常 用 积 分 公 式

（一）含有 $ax+b$ 的积分 $(a \neq 0)$

1. $\int \dfrac{\mathrm{d}x}{ax+b} = \dfrac{1}{a}\ln|ax+b| + C$

2. $\int (ax+b)^{\mu}\mathrm{d}x = \dfrac{1}{a(\mu+1)}(ax+b)^{\mu+1} + C$ （$\mu \neq -1$）

3. $\int \dfrac{x}{ax+b}\mathrm{d}x = \dfrac{1}{a^2}(ax+b-b\ln|ax+b|) + C$

4. $\int \dfrac{x^2}{ax+b}\mathrm{d}x = \dfrac{1}{a^3}\left[\dfrac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2\ln|ax+b|\right] + C$

5. $\int \dfrac{\mathrm{d}x}{x(ax+b)} = -\dfrac{1}{b}\ln\left|\dfrac{ax+b}{x}\right| + C$

6. $\int \dfrac{\mathrm{d}x}{x^2(ax+b)} = -\dfrac{1}{bx} + \dfrac{a}{b^2}\ln\left|\dfrac{ax+b}{x}\right| + C$

7. $\int \dfrac{x}{(ax+b)^2}\mathrm{d}x = \dfrac{1}{a^2}(\ln|ax+b| + \dfrac{b}{ax+b}) + C$

8. $\int \dfrac{x^2}{(ax+b)^2}\mathrm{d}x = \dfrac{1}{a^3}(ax+b - 2b\ln|ax+b| - \dfrac{b^2}{ax+b}) + C$

9. $\int \dfrac{\mathrm{d}x}{x(ax+b)^2} = \dfrac{1}{b(ax+b)} - \dfrac{1}{b^2}\ln\left|\dfrac{ax+b}{x}\right| + C$

（二）含有 $\sqrt{ax+b}$ 的积分

10. $\int \sqrt{ax+b}\,\mathrm{d}x = \dfrac{2}{3a}\sqrt{(ax+b)^3} + C$

11. $\int x\sqrt{ax+b}\,\mathrm{d}x = \dfrac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C$

12. $\int x^2\sqrt{ax+b}\,\mathrm{d}x = \dfrac{2}{105a^3}(15a^2x^2 - 12abx + 8b^2)\sqrt{(ax+b)^3} + C$

13. $\int \dfrac{x}{\sqrt{ax+b}}\mathrm{d}x = \dfrac{2}{3a^2}(ax-2b)\sqrt{ax+b} + C$

14. $\displaystyle\int \frac{x^2}{\sqrt{ax+b}}\mathrm{d}x = \frac{2}{15a^3}(3a^2x^2 - 4abx + 8b^2)\sqrt{ax+b} + C$

15. $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{b}}\ln\left|\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C & (b>0) \\[4mm] \dfrac{2}{\sqrt{-b}}\arctan\sqrt{\dfrac{ax+b}{-b}} + C & (b<0) \end{cases}$

16. $\displaystyle\int \frac{\mathrm{d}x}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}$

17. $\displaystyle\int \frac{\sqrt{ax+b}}{x}\mathrm{d}x = 2\sqrt{ax+b} + b\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}$

18. $\displaystyle\int \frac{\sqrt{ax+b}}{x^2}\mathrm{d}x = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{\mathrm{d}x}{x\sqrt{ax+b}}$

（三）含有 $x^2 \pm a^2$ 的积分

19. $\displaystyle\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

20. $\displaystyle\int \frac{\mathrm{d}x}{(x^2+a^2)^n} = \frac{x}{2(n-1)a^2(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2}\int \frac{\mathrm{d}x}{(x^2+a^2)^{n-1}}$

