

## ASSIGNMENT:- 01

Ans 01 :-

Asymptotic Notation:- It is the mathematical notation used to describe the running time of an algorithm.

Different types of Notations are:-

(i) Big O Notation (Big - O) → It represents the upper bound of the algorithm.  
 $f(n) = O(g(n))$  iff  $f(n) \leq C(g(n))$

(ii) Omega Notation ( $\Omega$ ) → It represents the lower bound of the algorithm.  
 $f(n) = \Omega(g(n))$  iff  $f(n) \geq c(g(n))$

(iii) Theta Notation ( $\Theta$ ) → It represents upper and lower bound of the algorithm.  
 $f(n) = \Theta(g(n))$  iff  $C_1 g(n) \leq f(n) \leq C_2 g(n)$

Ans 02 :-

for (i=1 to n)  
  {  
    i=i+2;  
  }

$i = 1, 2, 3, 4, 5, 6, 7$   
*i* value = 2, 4, 8, 16, ...

forms a GP;  $a_n = a(2)^{n-1}$

$a = 1$

$$n = a(2)^{k-1}$$

$$\log n = \log 2^{k-1}$$

$$\log n = k-1$$

$$k = \log n + 1$$

— taking log both sides

$$\Rightarrow T(n) = \Theta(\log n). \quad \boxed{\text{Ans}}$$

Ans 03 :-

$$T(n) = 3T(n-1) \quad \text{if } n > 0; \text{ otherwise } 1$$

$$T(1) = 3T(0) = 3$$

$$T(2) = 3T(1) = 3 \cdot 3T(0) = 3^2$$

$$\vdots T(k) = 3^k$$

~~$$T(n) = 3^n$$~~

~~$$T(n) = \Theta(3^n). \quad \boxed{\text{Ans}}$$~~

Ans 04  $T(n) = 2T(n-1) + 1 \quad \text{if } n > 0 ; \text{ otherwise } 1$

$$T(0) = 1$$

$$T(1) = 2T(0) + 1 = 1$$

$$T(2) = 2T(1) + 1 = 2(T(0) + 1) - 1 = 2^2 T(1) - 5$$

$$T(n-k) = 2^k T(n-k) - 2^{k-1} \dots - 2^0$$

Substituting  $k=n-1$

$$T(n) = 2^{n-1} T(1) - [2^0 + 2^1 + \dots + 2^{n-2}]$$

$$= 2^{n-1} \times 1 - [2^{n-1} - 1]$$

$$T(n) = 1$$

$T(n) = O(1)$  — Ans

Ans 05)

int i=1, s=1;  
while ( $s \leq n$ )

{  
 $s = s + i;$   
pointf("#");  
}

i = 1 2 3 ... loops end  
s = 1 1+2 1+2+3 ... where  $s \geq n$

Hence;

$s \geq n$

$$1+2+3+4+\dots+K \geq n$$

$$\frac{K(K+1)}{2} \geq n \Rightarrow K \geq \sqrt{n}$$

$T(n) = O(\sqrt{n})$  — Ans

Ans 06)

void function (int n)

{  
int i, count=0;  
for (i=1; i\*i <= n; i++)  
count++;  
}

i = 1, 4, 9, 16 ... till  $i^2 = n$

Hence;

$i \leq n$

$K \leq K \leq n$

$K^2 \leq n$

$K \leq \sqrt{n}$

$T(n) = O(\sqrt{n})$  — Ans

Ans. 7)

```

void function (int n)
{
    int i, count = 0;
    for (int i =  $\frac{n}{2}$ ; i <= n; i++)
    {
        for (j = 1; j <= n; j * 2 = j)
        {
            for (k = 1; k <= n; k * 2 = k)
                count++;
        }
    }
}

```

Hence:-

$$\begin{aligned}
T(n) &= T(i) \times T(j) \times T(k) \\
&= O(n) \times O(\log n) \times O(\log n) \\
&\boxed{T(n) = O(n \log^2 n)} \quad - Ans.
\end{aligned}$$

1st loop:-

$$i = \frac{n}{2} \text{ to } n; i++$$

$$T(i) = O(n)$$

2nd loop:-

$$j = 1 \text{ to } n; j \times 2$$

$$T(j) = O(\log n)$$

3rd loop:-

$$k = 1 \text{ to } n; k \times 2$$

$$T(k) = O(\log n)$$

Ans 8)

function (int n)

```

{
    if (n == 1) return;      — T(1)
    for (i = 1 to n)
    {
        for (j = 1 to n)
        {
            printf ("*");
            — T(n^2)
        }
        function (n - 3);     — T(n - 3)
    }
}

```

Hence:

$$\text{Reln: } T(n) = T(n - 3) + n^2; \quad T(1) = 1$$

$$T(4) = T(1) + (4)^2 = 1 + 4^2$$

$$T(7) = T(4) + n^2 = 1^2 + 4^2 + 7^2$$

$$T(n) = 1^2 + 1^2 + 7^2 + \dots + n^2$$

$$= \frac{n(n+1)(2n+1)}{6} = \boxed{n^3} - \text{nearest}$$

$$\boxed{T(n) = O(n^3)}.$$

L Ans

Ans. 9

NAME:- AYUSH RAJ

void function (int n)

{

for (int i = 1; i <= n; i++)

{

    for (int j = 1; j <= n; j++)

{

        printf("\*");

