",self.iter,
99                   "(Ave Abs,AveSumSq) training
                         =",self.evaluate(self.ratings),
100                  "test =",self.evaluate(self.test_ratings))
```

evaluate evaluates current predictions on the rating set:

reInCollFilt.py — (continued)

```
102     def evaluate(self,ratings):
103         """returns (avergage_absolute_error, average_sum_squares_error) for
                ratings
104         """
105         abs_error = 0
106         sumsq_error = 0
107         if not ratings: return (0,0)
108         for (user,item,rating,timestamp) in ratings:
```

```
109             error = self.prediction(user,item) - rating
110             abs_error += abs(error)
111             sumsq_error += error * error
112         return abs_error/len(ratings), sumsq_error/len(ratings)
```

Exercise 15.1 The above code updates the parameters after each example, but only regularizes after the whole batch. Change the program so that it implements stochastic gradient descent with a given batch size, and only updates the parameters after a batch.

Exercise 15.2 In the previous questions, can the regularization avoid iterating through the parameters for all users and items after a batch? Consider items that are in many batches versus those in a few or even no batches. (Warning: This is challenging to get right.)

15.1.1 Plotting

relnCollFilt.py — (continued)

```
114     def plot_predictions(self, examples="test"):
115         """
116         examples is either "test" or "training" or the actual examples
117         """
118         if examples == "test":
119             examples = self.test_ratings
120         elif examples == "training":
121             examples = self.ratings
122         plt.ion()
123         plt.xlabel("prediction")
124         plt.ylabel("cumulative proportion")
125         self.actuals = [[] for r in range(0,6)]
126         for (user,item,rating,timestamp) in examples:
127             self.actuals[rating].append(self.prediction(user,item))
128         for rating in range(1,6):
129             self.actuals[rating].sort()
130             numrat=len(self.actuals[rating])
131             yvals = [i/numrat for i in range(numrat)]
132             plt.plot(self.actuals[rating], yvals,
                         label="rating="+str(rating))
133         plt.legend()
134         plt.draw()
```

This plots a single property. Each $(user, item, rating)$ is plotted where the x-value is the value of the property for the user, the y-value is the value of the property for the item, and the rating is plotted at this (x, y) position. That is, *rating* is plotted at the (x, y) position $(p(user), p(item))$.

relnCollFilt.py — (continued)

```
136     def plot_property(self,
137                       p,              # property
138                       plot_all=False, # true if all points should be plotted
```

```
                    num_points=200 # number of random points plotted if not
                        all
                    ):
        """plot some of the user-movie ratings,
        if plot_all is true
        num_points is the number of points selected at random plotted.

        the plot has the users on the x-axis sorted by their value on
            property p and
        with the items on the y-axis sorted by their value on property p and
        the ratings plotted at the corresponding x-y position.
        """
        plt.ion()
        plt.xlabel("users")
        plt.ylabel("items")
        user_vals = [self.user_prop[u][p]
                    for u in self.users]
        item_vals = [self.item_prop[i][p]
                    for i in self.items]
        plt.axis([min(user_vals)-0.02,
                  max(user_vals)+0.05,
                  min(item_vals)-0.02,
                  max(item_vals)+0.05])
        if plot_all:
            for (u,i,r,t) in self.ratings:
                plt.text(self.user_prop[u][p],
                         self.item_prop[i][p],
                         str(r))
        else:
            for i in range(num_points):
                (u,i,r,t) = random.choice(self.ratings)
                plt.text(self.user_prop[u][p],
                         self.item_prop[i][p],
                         str(r))
        plt.show()
```

15.1.2 Creating Rating Sets

A rating set can be read from the Internet or read from a local file. The default is to read the Movielens 100K dataset from the Internet. It would be more efficient to save the dataset as a local file, and then set *local_file = True*, as then it will not need to download the dataset every time the program is run.

relnCollFilt.py — (continued)

```
class Rating_set(Displayable):
    def __init__(self,
                 date_split=892000000,
                 local_file=False,
                 url="http://files.grouplens.org/datasets/movielens/ml-100k/u.data",
                 file_name="u.data"):
        self.display(1,"reading...")
```