21. $\displaystyle\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

（四）含有 $ax^2 + b(a>0)$ 的积分

22. $\displaystyle\int \frac{\mathrm{d}x}{ax^2+b} = \begin{cases} \dfrac{1}{\sqrt{ab}}\arctan\sqrt{\dfrac{a}{b}}\,x + C & (b>0) \\[4mm] \dfrac{1}{2\sqrt{-ab}}\ln\left|\dfrac{\sqrt{a}\,x-\sqrt{-b}}{\sqrt{a}\,x+\sqrt{-b}}\right| + C & (b<0) \end{cases}$

23. $\displaystyle\int \frac{x}{ax^2+b}\mathrm{d}x = \frac{1}{2a}\ln\left|ax^2+b\right| + C$

24. $\displaystyle\int \frac{x^2}{ax^2+b}\mathrm{d}x = \frac{x}{a} - \frac{b}{a}\int \frac{\mathrm{d}x}{ax^2+b}$

25. $\displaystyle\int \frac{\mathrm{d}x}{x(ax^2+b)} = \frac{1}{2b}\ln \frac{x^2}{\left|ax^2+b\right|} + C$

26. $\displaystyle\int \frac{\mathrm{d}x}{x^2(ax^2+b)} = -\frac{1}{bx} - \frac{a}{b}\int \frac{\mathrm{d}x}{ax^2+b}$

27. $\displaystyle\int \frac{\mathrm{d}x}{x^3(ax^2+b)} = \frac{a}{2b^2}\ln \frac{\left|ax^2+b\right|}{x^2} - \frac{1}{2bx^2} + C$

28. $\displaystyle\int \frac{\mathrm{d}x}{(ax^2+b)^2} = \frac{x}{2b(ax^2+b)} + \frac{1}{2b}\int \frac{\mathrm{d}x}{ax^2+b}$

（五）含有 $ax^2+bx+c\ (a>0)$ 的积分

29. $\displaystyle\int \frac{\mathrm{d}x}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\arctan \dfrac{2ax+b}{\sqrt{4ac-b^2}} + C & (b^2<4ac) \\[4mm] \dfrac{1}{\sqrt{b^2-4ac}}\ln \left|\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + C & (b^2>4ac) \end{cases}$

30. $\displaystyle\int \frac{x}{ax^2+bx+c}\mathrm{d}x = \frac{1}{2a}\ln\left|ax^2+bx+c\right| - \frac{b}{2a}\int \frac{\mathrm{d}x}{ax^2+bx+c}$

（六）含有 $\sqrt{x^2+a^2}\ (a>0)$ 的积分

31. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \operatorname{arsh}\frac{x}{a} + C_1 = \ln(x+\sqrt{x^2+a^2}) + C$

32. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$

33. $\displaystyle\int \frac{x}{\sqrt{x^2+a^2}}\mathrm{d}x = \sqrt{x^2+a^2} + C$

34. $\displaystyle\int \frac{x}{\sqrt{(x^2+a^2)^3}}\mathrm{d}x = -\frac{1}{\sqrt{x^2+a^2}} + C$

35. $\int \dfrac{x^2}{\sqrt{x^2+a^2}}\,\mathrm{d}x = \dfrac{x}{2}\sqrt{x^2+a^2} - \dfrac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$

36. $\int \dfrac{x^2}{\sqrt{(x^2+a^2)^3}}\,\mathrm{d}x = -\dfrac{x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2}) + C$

37. $\int \dfrac{\mathrm{d}x}{x\sqrt{x^2+a^2}} = \dfrac{1}{a}\ln\dfrac{\sqrt{x^2+a^2}-a}{|x|} + C$

38. $\int \dfrac{\mathrm{d}x}{x^2\sqrt{x^2+a^2}} = -\dfrac{\sqrt{x^2+a^2}}{a^2 x} + C$

39. $\int \sqrt{x^2+a^2}\,\mathrm{d}x = \dfrac{x}{2}\sqrt{x^2+a^2} + \dfrac{a^2}{2}\ln(x+\sqrt{x^2+a^2}) + C$