}

}

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## ASSIGNMENT:- Q1

Ans 01

```
void fun (int n)
{
    int j=1, i=0;
    while (i < n)
    {
        i = i + j;
        j++;
    }
}
```

$$\boxed{T(n) = O(\sqrt{n})}$$

└ Ans ─

$$\begin{array}{ll} j=0; & i=0+1 \\ j=1; & j=0+1+2 \\ j=2; & j=0+1+2+3 \\ \vdots & \vdots \\ j=n; & \boxed{i=n} \downarrow \text{HW} \end{array}$$

$$\begin{aligned} 0+1+2+\dots+n &> n \\ \frac{k(k+1)}{2} &> n \\ k^2 &> n \quad k > \sqrt{n} \end{aligned}$$

Ans 02 Recurrence rel<sup>n</sup> for Fibonacci series:-

$$T(n) = T(n-1) + T(n-2)$$

$$T(0) = T(1) = 1$$

Assume  $T(n-1) \approx T(n-2)$

$$\begin{aligned} T(n) &= 2T(n-2) \\ &= 2[2T(n-4)] = 4T(n-4) \\ &= 2[2T(n-6)] = 8T(n-6) \end{aligned}$$

$$\therefore T(n) = 2^K T(n-2^K)$$

$$n-2^K = 0$$

$$n = 2^K$$

$$K = \frac{n}{2}$$

$$T(n) = 2^{n/2} T(0)$$

$$T(n) = 2^{n/2}$$

$$T(n) = 2^{n/2} (2^{n/2}) = 2^{n/2}$$

if  $T(n-2) \approx T(n-1)$

$$\begin{aligned} T(n) &= 2T(n-1) \\ &= 2(2T(n-2)) = 4T(n-2) \end{aligned}$$

$$\therefore T(K) = 2^K T(n-K)$$

$$n-1 \leq 0$$

$$\boxed{K=n}$$

$$T(n) = 2^K T(0)$$

$$T(n) = 2^K = 2^n$$

$$\boxed{O(n) = 2^n}$$

└ Ans ─

Aus 03

```

for (i=0; i<n; i++)
{
    for (j=1; j<n; j=j+2)
        {
            // same O(1)
        }
}

```

} -  $O(n \log n)$

```

for (i=0; i<n; i++)
{
    for (j=0; j<n; j++)
        {
            for (k=0; k<n; k++)
                {
                    // some O(1);
                }
}

```

} -  $O(n^3)$

```

for (i=1; i<=n; i=i*2)
{
    for (j=1; j<=n; j=j*2)
        {
            // some O(1);
}

```

} -  $O(\log(\log n))$

Aus 04

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + Cn^2$$

$$\text{lets assume } T\left(\frac{n}{2}\right) \geq T\left(\frac{n}{4}\right)$$

$$\text{so: } T = 2T\left(\frac{n}{2}\right) + Cn^2$$

Applying Master's theorem;

$$\begin{array}{l} a = 2 \\ b = 2 \end{array} \quad f(n) = n^2$$

$$c = \log_b a = 1.$$

$$n^c = n$$

Comparing:

$$f(n) > n^1$$

$$\text{so, } T(n) = \Theta(n^2). \quad \text{— Aus}$$

Ans 05)

sort fun (int n)

{

for (i=1; i<n; i++)

{ for (j=i; j<n; j+=1)

{

// some O(1);

}

}

}

i = 1	2	3	4
j = 1	1	1	1
	3	4	5
	5	7	8
	7	9	10

$\therefore n^2$

Hence:  $T(n) = O(n^2) + O(n^{2/2}) + O(n^{2/3}) + \dots$

$$\boxed{T(n) = O(n^2)}$$

Ans

Ans 06)

for (i=2; i<n, i = pow(i, k))

{

// some O(1)

}

$$pow(i, k) = O(\log_k)$$

$\Rightarrow \log k$

loop ends  $2^{k^n} > n$

$\log 2^{k^n} > \log n$  — taking log both sides.

$$k^n \log 2 > \log n$$

$$\log k^n > \log (\log n)$$

$$n \log k > \log \log n$$

$$n > \frac{\log (\log n)}{\log k}$$

Hence:

$$\boxed{T(n) = \log (\log_2 n)}$$

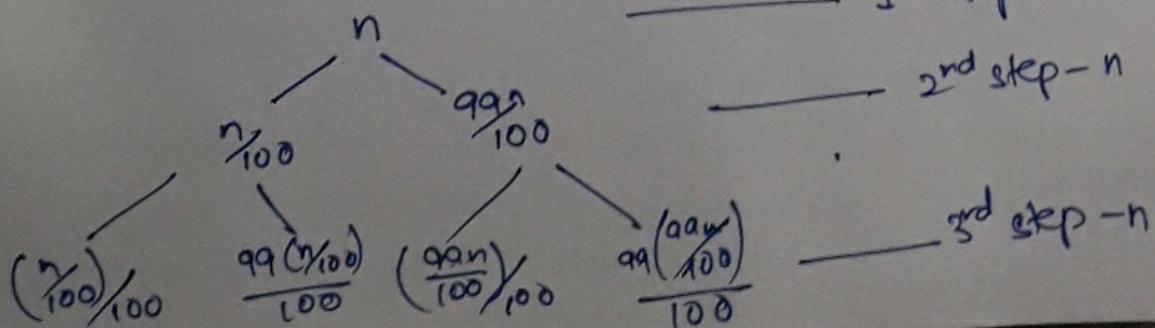
Ans

Ans 07)

Considering the statement;

$$T(n) = T(n/100) + T(99n/100) + O(n)$$

where  $n/100$  &  $99n/100$  are for parts &  $O(n)$  is partitioning algo.



(d); it will remain  $n$  at each step.