```
180         if local_file:
181             lines = open(file_name,'r')
182         else:
183             lines = (line.decode('utf-8') for line in
                    urllib.request.urlopen(url))
184         all_ratings = (tuple(int(e) for e in line.strip().split('\t'))
185                        for line in lines)
186         self.training_ratings = []
187         self.training_stats = {1:0, 2:0, 3:0, 4:0 ,5:0}
188         self.test_ratings = []
189         self.test_stats = {1:0, 2:0, 3:0, 4:0 ,5:0}
190         for rate in all_ratings:
191             if rate[3] < date_split: # rate[3] is timestamp
192                 self.training_ratings.append(rate)
193                 self.training_stats[rate[2]] += 1
194             else:
195                 self.test_ratings.append(rate)
196                 self.test_stats[rate[2]] += 1
197         self.display(1,"...read:", len(self.training_ratings),"training
                ratings and",
198             len(self.test_ratings),"test ratings")
199         tr_users = {user for (user,item,rating,timestamp) in
                self.training_ratings}
200         test_users = {user for (user,item,rating,timestamp) in
                self.test_ratings}
201         self.display(1,"users:",len(tr_users),"training,",len(test_users),"test,",
202                      len(tr_users & test_users),"in common")
203         tr_items = {item for (user,item,rating,timestamp) in
                self.training_ratings}
204         test_items = {item for (user,item,rating,timestamp) in
                self.test_ratings}
205         self.display(1,"items:",len(tr_items),"training,",len(test_items),"test,",
206                      len(tr_items & test_items),"in common")
207         self.display(1,"Rating statistics for training set:
                ",self.training_stats)
208         self.display(1,"Rating statistics for test set: ",self.test_stats)
```

Sometimes it is useful to plot a property for all $(user, item, rating)$ triples. There are too many such triples in the data set. The method *create_top_subset* creates a much smaller dataset where this makes sense. It picks the most rated items, then picks the users who have the most ratings on these items. It is designed for depicting the meaning of properties, and may not be useful for other purposes.

relnCollFilt.py — (continued)

```
210     def create_top_subset(self, num_items = 30, num_users = 30):
211         """Returns a subset of the ratings by picking the most rated items,
212         and then the users that have most ratings on these, and then all of
                the
213         ratings that involve these users and items.
214         """
```

```
        items = {item for (user,item,rating,timestamp) in
            self.training_ratings}

        item_counts = {i:0 for i in items}
        for (user,item,rating,timestamp) in self.training_ratings:
            item_counts[item] += 1

        items_sorted = sorted((item_counts[i],i) for i in items)
        top_items = items_sorted[-num_items:]
        set_top_items = set(item for (count, item) in top_items)

        users = {user for (user,item,rating,timestamp) in
            self.training_ratings}
        user_counts = {u:0 for u in users}
        for (user,item,rating,timestamp) in self.training_ratings:
            if item in set_top_items:
                user_counts[user] += 1

        users_sorted = sorted((user_counts[u],u)
                              for u in users)
        top_users = users_sorted[-num_users:]
        set_top_users = set(user for (count, user) in top_users)
        used_ratings = [ (user,item,rating,timestamp)
                        for (user,item,rating,timestamp) in
                            self.training_ratings
                        if user in set_top_users and item in set_top_items]
        return used_ratings

movielens = Rating_set()
learner1 = CF_learner(movielens, num_properties = 1)
#learner1.learn(50)
# learner1.plot_predictions(examples = "training")
# learner1.plot_predictions(examples = "test")
#learner1.plot_property(0)
#movielens_subset = movielens.create_top_subset(num_items = 20, num_users
    = 20)
#learner_s = CF_learner(movielens, rating_subset=movielens_subset,
    test_subset=[], num_properties=1)
#learner_s.learn(1000)
#learner_s.plot_property(0,plot_all=True)
```

15.2 Relational Probabilistic Models

The following implements relational belief networks – belief networks with plates. Plates correspond to logical variables.

relnProbModels.py — Relational Probabilistic Models: belief networks with plates

```
from display import Displayable
from probGraphicalModels import BeliefNetwork
```

```
from variable import Variable
from probRC import ProbRC
from probFactors import Prob
import random

boolean = [False, True]
```

A ParVar is a parametrized random variable, which consists of the name, a list of logical variables (plates), a domain, and a position. For each assignment of an entity to each logical variable, there is a random variable in a grounding.

relnProbModels.py — (continued)

