Spanning Star Forest Problem

Adam Starak

May 16, 2019

Star definition

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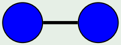

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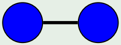

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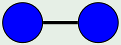

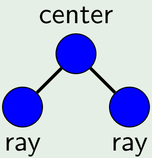

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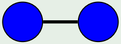
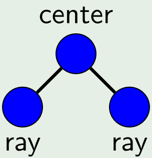
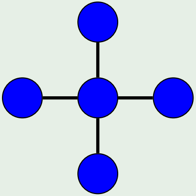




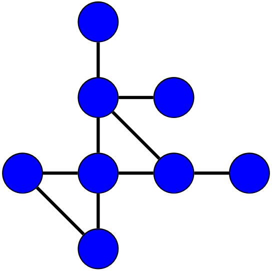


Spanning Star Forest Problem

Definition

Given a graph G decide whether it has a *Spanning Star Forest*.

Example



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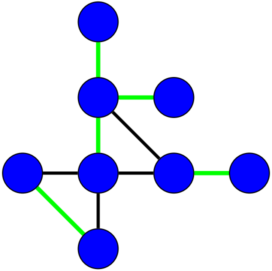

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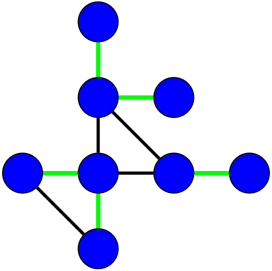

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G contains a Spanning Star Forest if and only if it does not contain any isolated vertices.

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Corollary

Spanning Star Forest problem can be solved in **linear time**.

Minimal Spanning Star Forest

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Given a pair (G, k) decide whether there exists a Spanning Star Forest of at most $\mathbf{k}$.

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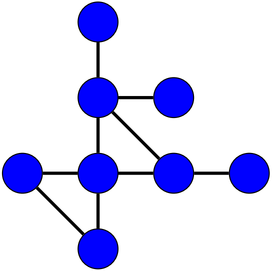
Given a pair (G, k) decide whether there exists a Spanning Star Forest of at most **k**.

Dominating Set

Given a pair (G, k) find a set $D \subseteq V(G)$ such that $|D| \leq k$ and every node is either in D or adjacent to D.

Examples

$(G, 3)$ for Dominating Set.



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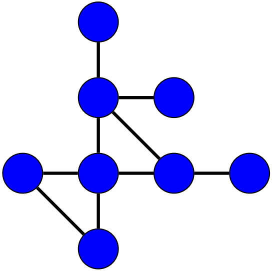

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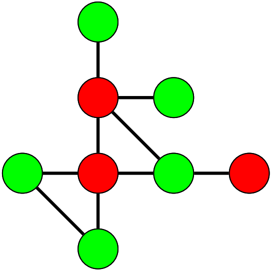

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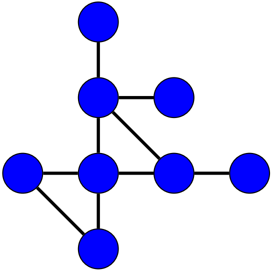

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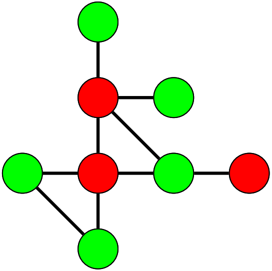

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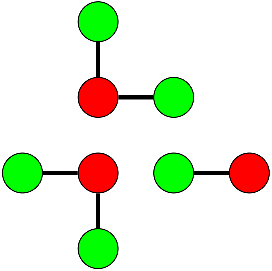

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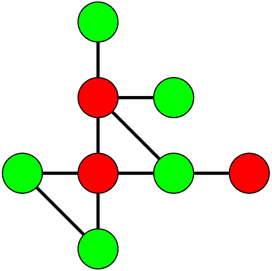

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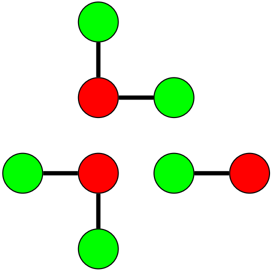

Observation

There is a relationship between:

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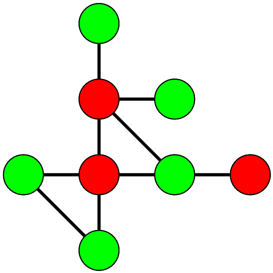

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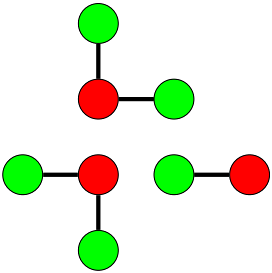

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There is a relationship between:

- **Centers** and **dominating** vertices.