(e); time complexity =  $O(n \# \log_{10} n)$  if we take longer branch  
=  $\underline{\underline{O(n \# \log_{10} n)}}$  TIME COMPLEXITY.

Question 08  
a

Order is :-

$$100 < \log n < \sqrt{n} < n < \log(\log n) < n \log n < \log n! \\ < n! < n^2 < \log 2n < 2^n < 4^n.$$

(b) Order is:-

$$1 < \sqrt{\log n} < \log n < 2 \log n < \log_2 N < N < 2N < 4N \\ < \log(\log N) < N \log N < \log N! < n! < n^2 < 2 \times 2^N.$$

(c) Order is:-

$$96 < \log_8 N < \log_2 N < n \log_6 N < n \log_2 N < \log n! \\ < n! < 5N < 8N^2 < 7N^3 < 8^{2n}.$$

## - ASSIGNMENT :- 03 -

Ans 1)

```
while (low <= high)
{
    mid = low + high / 2;
    if (arr[mid] == key)
        return true;
    else if (arr[mid] > key)
        high = mid - 1;
    else
        low = mid + 1;
}
return false;
```

Ans 2)

Iterative Insertion Sort

```
for (i = 1; i < n; i++)
{
    j = i - 1;
    x = A[i];
    while (j > -1 && A[j] > x)
    {
        A[j + 1] = A[j];
        j--;
    }
    A[j + 1] = x;
}
```

Recursive Insertion Sort

```
void insertionSort (int a[], int n)
{
    if (n <= 1)
        return;
    insertion (a, n - 1);
    int last = a[n - 1];
    i = n - 2;
    while (j >= 0 && arr[j] > last)
    {
        a[j + 1] = a[j];
        j--;
    }
    a[j + 1] = last;
}
```

It is a kind of ONLINE SORTING because whenever a new element comes, insertion sort defines its right place.

Apt 3/2

## Sorting

	Time Complexity
Bubble Sort	$O(n^2)$
Insertion Sort	$O(n^2)$
Selection Sort	$O(n^2)$
Merge Sort	$O(n \log n)$
Quick Sort	$O(n \log n)$
Count Sort	$O(n + k)$
Bucket Sort	$O(n)$

Apt 4/2

Online Sorting — Insertion sort

Stable sorting — Merge sort, Insertion sort, Bubble sort.

Inplace sorting — Bubble sort, Insertion sort, Selection sort

Apt 5/2

## Iterative Binary Search

while ( $\text{low} <= \text{high}$ )

{ int mid = low + high / 2;

if ( $\text{arr}[\text{mid}] == \text{key}$ )

return true;

else if ( $\text{arr}[\text{mid}] > \text{key}$ )

high = mid - 1;

else

low = mid + 1;

}

] -  $O(\log n)$ .

### Recursive Binary Search:-

```
while (low <= high)
{
    int mid = low + high / 2
    if (arr[mid] == key)
        return true;
    else if (arr[mid] > key)
        return BinarySearch(arr, low, mid - 1);
    else
        return BinarySearch(arr, mid + 1, high);
}
return false;
```

} -  $O(\log n)$

### Ans 6 :- Required RT :-

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + C$$

$$\boxed{T(n) = 2T\left(\frac{n}{2}\right) + C}$$

### Ans 7 :-

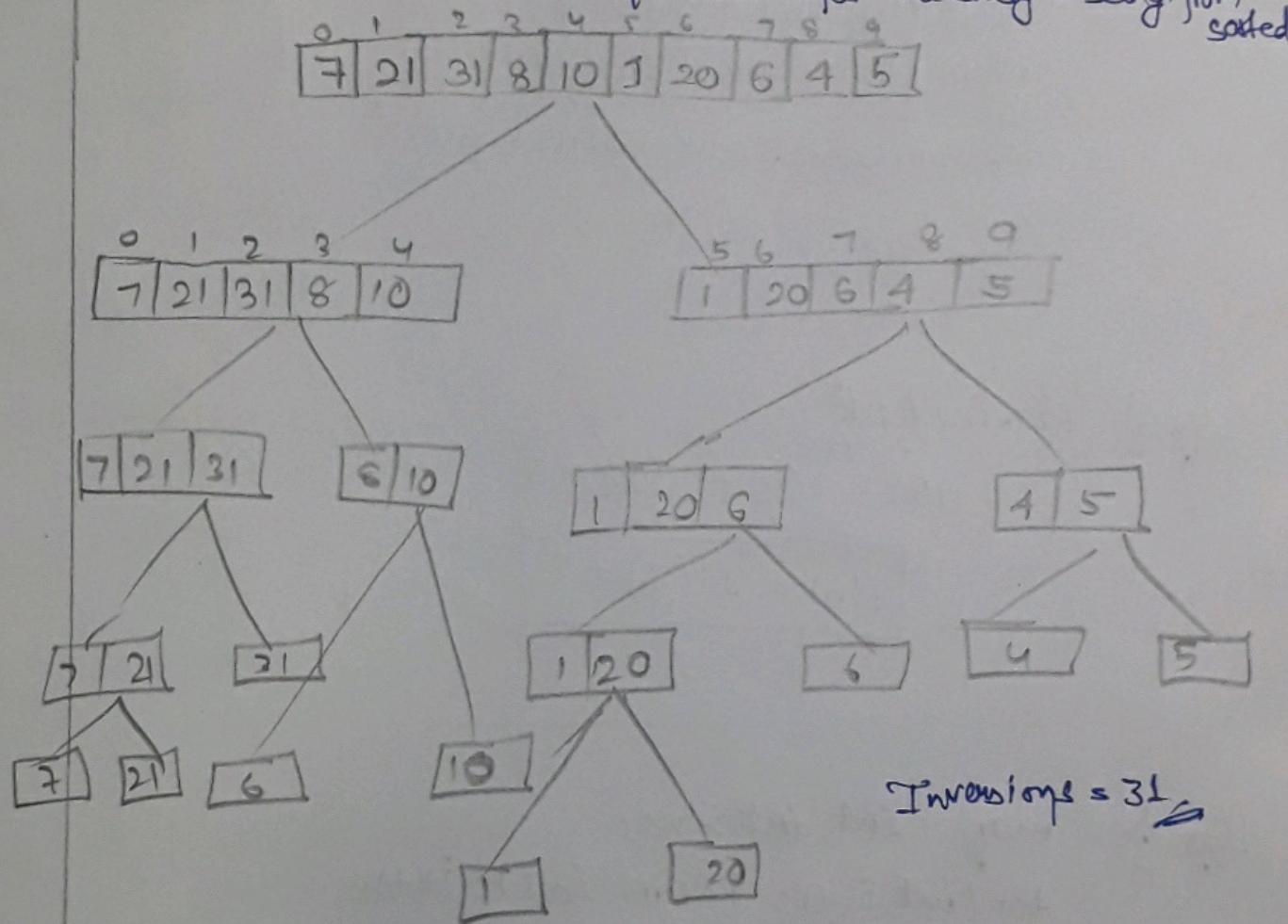
```
map<int, int> m;
for (int i = 0; i < arr.size(); i++)
{
    if (m.find(target - arr[i]) != m.end())
        m[arr[i]] = 1;
    else
    {
        cout << i << " " << m[arr[i]];
    }
}
```