```
class ParVar(object):
    """Parametrized random variable"""
    def __init__(self, name, log_vars, domain, position=None):
        self.name = name # string
        self.log_vars = log_vars
        self.domain = domain # list of values
        self.position = position if position else (random.random(),
            random.random())
        self.size = len(domain)
```

The class RBN is of relational belief networks.

relnProbModels.py — (continued)

```
class RBN(Displayable):
    def __init__(self, title, parvars, parfactors):
        self.title = title
        self.parvars = parvars
        self.parfactors = parfactors
        self.log_vars = {V for PV in parvars for V in PV.log_vars}

    def ground(self, populations):
        """Ground the belief network with the populations of the logical
            variables.
        populations is a dictionary that maps each logical variable to the
            list of individuals.
        Returns a belief network representation of the grounding.
        """
        assert all(lv in populations for lv in self.log_vars)
        self.cps = []   # conditional probabilities in the grounding
        self.var_dict = {}  # ground variables created
        for pp in self.parfactors:
            self.ground_parfactor(pp, list(self.log_vars), populations, {})
        return BeliefNetwork(self.title+"_grounded",
            self.var_dict.values(), self.cps)

    def ground_parfactor(self, parfactor, lvs, populations, context):
        """
        parfactor is the parfactor to get instances of
        lvs is a list of the logical variables in parfactor not assigned in
            context
```

```
        populations is {logical_variable: population} dictionary
        context is a {logical_variable:value} dictionary for
            logical_variable in parfactor
        """
        if lvs == []:
            if isinstance(parfactor, Prob):
                self.cps.append(Prob(self.ground_pvr(parfactor.child,context),
                                     [self.ground_pvr(p,context) for p in
                                         parfactor.parents],
                                     parfactor.values))
            else:
                print("Parfactor not implemented for",parfactor,"of
                    type",type(parfactor))
        else:
            for val in populations[lvs[0]]:
                self.ground_parfactor(parfactor, lvs[1:], populations,
                    {lvs[0]:val}|context)


    def ground_pvr(self, prv, context):
        """grounds a parametrized random variable with respect to a context
        prv is a parametrized random variable
        context is a logical_variable:value dictionary that assigns all
            logical variables in prv
        """
        if isinstance(prv,ParVar):
            args = tuple(context[lv] for lv in prv.log_vars)
            if (prv,args) in self.var_dict:
                return self.var_dict[(prv,args)]
            else:
                new_gv = GrVar(prv, args)
                self.var_dict[(prv,args)] = new_gv
                return new_gv
        else: # allows for non-parametrized random variables
            return prv
```

A `GrVar` is a variable constructed by grounding a parametrized random variable with respect to a tuple of values for the logical variables.

relnProbModels.py — (continued)

```
class GrVar(Variable):
    """Grounded Variable"""
    def __init__(self,parvar,args):
        (x,y) = parvar.position
        pos = (x + random.uniform(-0.2,0.2), y + random.uniform(-0.2,0.2))
        Variable.__init__(self,parvar.name+"("+",".join(args)+")",
            parvar.domain, pos)
        self.parvar= parvar
        self.args = tuple(args)
        self.hash_value = None

```

```
93      def __hash__(self):
94          if self.hash_value is None:
95              self.hash_value = hash((self.parvar, self.args))
96          return self.hash_value
97
98      def __eq__(self, other):
99          return isinstance(other,GrVar) and self.parvar == other.parvar and
                self.args == other.args
```

The following is a representation of Examples 17.5-17.7 of Poole and Mackworth [2023]. The plate model – represented here using `grades` – is shown in Figure 17.4. The observation in `obs` corresponds to the dataset of Figure 17.3. The grounding in `grades_gr` corresponds to Figure 17.5, but also includes the Grade variables no needed to answer the query (see exercise below).

Try the commented out queries to the Python shell:

relnProbModels.py — (continued)

