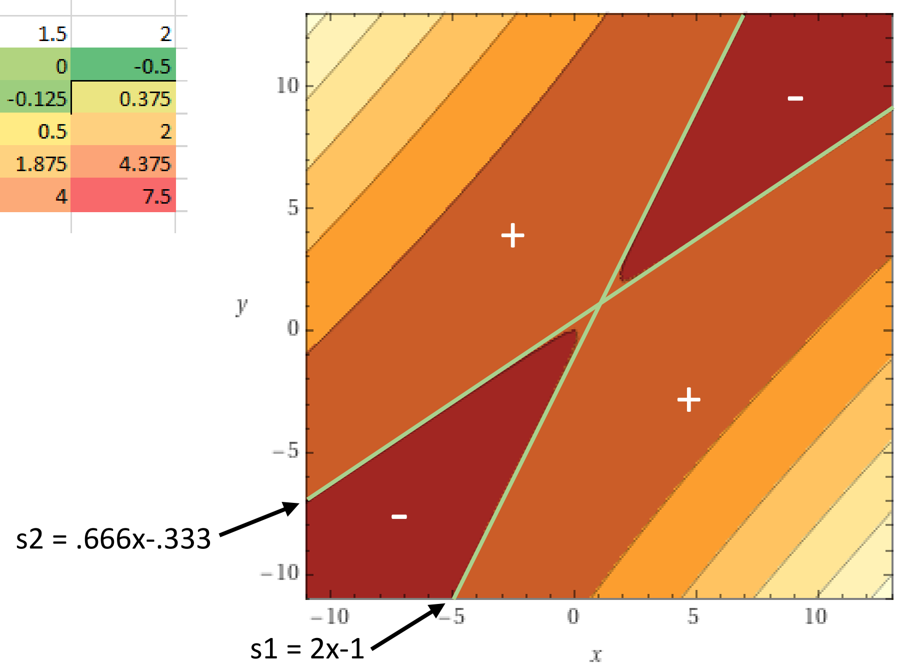
Problem 1

Grad = $[4x_1 - 4x_2, -4x_1 + 3x_2 + 1]$

Hess = [[4, -4], [-4, 3]]

```
First Eigenvalue = -0.5311288741492746 Second Eigenvalue = 7.531128874149275
Since the eigenvalues are both positive and negative this function is a saddle
The saddle point is: [1 1]
```

				x1		
		0	0.5	1	1.5	2
	2	7.5	4	1.5	0	-0.5
	1.5	4.375	1.875	0.375	-0.125	0.375
x2	1	2	0.5	0	0.5	2
	0.5	0.375	-0.125	0.375	1.875	4.375
	0	-0.5	0	1.5	4	7.5



Direction of downslopes away from the saddle:

s1 > 0 & s2 < 0 OR s1 < 0 & s2 > 0

also

$(ax_1+bx_2) < 0$ & $(cx_1+dx_2) > 0$ OR $(ax_1+bx_2) > 0$ & $(cx_1+dx_2) < 0$

Where a = .6666 b = -1.5 and c = 2 d = -.5

Problem 2

$x_1 + 2x_2 + 3x_3 = 1$ x = -1 +1t $(-1 + t) + 2(2t) + 3(1 + 3t) = 1$

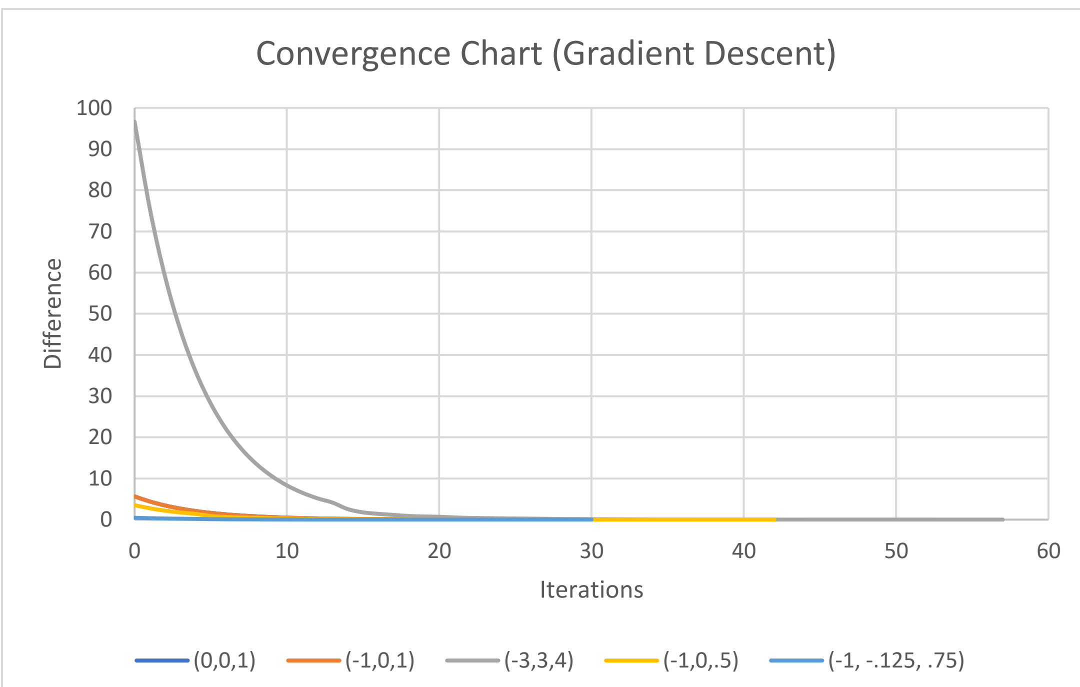
Point = (-1, 0, 1) y = 0 + 2t 14t = -1

n = <1, 2, 3> z = 1 + 3t t = -1/14

Point on plane closest to point: (-1.0714, -.1428, .7857)

Gradient Descent:

Gradient Descent		
Initial Guess	Solution	Iterations
(0,0,1)	(-1.07155,-.142837,.785744)	36
(-1,0,1)	(-1.07155,-.142837,.785744)	36
(-3,3,4)	(-1.0714, -.1424975, .785465)	58
(-1, 0, .5)	(-1.0714,-.1424658, .785444)	43
(-1, -.125, .75)	(-1.071396, -.1426583, .7854426)	31

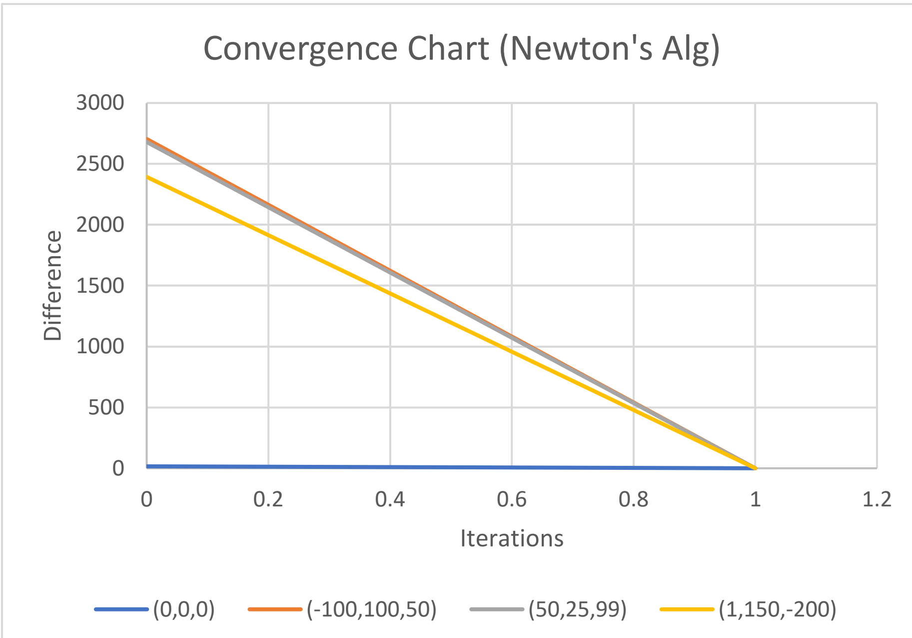


Example Output:

```
The point closest to the plane is: [-1.07139623 -0.14246583  0.78544263]
This point has an error of: 0.0009851010627638125
Algorithm ran for: 31 iteration(s)
```

Newton’s Algorithm:

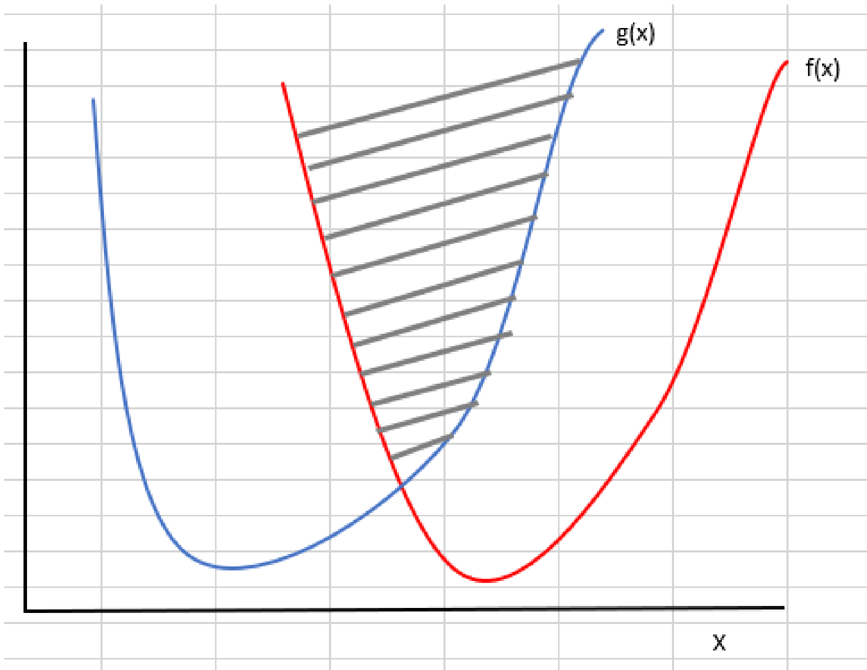
Newton's Method		
Initial Guess	Solution	Iterations
(0,0,0)	(-1.07142857,-.14285714, .78571429)	1
(-100,100,50)	(-1.07142857, -14285714, .78571429)	1
(50,25,99)	(-1.07142857, -14285714, .78571429)	1
(1,150,-200)	(-1.07142857, -14285714, .78571429)	1