Ans 8)

Quick Sort is the fastest general purpose. In most practical situations, Quick Sort is the method of choice. If stability is important and space is available, Merge Sort is best.

Ans 9)

Inverse indicators :- It indicates how close or far the array is being from sorted.



Inversions = 31

Ans 10)

Worst Case :-

It occurs when the pivot element is always an extreme (smallest or largest) element. This happens when input array is sorted and reversed and either first or last element is picked as pivot.

$$O(n) = n^2$$

Best Case :-

It occurs when pivot element is the middle element or near to the middle element.

$$O(n) = n \log n$$

Ans 11/2

Merge sort :-  $T(n) = 2T(\frac{n}{2}) + O(n)$

Quick sort :-  $T(n) = 2T(\frac{n}{2}) + n+1$

Parameter	Quick sort	Merge Sort
Partition	Splitting is done in any ratio.	Array is partitioned into just two halves.
Working	Smaller array	Fine on any size of array
Add'l space	Less (inplace)	More (not inplace).
Efficiency	Inefficient on larger array.	Effective on all types of array.
Sorting Method	Internal	External.
Stability	Not stable	stable.

Ans 12/2

stable selection sort :-

void stableSelectionSort (int a[], int n)

{

for (i = 0 to n-1; i++)

{

int min = i;

for (j = i+1 to n; j++)

if (a[min] > a[j])

min = j;

int key = a[min]

while (min > i)

{

a[min] = a[min-1];

min--;

? a[i] = key; ?

Ans 13)

```
void BubbleSort (int arr[], int n)
{
    int i, j;
    bool swapped;
    for (i = 0 to n - 1; i++)
    {
        swapped = false;
        for (j = 0 to n - i - 1; j++)
        {
            if (arr[j] > arr[j + 1])
            {
                swap(arr[j], arr[j + 1]);
                swapped = true;
            }
        }
        if (swapped == false)
            break;
    }
}
```

Ans 14)

We will use Mergesort because we can divide the 4GB data into 4 packets of 1GB and sort them separately and combine them later.

Internal Sorting :-  
All the data is sorted in memory at all times during sorting is in progress.

External Sorting :-

All the data is sorted is outside memory and loaded into memory in small chunks.

# ASSIGNMENT :- 04

Ans 1

$$T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$a=3; b=2 \quad f(n) = n^2$$

$$n^{\log_3 3} = n^{\log_2 3}$$

Comparing,  
 $n^{\log_2 3} < n^2$  [Case III]

According to master theorem;

$$T(n) = \Theta(n^2). \quad \text{Ans}$$

Ans 2

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$a=4; b=2$$

$$n^{\log_2 4} = n^{\log_2 2^2} = n^2 = f(n) \quad [\text{Case II}]$$

According to Master's theorem;

$$T(n) = \Theta(n^2 \log n). \quad \text{Ans}$$

Ans 3

$$T(n) = T\left(\frac{n}{2}\right) + 2^n$$

$$a=1; b=2$$

$$n^{\log_2 1} = n^0 = 1 < 2^n \quad (\text{Case III'})$$

According to Master's theorem;

$$T(n) = \Theta(2^n). \quad \text{Ans}$$

Ans 4

$$T(n) = 2T\left(\frac{n}{2}\right) + n^n$$

Master's theorem is not applicable as its  $\alpha$  is not a function of  $n$ .