```
101 Int = ParVar("Intelligent", ["St"], boolean, position=(0.25,0.75))
102 Grade = ParVar("Grade", ["St","Co"], ["A", "B", "C"], position=(0.5,0.25))
103 Diff = ParVar("Difficult", ["Co"], boolean, position=(0.75,0.75))
104
105 pg = Prob(Grade, [Int, Diff],
106                [[{"A": 0.1, "B":0.4, "C":0.5},
107                      {"A": 0.01, "B":0.09, "C":0.9}],
108                 [{"A": 0.9, "B":0.09, "C":0.01},
109                       {"A": 0.5, "B":0.4, "C":0.1}]])
110 pi = Prob( Int, [], [0.5, 0.5])
111 pd = Prob( Diff, [], [0.5, 0.5])
112 grades = RBN("Grades RBN", {Int, Grade, Diff}, {pg,pi,pd})
113
114
115 #grades_gr = grades.ground({"St":["s1", "s2", "s3", "s4"], "Co":["c1",
        "c2", "c3", "c4"]})
116
117 obs = {GrVar(Grade,["s1","c1"]):"A", GrVar(Grade,["s2","c1"]):"C",
        GrVar(Grade,["s1","c2"]):"B",
118           GrVar(Grade,["s2","c3"]):"B", GrVar(Grade,["s3","c2"]):"B",
                GrVar(Grade,["s4","c3"]):"B"}
119
120 # grades_rc = ProbRC(grades_gr)
121 # grades_rc.query(GrVar(Grade,["s3","c4"]), obs)
122 # grades_rc.query(GrVar(Grade,["s4","c4"]), obs)
123 # grades_rc.query(GrVar(Int,["s3"]), obs)
124 # grades_rc.query(GrVar(Int,["s4"]), obs)
```

Exercise 15.3 The grounding above creates a random variable for each element for each possible combination of individuals in the populations. Change it so that it only creates as many random variables as needed to answer a query. For example, for the observations and queries above, only the variables in Figure 17.5 need to be created.

Exercise 15.4 Displaying the ground network, e.g., using `grades_gr.show()`, creates a messy diagram. Make it so that the user can provide offsets for each individual and uses the position of the prv plus the offsets of all the individuals involved. Use this create to create a 2D grid of grades in the example above.

Chapter 16

Version History

- 2023-07-31 Version 0.9.7 includes relational probabilistic models and smaller changes
- 2023-06-06 Version 0.9.6 controllers are more consistent. Many smaller changes.
- 2022-08-13 Version 0.9.5 major revisions including extra code for causality and deep learning
- 2021-07-08 Version 0.9.1 updated the CSP code to have the same representation of variables as used by the probability code
- 2021-05-13 Version 0.9.0 Major revisions to chapters 8 and 9. Introduced recursive conditioning, simplified much code. New section on multiagent reinforcement learning.
- 2020-11-04 Version 0.8.6 simplified value iteration for MDPs.
- 2020-10-20 Version 0.8.4 planning simplified, and gives error if goal not part of state (by design). Fixed arc costs.
- 2020-07-21 Version 0.8.2 added positions and string to constraints
- 2019-09-17 Version 0.8.0 rerepresented blocks world (Section 6.1.2) due to bug found by Donato Meoli.

Bibliography

Chen, T. and Guestrin, C. (2016), Xgboost: A scalable tree boosting system. In *KDD '16: 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pages 785–794, URL https://doi.org/10.1145/2939672.2939785. 166

Chollet, F. (2021), *Deeep Learning with Python*. Manning. 169

Dua, D. and Graff, C. (2017), UCI machine learning repository. URL http://archive.ics.uci.edu/ml. 131

Glorot, X. and Bengio, Y. (2010), Understanding the difficulty of training deep feedforward neural networks. In *Thirteenth International Conference on Artificial Intelligence and Statistics*, pages 249–256, URL https://proceedings.mlr.press/v9/glorot10a.html. 170

Ke, G., Meng, Q., Finley, T., Wang, T., Chen, W., Ma, W., Ye, Q., and Liu, T.-Y. (2017), LightGBM: A highly efficient gradient boosting decision tree. In *Advances in Neural Information Processing Systems 30*. 166

Lichman, M. (2013), UCI machine learning repository. URL http://archive.ics.uci.edu/ml. 131

Pérez, F. and Granger, B. E. (2007), IPython: a system for interactive scientific computing. *Computing in Science and Engineering*, 9(3):21–29, URL https://ipython.org. 10

Poole, D. L. and Mackworth, A. K. (2017), *Artificial Intelligence: foundations of computational agents*. Cambridge University Press, 2nd edition, URL https://artint.info. 191

Poole, D. L. and Mackworth, A. K. (2023), *Artificial Intelligence: foundations of computational agents*. Cambridge University Press, 3rd edition, URL `https://artint.info`. 9, 23, 25, 43, 44, 60, 105, 197, 198, 276, 277, 282, 286, 316

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