40. $\int \sqrt{(x^2+a^2)^3}\,\mathrm{d}x = \dfrac{x}{8}(2x^2+5a^2)\sqrt{x^2+a^2} + \dfrac{3}{8}a^4\ln(x+\sqrt{x^2+a^2}) + C$

41. $\int x\sqrt{x^2+a^2}\,\mathrm{d}x = \dfrac{1}{3}\sqrt{(x^2+a^2)^3} + C$

42. $\int x^2\sqrt{x^2+a^2}\,\mathrm{d}x = \dfrac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \dfrac{a^4}{8}\ln(x+\sqrt{x^2+a^2}) + C$

43. $\int \dfrac{\sqrt{x^2+a^2}}{x}\,\mathrm{d}x = \sqrt{x^2+a^2} + a\ln\dfrac{\sqrt{x^2+a^2}-a}{|x|} + C$

44. $\int \dfrac{\sqrt{x^2+a^2}}{x^2}\,\mathrm{d}x = -\dfrac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2}) + C$

（七）含有 $\sqrt{x^2-a^2}$ $(a>0)$ 的积分

45. $\int \dfrac{\mathrm{d}x}{\sqrt{x^2-a^2}} = \dfrac{x}{|x|}\operatorname{arch}\dfrac{|x|}{a} + C_1 = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

46. $\int \dfrac{\mathrm{d}x}{\sqrt{(x^2-a^2)^3}} = -\dfrac{x}{a^2\sqrt{x^2-a^2}} + C$

47. $\int \dfrac{x}{\sqrt{x^2-a^2}}\,\mathrm{d}x = \sqrt{x^2-a^2} + C$

48. $\int \dfrac{x}{\sqrt{(x^2-a^2)^3}}\,dx = -\dfrac{1}{\sqrt{x^2-a^2}} + C$

49. $\int \dfrac{x^2}{\sqrt{x^2-a^2}}\,dx = \dfrac{x}{2}\sqrt{x^2-a^2} + \dfrac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

50. $\int \dfrac{x^2}{\sqrt{(x^2-a^2)^3}}\,dx = -\dfrac{x}{\sqrt{x^2-a^2}} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

51. $\int \dfrac{dx}{x\sqrt{x^2-a^2}} = \dfrac{1}{a}\arccos\dfrac{a}{|x|} + C$

52. $\int \dfrac{dx}{x^2\sqrt{x^2-a^2}} = \dfrac{\sqrt{x^2-a^2}}{a^2 x} + C$

53. $\int \sqrt{x^2-a^2}\,dx = \dfrac{x}{2}\sqrt{x^2-a^2} - \dfrac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

54. $\int \sqrt{(x^2-a^2)^3}\,dx = \dfrac{x}{8}(2x^2-5a^2)\sqrt{x^2-a^2} + \dfrac{3}{8}a^4\ln\left|x+\sqrt{x^2-a^2}\right| + C$

55. $\int x\sqrt{x^2-a^2}\,dx = \dfrac{1}{3}\sqrt{(x^2-a^2)^3} + C$

56. $\int x^2\sqrt{x^2-a^2}\,dx = \dfrac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \dfrac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

57. $\int \dfrac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\arccos\dfrac{a}{|x|} + C$

58. $\int \dfrac{\sqrt{x^2-a^2}}{x^2}\,dx = -\dfrac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

（八）含有 $\sqrt{a^2-x^2}\ (a>0)$ 的积分

59. $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \arcsin\dfrac{x}{a} + C$

60. $\int \dfrac{dx}{\sqrt{(a^2-x^2)^3}} = \dfrac{x}{a^2\sqrt{a^2-x^2}} + C$

61. $\int \dfrac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}+C$

62. $\int \dfrac{x}{\sqrt{(a^2-x^2)^3}}dx = \dfrac{1}{\sqrt{a^2-x^2}}+C$

63. $\int \dfrac{x^2}{\sqrt{a^2-x^2}}dx = -\dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}\arcsin\dfrac{x}{a}+C$

