

Introduction to AI /ML

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Question No. 22 (Geometric form)

22. If an equilateral triangle ,having centroid at the origin , has a side along the line

$$x + y = 2,$$

then find the area of the triangle.

Question No. 22 (Matrix form)

22. If an equilateral triangle, having centroid at the origin, has a side along the line

$$(1 \ 1) \begin{pmatrix} x \\ y \end{pmatrix} = 2$$

then find the area of the triangle .

Solution

Given the equation of the line along which the side of the equilateral triangle is,

$$x + y = 2, \quad \dots \dots \dots (1)$$

So, let $A = (2, 0)$, $B = (0, 2)$ be two points on $x + y = 2$.

So, we have the direction vector of the line is ,

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = d(A)$$

Now, let $Q(x, y)$ be the point of intersection of median CQ passing through origin and line AB ($x + y = 2$).

So, we have the direction vector of OQ,

$$OQ = d(B) = Q - O = \begin{pmatrix} x \\ y \end{pmatrix}$$

Since, the centroid and orthocenter of a equilateral triangle are same.
we know that $d(A)$ and $d(B)$ are perpendicular

So,

$$\begin{aligned} d(A)^T d(B) &= 0 \\ 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} &= 0 \end{aligned}$$

$$x - y = 0 \quad \dots \dots \dots (2)$$

Combining the equations (1) and (2),

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Now we have ,

$$length = OQ = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$$

Now, in an equilateral triangle with side 'a' and height 'h' ,

$$h = \sqrt{3}a/2$$

We also know that the centroid divides the median in the ratio 2:1

$$CO : OQ = 2 : 1$$

So,

$$OQ = (1/3)h = a/2\sqrt{3}$$

.

$$a = OQ(2\sqrt{3})$$

We know that the area of a equilateral triangle with side 'a' is,

$$A = \sqrt{3}a^2/4$$

$$A = \sqrt{3} * 4 * 3 * (OQ)^2$$

$$A = 3\sqrt{3}(OQ)^2$$

Substituting the value of OQ

$$A = 3\sqrt{3}(\sqrt{2})^2$$

$$A = 6\sqrt{3}sq.units$$

Plotting and computed values

We can calculate the three vertices of a triangle using the equations for parametric co-ordinates and distance formula.

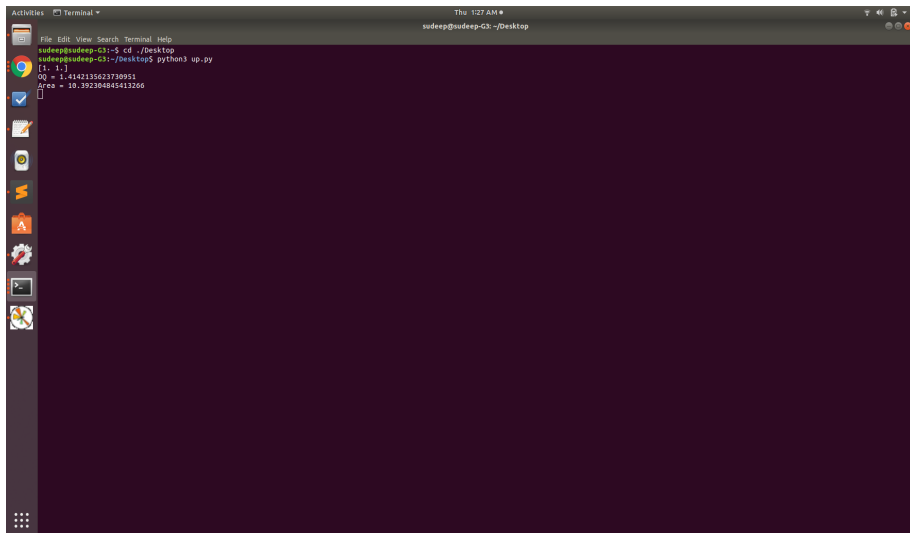
So, we have

$$A = (2\sqrt{2} \cos(\pi/12), -2\sqrt{2} \sin(\pi/12))$$

$$B = (-2\sqrt{2} \sin(\pi/12), 2\sqrt{2} \cos(\pi/12)),$$

$$C = (1 - \sqrt{12}, 1 - \sqrt{12})$$

Computation

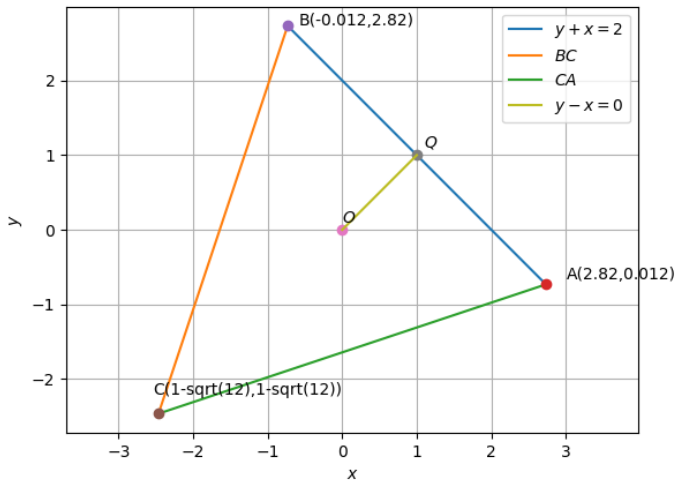


A terminal window titled "Terminal" is open, showing the execution of a Python script. The window has a menu bar with "File", "Edit", "View", "Search", "Terminal", and "Help". The terminal output is as follows:

```
sudeep@sudeep-G3:~$ cd ~/Desktop
sudeep@sudeep-G3:~/Desktop$ python3 up.py
[1. 1.]
OQ = 1.4142135623730951
Area = 10.392304845413266
```

The terminal window is part of a desktop environment with a sidebar on the left containing various application icons. The top status bar shows the time as "Thu 1:27 AM" and the user as "sudeep@sudeep-G3: ~/Desktop".

graph



THE END