

# Row/Column Dominance

# Implementation Details

```
class Record:
    binary: str
    used: bool = False
    one_count: int
    representing_numbers: Set[int]

    def __init__(self, binary, representing_numbers):
        self.binary = binary
        self.one_count = self.binary.count("1")
        self.representing_numbers = representing_numbers

    def xor(self, x: str):
        # equivalent to hamming distance
        lhs = self.binary[:]
        rhs = x[:]
        ret_string = ""
        for lhs_char, rhs_str in zip(lhs, rhs):
            if lhs_char == rhs_str:
                ret_string += "0"
            else:
                ret_string += "1"
        return ret_string

    def __repr__(self):
        return f"({self.binary}: {' '.join([str(x) for x in sorted(self.representing_numbers)]))}"

    def __str__(self):
        return_string = ""
        for index, char in enumerate(self.binary):
            if char == "-": continue
            elif char == "0":
                return_string += f"x{index + 1}"
            else:
                return_string += f"x{index + 1}"
        return return_string

    def __eq__(self, other):
        return self.binary == other.binary
```

- binary: string representation
- used: was this term used during combination?
- one\_count: how many '1's are present?
- representing\_numbers: what terms does this cover?
- xor: hamming distance
- rest are for debugging and printing

# Function details

```
n_vars = 4; minterms = {0, 2, 5, 6, 7, 8, 10, 12, 13, 14, 15}; dontcares = set()
keep_running = True
n_iterations = 0

pis = find_pi(n_vars, minterms, dontcares)
epis = set()
while keep_running:
    print("Iteration", n_iterations)

    #pis = pis.intersection(find_pi(n_vars, minterms, dontcares))

    epis = epis.union(find_epi(pis, dontcares)) # calculate epi

    pis = pis - epis

    epi_covered_numbers = set()
    for epi in epis:
        epi_covered_numbers.update(epi.representing_numbers)

    if (minterms - dontcares).issubset(epi_covered_numbers): # check if EPI covers all implicants
        pis = set()
        break

    excluded_columns = column_dominance(pis, epis, minterms, dontcares) # column dominance
    if not excluded_columns: keep_running = False
    dontcares = dontcares.union(excluded_columns)
    excluded_terms = row_dominance(pis, epis, minterms, dontcares) # row dominance
    for binary in excluded_terms:
        remove_something = None
        for record in pis:
            if record.binary == binary:
                remove_something = record
                break

        if remove_something:
            pis.remove(remove_something)

    if not excluded_terms:
        keep_running = keep_running | False
```

- 1) Find PI and EPI
- 2) Check if EPI covers all minterms
- 3) Perform column dominance, removing columns
- 4) Perform row dominance, removing Pis
- 5) Run until no further updates are possible

# Column Dominance

```
def column_dominance(pi_records: Set[Record], epi_records: Set[Record], minterms: Set[int], dontcares: Set[int]) -> Set[int]:
    # returns columns to exclude
    epi_covered_terms = set()
    for epi in epi_records:
        epi_covered_terms.update(epi.representing_numbers)

    target_columns = (minterms - dontcares) - epi_covered_terms
    column_dict = {}
    for column in target_columns:
        column_dict[column] = set()
    for pi in pi_records:
        if pi in epi_records: continue
        for number in pi.representing_numbers:
            if number in column_dict:
                column_dict[number].update(pi.representing_numbers)
            else:
                column_dict[number] = set(pi.representing_numbers)

    for col1, cover1 in column_dict.items():
        if not col1: continue
        for col2, cover2 in column_dict.items():
            if col1 == col2: continue
            if not cover2: continue
            if cover2.issubset(cover1):
                print(f"minterm {col1} dominates minterm {col2}")
                if col1 in target_columns:
                    target_columns.remove(col1)

    return (minterms - dontcares) - epi_covered_terms - target_columns
```

Record data structure makes it easy to compare implicants. Just check if representing numbers of an implicant is a subset of another implicant.

# Row Dominance

```
def row_dominance(pi_records: Set[Record], epi_records: Set[Record], minterms: Set[int], dontcares: Set[int]) -> Set[str]:
    # returns binary strings of records to exclude
    epi_covered_numbers = set()
    for epi in epi_records:
        epi_covered_numbers.update(epi.representing_numbers)

    excluded_terms = set()

    for c1_record in pi_records - epi_records:
        for c2_record in pi_records - epi_records:
            if c1_record.binary == c2_record.binary: continue
            c1_cover = (c1_record.representing_numbers - dontcares) - epi_covered_numbers
            c2_cover = (c2_record.representing_numbers - dontcares) - epi_covered_numbers

            if c2_cover.issubset(c1_cover) and c1_record.binary not in excluded_terms:
                print(f"{c1_record} dominates {c2_record}")
                excluded_terms.add(c2_record.binary)

    return excluded_terms
```

Even easier for row dominance!

# Example 1

- From the lecture notes
- –  $f(x_1, \dots, x_4) = \sum (m(0,4,8,10,11,12) + d(13,15))$
- Result:

```
Iteration 0
x1x2'x3 dominates x1x2x4
x1x2'x3 dominates x1x2'x4'
x1x2'x3 dominates x1x2x3'
x1x2'x3 dominates x1x3x4
pi: {(101-: 10,11)}
epi: {(--00: 0,4,8,12)}
final terms: {(--00: 0,4,8,12), (101-: 10,11)}
standard form: f = x3'x4' + x1x2'x3
```

```
Process finished with exit code 0
```

```
|
```

# Example 2

- $F = m(2, 3, 7, 9, 11, 13) + d(1, 10, 15)$

```
Iteration 0  
final terms: {(-01-: 2,3,10,11), (--11: 3,7,11,15), (1--1: 9,11,13,15)}  
standard form: f = x2'x3 + x3x4 + x1x4  
  
Process finished with exit code 0
```

Verify:

## Essential Prime Implicants

Using column/row dominance methods.

[Tutorial](#) 

$$\boxed{AD} + \boxed{CD} + \boxed{B'C}$$

# Example 3

- $F = m(0, 2, 5, 6, 7, 8, 10, 12, 13, 14)$

```
Iteration 0
minterm 14 dominates minterm 12
minterm 14 dominates minterm 6
x1x4' dominates x1x2
x2x3 dominates x3x4'
pi: {(1--0: 8,10,12,14), (-11-: 6,7,14,15)}
epi: {(-0-0: 0,2,8,10), (-1-1: 5,7,13,15)}
Iteration 1
final terms: {(1--0: 8,10,12,14), (-0-0: 0,2,8,10), (-11-: 6,7,14,15), (-1-1: 5,7,13,15)}
standard form: f = x1x4' + x2'x4' + x2x3 + x2x4
Process finished with exit code 0
```

Verify:

## Essential Prime Implicants

Using column/row dominance methods.

[Tutorial](#) 

$$BD + AD' + CD' + B'D'$$

or

$$BC + BD + AD' + B'D'$$

or

$$AB + BD + CD' + B'D'$$

or

$$AB + BC + BD + B'D'$$