Aus 5)

$$T(n) = 16T(n/4) + n$$

$a=16, b=4; f(n)=n$

$$n^{\log_b a} = n^{\log_4 16} = n^4 > f(n) - (\text{Case I})$$
$$T(n) = \Theta(n^4) - \text{Ans}$$

Aus 6)

$$T(n) = 2T(n/2) + n \log n$$

$a=2, b=2; f(n)=n \log n$

$$n^{\log_b a} = n^{\log_2 2} = n < f(n) - (\text{Case III})$$
$$T(n) = \Theta(n \log n) - \text{Ans.}$$

Aus 7)

$$T(n) = 2T(n/2) + n \log n$$

$a=2, b=2; f(n)=n \log n$

$$n^{\log_b a} = n^{\log_2 2} = n > f(n) - (\text{Case II})$$
$$T(n) = \Theta(n) - \text{Ans.}$$

Aus 8)

$$T(n) = 2T(n/4) + n^{0.5}$$

$a=2, b=4; f(n)=n^{0.5}$

$$n^{\log_b a} = n^{\log_4 2} = n^{0.5} < f(n) - (\text{Case III})$$
$$T(n) = \Theta(n^{0.5}) - \text{Ans.}$$

Aus 9)

$$T(n) = 0.8T(n/2) + kn$$

Master's theorem is not applicable as  $\boxed{a < 1}$ .

Aus 10)

$$T(n) = 16T(n/4) + n!$$
$$n^{\log_b a} = n^{\log_4 16} = n^4 < n! - (\text{case III})$$
$$T(n) = \Theta(n!) - \text{Ans.}$$

Aus 11)

$$T(n) = 4T(n/2) + \log n$$
$$n^{\log_b a} = n^2 > f(n) - (\text{case I})$$
$$T(n) = \Theta(n^2) - \text{Ans.}$$

Aus 12)

Master's method not applicable

$$T(n) = \text{constant} T(n/2) + \log n$$

Master's theory is not applicable as  $\alpha$  is not a constant.

Aus 13)

$$T(n) = 3T(n/2) + n$$

$$\alpha = 3; b = 2; f(n) = n$$

$$n \log_b \alpha = n \log_2 3 > n \quad (\text{Case I})$$

$$\text{Hence;} T(n) = \underline{\Theta(n \log_2 3)}. \quad \text{Ans}$$

Aus 14)

$$T(n) = 3T(n/3) + \sqrt{n}$$

$$\alpha = 3; b = 3; f(n) = \sqrt{n}$$

$$n \log_b \alpha = n \log_3 3 = n > \sqrt{n} \quad (\text{Case I})$$

$$\text{Hence;} T(n) = \underline{\Theta(n)} \quad \text{Ans}$$

Aus 15)

$$T(n) = 4T(n/2) + cn$$

$$\alpha = 4, b = 2; f(n) = cn$$

$$n \log_b \alpha = n \log_2 4 = n^2 > cn \quad [\text{Case I}]$$

$$\text{Hence;} T(n) = \underline{\Theta(n^2)} \quad \text{Ans}$$

Aus 16)

$$T(n) = 3T(n/4) + n \log n$$

$$\alpha = 3, b = 4; f(n) = n \log n$$

$$n \log_b \alpha = n \log_4 3 < n \log n \quad [\text{Case III}]$$

$$\text{Hence;} T(n) = \underline{\Theta(n \log n)} \quad \text{Ans}$$

Aus 17)

$$T(n) = 3T(n/3) + n/2$$

$$\alpha = 3, b = 3; f(n) = n/2$$

$$n \log_b \alpha = n \log_3 3 = n \Leftrightarrow n/2 \quad (\text{Case II})$$

$$\text{Hence;} T(n) = \underline{\Theta(n \log n)} \quad \text{Ans}$$

Aus 18)  $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$   
 $a=6; b=3; f(n)=n^2 \log n$   
 $n \log_b a = n \log_3 6 < f(n) - (\text{Case III})$   
 $T(n) = \Theta(n^2 \log n) - (\text{Ans})$

Aus 19)  $T(n) = 4T\left(\frac{n}{2}\right) + n \log n$   
 $a=4; b=2; f(n)=n \log n$   
 $n \log_b a = n \log_2 4 = n^2 > f(n) - (\text{Case I})$   
 $T(n) = \Theta(n^2) - (\text{Ans})$

Aus 20)  $T(n) = 6T\left(\frac{n}{8}\right) + n^2 \log n$   
Master's theory not applicable as it is not a INCREASING FUNCTION.

Aus 21)  $T(n) = 7T\left(\frac{n}{3}\right) + n^2$   
 $a=7; b=3; f(n)=n^2$   
 $n \log_3 7 = n^{1.1} < f(n) - (\text{Case III})$   
 $T(n) = \Theta(n^2) - (\text{Ans})$

Aus 22)  $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$   
Master's theory is not applicable since regular condition is isolated.

# ASSIGNMENT:- 05

Ans 1)

## BFS

- uses queue data structure.
- Stands for Breadth First Search
- can be used to find single source shortest path in an unweighted path in an unweighted graph and we reach a vertex with min. no. of edges from a source vertex.
- Siblings are visited before children.

## Applications:-

- Shortest Path & minimum spanning tree in unweighted graph.
- Peer to peer networks.
- Social Networking Websites.
- GPS Navigation Systems.

## DFS

- uses stack data structure.
- Stands for Depth First Search
- we might traverse through more edges to reach a destination vertex from source.
- Children are visited before siblings.

## Applications:-

- Detecting cycle in graph.
- Path finding.
- Topographical Sorting.
- Solving puzzles with only 1 solution.

Ans 2)

In BFS; we use queue data structure as queue is used when things don't have to be processed immediately, but have to be processed immediately, but have to be processed in FIFO order like BFS.

In DFS; stack data structure is used as it is beneficial for backtracking. For DFS, we retrieve it from root to the farthest node as much as possible; giving it a LIFO like approach.

