Number Base Case
$N = d_n R^n + d_1 R^1 + d_0 R^0$
the d specifies the Number system -> $d_2 == $ binary
can also be written as R_2
This can also be used to expand numbers:
$N_{10} 255 = 2 * 10^2 + 5 * 10^1 + 5 * 10^0$
$N_2 110 = 1 * 2^2 + 1 * 2^1 + 0 * 2^0 => N_{10} 6$

Quantities:
N -> natural numbers | | **Z** -> full numbers
Q -> rational numbers | | **R** -> real numbers

Common number systems:
Decimal: $N_{10} = n * 10^n..0 * 10^0$
Binary: $N_2 = n * 2^n..0 * 2^0$
$2^{10} = 1024, 2^9 = 512, 2^8 = 256, 2^7 = 128, 2^6 = 64,$
$2^5 = 32, 2^4 = 16, 2^3 = 8, 2^2 = 4, 2^1 = 2, 2^0 = 1$
Hexadecimal: $N_1 6 = n * n^{16}..0 * 16^0$
notation: 0 1 2 3 4 5 6 7 8 9 A B C D E F
$16^5 = 1048576, 16^4 = 65536, 16^3 = 4096, 16^2 = 256,$
$16^1 = 16, 16^0 = 1$

Modulo
8 mod 4 = (8) -> 0 , 8 mod 3 = (6) -> 2 , 8 mod 5 = (5) -> 3
if x<y in x mod y then the result will always be x!
any negative numbers can be considered as NOTnegative
aka only absolute values! modulo deals with |x|
many programming languages actually do not follow this!
they have their own implementation of modulo.
$5 \equiv 3$ mod 2 -> as 5 mod 2 = 1 and 3 mod 2 = 1

Codeword length
Byte = 8 bit || Word = 16 or 32 bit
TCP packet = 1024 bit

Cyclic group

Es sei $F(a) = a^3 + a + 1 = 0$,
- Dann können wir zunächst festhalten
- a = a
- $a^2 = a^2$ aber
- $a^3 = \boldsymbol{a+1}$
- $a^4 = a(\boldsymbol{a+1}) = a^2 + a$
- $a^5 = a(a^2 + a) = a^3 + a^2 = a^2 + \boldsymbol{a+1}$
- $a^6 = a(a^2 + \boldsymbol{a} + 1) = a^3 + \boldsymbol{a^2} + \boldsymbol{a} = \boldsymbol{a+1} + \boldsymbol{a^2} + \boldsymbol{a} = \boldsymbol{a^2} + 1$
- $a^7 = a(\boldsymbol{a^2} + 1) = a^3 + a = \boldsymbol{a+1} + a = 1$
- a^8 = a : der Zyklus beginnt von vorne!
- **$\{0, 1, a, a^2, a+1, a^2 + a, a^2 + a+1, a^2 + 1\}$**
- ***{000, 001, 010, 100, 011, 110, 111, 101}***

WHAT THE FUCK

Result Quantity the result of all possible outcomes
it is denoted with: Ω
A single element of the result list is: ω -> $\omega \in \Omega$
The list of results is $|\Omega|$
Example Dice roll: $\Omega = \{1, 2, 3, 4, 5, 6\}$

Probability: $P(A) = \frac{\text{best results}}{\text{all results}} = \frac{|A|}{|\Omega|} = \frac{|A|}{n}$

So what is the probability of rolling a 6?
$P(\text{desired number to roll}) = \frac{\text{only 1 good result!}}{\text{6 possible results}} = \frac{1}{6}$
hence the chance is 1 in 6
Why this complicated method? You can modify desired results!
just change the A in P(A)!

Inverse Probability: P(inverse) = 1 - P(A)
dice -> $1 - \frac{1}{6} = \frac{5}{6}$

Addition rule:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
!!The last part is needed, as otherwise the number
would exceed the possible states!!
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

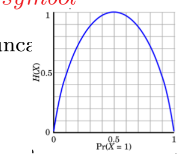
Amount of possibilities:
ordered probes with replication:
2 coins, head and tail, possibilities? **k**=head/tail=2 **n**=coins=2
$$\Omega = n^k = 2^2$$
ordered probes without replication:
5 dices. How many combinations?
dice numbers = **n** = 6 (1-6), dice amount = **k** = 5
possibilities = $\Omega = \frac{n!}{(n-k)!} = \Omega = \frac{6!}{(6-5)!} = 720$
Or this:
$$\Omega = \Pi_n^{n-k+1} n = \Pi_6^{6-5+1} 6 = 2 * 3...5 * 6 = 720$$
unordered probes whitout replication:
25 players, each should only play once with the other.
$\Omega = \frac{n!}{k!(n-k)!} \text{->} \frac{25!}{2!(25-2)!} \text{->} \frac{\text{too big}}{\text{too big}} = 300$
as you can see the bottom is a BIG calculation, so
$$\Omega = \frac{\Pi_n^{n-k+1} n}{k!} \text{->} \frac{\Pi_{25}^{25-2+1} 25}{2!} \text{->} \frac{24 * 25}{2} = 300$$
Note that **k** can also be defined as the
length of the tuple we want to receive.
-> (Player,Player) - > 2

Source to Sink Information

Nachricht (Darstellung & Bedeutung)	redundant	nicht-redundant
irrelevant	Zeichenvorrat bei Quelle und Senke verschieden	
relevant	vorhersagbar	Information

