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Optional overhead

```
clear; % Clear the workspace
close all; % Close all windows
```

Optimization settings

Here we specify the objective function by giving the function handle to a variable, for example:

```
f = @(x)(x(1)^2+(x(2)-3)^2);
df = @(x)([2*x(1); 2*(x(2)-3)]);
g = @(x)([x(2)^2-2*x(1); (x(2)-1)^2+5*x(1)-15]);
dg = @(x)([-2, 2*x(2); 5, 2*(x(2)-1)^2]);

% % Specify algorithm
opt.alg = 'matlabqp'; % 'myqp' or 'matlabqp'

% Turn on or off line search
opt.linesearch = true; % false or true

% Set the tolerance to be used as a termination criterion:
opt.eps = 1e-3;

% Set the initial guess:
x0 = [1;1];

% Feasibility check for the initial point.
if max(g(x0)>0)
    errordlg('Infeasible intial point! You need to start from a feasible
 one!');
    return
end
```

Run optimization

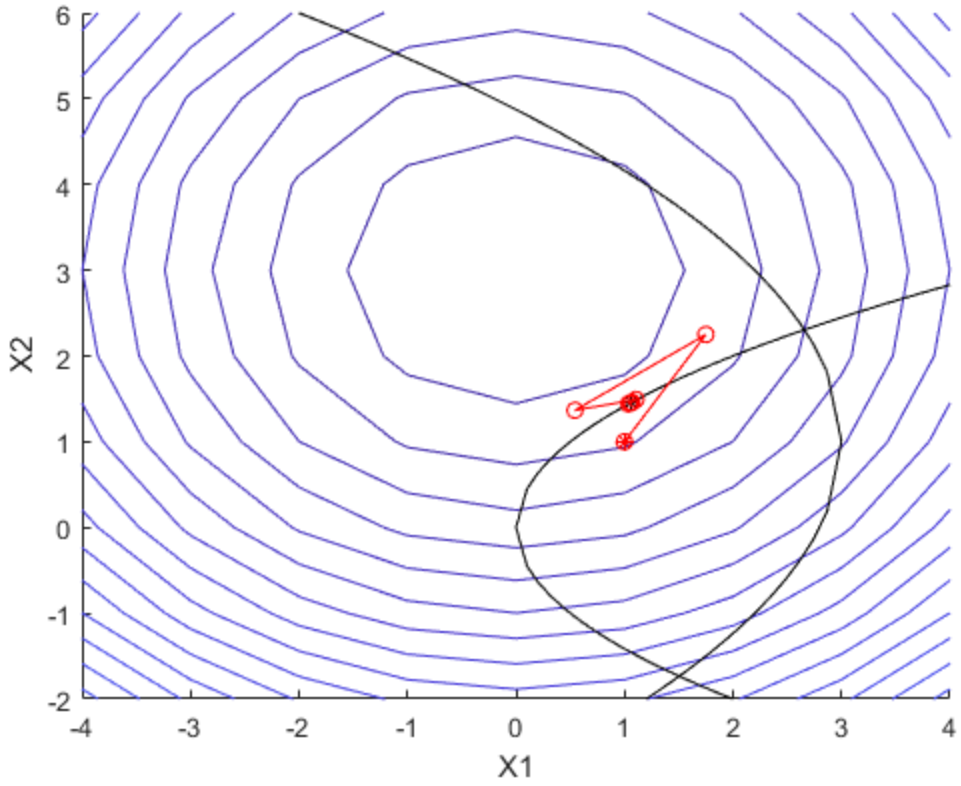
Run your implementation of SQP algorithm. See mysqp.m