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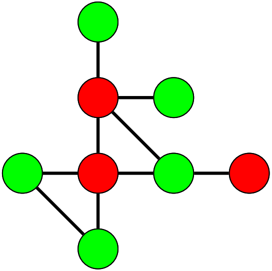

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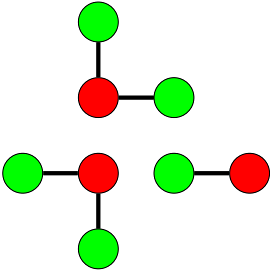

Observation

There is a relationship between:

- **Centers** and **dominating** vertices.
- **Rays** and **dominated** vertices.

Minimal Spanning Star Forest is NP-complete

Theorem

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Replace every isolated vertex $\mathbf{v}$ with an isolated edge $(\mathbf{v}, \mathbf{v}')$.

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(G, k) has a solution if and only if (G', k) has one.

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Proof on the board.

Reverse reduction

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$$(G', k') = \begin{cases} (G, 0), & \text{if } G \text{ contains an isolated vertex.} \\ (G, k), & \text{otherwise.} \end{cases}$$

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Corollary

Dominating Set and Minimal Spanning Star Forest are interreductible.

Transfering theorems

Theorem

Unless SETH fails, there is no algorithm solving Dominating Set in time $\mathcal{O}^*(n^{k-\epsilon})$ for $\epsilon > 0$.

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(G, k) - Dominating Set instance.

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Contradiction.

Spanning Star Forest Extension Problem

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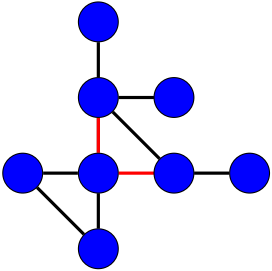
Given a graph G and a set of **forced edges** $M \subseteq E(G)$, find a Spanning Star Forest S such that $M \subseteq E(S)$.



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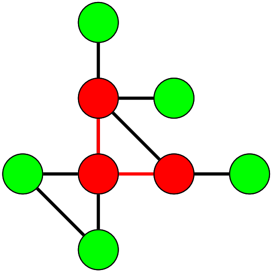
$R = V \setminus F$



Instance normalization

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Corollary

After exhaustive application of the R3, M is a matching.

Instance normalization 2

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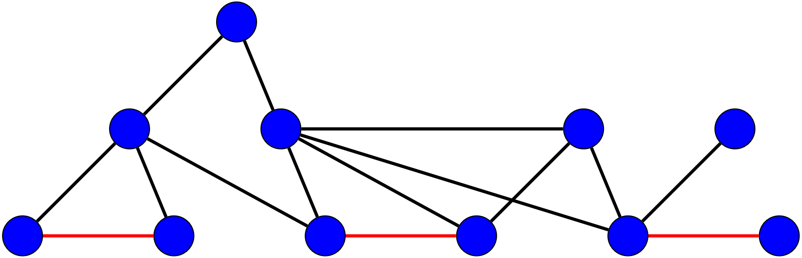

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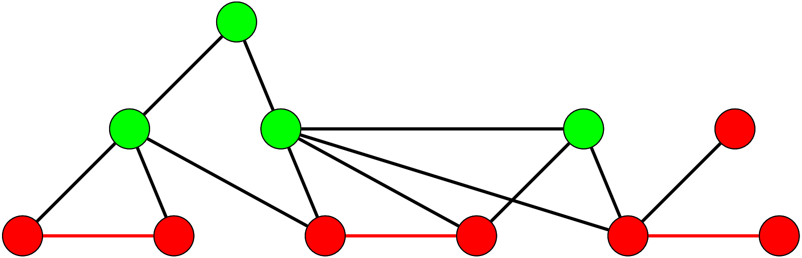

