Project 2
Date due: 3/31/22 (submitted via Moodle)

Project objective:
The objectives of this project include:
- Estimating the coefficient of restitution of an object and characterizing the uncertainty in the result
- Providing an explanation for the observed results
- Using computer software for data collection
- Conducting an absolute motion analysis of a mechanism so that speed of the slider at a given θ is maximized
- Describing the design tradespace by generating a contour plot for a defined objective
- Evaluating constraints and understanding how they limit the available solution space
- Estimating the optimum solution graphically and determining the exact values analytically

Reporting requirements:
You will work is assigned groups for Part I of the project. Your group will submit a single report as a .pdf file to Moodle. You will complete Part II individually and submit your own .pdf file to Moodle. Submission folders for both have been created.

Part I - Measuring an object's coefficient of restitution and exploring uncertainty (Group submission):

Motivation for having you solve this problem:
We have solved problems in our dynamics class where we either know the coefficient of restitution (e) in an impact problem, or the pre- and post-conditions of the impact have been specified. There will be times in an engineering career where you will have to run an experiment so that you can characterize the coefficient of restitution for an impact between two objects. Conducting such a set of experiments so that you can estimate the coefficient of restitution is one goal for this part of the project.

We also must acknowledge that uncertainty is a part of every experiment. There are three main types of experimental uncertainties (or 'errors') that we must think about:
1. Limited accuracy associated with our measurement tools
2. Limits and simplifications of the experimental procedure
3. Uncontrolled changes to the environment (noise factors)

The second goal for this part of the project is exploring the variability in your coefficient of restitution (e) estimate. You will do this by repeating the same experiment multiple times "under the same conditions". The data that you collect will allow you to observe the range of values and how they are distributed.

Experimental setup:
You and your group will estimate the coefficient of restitution between two objects undergoing impact. One of these objects will be a stationary surface that does not move when impacted. The other object should be a ball of some type, and I leave this selection up to you. Whatever you have available can be used, so long as it bounces upon impact.

You and your group will estimate the coefficient of restitution between the ball you have selected and the surface of your choice **at two different heights**. We do this so that we can understand how the coefficient of restitution changes with respect to the velocity of the impact. The second height you select should be at least double the first height. You are free to select both heights so long as this constraint is satisfied.

Data collection:
I would like you to use the open-source software package Tracker. You can download it here:

https://physlets.org/tracker/

We are using Tracker because our results will be more consistent (and hopefully more accurate) if we pull the data from a video file rather than "eyeballing" it. You will need to take video of each trial that you run. When I have done this type of experiment in the past, we have found that videos were more effective if recorded at 60 frames per second.

You will want to record your video with an object of known dimension in the frame so that you can use it for setting a reference measurement.

In this project you will collect data from 30 total trials. A trial involves you dropping the ball from the height that you specified, the ball impacting the surface, and the ball rebounding to its new peak height. As shown in table below, the first 15 trials are from your 'low height' position. The second 15 trials are from your 'high height' position. You will calculate a coefficient of restitution for each impact.

Trial number	**Initial position**
Trial 1	Low Height
.	.
.	.
Trial 15	Low Height
Trial 16	High Height
.	.
.	.
Trial 30	High Height

<u>Data analysis and reporting:</u>

1. For one trial, I would like you to include screenshots form the Tracker program that show:
    - The loaded video with position markets identified
    - A plot of position as a function of time (in the direction of motion)
    - A plot of velocity as a function of time (in the direction of motion)

2. Create a separate data table for each of the two heights. For each trial, the data tables should contain the following information:
    - Initial height
    - Velocity before first impact
    - Velocity after first impact
    - Rebound height from first impact
    - Estimated coefficient of restitution

3. Create a histogram of the estimated coefficient of restitution values for each height. This means that you will get two histograms. Describe how you define the bin size. Also, report the average value and the standard deviation for each height.

    https://support.microsoft.com/en-us/office/create-a-histogram-85680173-064b-4024-b39d-80f17ff2f4e8

4. Create a box plot showing the distribution of the data of your trials. There should be two entries on the x-axis: the low height and the high height. The y-axis will be your estimated coefficient of restitution. You can find information on how to create a box plot in Excel here:

https://support.microsoft.com/en-us/office/create-a-box-and-whisker-chart-62f4219f-db4b-4754-aca8-4743f6190f0d

<u>Questions that I would like you to answer:</u>

Additionally, I would like your group to provide answers to the following questions:

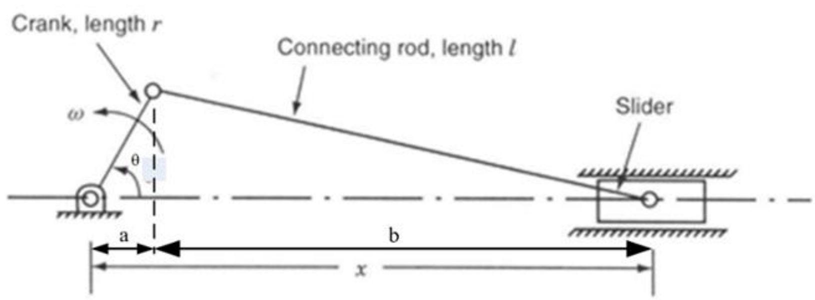
1. Discuss the variability in your estimated coefficient of restitution for each height.
    - Describe the variability of the results in mathematical terms. For example, how might you represent the distribution of estimated result?
    - Discuss whether the variability differs across the independent variable of height.
    - Discuss whether the variability in your estimates is significant and describe why you make this argument.

2. Describe how changing the impact height changes the coefficient of restitution. Explain, using concepts from our class, why this happens (or doesn't happen).

Part II – Optimization of a slider-crank (Individual submission):

Motivation for having you solve this problem:
In this part of the project, you are going to explore how the decisions we make about the geometry of a device impact outputs that we care about. What becomes interesting here is that we can pose this as an optimization problem where we are interested in maximizing a desired output by changing two parameters, subject to constraints. This moves us from a 'pure analysis' to a 'selection problem' case as there are multiple feasible answers, but only one best answer.

Problem setup:
Consider the slider-crank mechanism shown in the following figure. The slider-crank mechanism is made of a crank (of length r), a connecting rod (of length l) and a slider that is constrained so that it only moves in the x-direction. A motor is attached to the crank, and the crank rotates with a constant angular velocity ($\omega = 70\ rad/s$). The distance from the motor to the connection between the connecting rod and the slider is given by x. We have also defined the terms a and b, which are the x-axis projections of the crank length and the connecting rod length, respectively. The angle that the crank makes with the horizontal is given by θ.



You will develop an expression, using absolute motion analysis, that describes the velocity of the slider.

- Hint 1: Derive an expression for a and b in terms of the variables r, l, and θ.
- Hint 2: Use geometry to relate position (x) and the expression that you have derived for a and b.
- Hint 3: Think about how velocity (v) can be determined from position (x) with respect to changes in time.

Exploring the range of possible slider speeds:
Our goal for this problem is maximizing the speed (magnitude) of the slider when $\theta = 40°$ by selecting lengths of the crank (r) and the connecting rod (l). However, the design space for this problem is not unbounded. The lower and upper bounds for the crank and connecting rod are given by:

$$0.5 \leq r \leq 10 \quad \text{and} \quad 2.5 \leq l \leq 25$$

Before we can maximize, we need to understand the possible solution space. In this part of the project, I want you to use Matlab to create a contour plot. Use the meshgrid command to create the values of r and l that you will sample. This contour plot should have the following structure:

x-axis:	r over the range of `(.3:.1:11)`
y-axis	l over the range of `(2.5:1:26)`
z-axis:	Values of slider **speed** (absolute of velocity)

When plotting contours of speed, I want you to plot the following contours: [100 200 300 400 500 600 700 800]

Identifying constraints within the design space:
There are additional constraints that are placed upon our system. First, to ensure 360° rotation of the crank, the mechanism must satisfy Groshof's criterion:

$$l \geq 2.5r$$

Also, to ensure that the slider stays within its confined channel, there is a constraint that:

$$10 \leq x \leq 20$$

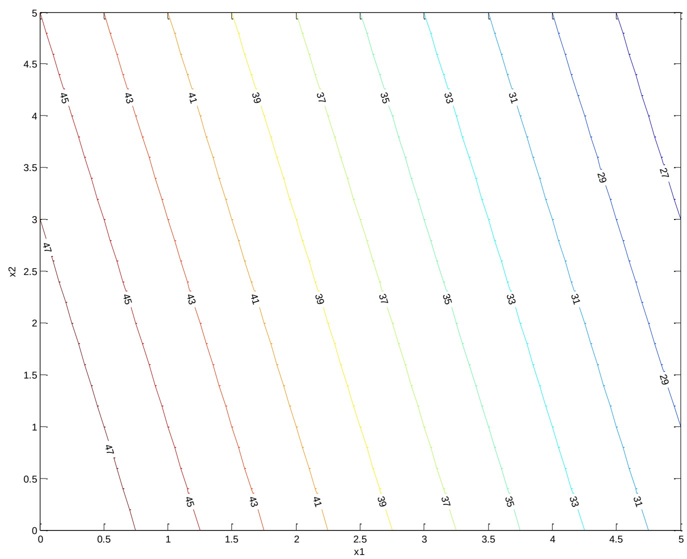
Before this constraint is usable, however, it must be re-written in terms of the lengths of the crank (r) and the connecting rod (l).

