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Hereby I confirm that the presented thesis was prepared under my supervision and that it fulfils the requirements for the degree of Master of Computer Science.

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Abstract

W pracy przedstawiono prototypową implementację blabalizatora różnicowego bazującą na teorii fetorów σ-ρ profesora Fifaka. Wykorzystanie teorii Fifaka daje wreszcie możliwość efektywnego wykonania blabalizy numerycznej. Fakt ten stanowi przełom technologiczny, którego konsekwencje trudno z góry przewidzieć.

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Introduction

Blabalizator różnicowy jest podstawowym narzędziem blabalii fetorycznej. Dlatego naukowcy z całego świata prześcigają się w próbach efektywnej implementacji. Opracowana przez prof. Fifaka teoria fetorów σ-ρ otwiera w tej dziedzinie nowe możliwości. Wykorzystujemy je w niniejszej pracy.

Chapter 1

Basic definitions

1.1. Structures

A simple graph G is a pair (V, E) where V denotes a set of vertices and E denotes a set of undirected edges. Let $deg_G(v)$ denote a degree of vertex v in graph G. Let $G \setminus \{v\}$ be the abbreviation for $G' = (V(G) \setminus \{v\}, E(G) \setminus \{(u, v) : u \in V(G)\})$. For a set $X \subset V(G)$ we denote $G[X]$ as a graph induced by vertices from X. A *tree* T is a graph where two vertices are connected by excatly one path. A *spanning tree* T of a graph G is a graph which includes all of the vertices of G, with minimum possible number of edges. A *star* S is a tree of size at least 2 for which at most 1 vertex has a degree greater than 1. A vertex in a *star* that has the greatest degree is called a *center* while the others are called *rays*.

All the further definitions are taken from

1.2. Parameterized complexity

Definition 1.1. *A parameterized problem is a language* $L \subseteq \Sigma^* \times \mathbf{N}$*, where* Σ *is a fixed, finite alphabet. For an instance* $(x, k) \in L$*,* k *is called the parameter.*

Definition 1.2. *For a parameterized problem* Q*, an FPT algorithm is an algorithm* $\mathcal{A}$ *which, for any input* (x, k)*, decides whether* $(x, k) \in Q$ *in time* $\mathcal{O}(f(k) \cdot n^c)$ *where* c *is independent of* n, c *and* f *is a computable function.*

Definition 1.3. *A kernel for a parameterized problem* Q *is an algorithm* $\mathcal{A}$ *that, given an instance* $(x, k) \in Q$*, works in polynomial time and returns an equivalent instance* $(x', k') \in Q$ *such that* $|x'| + k' \leq g(k)$ *for a computable function* g*.*

1.3. Graph decomposition

1.3.1. Path decomposition

A path decomposition of a graph G is a sequence $\mathcal{P} = (X_1, X_2, ..., X_r)$ of bags where $X_i \subseteq V(G)$ for each $i \in \{1, 2, ..., r\}$ such that the following equations hold:

(P1) $\bigcup_{i=1}^{r} X_i = V(G)$.

(P2) For every edge $(v, u) \in E(G)$ there exists a bag X_p such that $u, v \in X_p$.

(P3) For every $u \in V(G)$, if $u \in X_i \cap X_j$ for some $i < k$, then $u \in X_k$ for every $i < k < j$.

The width of a decomposition $\mathcal{P}$ is denoted as $w(P) = \max_{1 \leq i \leq r} |X_i| - 1$. The pathwidth of a graph G is the minimum possible width of a path decomposition i.e. $pw(G) = \min_{\mathcal{P}} w(P)$.

A nice path decomposition of a graph G is a path decomposition $\mathcal{P}$ that satisfies:

- $X_0 = \emptyset$
- For every $i \in \{1, 2, ..., r\}$ there is either a vertex $v \notin X_i$ such that $X_{i+1} = X_i \cup \{v\}$ or there is a vertex $w \in X_i$ such that $X_{i+1} = X_i \setminus \{w\}$.

Nice path decomposition form is useful for designing dynamic-programming algorithm. It is worth to mention that any path decomposition $\mathcal{P}$ can be transformed into a nice path decomposition $\mathcal{P}'$ is polynomial time.

1.3.2. Tree decomposition

A tree decomposition is a generalization of a path decomposition. Formally, a tree decomposition of a graph G is a pair $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ where $\mathcal{T}$ is a tree whose every node t is assigned to a vertex subset $X_t \subseteq V(G)$, called a bag, such that the following three conditions hold:

(T1) $\bigcup_{t \in V(T)} X_t = V(G)$.

