# Introduction to AI and ML

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February 14, 2019

# IIT JEE, 1980 (Geometric form)

Q. The point (4 , 1) undergoes the following three transformations successively .

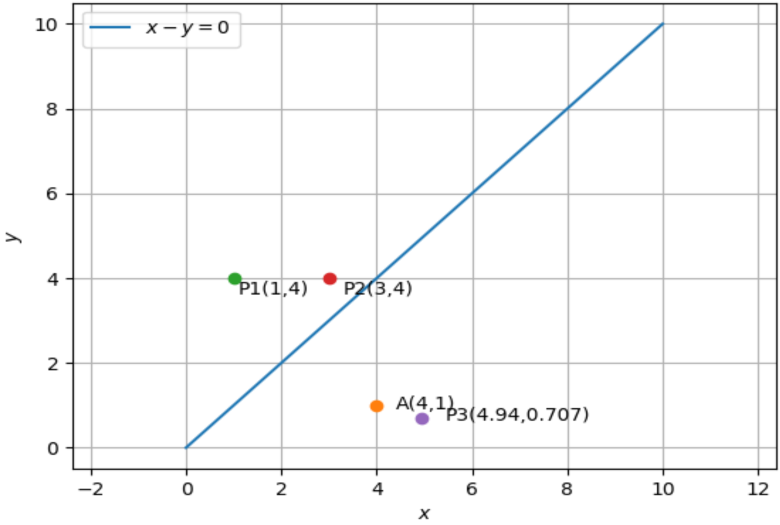
1. Reflection about the line $y = x$.
2. Translation through a distance 2 units along the positive direction of x-axis .
3. Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

# IIT JEE mains, 1980 (Matrix form)

Q. The point $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ undergoes the following three transformations successively .

1. Reflection about the line $\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$
2. Translation through a distance 2 units along the positive direction of x-axis .
3. Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

# Graph



# Solution

Let the given point be A (4,1) be represented as,

$$A = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Now, the first transformation is reflection about the line $y = x$.
So , the matrix associated with this reflection is given by ,

$$Z_{ref} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Right multiplication of any matrix , $A = \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ with Z is equivalent to the reflection of the point A(x,y) with respect to the given line,
$\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$ .

# Solution.

So, we have A transformed to $P_1$ ,where

$$P_1 = Z_{ref} A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

# Solution..

The next transformation is shifting the point '$P_1$' by 2 units to the positive x-axis
Let $Q = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ be the column matrix associated with the current transformation.
After applying the transformation , the new transformed matrix $P_2$ corresponding to the point Q is ,

$$P_2 = Q + P_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

# Solution...

Now, the last transformation is rotating the point $P_2$ by an angle $\frac{\pi}{4}$ in the counter clockwise direction about the origin.
Now, we have the
rotation matrix,where $\theta$ is the angle of the rotation of the point about the origin.

$$Z_{rotate} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now, substituting $\theta = \frac{\pi}{4}$

$$Z_{rotate} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Right multiplication of the matrix , $P_2 = \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ with $Z_{rotate}$ is equivalent to the rotation of the point $P_2$ by an angle $\theta$ about the origin.
So, we have $P_2$ transformed to $P_3$ , where

$$P_3 = Z_{rotate} P_2 = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 7/\sqrt{2} \end{bmatrix}$$

Therefore the final transformed matrix is ,

$$P_3 = \begin{bmatrix} -1/\sqrt{2} \\ 7/\sqrt{2} \end{bmatrix}$$

Hence, A(4,1) transforms to $P_3(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}})$

THE END