Ans 3)

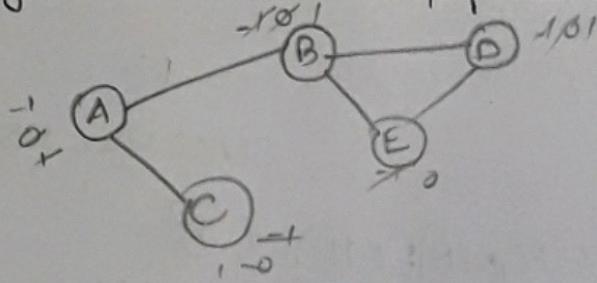
Dense Graph is a graph in which no of edges is close to the maximal no. of edges.

Sparse Graph is a graph in which the no. of edges is close to the minimal no. of edges. It can be disconnected graph.

Adjacency lists are preferred for sparse Graph & Adjacency Matrix for Dense Graph.

Aus 4.)

### Cycle detection in directed graph (BFS)



-1 - unvisited.  
0 - into queue.  
1 - traversed.

Queue: 

A	B	C	D	E
---	---	---	---	---

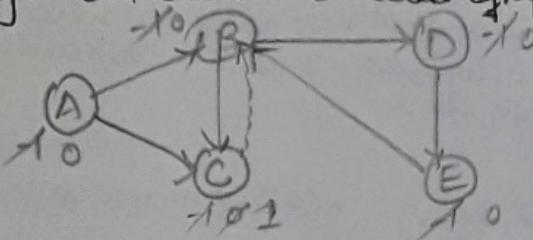
When D checks its adjacent vertices it finds E with 0.

Visited set: 

A	B	C	D
---	---	---	---

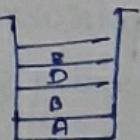
If any vertex finds the adjacent vertex with flag 0, then it contains cycle.

### Cycle detection in directed graph (DFS)



-1 - unvisited.  
0 - visited & stack push.  
1 - visited & popped.

Stack:-



visited:-  
ABCDE  
 $\rightarrow D \rightarrow D \rightarrow E \rightarrow B$

Here; E find B with 0.  
 $\Rightarrow$  It's a cycle.

Parent Map	
Vertex	Parent
A	-
B	A
C	B
D	B
E	D

Aus 5)

The DISJOINT SETS data structure is also known as Union-find data structure and merge-find set. It is a data structure that contains a collection of disjoint or non-overlapping sets.

The DISJOINT SETS means when the set of partitioned into the disjoint subsets, various operations are performed on it.

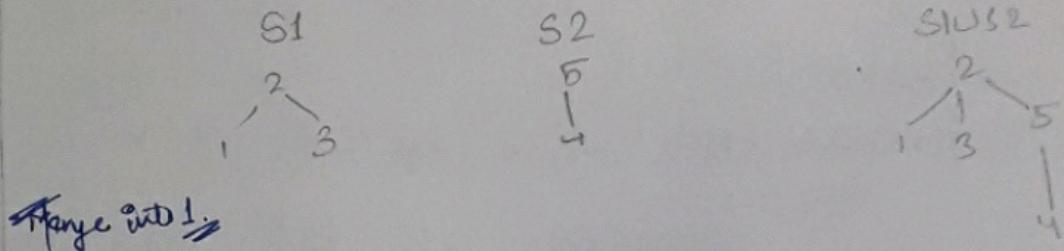
In this case, we can add new set, merge them & also find the representative member of a set. It also allows to find out whether the two elements are at same set or no effectively.

### Operations on Disjoint sets:-

(1) Union:-

(a) If sets  $S_1$  &  $S_2$  are 2 disjoint sets, then union  $S_1 \cup S_2$  is a set of all elements  $\alpha$  such that  $\alpha$  is in either  $S_1$  or  $S_2$ .

- (b) As the sets should be disjoint  $S_1 \cup S_2$  replace  $S_1$  &  $S_2$  which no longer exists  
 (c) It is achieved by simply making one of the trees as a subtree of other, that is; set parent field of one of the roots of the trees to other root.



(2) Find:-

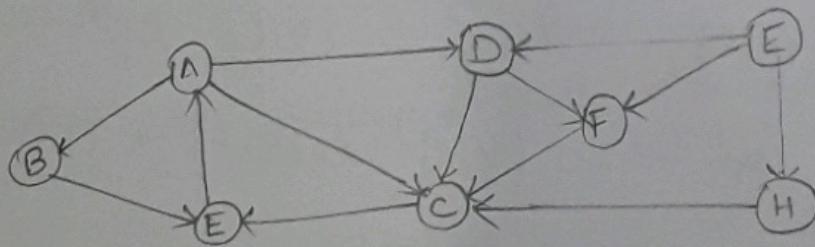
Given an element of; to find the set containing it.

Eg:-  $\text{find}(3) \Rightarrow S_1$   
 $\text{find}(4) \Rightarrow S_2$

(3) Make Set(x):-

Create a set containing &  
 $\text{makeset}(1) = \{1\}$

Ans:-



BFS:- child    G    D    F    H    C    E    A    B  
 Parent -    G    G    G    H    C    E    A

Path:-  
 $G \rightarrow H \rightarrow C \rightarrow E \rightarrow A \rightarrow B$

DFS:- Node Visited

G  
D  
H  
F  
C  
F  
A  
B

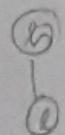
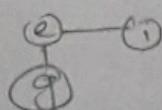
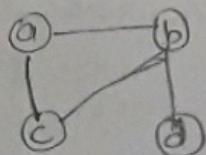
Stack