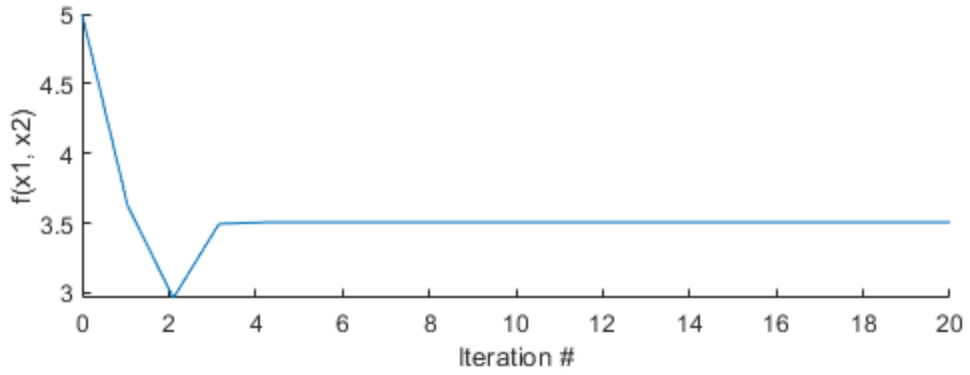
```
solution = mysqp(f, df, g, dg, x0, opt);
```

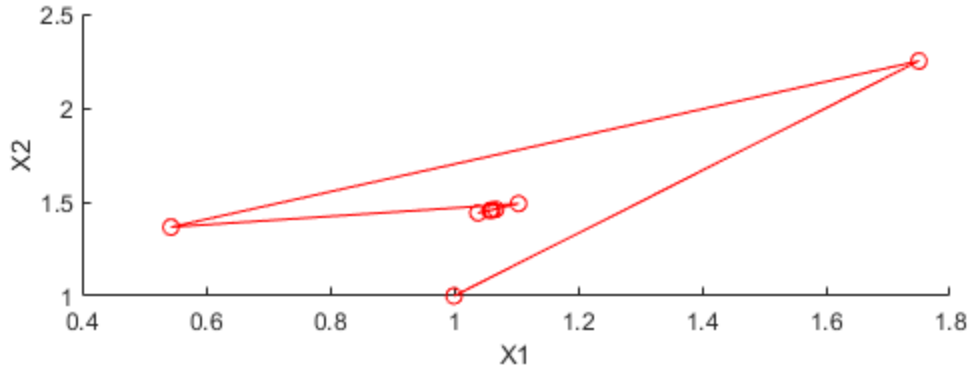
Plot Solution

```
x1_ub = 10;
x1_lb = -10;
x2_ub = 10;
x2_lb = -10;

% Plot objective function
x1 = x1_lb:x1_ub;
x2 = x2_lb:x2_ub;
[X1, X2] = meshgrid(x1,x2);
z = X1.^2+(X2-3).^2;
hold on;
contour(X1,X2,z,100);
xlabel('X1');
ylabel('X2');

% Plot inequality constraints
x1_var = linspace(0,x1_ub);
x1_var2 = linspace(x1_lb,3);
g_ans11 = sqrt(2*x1_var);
g_ans12 = -sqrt(2*x1_var);
g_ans21 = 1+sqrt(15-5*x1_var2);
g_ans22 = 1-sqrt(15-5*x1_var2);
plot(x1_var, g_ans11,'k', x1_var, g_ans12, 'k');
plot(x1_var2, g_ans21,'k', x1_var2, g_ans22, 'k');
axis([(x1_lb+6) (x1_ub-6) (x2_lb+8) (x2_ub-4)]);

% Plot test points
plot(1,1,'r*');
plot(1.0602, 1.4562, 'k*');
sol_ans = solution.x;
sol_x1 = sol_ans(1,:);
sol_x2 = sol_ans(2,:);
plot(sol_x1, sol_x2, 'ro-');
hold off;

% Plot Testing Points Over Time
figure;
subplot(2,1,1);
hold on;
x1_val = sol_ans(1,:);
x2_val = sol_ans(2,:);
func_val = x1_val.^2 + (x2_val-3).^2;
iter_num = linspace(0, length(x1_val), length(x1_val));
plot(iter_num, func_val);
xlabel('Iteration #');
ylabel('f(x1, x2)');
hold off;

% Plot How X1 and X2 Change w/ Time
subplot(2,1,2);
hold on;
```

```
plot(x1_val, x2_val, 'ro-');
xlabel('X1');
ylabel('X2');
hold off;
```







