

# Intro to modern neural networks

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COMP 4630 | Winter 2024

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# Overview

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- More decisions when making a neural network
  - Weight initialization
  - Number of neurons and layers
  - Optimization algorithms
- References and suggested reading:
  - [Scikit-learn book](#): Chapters 10-11
  - [Deep Learning Book](#): Chapter 8
  - [Understanding Deep Learning](#): Chapter 7

# Revisiting Backpropagation

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- For a network with  $l$  layers, the gradients of the loss function with respect to the weights in the last layer are given by:

$$\frac{\partial L}{\partial W^{(l)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial f^{(l)}} \frac{\partial f^{(l)}}{\partial z^{(l)}} \frac{\partial z^{(l)}}{\partial W^{(l)}}$$

assuming that the output  $\hat{y} = f^{(l)}(z^{(l)})$  is a function of layer  $l$ 's input  $z^{(l)} = W^{(l)} f^{(l-1)}(z^{(l-1)}) + b^{(l)}$ .

- At layer  $l - 1$ , the gradients are computed as:

$$\frac{\partial L}{\partial W^{(l-1)}} = \left( \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial f^{(l)}} \frac{\partial f^{(l)}}{\partial z^{(l)}} \right) \frac{\partial z^{(l)}}{\partial f^{(l-1)}} \frac{\partial f^{(l-1)}}{\partial z^{(l-1)}} \frac{\partial z^{(l-1)}}{\partial W^{(l-1)}}$$

- At layer  $l - 2$ , this becomes:

$$\frac{\partial L}{\partial W^{(l-2)}} = \left( \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial f^{(l)}} \frac{\partial f^{(l)}}{\partial z^{(l)}} \frac{\partial z^{(l)}}{\partial f^{(l-1)}} \frac{\partial f^{(l-1)}}{\partial z^{(l-1)}} \right) \frac{\partial z^{(l-1)}}{\partial f^{(l-2)}} \frac{\partial f^{(l-2)}}{\partial z^{(l-2)}} \frac{\partial z^{(l-2)}}{\partial W^{(l-2)}}$$

- And so on, until we reach the first layer.
- We are **recursively** applying the chain rule and re-using the gradients computed at the previous layer
- This is great for computational efficiency, but it can also lead to **vanishing** or **exploding gradients**


# Vanishing and Exploding Gradients

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- **Vanishing/exploding gradients** are where the gradients become near zero or near infinity as they are propagated back through the network
- Particularly problematic for **recurrent** neural networks, where the same weights are multiplied by themselves repeatedly
- Also a problem for very deep networks, and part of the reason that deep learning was not popular until the 2010s
- **?** What changed?

# Consider the variance

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- At the input layer,  $Z^{(0)} = W^{(0)}X + b^{(0)}$ , and  $X$  has some variance  $\sigma^2$
- Assume  $W^{(0)}$  and  $b^{(0)}$  are initialized to 0
-  What is the variance of  $Z^{(0)}$ ?
- What about after the activation function  $f^{(0)}(Z^{(0)})$ ?

# Initialization strategies

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- In 2010, [Glorot and Bengio](#) proposed the **Xavier** initialization for a layer with  $m$  inputs and  $n$  outputs:

$$W_{i,j} \sim U \left( -\sqrt{\frac{6}{m+n}}, \sqrt{\frac{6}{m+n}} \right)$$

- Goal is to preserve the variance of the input and output in both directions
- Similar to LeCun initialization, and apparently an overlooked feature of networks from the 1990s

# Initialization for ReLU

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- Glorot initialization was derived under the assumption of **linear** activation functions (even though they knew this wasn't the case)
- In 2015, [He et al.](#) proposed the **He** initialization specifically for ReLU activations:

$$W_{i,j} \sim N \left( 0, \sqrt{\frac{2}{m}} \right)$$

- This can be defined in Keras as `kernel_initializer='he_normal'` or `kernel_initializer='he_uniform'`
- The choice of normal vs uniform is apparently not very important



# Batch normalization

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- Also in 2015, [Ioffe and Szegedy](#) proposed **batch normalization** as a way to mitigate vanishing/exploding gradients
- This is simply a normalization **at each layer**, shifting and scaling the inputs to have a mean of 0 and a variance of 1 (across the batch)
- A **moving average** of the mean and variance is maintained **during training**, and used for normalization during inference
- It also ends up acting as **regularization**, magic!
- **?** Why wouldn't you want to use batch normalization?

# RELU and its variants

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- In early works, the sigmoid or tanh functions were popular
- Both have a small range of non-zero gradients
- ReLU has a stable gradient for positive inputs, but can lead to the **dying ReLU** problem whereby certain neurons are "turned off"
- **?** How can we prevent dying ReLUs?

*Note: this may not be a problem, and ReLU is cheap. Don't optimize prematurely unless you're seeing lots of "dead" neurons.*

# Number of neurons and layers

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- Number of neurons in the **input layer** is defined by number of features
- Number of neurons in the **output layer** is defined by prediction task
- In between is a **design choice**
- Common early choice was a pyramid shape, but it turns out that a stack of layers with the same number of neurons works well too
- Deeper networks can solve more complex problems with the same number of total parameters, but are also prone to vanishing/exploding gradients
- Ultimately a hyperparameter to be tuned

# Optimization algorithms: variations on gradient descent

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- Gradient descent takes small regular steps, constant or otherwise
- Many variations exist! For example, **momentum** keeps track of the previously computed gradient and uses it to inform the new step:

$$\begin{aligned}\mathbf{m} &= \beta\mathbf{m} - \eta\nabla_{\mathbf{W}}J(\mathbf{W}) \\ \mathbf{W} &= \mathbf{W} + \mathbf{m}\end{aligned}$$

where  $\beta$  is a hyperparameter between 0 and 1

- Adaptive moment estimation (**Adam**) is a popular choice that adds on an exponentially decaying average of the squared gradients

# Implementation time

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