**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- Given

Mean ( ) =45

Std( )=8

The time the service manager has committed to is 1 hour, which is 60 minutes. The time from drop-off to start of work is 10 minutes. Therefore, the total time available for servicing is

60−10=50 minutes.

To fined the Z test=(x-)/

Z=(50-45)/8

to find the probability that a standard normal random variable is greater than

5/8. This corresponds to the probability that the service time exceeds 50 minutes

P(Z>85)≈0.2676

(B) 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:- TRUE

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- The result will give the expected number of employees under the age of 30. If this number is close to 36, the statement is considered true; otherwise, it is false.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:-

• For 2X1, the distribution has a higher variance (spread) compared to X1+X2.

• Both have the same mean (2μ), but the variance of 2X1 is twice that of X1+X2.

This highlights the fact that the spread of the distribution is affected differently when you scale a random variable by a constant (as in 2X1) compared to when you add two independent random variables (as in X1+X2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:-

For a standard normal distribution (mean = 0, standard deviation = 1), the z-scores corresponding to the tails where the cumulative probability is 0.005 (for the lower tail) and 0.995 (for the upper tail) will give us the values of a and b.

Zlower=−2.576 Zupper=2.576

Now, we convert these z-scores back to the original scale:

a=μ+Zlower×σ b=μ+Zupper×σ

Substitute the given values:

a=100+(−2.576)×20 b=100+(2.576)×20

Calculating these values will give us the range [a,b]:

a≈56.48 b≈143.52

The closest option from the provided choices is:

D. 48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:-

For a normal distribution, approximately 95% of the probability lies within 2 standard deviations from the mean. Therefore, the Rupee range can be calculated as follows:

Rupee Range=[μX−2σX,μX+2σX]

Substitute the values:

Rupee Range=[12−2(5),12+2(5)]=[2,22]

So, the Rupee range centered on the mean that contains 95% probability for the annual profit of the company is [2, 22] million Rupees.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:-

The formula for finding the p-th percentile for a normal distribution with mean μ and standard deviation σ is:

Percentile=μ+Z⋅σ

whereZ is the Z-score corresponding to the desired percentile. For the 5th percentile, Z is approximately -1.645.

Percentile=12+(−1.645)⋅5Percentile=12+(−1.645)⋅5

Percentile≈12−8.225Percentile≈12−8.225

Percentile≈3.775Percentile≈3.775

So, the 5th percentile of the profit for the company is approximately 3.7753.775 million Rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-

Using a standard normal distribution table or a calculator, we can find these probabilities.

For Profit1: P(Profit1 < 0)≈P(Z<−1.667)

(Profit1 < 0)≈0.0475

For Profit2: P(Profit2 < 0)≈P(Z<−1.75) (Profit2 < 0)≈0.0401

Therefore, Profit1 has a larger probability of making a loss in a given year (4.75%) compared to Profit2 (4.01%).