Print the results

```
init_val = [1, 1];
fx1 = x1_val(end);
fx2 = x2_val(end);
final_val = [fx1, fx2];
final_sol = f(final_val);
final_g = g(final_val);
final_g1 = final_g(1);
final_g2 = final_g(2);
Ans = ['With an initial guess of [x1, x2] = ', num2str(init_val)];
Iter = ['After ', num2str(length(x1_val)), ' iterations'];
Final = ['The final most optimal values = ', num2str(final_val)];
Func = ['The ohjectibe solution at this point = ', num2str(final_sol)];
G1 = ['The first inequality constraint = ', num2str(final_g1)];
G2 = ['The second inequality constraint = ', num2str(final_g2)];
disp(Ans);
disp(Iter);
disp(Final);
disp(Func);
disp(G1);
disp(G2);
```

```
With an initial guess of [x1, x2] = 1  1
After 20 iterations
The final most optimal values = 1.0602      1.4562
```

The ohjectibe solution at this point = 3.5075
The first inequality constraint = 0
The second inequality constraint = -9.4909

Report

```
function solution = mysqp(f, df, g, dg, x0, opt)
    % Set initial conditions

    x = x0; % Set current solution to the initial guess
    k = 0;

    % Initialize a structure to record search process
    solution = struct('x',[]);
    solution.x = [solution.x, x]; % save current solution to solution.x

    % Initialization of the Hessian matrix
    W = eye(numel(x));             % Start with an identity Hessian matrix
    % Initialization of the Lagrange multipliers
    mu_old = zeros(size(g(x)));    % Start with zero Lagrange multiplier
 estimates
    % Initialization of the weights in merit function
    w = zeros(size(g(x)));         % Start with zero weights

    % Set the termination criterion
    gnorm = norm(df(x) + mu_old'*dg(x)); % norm of Largangian gradient

    while ((gnorm>opt.eps) && (k < 19))  % if not terminated

        % Implement QP problem and solve
        if strcmp(opt.alg, 'myqp')
            % Solve the QP subproblem to find s and mu (using your own method)
            [s, mu_new] = solveqp(x, W, df, g, dg);
        else
            % Solve the QP subproblem to find s and mu (using MATLAB's solver)
            qpalg = optimset('Algorithm', 'active-set', 'Display', 'off');
            [s,~,~,~,lambda] = quadprog(W,[df(x)]',dg(x),-g(x),[], [], [], [],
 x,  qpalg);
            mu_new = lambda.ineqlin;
        end

        % opt.linesearch switches line search on or off.
        if opt.linesearch
            [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w);
        else
            a = 0.1;
        end

        % Update the current solution using the step
        dx = a*s;               % Step for x
        x = x + dx;             % Update x using the step

        % Update Hessian using BFGS. Use equations (7.36), (7.73) and (7.74)
```

```
        % Compute y_k
        y_k1 = df(x);
        y_k2 = dg(x)*mu_new;
        y_k3 = df(x-dx);
        y_k4 = dg(x-dx)*mu_new;
        y_k = [y_k1 + y_k2 - y_k3 - y_k4];
        % Compute theta
        if dx'*y_k >= 0.2*dx'*W*dx
            theta = 1;
        else
            theta = (0.8*dx'*W*dx)/(dx'*W*dx-dx'*y_k);
        end
        % Compute  dg_k
        dg_k = theta*y_k + (1-theta)*W*dx;
        % Compute new Hessian
        W = W + (dg_k*dg_k')/(dg_k'*dx) - ((W*dx)*(W*dx)')/(dx'*W*dx);

        % Update termination criterion:
        gnorm = norm(df(x) + mu_new'*dg(x)); % norm of Largangian gradient
        mu_old = mu_new;

        % save current solution to solution.x
        solution.x = [solution.x, x];
        k = k + 1;
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The following code performs line search on the merit function

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Armijo line search
function [a, w] = lineSearch(f, df, g, dg, x, s, mu_old, w_old)
    t = 0.1; % scale factor on current gradient: [0.01, 0.3]
    b = 0.8; % scale factor on backtracking: [0.1, 0.8]
    a = 1; % maximum step length

    D = s;                  % direction for x

    % Calculate weights in the merit function using eaution (7.77)
    w = max(abs(mu_old), 0.5*(w_old+abs(mu_old)));
    % terminate if line search takes too long
    count = 0;
    while count<100
        % Calculate phi(alpha) using merit function in (7.76)
        phi_a = f(x + a*D) + w'*abs(min(0, -g(x+a*D)));

        % Caluclate psi(alpha) in the line search using phi(alpha)
        phi0 = f(x) + w'*abs(min(0, -g(x)));        % phi(0)
        dphi0 = [df(x)]'*D + w'*((dg(x)*D).*(g(x)>0)); % phi'(0)