64. $\int \dfrac{x^2}{\sqrt{(a^2-x^2)^3}}dx = \dfrac{x}{\sqrt{a^2-x^2}}-\arcsin\dfrac{x}{a}+C$

65. $\int \dfrac{dx}{x\sqrt{a^2-x^2}} = \dfrac{1}{a}\ln\dfrac{a-\sqrt{a^2-x^2}}{|x|}+C$

66. $\int \dfrac{dx}{x^2\sqrt{a^2-x^2}} = -\dfrac{\sqrt{a^2-x^2}}{a^2 x}+C$

67. $\int \sqrt{a^2-x^2}\,dx = \dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}\arcsin\dfrac{x}{a}+C$

68. $\int \sqrt{(a^2-x^2)^3}\,dx = \dfrac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2}+\dfrac{3}{8}a^4\arcsin\dfrac{x}{a}+C$

69. $\int x\sqrt{a^2-x^2}\,dx = -\dfrac{1}{3}\sqrt{(a^2-x^2)^3}+C$

70. $\int x^2\sqrt{a^2-x^2}\,dx = \dfrac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2}+\dfrac{a^4}{8}\arcsin\dfrac{x}{a}+C$

71. $\int \dfrac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2}+a\ln\dfrac{a-\sqrt{a^2-x^2}}{|x|}+C$

72. $\int \dfrac{\sqrt{a^2-x^2}}{x^2}dx = -\dfrac{\sqrt{a^2-x^2}}{x}-\arcsin\dfrac{x}{a}+C$

（九）含有 $\sqrt{\pm ax^2+bx+c}$ $(a>0)$ 的积分

73. $\int \dfrac{dx}{\sqrt{ax^2+bx+c}} = \dfrac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right|+C$

74. $\displaystyle\int \sqrt{ax^2+bx+c}\,\mathrm{d}x = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c}$

$$+ \frac{4ac-b^2}{8\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C$$

75. $\displaystyle\int \frac{x}{\sqrt{ax^2+bx+c}}\,\mathrm{d}x = \frac{1}{a}\sqrt{ax^2+bx+c}$

$$- \frac{b}{2\sqrt{a^3}}\ln\left|2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}\right| + C$$

76. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{c+bx-ax^2}} = -\frac{1}{\sqrt{a}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$

77. $\displaystyle\int \sqrt{c+bx-ax^2}\,\mathrm{d}x = \frac{2ax-b}{4a}\sqrt{c+bx-ax^2} + \frac{b^2+4ac}{8\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$

78. $\displaystyle\int \frac{x}{\sqrt{c+bx-ax^2}}\,\mathrm{d}x = -\frac{1}{a}\sqrt{c+bx-ax^2} + \frac{b}{2\sqrt{a^3}}\arcsin\frac{2ax-b}{\sqrt{b^2+4ac}} + C$

（十）含有 $\sqrt{\pm\dfrac{x-a}{x-b}}$ 或 $\sqrt{(x-a)(b-x)}$ 的积分

79. $\displaystyle\int \sqrt{\frac{x-a}{x-b}}\,\mathrm{d}x = (x-b)\sqrt{\frac{x-a}{x-b}} + (b-a)\ln(\sqrt{|x-a|}+\sqrt{|x-b|}) + C$

80. $\displaystyle\int \sqrt{\frac{x-a}{b-x}}\,\mathrm{d}x = (x-b)\sqrt{\frac{x-a}{b-x}} + (b-a)\arcsin\sqrt{\frac{x-a}{b-x}} + C$

81. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-x}} + C \quad (a<b)$

82. $\displaystyle\int \sqrt{(x-a)(b-x)}\,\mathrm{d}x = \frac{2x-a-b}{4}\sqrt{(x-a)(b-x)} + \frac{(b-a)^2}{4}\arcsin\sqrt{\frac{x-a}{b-x}} + C$

$$(a<b)$$

（十一）含有三角函数的积分

83. $\displaystyle\int \sin x\,\mathrm{d}x = -\cos x + C$

84. $\int \cos x \mathrm{d}x = \sin x + C$

85. $\int \tan x \mathrm{d}x = -\ln|\cos x| + C$

86. $\int \cot x \mathrm{d}x = \ln|\sin x| + C$

87. $\int \sec x \mathrm{d}x = \ln\left|\tan(\frac{\pi}{4} + \frac{x}{2})\right| + C = \ln|\sec x + \tan x| + C$