G  
F  
C  
E  
A  
B

Path

$G \rightarrow F \rightarrow C \rightarrow E \rightarrow A \rightarrow B$

Aus7



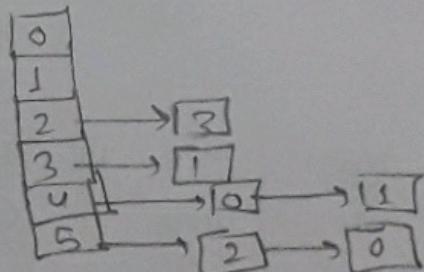
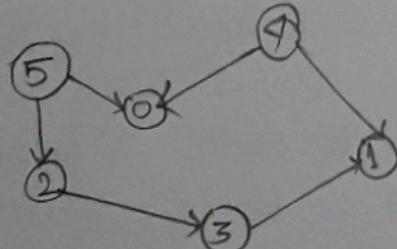
$$V = \{a, b, c, d, e, f, g, h, i, j, l, m\}$$

$$E = \{(a, b), (a, c), (b, c), (b, d), (e, i), (e, g), (h, l), (j, m)\}$$

	<u>vertices</u>	$\{a\}$	$\{b\}$	$\{c\}$	$\{d\}$	$\{e\}$	<del><math>\{f\}</math></del>	$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(a, b)$	$\{a, b\}$	$\{c\}$	$\{d\}$	$\{e\}$	—			$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(a, c)$	$\{a, b, c\}$		$\{d\}$	$\{e\}$				$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(b, d)$	$\{a, b, c\}$		$\{d\}$	$\{e\}$				$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(b, d)$	$\{a, b, c, d\}$		$\{e\}$					$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(e, i)$								$\{g\}$	$\{h\}$	$\{i\}$	$\{j\}$	$\{l\}$
$(e, g)$												
$(h, l)$												
$(j, m)$												

Finally -  $\{a, b, c, d\}$ ,  $\{e, g\}$ ,  $\{h, l\}$  &  $\{j\}$

Aus8



algo:-

- (1) Go to node 0, it has no outgoing edges and push node 0 into the stack & mark as visited.
- (2) Go to node 1, again it has no outgoing edges so push it in stack & mark as visited.
- (3) Go to node 2, process all adjacent and mark it visited.
- (4) Node 3 is visited, so continue with next node.
- (5) Go to node 4, all the adjacent nodes are already visited, so push it in stack & mark as visited.
- (6) Go to node 5, push it in stack stop.

Output:-  
5 4 2 3 1 0

Ans 9.)

Heap is generally preferred for priority queue implementation because heap provide better performance compared to array & linked list.

Algorithms when priority queue is used:-

(1) Dijkstra's Algo :-

shortest path algorithm ; when the graph is stored in the form of adjacency list or matrix ; priority queue can be used to extract minimum effectively when implementing it.

(2) Prim's Algo :-

To store keys of Nodes & extract minimum key node at every step.

Ans 10.)

Min Heap

- For every pair of parent & child nodes, parent node always has <sup>lower</sup> value than descendent child node.
- Value of nodes increases as we traverse from root to leaf node.
- Root node has the lowest value.

Max Heap

- For every pair of the parent and descendant child node, the parent has higher value than child node.
- Value of nodes decreases as we traverse from root to leaf node.
- Root node has the largest value.

# ASSIGNMENT:- 06

Aus 1)

## MINIMUM SPANNING TREE:-

It is a subset of the edges of a connected edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

## Applications:-

- Consider  $n$  stations are to be linked using a communication network, and laying of communication link b/w any 2 stations involves a cost; so we use MST for a better output.
- Same goes with roadways & highways & airlines.
- Design LAN
- Laying Pipelines connecting off-shore drilling sites, refineries & markets.

Aus 2)

## Algorithm

Prim's Algorithm

Time Complexity

$O(V)$

Dijkstra's Algorithm

$O(V^2)$

$O(V^2)$

Kruskal's Algorithm

$O(E \log V)$

$O(VI)$

Bellman Ford

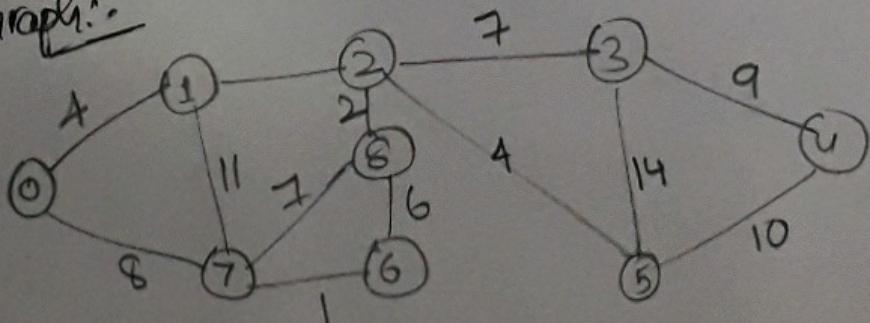
$O(VE)$

$O(V)$

Space Complexity

Aus 3)