(T2) For every $(v, u) \in E(G)$ there exists a bag t of $\mathcal{T}$ such that $v, u \in X_t$.

(T3) For every $v \in V(G)$ the set $T_v = \{t \in V(T) : v \in X_t\}$ induces a connected subtree of T.

Similarly, the width of a tree decomposition $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$, denoted as $w(\mathcal{T})$, is equal to $\max_{t \in V(T)} |X_t| - 1$. The treewidth of a graph G, denoted as $pw(G)$, is the minimal width over all tree decompositions of G.

A nice tree decomposition of a graph G is a tree decomposition $(T, \{X_t\}_{t \in V(T)})$ such that

- $X_i = \emptyset$ is i is either root or leaf.
- Every non-leaf node is of one of the three following types:
  - **Introduce node**: a node t with exactly one child t' such that $X_t = X_{t'} \cup \{v\}$ for some vertex $v \notin X_{t'}$.
  - **Forget node**: a node t with exactly one child t' such that $X_t = X_{t'} \setminus \{w\}$ for some vertex $w \in X_{t'}$
  - **Join node**: a node t with exactly two children t_1, t_2 such that $X_t = X_{t_1} = X_{t_2}$.

Chapter 2

Spanning Star Forest Problem

For a given graph G, we say that S is a *Spanning Star Forest* if every connected component C is a star. In *Spanning Star Forest Problem* given a graph G, the objective is to determine whether there exists a *Spanning Star Forest*.

It turns out that the problem formulated in such a way is relatively simple. Although, various parametrizations described in this paper make it more complex. The following lemma easily clarifies all the concerns about it's hardness.

Lemma 2.1. *A graph G has a* Spanning Star Forest *if and only if it does not contain any isolated vertices.*

Proof. If G has a *Spanning Star Forest* S, then trivially for all $v \in V(G)$ $1 \leq deg_S(v) \leq deg_G(v)$. Thus, none of the vertices is isolated.

For the opposite direction, we prove the lemma by induction on $|V(G)|$. Assume $|V(G)| = 2$. The statement trivially holds because a graph representing an edge is a correct *Spanning Star Forest*. Let $|V(G)| > 2$. Suppose that there does not exist a vertex v such that $G \setminus \{v\}$ has no isolated vertices. Then, it holds that for all $v \in V(G)$ $deg_G(v) = 1$ so G itself is a correct *Spanning Star Forest*. Now, suppose that v is a vertex such that $G \setminus \{v\}$ has no isolated vertices. From the inductive assumption, let S be a *Spanning Star Forest* of a graph $G \setminus \{v\}$, u be a vertex such that $(u, v) \in E(G)$ and w be a vertex such that $w \in N_S(u)$. Consider the 3 following cases:

1. $deg_S(u) > 1$. Then, $S' = \big(V(S) \cup \{v\}, E(S) \cup \{(u, v)\}\big)$ is a correct solution for graph G.
2. $deg_S(u) = deg_S(w) = 1$. Then, $S' = \big(V(S) \cup \{v\}, E(S) \cup (u, v)\big)$ is a correct solution for graph G.
3. $deg_S(w) > 1$. Then, $S' = \big(V(S) \cup \{v\}, (E(S) \cup \{(u, v)\}) \setminus \{(u, w)\}\big)$ is a correct solution for graph G.

Observe that in graph G there are no isolated vertices. Thus, one can always extend a solution inductively.

□

Application of Lemma 2.1 yields the following result for *Spanning Star Forest Problem*.

Theorem 2.1. *Decision version of* Spanning Star Forest Problem *can be solved in linear time.*

Proof. Given an input $G = (V, E)$ the answer is YES if for all $v \in V(G)$ $deg_G(v) \neq 0$ and NO otherwise. □

2.1. Obtaining a solution

In this section we focus on obtaining an arbitrary solution for a given instance of *Spanning Star Forest Problem*. Firstly, let us introduce 2 claims in order to normalize the instance and make the algorithm look more clear.

Claim 2.1. *Family of disjoint* Spanning Star Forests *is a* Spanning Star Forest.

Claim 2.2. *G has a* Spanning Star Forest *if and only if it's spanning tree T has.*

The first claim can be trivially proven by the definition of *Spanning Star Forest Problem* while the second one follows directly from Lemma 2.1. Equipped with this information, all that is left to do, is to design an algorithm which solves *Spanning Star Forest Problem* for trees.