88. $\int \csc x \mathrm{d}x = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C$

89. $\int \sec^2 x \mathrm{d}x = \tan x + C$

90. $\int \csc^2 x \mathrm{d}x = -\cot x + C$

91. $\int \sec x \tan x \mathrm{d}x = \sec x + C$

92. $\int \csc x \cot x \mathrm{d}x = -\csc x + C$

93. $\int \sin^2 x \mathrm{d}x = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

94. $\int \cos^2 x \mathrm{d}x = \frac{x}{2} + \frac{1}{4}\sin 2x + C$

95. $\int \sin^n x \mathrm{d}x = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \mathrm{d}x$

96. $\int \cos^n x \mathrm{d}x = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x \mathrm{d}x$

97. $\int \frac{\mathrm{d}x}{\sin^n x} = -\frac{1}{n-1} \cdot \frac{\cos x}{\sin^{n-1} x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\sin^{n-2} x}$

98. $\int \frac{\mathrm{d}x}{\cos^n x} = \frac{1}{n-1} \cdot \frac{\sin x}{\cos^{n-1} x} + \frac{n-2}{n-1}\int \frac{\mathrm{d}x}{\cos^{n-2} x}$

99. $\int \cos^m x \sin^n x \mathrm{d}x = \frac{1}{m+n}\cos^{m-1} x \sin^{n+1} x + \frac{m-1}{m+n}\int \cos^{m-2} x \sin^n x \mathrm{d}x$

$$= -\frac{1}{m+n}\cos^{m+1} x \sin^{n-1} x + \frac{n-1}{m+n}\int \cos^m x \sin^{n-2} x \mathrm{d}x$$

100. $\int \sin ax \cos bx \mathrm{d}x = -\frac{1}{2(a+b)}\cos(a+b)x - \frac{1}{2(a-b)}\cos(a-b)x + C$

101. $\int \sin ax \sin bx \mathrm{d}x = -\dfrac{1}{2(a+b)}\sin(a+b)x + \dfrac{1}{2(a-b)}\sin(a-b)x + C$

102. $\int \cos ax \cos bx \mathrm{d}x = \dfrac{1}{2(a+b)}\sin(a+b)x + \dfrac{1}{2(a-b)}\sin(a-b)x + C$

103. $\int \dfrac{\mathrm{d}x}{a+b\sin x} = \dfrac{2}{\sqrt{a^2-b^2}}\arctan\dfrac{a\tan\dfrac{x}{2}+b}{\sqrt{a^2-b^2}} + C \quad (a^2 > b^2)$

104. $\int \dfrac{\mathrm{d}x}{a+b\sin x} = \dfrac{1}{\sqrt{b^2-a^2}}\ln\left|\dfrac{a\tan\dfrac{x}{2}+b-\sqrt{b^2-a^2}}{a\tan\dfrac{x}{2}+b+\sqrt{b^2-a^2}}\right| + C \quad (a^2 < b^2)$

105. $\int \dfrac{\mathrm{d}x}{a+b\cos x} = \dfrac{2}{a+b}\sqrt{\dfrac{a+b}{a-b}}\arctan\left(\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{x}{2}\right) + C \quad (a^2 > b^2)$

106. $\int \dfrac{\mathrm{d}x}{a+b\cos x} = \dfrac{1}{a+b}\sqrt{\dfrac{a+b}{b-a}}\ln\left|\dfrac{\tan\dfrac{x}{2}+\sqrt{\dfrac{a+b}{b-a}}}{\tan\dfrac{x}{2}-\sqrt{\dfrac{a+b}{b-a}}}\right| + C \quad (a^2 < b^2)$

