# Introduction to AI /ML

Ujjwal (ee18btech11010), Pranay (ee18btech11009)

Indian Institute of Technology

*hyderabad*

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# Question No. 22 (Geometric form)

22. If an equilateral triangle ,having centroid at the origin , has a side along the line

$$x + y = 2,$$

then find the area of the triangle.

22. If an equilateral triangle , having centroid at the origin, has a side along the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2$$

then find the area of the triangle .

# Solution

Given the equation of the line along which the side of the equilateral triangle is,

$$x + y = 2 , \qquad \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ .(1)$$

So, let A = (2 , 0) , B = (0 , 2 ) be two points on x+y = 2 .
So, we have the direction vector of the line is ,

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = d(A)$$

Now, let Q(x , y) be the point of intersection of median CQ passing through origin and line AB $(x + y = 2)$ .

.

So, we have the direction vector of OQ,

$$OQ = d(B) = Q - O = \begin{pmatrix} x \\ y \end{pmatrix}$$

Since, the centroid and orthocenter of a equilateral triangle are same.
we know that d(A) and d(B) are perpendicular

So,

$$d(A)^T d(B) = 0$$
$$2 \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0$$

.

$$x - y = 0 \qquad \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ .(2)$$

Combining the equations (1) and (2),

.

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

.

Now we have ,

$$length = OQ = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$$

Now, in an equilateral triangle with side 'a' and height 'h' ,

$$h = \sqrt{3}a/2$$

We also know that the centroid divides the median in the ratio 2:1

$$\text{CO} : \text{OQ} = 2 : 1$$

So,

$$OQ = (1/3)h = a/2\sqrt{3}$$

.

$$a = OQ(2\sqrt{3})$$

We know that the area of a equilateral triangle with side 'a' is,

$$A = \sqrt{3}a^2/4$$
$$A = \sqrt{3} * 4 * 3 * (OQ)^2$$
$$A = 3\sqrt{3}(OQ)^2$$

Substituting the value of OQ

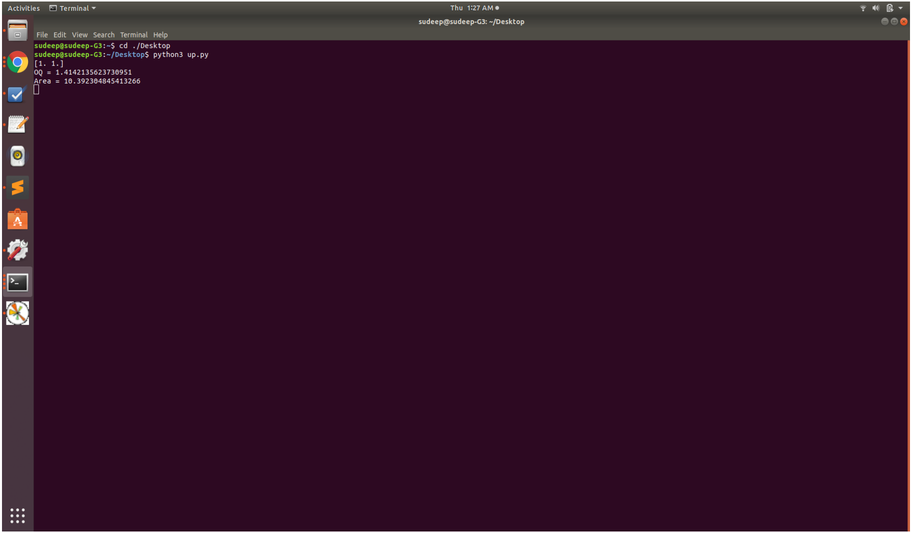
$$A = 3\sqrt{3}(\sqrt{2})^2$$
$$A = 6\sqrt{3} sq.units$$

# Plotting and computed values

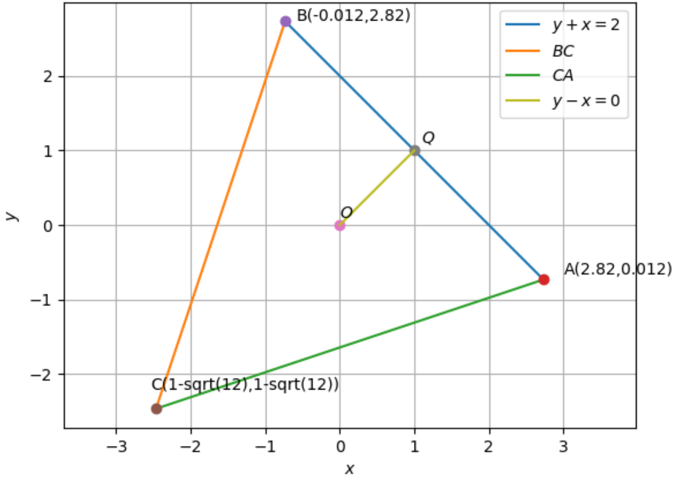
We can calculate the three vertices of a triangle using the equations for parametric co-ordinates and distance formula.
So, we have

$$A = (2\sqrt{2}\cos(\pi/12), -2\sqrt{2}\sin(\pi/12))$$
$$B = (-2\sqrt{2}\sin(\pi/12), 2\sqrt{2}\cos(\pi/12)),$$
$$C = (1-\sqrt{12}, 1-\sqrt{12})$$

# Computation



# graph


B(-0.012,2.82)
$y+x=2$
$BC$
$CA$
$y-x=0$
$Q$
$O$
A(2.82,0.012)
C(1-sqrt(12),1-sqrt(12))
$x$
$y$
−3
−2
−1
0
1
2
3

THE END