Example Output:

```
The point closest to the plane is: [-1.07142857 -0.14285714  0.78571429]
This point has an error of: 1.1647215137477994e-11
Algorithm ran for: 1 iteration(s)
```

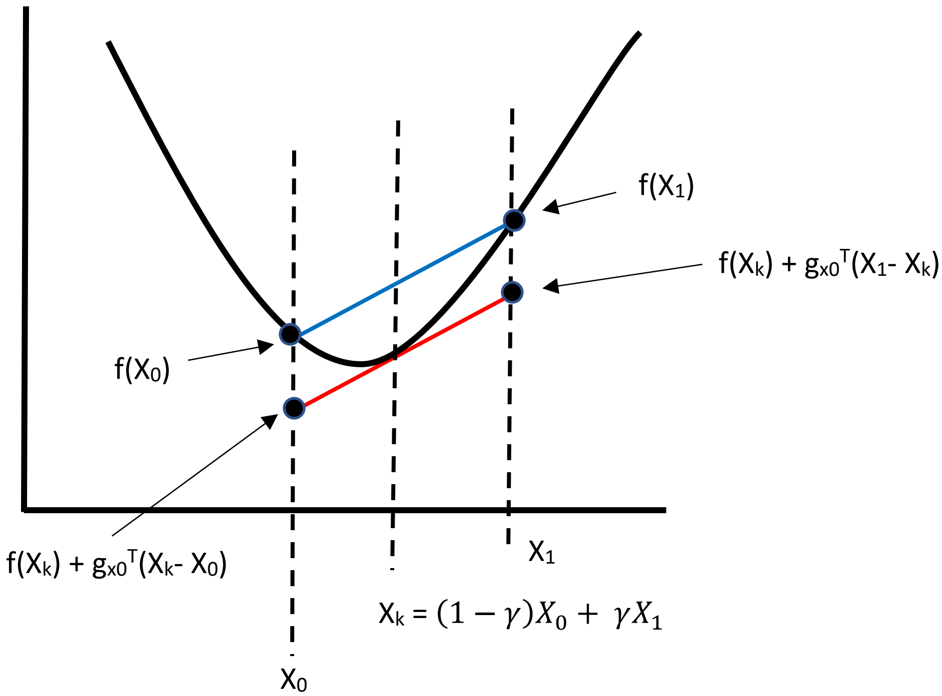
Problem 3



A) Determining the hessian will tell you what kind of functions these are. If the hessian comes out to either positive definite or positive semidefinite then the functions are convex as long as a>0 and b>0

B) The locations of the gradients cant overlap

Problem 4



Show that $f(\mathbf{x}_1) \geq f(\mathbf{x}_0) + \mathbf{g}_{\mathbf{x}_0}^T(\mathbf{x}_1 - \mathbf{x}_0)$

$$g_{x0}^T = \lim_{x \to x0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f(x_1) \geq f(x_0) + \frac{f(x) - f(x_0)}{x - x_0}(x_1 - x_0)$$

$$f(x_1)(x - x_0) \geq f(x_0)(x - x_0) + (f(x) - f(x_0))(x_1 - x_0)$$

$$(f(x_1) - f(x_0))((1-\gamma)x_0 + \gamma x_1 - x_0) \geq (f(x) - f(x_0))(x_1 - x_0)$$

$$(f(x_1) - f(x_0))(\gamma x_1 - \gamma x_0) \geq (f(x) - f(x_0))(x_1 - x_0)$$

$$(f(x_1) - f(x_0))(\gamma)(x_1 - x_0) \geq (f(x) - f(x_0))(x_1 - x_0)$$

$$(f(x_1) - f(x_0))(\gamma) \geq (f(x) - f(x_0))$$

Problem 5

A)

Minimize: P $\sum_k (a_k^T p - I)^2$

Gradient = $\sum_k [2(a_k^T p - I) a_k]$ = $\sum_k [2 a_k^T p a_k - 2 I a_k]$

Hessian = $\sum_k [2 a_k^T a_k]$

B)

If $d^T H d\ \geq 0$ then: H is Positive Semidefinite (p.s.d)

$\sum_k [2 * d a_k^T * a_k d^T]$ = $\sum_k [2 * X_k^2]$

Can only be zero if d is perpendicular to all vectors of a_k

If a_k spans the entire n-dimensional space, then the hessian will be positive definite

If that's the case then this problem is strictly convex, otherwise it's just convex

C)

There can't exist a solution where the power is reduced below the optimal value, a value could still be calculated but it wouldn't be a unique solution

D)

Only if the number of mirrors is equal to or greater than the number of lamps, if this is not the case then there is no unique solution