107. $\int \dfrac{\mathrm{d}x}{a^2\cos^2 x + b^2\sin^2 x} = \dfrac{1}{ab}\arctan\left(\dfrac{b}{a}\tan x\right) + C$

108. $\int \dfrac{\mathrm{d}x}{a^2\cos^2 x - b^2\sin^2 x} = \dfrac{1}{2ab}\ln\left|\dfrac{b\tan x + a}{b\tan x - a}\right| + C$

109. $\int x\sin ax \mathrm{d}x = \dfrac{1}{a^2}\sin ax - \dfrac{1}{a}x\cos ax + C$

110. $\int x^2\sin ax \mathrm{d}x = -\dfrac{1}{a}x^2\cos ax + \dfrac{2}{a^2}x\sin ax + \dfrac{2}{a^3}\cos ax + C$

111. $\int x\cos ax \mathrm{d}x = \dfrac{1}{a^2}\cos ax + \dfrac{1}{a}x\sin ax + C$

112. $\int x^2\cos ax \mathrm{d}x = \dfrac{1}{a}x^2\sin ax + \dfrac{2}{a^2}x\cos ax - \dfrac{2}{a^3}\sin ax + C$

（十二）含有反三角函数的积分（其中 $a > 0$）

113. $\int \arcsin\dfrac{x}{a}\mathrm{d}x = x\arcsin\dfrac{x}{a} + \sqrt{a^2-x^2} + C$

114. $\int x \arcsin \dfrac{x}{a} \mathrm{d}x = (\dfrac{x^2}{2} - \dfrac{a^2}{4}) \arcsin \dfrac{x}{a} + \dfrac{x}{4}\sqrt{a^2 - x^2} + C$

115. $\int x^2 \arcsin \dfrac{x}{a} \mathrm{d}x = \dfrac{x^3}{3} \arcsin \dfrac{x}{a} + \dfrac{1}{9}(x^2 + 2a^2)\sqrt{a^2 - x^2} + C$

116. $\int \arccos \dfrac{x}{a} \mathrm{d}x = x \arccos \dfrac{x}{a} - \sqrt{a^2 - x^2} + C$

117. $\int x \arccos \dfrac{x}{a} \mathrm{d}x = (\dfrac{x^2}{2} - \dfrac{a^2}{4}) \arccos \dfrac{x}{a} - \dfrac{x}{4}\sqrt{a^2 - x^2} + C$

118. $\int x^2 \arccos \dfrac{x}{a} \mathrm{d}x = \dfrac{x^3}{3} \arccos \dfrac{x}{a} - \dfrac{1}{9}(x^2 + 2a^2)\sqrt{a^2 - x^2} + C$

119. $\int \arctan \dfrac{x}{a} \mathrm{d}x = x \arctan \dfrac{x}{a} - \dfrac{a}{2}\ln(a^2 + x^2) + C$

120. $\int x \arctan \dfrac{x}{a} \mathrm{d}x = \dfrac{1}{2}(a^2 + x^2) \arctan \dfrac{x}{a} - \dfrac{a}{2}x + C$

121. $\int x^2 \arctan \dfrac{x}{a} \mathrm{d}x = \dfrac{x^3}{3} \arctan \dfrac{x}{a} - \dfrac{a}{6}x^2 + \dfrac{a^3}{6}\ln(a^2 + x^2) + C$

（十三）含有指数函数的积分

122. $\int a^x \mathrm{d}x = \dfrac{1}{\ln a} a^x + C$

123. $\int \mathrm{e}^{ax} \mathrm{d}x = \dfrac{1}{a} \mathrm{e}^{ax} + C$

124. $\int x\mathrm{e}^{ax} \mathrm{d}x = \dfrac{1}{a^2}(ax - 1)\mathrm{e}^{ax} + C$