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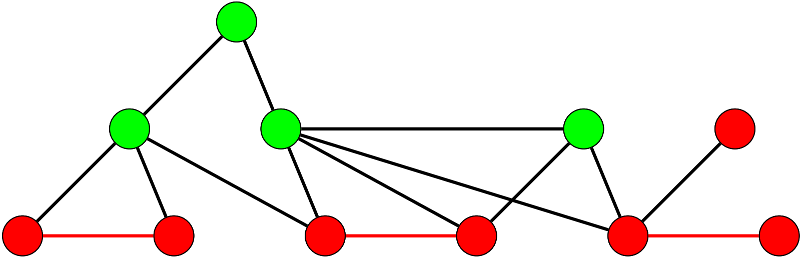

Corollary

G_P has always a solution.

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Let S be a solution for G_{NP}.

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Let S be a solution for G_{NP}.

Partition G into (G_P, G_{NP}).

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Reduction Rules

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(R4) Apply $G = G_{NP}$.

SSFE is NP-complete

Theorem

There exists a reduction from CNF-SAT to SSFE.

SSFE is NP-complete

ϕ CNF-formula.

$$(x_1 \lor x_2 \lor \neg x_4) \land (\neg x_1 \lor x_3 \lor \neg x_5) \land (x_4 \lor x_5 \lor \neg x_1)$$

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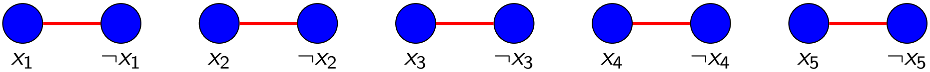
Variable $\mathbf{x_i}$ $\rightarrow$ **vertices** $x_i, \neg x_i$ and **marked edge** $(x_i, \neg x_i)$.

$$(x_1 \vee x_2 \vee \neg x_4) \wedge (\neg x_1 \vee x_3 \vee \neg x_5) \wedge (x_4 \vee x_5 \vee \neg x_1)$$



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Clause C_i $\rightarrow$ **vertex** c_i.

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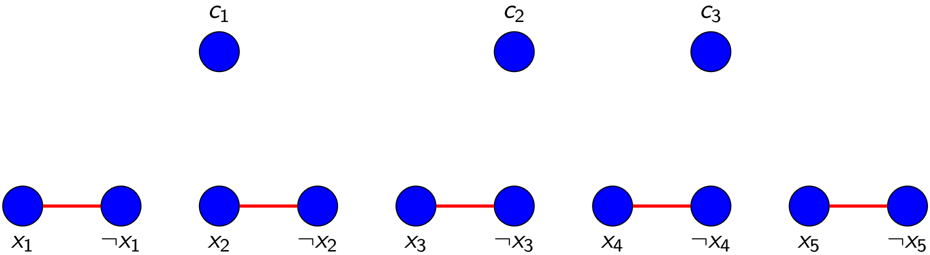

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literal $\neg x_k$ in clause c_i $\rightarrow$ **edge** $(\neg x_k, c_i)$.

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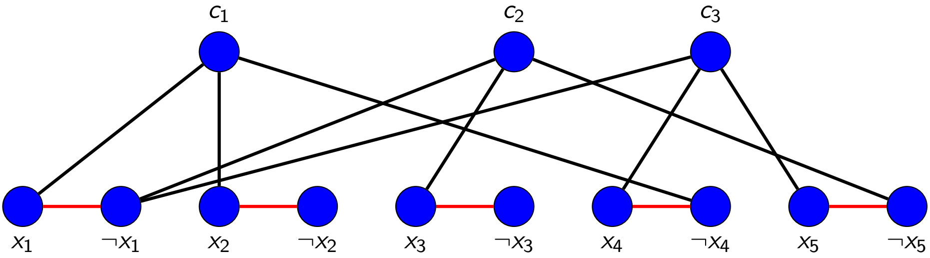

SSFE is NP-complete

Lemma

ϕ is satisfiable if and only there exists a SSFE.

