Derivation Rules

$\frac{d}{dx}(x^a) = a * x^{a-1}$ given: $x, a \in \mathbb{R}$ & $x > 0$

subexamples:

$\frac{d}{dx}x = 1 \rightarrow \frac{d}{dx}(x^1) = 1 * x^{1-1}$

$\frac{d}{dx}x^2 = 2x \rightarrow \frac{d}{dx}(x^2) = 2 * x^{2-1}$

$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2} \rightarrow \frac{d}{dx}(x^{-1} = -1 * x^{-1-1})$

$\frac{d}{dx}\sqrt{x} = \frac{1}{2*\sqrt{x}} \rightarrow \frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2*x^{\frac{1}{2}}}$

$\frac{d}{dx}(c) = 0$ given: $c \in \mathbb{R}$ & c is constant & c != factor

$\frac{d}{dx}(e^x) = e^x \rightarrow \frac{d}{dx}(e^x) = ln(e) * e^x * x' = 1 * 1 * e^x$

$\frac{d}{dx}(a^x) = ln(a) * a^x \rightarrow \frac{d}{dx}(a^x) = x' * ln(x) * a^x$ because: $e^{x*ln(a)} = a^x$

Note for this rule:

$\frac{d}{dx}(2^{2x+1}) = ln(2x+1) * 2^{2x+1} * (2x+1)' = ln(2x+1) * 2^{2x+1} * 2$

$\frac{d}{dx}(ln(x)) = \frac{1}{x}$

$\frac{d}{dx}log_b(x) = \frac{1}{ln(b)*x} \rightarrow$ special case for $\frac{d}{dx}(\frac{ln(x)}{ln(b)})$

this is the case because of base change in lograrithmic functions!

$log_a(x) = \frac{ln(x)}{ln(a)} = \frac{log_c(x)}{log_c(a)} \rightarrow \frac{d}{dx}log_a(x) = \frac{ln(x)}{ln(a)} -> \frac{d}{dx}ln(x) = \frac{\frac{1}{x}}{ln(a)} = \frac{1}{ln(a)*x}$

c can be any number!

$\frac{d}{dx}sin(x) = cos(x)$

$\frac{d}{dx}cos(x) = -sin(x)$

$\frac{d}{dx}tan(x) = \frac{1}{cos^2(x)}$

$\frac{d}{dx}tan(x) = 1 + tan^2(x)$

$\frac{d}{dx}(ax) = a \rightarrow \frac{d}{dx}(ax) = a * 1 \rightarrow$ we derive x NOT a!!

$\frac{d}{dx}(3x) = a \rightarrow \frac{d}{dx}(3x) = 3 * 1 \rightarrow$ 3 is a factor!

All of these derive from:

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Sum Rule

$(f+g)' = f' + g'$

Difference Rule

$(f-g)' = f' - g'$

Product Rule

$(f*g)' = f*g' + f'*g$

Quotient Rule

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f' * g - f * g'}{g^2}$$

Chain Rule

$[f(g(x))]' = f'(g(x)) * g'(x)$

Example:

$\frac{d}{dx}\sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}} = \frac{1}{2} * (x^2+1)^{-\frac{1}{2}} * 2x$

$\frac{1}{2} * \frac{1}{\sqrt{x^2+1}} * 2x = \frac{2x}{2*\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$

Note: (x^2+1) is g(x), while f is the exponent function

More examples:

$\frac{d}{dx}sin(x^2) = sin(x^2)' * (x^2)' = cos(x^2) * 2x$

$\frac{d}{dx}sin^2(x) = \frac{d}{dx}(sin(x))^2 = 2 * sin(x) * cos(x)$

$$\frac{d}{dx}\left(\frac{x-1}{x+1}\right)^2 = 2 * \left(\frac{x-1}{x+1}\right) * \left(\frac{x-1}{x+1}\right)'$$

$\frac{d}{dx}(x+2)^3(x)^4 = (x+2)^3 * ((x)^4)' + ((x+2)^3)' * (x)^4$

$((x+2)^3)' = 3 * (x+2)^2 * (x+2)' = 3 * (x+2)^2 * 1$

$$\frac{d}{dx}sin(cos[tan(x)]) = cos(cos[tan(x)]) * -sin(tan(x)) * \frac{1}{cos^2(x)}$$

Implicit Differentiation

$\frac{d}{dx}(x^2+y^2=9) \rightarrow 2x + \frac{d}{dx}((y)^2) * \frac{dy}{dx}(y) = 0 \rightarrow 2x + (2y * y') = 0 \rightarrow y' = \frac{-2x}{2y}$

!! Remember that this is only necessary if y needs to be derived !!

Higher Derivatives

The best idea for higher derivatives is distance s, velocity v and acceleration a.

$\frac{d}{dt}(s(t)) = v(t) = s'(t) \;||\; \frac{d}{dt}(v(t)) = a(t) = s''(t) = v'(t)$

This is why the acceleration on earth -> gravity is constant!! HOLY FUCK

Taking Derivations higher than 3

$1: f' \rightarrow 2: f'' \rightarrow 3: f''' \rightarrow 4: f^{(4)} \rightarrow n: f^{(n)}$