Introduction to AI /ML

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Question No. 22 (Geometric form)

22. If an equilateral triangle ,having centroid at the origin , has a side along the line

$$x + y = 2,$$

then find the area of the triangle.

Question No. 22 (Matrix form)

22. If an equilateral triangle , having centroid at the origin, has a side along the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2$$

then find the area of the triangle .

Solution

Given the equation of the line along which the side of the equilateral triangle is,

$$x + y = 2, \qquad \qquad ....(1)$$

So, let A = (2 0) , B = (0 2) be two points on x+y = 2 .
So, we have the direction vector of the line is ,

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \text{d (A)}$$

Now, let Q(x y) be the foot of the perpendicular from the origin O (0 ,0) on the line x+y = 2.

.

.

So, we have the direction vector of OQ,

$$\text{OQ} = \text{d(B)} = \text{Q - O} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Since, the centroid and orthocenter of a equilateral triangle are same. we know that d(A) and d(B) are perpendicular .

So,

$$\implies d(A)^T d(B) = 0$$

.

$$\implies 2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

.

$$\text{x - y} = 0 \qquad \ldots\ldots\ldots\ldots.(2)$$

Combining the equations (1) and (2),

.

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\implies Q = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

.

Now we have ,

$$\text{lenght} = \text{OQ} = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$$

Now, in an equilateral triangle with side 'a' and height 'h' ,

$$\text{h} = \sqrt{3}a/2$$

We also know that the centroid divides the median in the ratio 2:1

$$\implies CO : OQ = 2 : 1$$

So,

$$OQ = (1/3)h = a/2\sqrt{3}$$

.

$$\implies a = OQ(2\sqrt{3})$$

We know that the area of a equilateral triangle with side 'a' is,

$$A = \sqrt{3}a^2$$

$$\implies A = \sqrt{3} * 4 * 3 * (OQ)^2$$

$$\implies A = 3\sqrt{3}(OQ)^2$$

Substituting the value of OQ

$$A = 3\sqrt{3}(\sqrt{2})^2$$

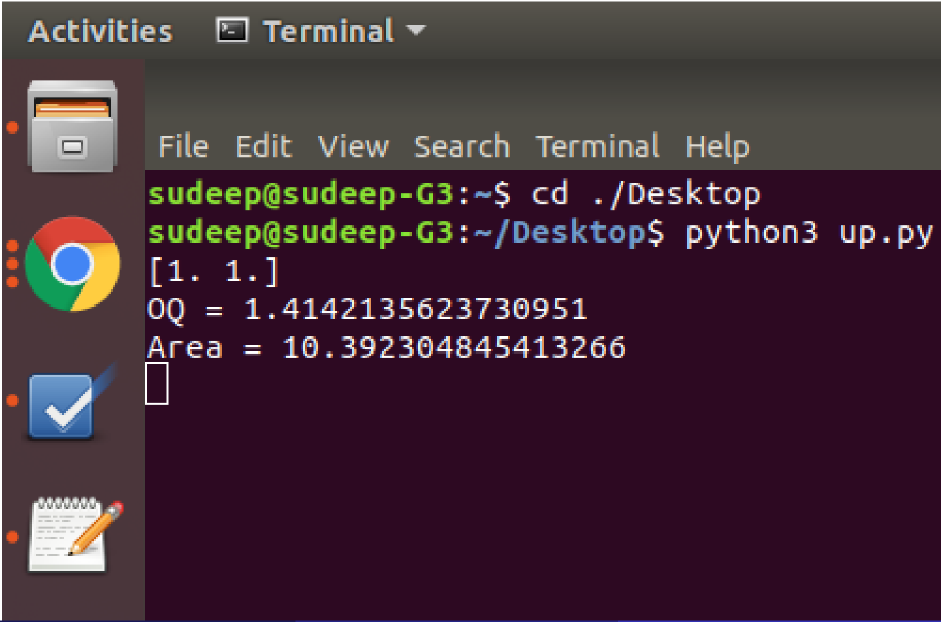
$$\implies A = 6\sqrt{3} sq.units$$

Plotting and computed values

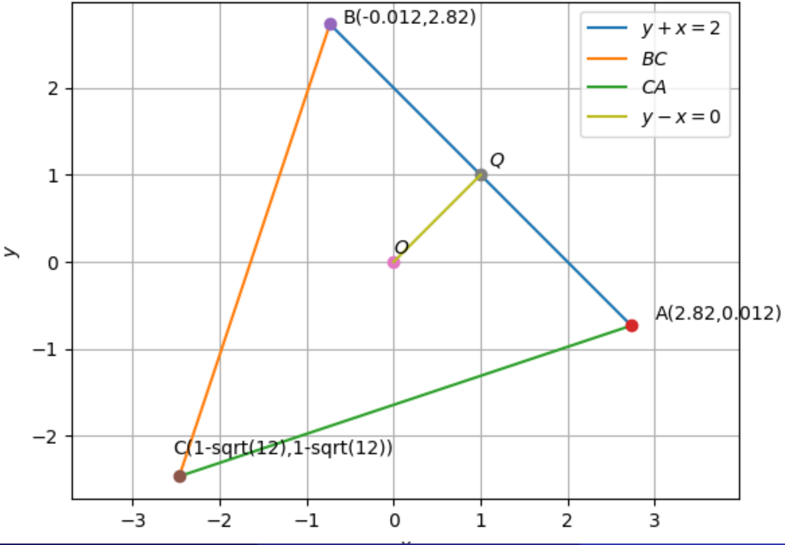
We can calculate the three vertices of a triangle using the equations for parametric co-ordinates and distance formula.
So, we have

$$A=(2\sqrt{2}\cos(\pi/12), -2\sqrt{2}\sin(\pi/12))$$
$$B = (-2\sqrt{2}\sin(\pi/12), 2\sqrt{2}\cos(\pi/12),)$$
$$C =(1- \sqrt{12}, 1-\sqrt{12})$$

Comptation


Activities
Terminal
File Edit View Search Terminal Help
sudeep@sudeep-G3:~$ cd ./Desktop
sudeep@sudeep-G3:~/Desktop$ python3 up.py
[1. 1.]
OQ = 1.4142135623730951
Area = 10.392304845413266

graph



B(-0.012,2.82)
Q
O
A(2.82,0.012)
C(1-sqrt(12),1-sqrt(12))
$y+x=2$
BC
CA
$y-x=0$
y
−3
−2
−1
0
1
2
3

THE END