For this part of the project, I would like you to add all constraints and bounds to your contour plot. I recommend drawing the constraints by hand or by a digital pen.

How do we add constraints to a contour plot? Consider the following problem with two independent variables x_1 and x_2. Let's say that they are the independent variables of the following function:

$$-4x_1 - x_2 + 50$$

We get the following contour plot if we consider x_1 and x_2 on the range of 0 to 5:



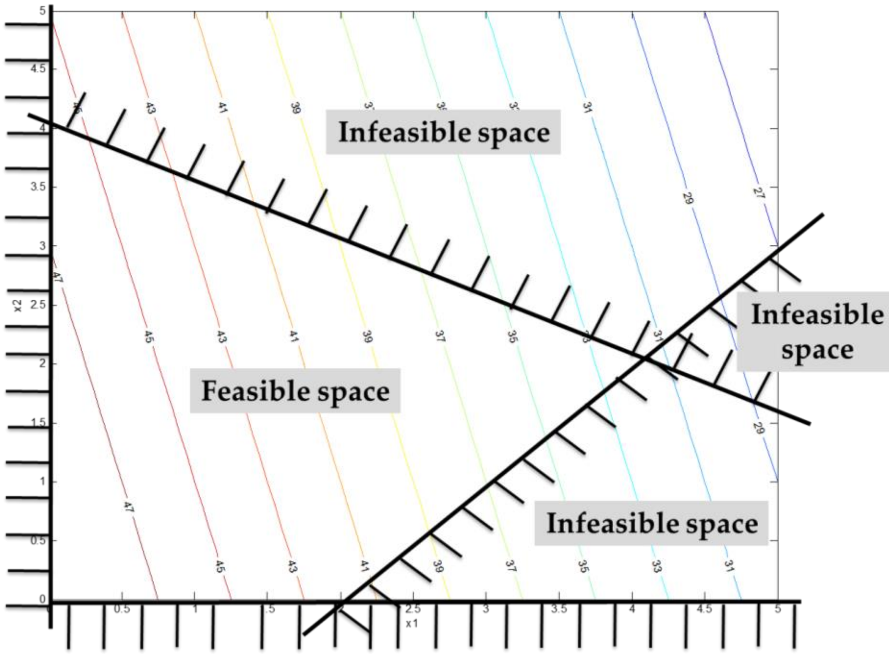
Now, let's say that we have two constraints and two bounds on our design variables:

Constraint 1:	$x_1 - x_2 \leq 2$
Constraint 2:	$x_1 + 2x_2 \leq 8$
Bound on x_1	$x_1 \geq 0$
Bound on x_2	$x_2 \geq 0$

Drawing each constraint and bound on a contour plot requires two different steps:

1) Drawing the line where each constraint/bound is exactly satisfied
2) Placing hash marks on the side of the line where the constraint or bound is **not satisfied**

For example, the first constraint requires a line where $x_1 - x_2 = 2$. Then, you hash the side of the line where the constraint is violated. The location (4,1), for instance, gives a value of 3, which is **not** less than 2. Our updated contour plot now looks like:



For our slider-crank problem, you will have 4 bounds and 3 constraints that must be included on your contour plot.

<u>Identifying the optimal solution in the constrained space:</u>
Working within the feasible space, identify the lengths of the crank (r) and the connecting rod (l) that maximize the speed of the slider crank when $\omega = 70\ rad/s$ and $\theta = 40°$. Identify this location on your figure.

<u>Calculating the exact value of the optimal solution:</u>
Finally, get the exact values for the lengths of the crank (r) and the connecting rod (l) that maximize the speed of the speed of the slider crank when $\omega = 70\ rad/s$ and $\theta = 40°$. Your optimum will exist at the intersection of constraints and/or bounds. Using the intersection point that you have identified, solve for the exact values of r and l.

<u>What you need to report:</u>
Your single pdf file should contain the results of each part of this project. Show your work when writing out equations and calculating the values of the optimal solution.

- An expression, using absolute motion analysis, that describes the velocity of the slider
- A contour plot showing slider speeds
- The contour plot with constraints and bounds identified
- The contour plot, with constraints and bounds, and optimal solution identified
- The exact values of the optimum