Data: Graph G
Result: Spanning Star Forest of T
$T \leftarrow SpanningTree(G)$;
$S \leftarrow \emptyset$;
for *v: postorder(T) and* $v \notin V(S)$ **do**
  if *v is not a root* **then**
    S $\leftarrow S \cup \{(u, v)\}$ where $u = parent(v)$
  else
    S $\leftarrow S \cup \{(u, v)\}$ where u is any of the root's children
  end
end
return S

Algorithm 1: Obtaining a Spanning Star Forest from a tree.

Lemma 2.2. *Algorithm 1 is correct.*

Proof. Assume contrary, that the algorithm yields an incorrect solution S. Consider the first case: a path $(u, v), (v, w), (w, z)$ exists in S where u is v's child, v is w's child and w is z's child. But, if u is w's grandchild and $(u, v), (v, w) \in S$, then it means that w is a root. Contradiction because w cannot be z's child. Now, suppose the alternative relationship: u is v's child, v and z are w's children. Provided that vertices were visited in postorder, edge (v, w) should not have been added because v was introduced by u and w was introduced by z. □

Theorem 2.2. *A solution for Spanning Star Forest Problem can be found in linear time.*

Proof. Spanning tree of any graph can be found in linear time. The main loop has n iterations (every vertex is visited once), each of which takes constant time. Thus, the total runtime is linear. □

2.2. Spanning Star Forest parameterized by the number of stars

In *Spanning Star Forest Problem* parameterized by the number of stars, given a graph G and a natural number k, the objective is to determine whether there exists a *Spanning Star Forest* S such that the number of connected components is at most k.

It is natural to ask whether one can find a solution that minimizes the number of connected components. The problem formulated in that way looks slightly different than the previous one. From the other hand, the problem resembles *Dominating Set Problem*, which is defined as follows:

Definition 2.1. Dominating Set Problem*: Given a graph G and a positive integer k find a set D such that $|D| \leq k$ and every vertex from the graph is adjacent to one of the vertices from D.*

It turns out, that the second comparison is true and *Spanning Star Forest Problem* parameterized by the number of stars is NP-Complete. But, before we begin, let us introduce one more definition and a lemma that supports a reduction.

Definition 2.2. *Given an instance (G, k) of Dominating Set Problem that does not contain any isolated vertices and a solution D, a dominating mapping is a function $m : V(G) \setminus D \to D$ such that satisfies $(x, m(x)) \in E(G)$ for all $x \in Dom(m)$.*

Lemma 2.3. *Let (G, k) be an instance of* Dominating Set Problem *without isolated vertices and let D be a solution of minimal size. Then, there exists a dominating mapping m such that m is surjective.*

Proof. Suppose contrary that such a mapping does not exist i.e. for every mapping m there exists a vertex $v \in D$ such that $v \notin im(m)$. Let us break the proof into 4 cases:

1. Suppose $N_G(v) = \emptyset$. Contradiction, G has no isolated vertices.
2. Suppose $u \in N_G(v) \cap D$. Contradiction, D was said to be a solution of minimal size whereas $D \setminus \{u\}$ is a valid, smaller solution.
3. Suppose $u \in N_G(v) \setminus D$ and $w \in N_G(u) \cap im(m)$. If $|m^{-1}(w)| = 1$, then $((D \setminus \{v, w\}) \cup u)$ is a valid, smaller solution for a graph G. Contradiction.
4. Suppose $u \in N_G(v) \setminus D$ and $w \in N_G(u) \cap im(m)$. If $|m^{-1}(w)| > 1$ then a mapping:

$$m'(x) = \begin{cases} v, & \text{if x = u} \\ m(x), & \text{otherwise} \end{cases}$$

   is a valid mapping that satisfies $im(m) \subset im(m')$. Thus, one can create a new mapping m'' inductively such that m'' is surjective. Contradiction, we assumed that no such mapping exists.

Since all the possible cases led to a contradiction, we may conclude that there exists a dominating mapping f such that f is surjective. □

Armed with the lemma, we are ready to prove the main theorem of the section.

Theorem 2.3. Spanning Star Forest Problem *parameterized by the number of stars is NP-Complete.*

Proof. Membership in NP: given an oracle (O, k), we check whether the number of components in O is less than k and whether every connected component forms a star. The task can be easily done in polynomial time.

We show hardness by a reduction from *Dominating Set Problem* that completes the proof. Let (G, k) be an instance of it. We create a graph G' as follows: for every isolated vertex $v \in V(G)$ introduce a vertex v' and an edge (v, v'). Now, we claim that (G, k) is a YES-instance for *Dominating Set Problem* if and only if $(G'.k)$ is a YES-instance for *Spanning Star Forest Problem* parameterized by the number of stars.