125. $\int x^n \mathrm{e}^{ax} \mathrm{d}x = \dfrac{1}{a} x^n \mathrm{e}^{ax} - \dfrac{n}{a} \int x^{n-1}\mathrm{e}^{ax} \mathrm{d}x$

126. $\int xa^x \mathrm{d}x = \dfrac{x}{\ln a} a^x - \dfrac{1}{(\ln a)^2} a^x + C$

127. $\int x^n a^x \mathrm{d}x = \dfrac{1}{\ln a} x^n a^x - \dfrac{n}{\ln a} \int x^{n-1}a^x \mathrm{d}x$

128. $\int \mathrm{e}^{ax} \sin bx \mathrm{d}x = \dfrac{1}{a^2 + b^2} \mathrm{e}^{ax}(a \sin bx - b \cos bx) + C$

129. $\int \mathrm{e}^{ax} \cos bx \mathrm{d}x = \dfrac{1}{a^2 + b^2} \mathrm{e}^{ax}(b \sin bx + a \cos bx) + C$

130. $\int e^{ax} \sin^n bx dx = \dfrac{1}{a^2 + b^2 n^2} e^{ax} \sin^{n-1} bx (a \sin bx - nb \cos bx)$

$$+ \dfrac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \sin^{n-2} bx dx$$

131. $\int e^{ax} \cos^n bx dx = \dfrac{1}{a^2 + b^2 n^2} e^{ax} \cos^{n-1} bx (a \cos bx + nb \sin bx)$

$$+ \dfrac{n(n-1)b^2}{a^2 + b^2 n^2} \int e^{ax} \cos^{n-2} bx dx$$

（十四）含有对数函数的积分

132. $\int \ln x dx = x \ln x - x + C$

133. $\int \dfrac{dx}{x \ln x} = \ln |\ln x| + C$

134. $\int x^n \ln x dx = \dfrac{1}{n+1} x^{n+1} (\ln x - \dfrac{1}{n+1}) + C$

135. $\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$

136. $\int x^m (\ln x)^n dx = \dfrac{1}{m+1} x^{m+1} (\ln x)^n - \dfrac{n}{m+1} \int x^m (\ln x)^{n-1} dx$

（十五）含有双曲函数的积分

137. $\int \text{sh} x dx = \text{ch} x + C$

138. $\int \text{ch} x dx = \text{sh} x + C$

139. $\int \text{th} x dx = \ln \text{ch} x + C$

140. $\int \text{sh}^2 x dx = -\dfrac{x}{2} + \dfrac{1}{4} \text{sh} 2x + C$

141. $\int \text{ch}^2 x dx = \dfrac{x}{2} + \dfrac{1}{4} \text{sh} 2x + C$

（十六）定积分

142. $\int_{-\pi}^{\pi} \cos nx dx = \int_{-\pi}^{\pi} \sin nx dx = 0$

143. $\int_{-\pi}^{\pi} \cos mx \sin nx dx = 0$

144. $\int_{-\pi}^{\pi} \cos mx \cos nx dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

145. $\int_{-\pi}^{\pi} \sin mx \sin nx \mathrm{d}x = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$

146. $\int_0^{\pi} \sin mx \sin nx \mathrm{d}x = \int_0^{\pi} \cos mx \cos nx \mathrm{d}x = \begin{cases} 0, & m \neq n \\ \dfrac{\pi}{2}, & m = n \end{cases}$

147. $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \cos^n x \mathrm{d}x$

$I_n = \dfrac{n-1}{n} I_{n-2}$

$I_n = \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \cdots \cdot \dfrac{4}{5} \cdot \dfrac{2}{3}$ （n 为大于 1 的正奇数），$I_1 = 1$

$I_n = \dfrac{n-1}{n} \cdot \dfrac{n-3}{n-2} \cdot \cdots \cdot \dfrac{3}{4} \cdot \dfrac{1}{2} \cdot \dfrac{\pi}{2}$ （n 为正偶数），$I_0 = \dfrac{\pi}{2}$