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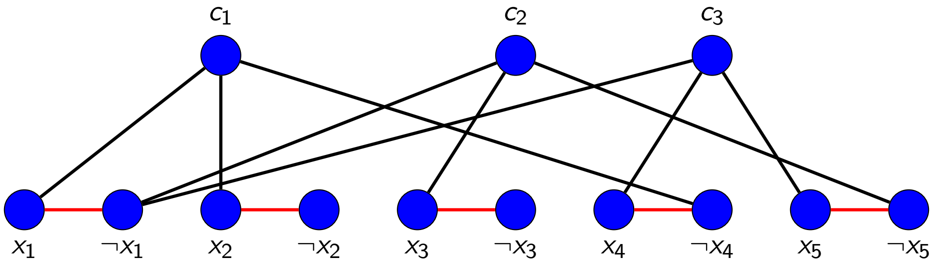

Reverse reduction

Observation

There is a relationship between:

- **M** and **variables**.
- **R** and **clauses**.

Reverse reduction

Theorem

There is a reduction from SSFE to CNF-SAT

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Construction.

Reverse reduction

Theorem

There is a reduction from SSFE to CNF-SAT

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$\phi \rightarrow$ formula of $|M'|$ variables and $|R|$ clauses.

Lemma

(G, M) has a SSF $\iff \phi$ is satisfiable.

Proof on the board.

Wrap up

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Corollary

Spanning Star Forest Extension and CNF-SAT are interreductible.

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n variables	n marked edges
m clauses	$\lvert R \rvert = m$

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Observation 1

Theorems for CNF-SAT paramterized by the number of **variables** can be transfered to Spanning Star Forest Extension parameterized by the number of **marked edges**.

Wrap up

Corollary

Spanning Star Forest Extension and CNF-SAT are interreductible.

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Observation 2

Theorems for CNF-Sat parameterized by the number of **clauses** can be transfered to Spanning Star Forest Extension paramtereized by $\lvert \mathbf{R} \rvert$.

Summary

So far we proved that the following problems are interreductible:

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Dominating Set	Minimal SSF
CNF-Sat n variables	SSFE $\|M\|$
CNF-Sat m clauses	SSFE $\|R\|$

$|M|$ parametrization

Theorem

CNF-Sat with n variables doesn't admit any kernel.

$|M|$ parametrization

Theorem

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Consequently, nor does SSFE parameterized by $|M|$.

$|M|$ parametrization

Proving nonexistence of a kernel for a problem $\mathcal{P}$

Design an algorithm $\mathcal{A}$:

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$|R|$ parametrization

Theorem

CNF-Sat parameterized with m clauses has a linear kernel.

$|R|$ parametrization

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CNF-Sat parameterized with m clauses has a linear kernel.

Consequently, so does SSFE parameterized by $|R|$.

$|R|$ parametrization

Quadratic kernel

If there exists a vertex $v \in R$ such that $deg_G(v) > k$, then remove v.

Proof on the board.

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After exhaustive application of the rule, an instance contains:

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After exhaustive application of the rule, an instance contains:

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$\leq k + 2k^2$ vertices.

SSFE parameterized by $tw(G)$

Dynamic table. $dp[t, f]$ where t is node from a tree decomposition and $f : X_t \to \{true, false\}$ defined as follows:

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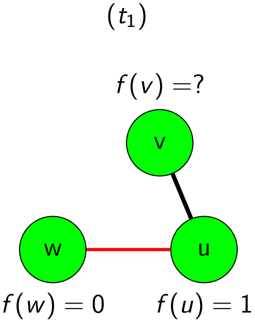
Assume we have everything finished **except for a join node**.

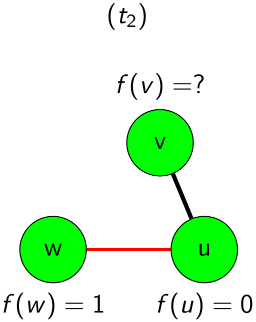
SSFE parameterized by $tw(G)$

Join nodes t_1, t_2

SSFE parameterized by $tw(G)$

Join nodes t_1**,** t_2

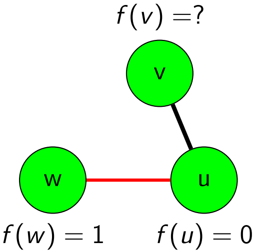




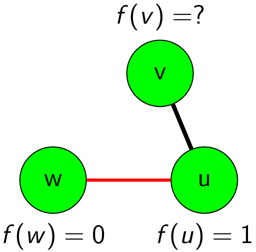
SSFE parameterized by $tw(G)$

Join nodes t_1**,** t_2

(t_1)