The backward implication is simple. Suppose S is a solution for (G', k). We claim that a set D representing centers of stars is a correct *Dominating Set*. Obviously $|D| \leq k$ because there are at most k connected components. Every vertex from G' is adjacent to one of the centers. If there exists a vertex $v' \in D$ such that $v' \notin V(G)$ we transform the solution as follows: $D := (D \setminus \{v'\}) \cup \{v\}$.

To prove the forward implication, let D be a solution of minimal size for (G, k). Obviously, D is also a minimal solution for a graph G'. Thus, by lemma 2.3. there exists a mapping m that is surjective. Now, we claim that a graph $S = (V(G'), \{(x, m(x)) : x \in Dom(m)\})$ is a correct solution for *Spanning Star Forest Problem*. Trivially, there are no isolated vertices in S. Moreover, there is no path of length 4 because S consists of edges (v, u) such that $v \in D$, $u \notin D$ and for all $u \in V(S) \setminus D$ $deg_S(u) = 1$.

□

The theorem implies that *Spanning Star Forest Problem* parameterized by the number of stars is as hard as *Dominating Set Problem*. Thus, we can immediately obtain the following corollary.

Corollary 2.1. Spanning Star Forest Problem *parameterized by the number of stars is* W[2]-complete.

The problems look so similar that one could ask whether the reverse reduction is true. Indeed, with a small twist to the previous idea one can prove the reverse reduction instantly.

Theorem 2.4. *There exists a reduction from* Spanning Star Forest Problem *parameterized by the number of stars to* Dominating Set Problem.

Proof. Let (G, k) be an instance of *Spanning Star Forest Problem*. We create an instance (G', k') for *Dominating Set* as follows: let $G' = G$ and if G contains an isolated vertex, then $k' = 0$. Otherwise, the value remains the same. Now, we claim that (G, k) is a YES-instance for *Spanning Star Forest Problem* if and only if (G', k') is a YES-instance for *Dominating Set*.

To prove the following reduction one can use the method which was described in Theorem 2.4 with a little remark: if an instance (G, k) contains an isolated vertex, then obviously it is a NO-instance for *Spanning Star Forest Problem* and so is (G', k') for *Dominating Set* because G' is not an empty graph. □

One can observe now the immediate corollary of the theorem 2.3 and theorem 2.4.

Corollary 2.2. *Every theorem is true for* Dominating Set Problem *if and only if it is true for a* Spanning Star Forest Problem *parameterized by the number of stars.*

As an example, the following theorem described in can be transfered to *Spanning Star Forest Problem* parameterized by the number of stars.

Theorem 2.5. *Unless* CNF-SAT *can be solved in time* $\mathcal{O}^*((2 - \epsilon')^n)$ *for some* $\epsilon' > 0$ *there do not exist constant* $\epsilon > 0$, $k \geq 3$ *and an algorithm solving* Dominating Set Problem *parameterized by the number of stars in time* $\mathcal{O}^*(N^{k-\epsilon})$, *where* N *is the number of vertices of the input graph.*

Chapter 3

Spanning Star Forest Extension Problem

In this chapter, we significantly change the problem. Let G be a graph and $F \subseteq E(G)$. In the *Spanning Star Forest Extension Problem* the question that we want to answer now is that whether there exists a *Spanning Star Forest* S such that $F \subseteq E(S)$. Hardness of the problem lays in deciding which end of every isolated edge is a center and which one is a ray. We used three different parameters: number of isolated edges, number of non-isolated edges and treewidth.

3.1. General overview

3.1.1. Instance normalization

Observe that a star is a primitive structure. The star's maximal radius is equal to 3. It means that we can look at the problem rather locally than globally. Notice that this time we do not have any limit on the number of connected components. As it was said before, the hardness of the problem lays in choosing which of the endings of an isolated edge is a center. Thus, it might be possible to normalize instances i.e. try to remove vertices that are "far enough" from isolated vertices.

Let (G, F) be an arbitrary instance of *Spanning Star Forest Extension Problem*. Firstly, consider trivial cases.

Reduction 1 If graph G contains an isolated vertex, it is a NO-instance.

Reduction 2 If in graph G there exists a path of size at least 3 made from isolated edges, it is a NO-instance.

Suppose that isolated edges form a star of size at least 3. Then, the center is already set. Thus, we can remove from the instance all the vertices that are adjacent to the pre-created centers.

Reduction 3 $C = \{v : |N_G(v) \cap V(F)| > 1\}$. Update $G = G \setminus (N_G(C) \cup C)$.