        psi_a = phi0 +  t*a*dphi0;                  % psi(alpha)
        % stop if condition satisfied
```

```
        if phi_a<psi_a
            break;
        else
            % backtracking
            a = a*b;
            count = count + 1;
        end
    end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The following code solves the QP subproblem using active set strategy

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [s, mu0] = solveqp(x, W, df, g, dg)
    % Implement an Active-Set strategy to solve the QP problem given by
    % min     (1/2)*s'*W*s + c'*s
    % s.t.    A*s-b <= 0
    %
    % where As-b is the linearized active contraint set

    % Strategy should be as follows:
    % 1-) Start with empty working-set
    % 2-) Solve the problem using the working-set
    % 3-) Check the constraints and Lagrange multipliers
    % 4-) If all constraints are staisfied and Lagrange multipliers are
 positive, terminate!
    % 5-) If some Lagrange multipliers are negative or zero, find the most
 negative one
    %     and remove it from the active set
    % 6-) If some constraints are violated, add the most violated one to the
 working set
    % 7-) Go to step 2


    % Compute c in the QP problem formulation
    c = [df(x)]';

    % Compute A in the QP problem formulation
    A0 = dg(x);

    % Compute b in the QP problem formulation
    b0 = -g(x);

    % Initialize variables for active-set strategy
    stop = 0;           % Start with stop = 0
    % Start with empty working-set
    A = [];         % A for empty working-set
    b = [];         % b for empty working-set
    % Indices of the constraints in the working-set
    active = [];    % Indices for empty-working set
```

```
while ~stop  % Continue until stop = 1
    % Initialize all mu as zero and update the mu in the working set
    mu0 = zeros(size(g(x)));

    % Extact A corresponding to the working-set
    A = A0(active,:);
    % Extract b corresponding to the working-set
    b = b0(active);

    % Solve the QP problem given A and b
    [s, mu] = solve_activeset(x, W, c, A, b);
    % Round mu to prevent numerical errors (Keep this)
    mu = round(mu*1e12)/1e12;

    % Update mu values for the working-set using the solved mu values
    mu0(active) = mu;

    % Calculate the constraint values using the solved s values
    gcheck = A0*s-b0;

    % Round constraint values to prevent numerical errors (Keep this)
    gcheck = round(gcheck*1e12)/1e12;

    % Variable to check if all mu values make sense.
    mucheck = 0;        % Initially set to 0

    % Indices of the constraints to be added to the working set
    Iadd = [];              % Initialize as empty vector
    % Indices of the constraints to be added to the working set
    Iremove = [];           % Initialize as empty vector

    % Check mu values and set mucheck to 1 when they make sense
    if (numel(mu) == 0)
        % When there no mu values in the set
        mucheck = 1;         % OK
    elseif min(mu) > 0
        % When all mu values in the set positive
        mucheck = 1;         % OK
    else
        % When some of the mu are negative
        % Find the most negaitve mu and remove it from acitve set
        [~,Iremove] = min(mu);  % Use Iremove to remove the constraint
    end

    % Check if constraints are satisfied
    if max(gcheck) <= 0
        % If all constraints are satisfied
        if mucheck == 1
            % If all mu values are OK, terminate by setting stop = 1
            stop = 1;
        end
    else
        % If some constraints are violated
        % Find the most violated one and add it to the working set
```

```
            [~,Iadd] = max(gcheck); % Use Iadd to add the constraint
        end
        % Remove the index Iremove from the working-set
        active = setdiff(active, active(Iremove));
        % Add the index Iadd to the working-set
        active = [active, Iadd];

        % Make sure there are no duplications in the working-set (Keep this)
        active = unique(active);
    end
end

function [s, mu] = solve_activeset(x, W, c, A, b)
    % Given an active set, solve QP

    % Create the linear set of equations given in equation (7.79)
    M = [W, A'; A, zeros(size(A,1))];
    U = [-c; b];
    sol = M\U;          % Solve for s and mu

    s = sol(1:numel(x));                % Extract s from the solution
    mu = sol(numel(x)+1:numel(sol));    % Extract mu from the solution

end
```

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