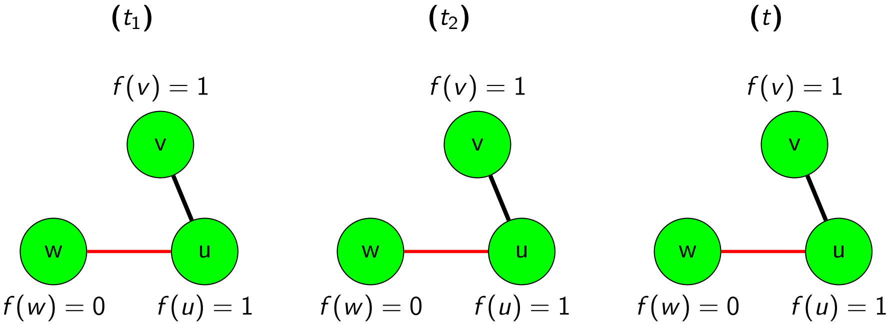


(t_2)



$C_1 =$ For every vertex $v \in (F \cap X_t)$ it holds $f(v) = f_1(v) = f_2(v)$

Join nodes t_1, t_2



SSFE parameterized by $tw(G)$

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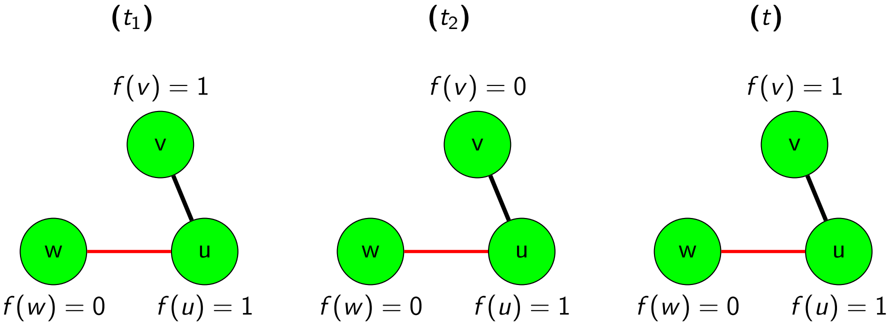

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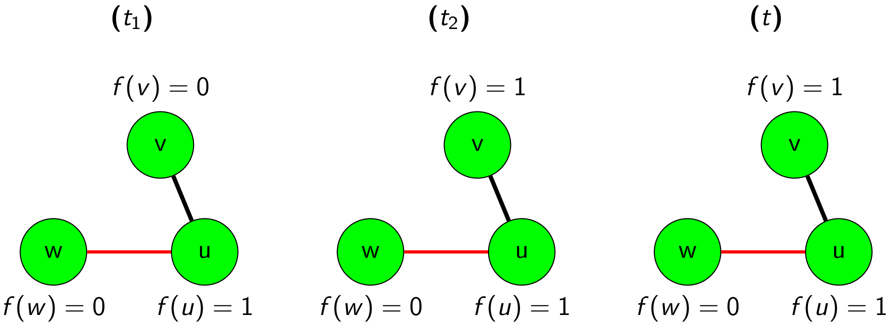

$C_2 =$ For all $v \in R$ it holds $f(v) = f_1(v) \vee f_2(v)$.

SSFE parameterized by $tw(G)$

SSFE parameterized by $tw(G)$

$$dp[t, f] = \begin{cases} \bigvee_{f_1, f_2} dp[t_1, f_1] \wedge dp[t_2, f_2], & \text{if } C_1 \text{ and } C_2 \text{ holds.} \\ \mathit{false}, & \text{otherwise.} \end{cases}$$

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Assuming leaf, introduce and forget are easy:

Lower bound

SSFE parameterized by $tw(G)$

SSFE parameterized by treewidth can be solved in $\mathcal{O}^*(2^{tw(G)})$.

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Lower bound

Assuming SETH, $\mathcal{O}^*(2^{tw(G)})$ is optimal.

Proof on the board.