Now, let $V_P = \{v : (v, u) \in (E(G) \setminus F)$ and $v \notin V(F)\}$ and $V_{NP} = V(G) \setminus V_P$. Finally, $G_{NP} = G[V_{NP}]$ and $G_P = G[V_P]$. Notice an immediate consequence of the partitioning of graph G.

Claim 3.1. *G_P always has a solution*

To prove the claim we can apply lemma 2.1. G_P does not have any isolated edges nor isolated vertices. All that is left to do is to prove that edges between G_P and G_{NP}, that were lost during partitioning, does not have any effect on the solution. The following theorem proves the intuition.

Lemma 3.1. *An instance (G, F) has a solution if and only if (G_{NP}, F) has one.*

Proof. The backward implication is trivial. Suppose S is a solution for an instance (G_{NP}, F). We can partition G into G_P and G_{NP} and find a solution, say S', for a graph G_P. Then, $S \cup S'$ is a correct solution for G.

Now, consider the forward implication. Let S be a solution for an instance (G, F). Assume contrary that there exists a vertex $v \in V(G_{NP})$ does not belong to any star. Trivially, vertices from $V(F)$ are covered. Thus, $v \in V(G_{NP}) \setminus V(F)$. If $v \in V(G_{NP}) \setminus V(F)$ it follows that for all $(v, u) \in E(G)$ $(v, u) \in E(G_{NP}))$. If it was not true, the vertex v would have been place in the graph G_P. Thus, there exists an edge $(v, u) \in E(S)$ such that $(v, u) \in E(G_{NP})$. Contradiction.

□

Ultimately, we can state the last rule.

Reduction 4 Update $G = G \setminus G_P$.

3.1.2. NP-completeness

After exhaustive application of rules, any graph has a simple structure. Let (G, F) be an instance of *Spanning Star Forest Extension Problem* after normalization. Then, G consists of vertices of two types: ones that have only edges to vertices from $V(F)$ and ones that have exactly one isolated edge (and potentially many non-isolated). Such a representation substantially simplifies further investigations. Indeed, we can prove that the problem parameterized by the number of isolated edges is NP-complete.

Theorem 3.1. Spanning Star Forest Extension Problem *is NP-complete.*

Proof. Membership in NP: Given a solution S for instance (G, F) check whether $F \subseteq S$ and whether S is a correct *Spanning Star Forest*.

To prove hardness, we show a reduction from *3-SAT*. Let ϕ be an arbitrary instance. We create a graph G as follows: for every variable x_i we introduce vertices $x_i, \neg x_i$ and an isolated edge $(x_i, \neg x_i)$. For every clause c_i we introduce a vertex c_i. For every x_k (symmetrically $\neg x_k$) we introduce an edge (x_k, c_p) if and only if x_k is present in p'th clause. Now, we claim that ϕ is satisfiable if and only if (G, F) has a *Spanning Star Forest*.
Backward implication: Let S be a solution for (G, F). Then, a set of centers is a correct evaluation that satisfies the formula ϕ because every clause S has a witness.
To prove the forward implication, assume there exists an evaluation σ of variables that satisfies the formula. Let $C = \{l_i : \sigma(l_i) = 1\}$. Note that C contains either x_i or $\neg x_i$. Now, let us construct a solution S. Firstly, include all the isolated edges. Then, for every vertex representing a clause c_i take a random l_j such that $l_j \in N(c_i) \cap C$ and include edge (c_i, l_j) into the solution. The operation is safe. All sets $N(c_i) \cap C$ are nonempty because there exists a witness l_k that satisfies the clause and there exists an edge between a literal and a clause. □

Surprisingly, *3-SAT* is trivially encoded in *Spanning Star Forest Extension Problem*.

Definition 3.1. 3-SAT

Lemma 3.2. *Any CNF formula can be represented as a formula where each clause has size at most 3*

Lemma 3.3. *There exists a reduction from* Spanning Star Forest Extension Problem *to* 3-SAT.

Proof. Let (G, F) be an arbitrary normalized instance of *Spanning Star Forest Extension Problem*. We create a formula ϕ as follows: for every isolated edge (u, v) we introduce literals x and $\neg x$. For every vertex $v \in V(G) \setminus V(F)$ we introduce a clause c_v. Literal l is present in a clause c_j if and only if there exists an edge between vertices corresponding to c_i and l. Application of lemma 3.2 to ϕ produces a formula ϕ' that is a correct instance of *3-SAT*.

Proof of correctness was previously described in the theorem. We omit it for the sake of clarity. □

3.2. Parametrization by the number of isolated edges

3.3. Parametrization by the number of non-isolated edges

3.4. Parametrization by treewidth

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