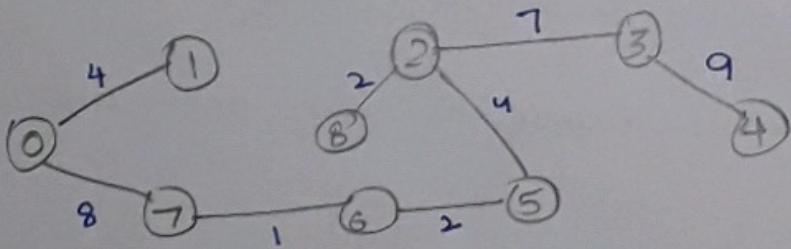
## Graph:-



0  
6  
5  
2  
0  
2  
6  
2  
7  
0  
1  
4  
4  
1  
3

v  
7  
6  
8  
1  
5  
8  
3  
8  
7  
2  
3  
3  
7  
5

w  
1  
2  
2  
4  
4  
6  
7  
7  
8  
8  
9  
10  
11  
14



$$\text{wgt.} = [37] - 4v_2$$

Prims Algo:-

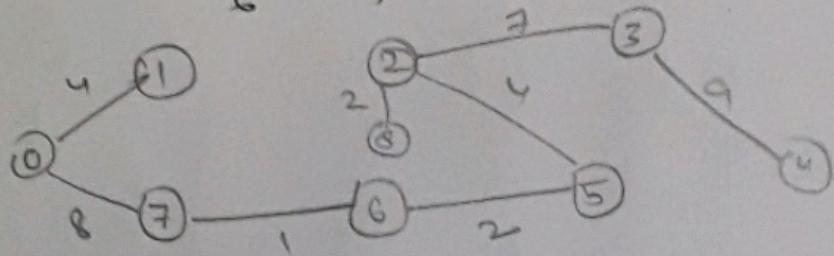
Weight:-



1	2	3	4	5	6	7	8
2	∞	∞	∞	∞	∞	2	2
4	∞	∞	∞	∞	∞	3	6
4	8	∞	∞	∞	1	8	7
11	8	7	∞	1	1	8	2
11	8	7	2	2	1	8	6
4	14	1	10	2	1	8	5
4	7	1	10	2	1	8	6
4	7	1	9	2	1	8	6

Breadth:-

0	1	2	3	4	5	6	7	8
-1	-1	-1	-1	-1	-1	-1	-1	-1
6	1	1	1	1	1	1	1	1



Wgt. = 37.

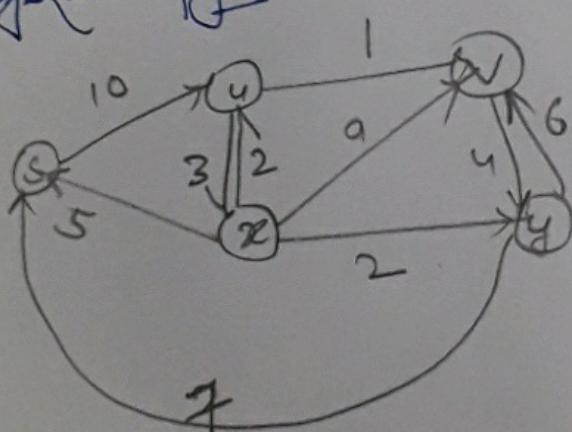
Ans 4)

i) Shortest path may change. The reason is that there may be different paths from 's' to 't'. For example:- let shortest path be of weight 15 & edge 5. Let there be another path with 2 edges and total weight 25. Hence, the shortest path increases. Max weight of other paths are also increased.

ii) If we multiply all edges wgt by 10, shortest path won't change. Only the weight will be increased by 10x otherwise it will follow the same path.

Ans 5)

Dijkstra's Algo:-



Node

U

X

Y

Z

Shortest distance from source node

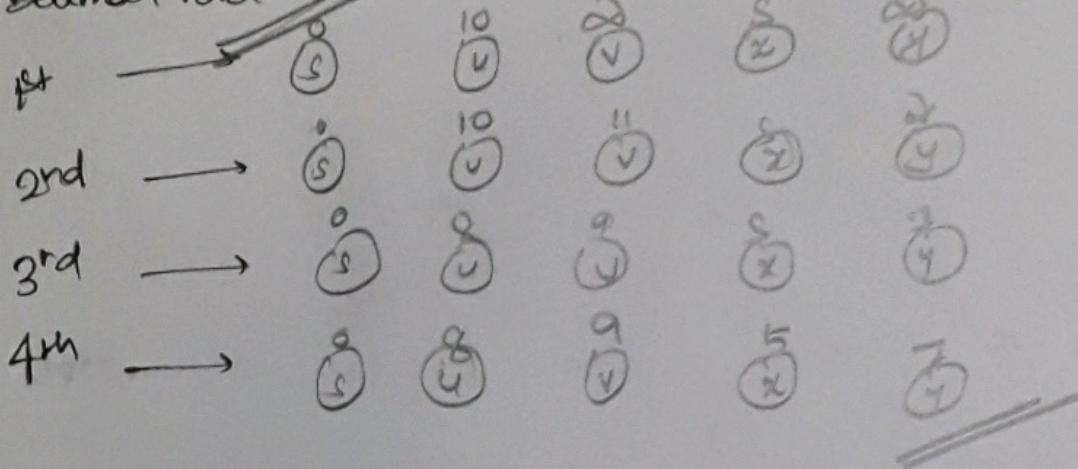
8

5

9

7

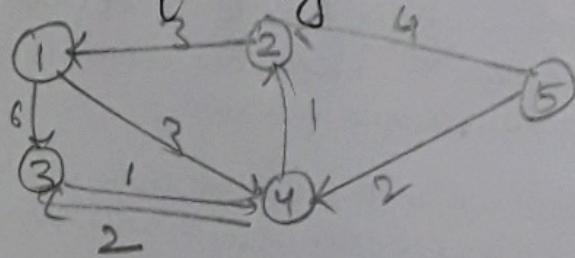
~~Bellman Ford:-~~



No cycle exists.

~~Question~~

Floyd Warshall Algo:-



$$TC = O(V^3)$$

$$SC = O(V^2)$$