Entropy
information content
this essentially just us how many bits are needed
k is base state count -> bit = 2
and **N** is the full number of states
example: list True,False,True,False 4 states total, base 2.
$$H_0 = log_k(N)[k] \text{->} H_0 = log_2(4)[bit] = 2$$

information flow
essentially information content over time
$$H_0^* = \frac{log_2(N)}{\tau}[\frac{bit}{s}]$$
information quantity / Surprise
$$I(x_k) = -log_2(P(x_k))[bit]$$
Entropy (Surprise per element)
0 means no symbols. 1 means perfect balance 50-50
$$H(X) = \Sigma_{k=1}^N P(x_k) * I(x_k)[\frac{bit}{symbol}]$$
where X is the list of symbols



Sink Redundance / Code Redunca
$$R_Q = H_0 - H(X)[\frac{bit}{symbol}]$$
$$R_c = L - H(X)[\frac{bit}{symbol}]$$
Code Word Length
$$L(x_k) = \text{rounded}(I(x_k))[bit]$$
Median Code Word Length
$$L = \Sigma_{k=1}^N P(x_k) * L(x_k)[\frac{bit}{symbol}]$$
Entropy of the entire Code
$$H_c(X) = \Sigma_{k=1}^N P(x_k) * L(x_k)[\frac{bit}{symbol}]$$
H_c can be a real number -> $H_c \in \mathbb{R}$

Für jede beliebige zugehörige Binärcodierung mit Präfixeigenschaft ist die mittlere Codewortlänge nicht kleiner als die Entropie $H(X)$:	Für jede beliebige Quelle kann eine Binärcodierung gefunden werden, so dass die folgende Ungleichung gilt:
$H(X) \le L$	$H(X) \le L \le H(X) + 1$

Sink without memory
$$P(x_k, y_k) = P(x_k) + P(y_i)$$
Sink with memory
$$P(x_k, y_i) = P(x_k) + P(x_k|y_i)$$
Entropy without memory / Combined Entropy
$$H(H, Y) = \Sigma_{x_k}^N \Sigma_{y_i}^N P(x_k, y_i) * (-log_2(P(x_k, y_i))$$
or: $H(X, Y) = H(X) + H(Y)$
Entropy with memory
$$H(H, Y) = \Sigma_{x_k}^N \Sigma_{y_i}^N P(x_k) * P(x_k, y_i) * (-log_2(P(x_k) * P(x_k|y_i))$$

Encoding of Symbols
- **Ordne die Zeichen gemäss ihrer Auftrittswahrscheinlichkeit**
- **Die beiden Zeichen mit der kleinsten Auftrittswahrscheinlichkeit haben die gleiche CW-Länge L_N**
- **Sei L_N die mittlere CW-Länge für eine Quelle mit N Zeichen und L_{N-1} die mittlere CW-Länge für den Fall, dass die beiden letzten zu einem einzigen Zeichen zusammengefasst werden, dann gilt:**

$$L_N - (p(x_{N-1}) + p(x_N)) \cdot L(x_N) = L_{N-1} - (p(x_{N-1}) + p(x_N)) \cdot (L(X_N) - 1)$$
$$\Rightarrow L_N = L_{N-1} + p(x_{N-1}) + p(x_N)$$

1	2	3	4	5	6	7	8	9
0.22	0.19	0.15	0.12	0.08	0.07	0.07	0.06	0.04
1	2	3	4	8	9	5	6	7
				0	1			
0.22	0.19	0.15	0.12	0.1		0.08	0.07	0.07
1	2	3	6	7	4	8	9	5
			0	1		0	1	
0.22	0.19	0.15	0.14		0.12	0.1		0.08
1	2	8	9	5	3	6	7	4
		00	01	1		0	1	
0.22	0.19	0.18			0.15	0.14		0.12

continue this pattern until every symbol has a code
note the extra 0 on every step

Run Length Encoding RLE/RLC
- Quelltext w: Agggbbehfffgggg => |w|=15
- Codiert w_c : A3g2beh3f4g => | w_c |= 11

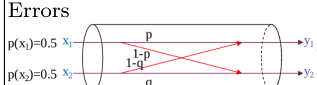
A + 3 x g + 2 x b + e + h + 3 x f + 4 x g

shortening of length
by compressing repetition.

Encoder and Decoder
You need to either choose 1 or 0 as the starting
bit. After that the decoder can print out the correct code.

Chiffre text
You can "encrypt" your data by
shifting the codes by a certain amount.
In the caesar chiffre this is done with the number 4. a -> e
Please do not use this, use RSA or other algorithms.

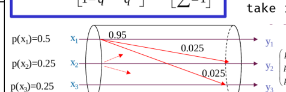
Errors



$p(y_1) = p(x_1) \cdot p + p(x_2) \cdot (1-q)$
$p(y_2) = p(x_1) \cdot (1-p) + p(x_2) \cdot (q)$

$$\boxed{p(Y|X) = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \mapsto \begin{bmatrix} \sum = 1 \\ \sum = 1 \end{bmatrix}}$$

1-p and 1-q are the chance for
error. Which we of course have to
take into account.

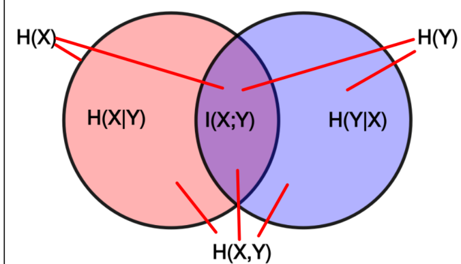


$$\begin{pmatrix} p(y_1) \\ p(y_2) \\ p(y_3) \end{pmatrix} = \begin{bmatrix} p(x_1) \cdot p(y_1|x_1) + p(x_2) \cdot p(y_1|x_2) + p(x_3) \cdot p(y_1|x_3) \\ p(x_1) \cdot p(y_2|x_1) + p(x_2) \cdot p(y_2|x_2) + p(x_3) \cdot p(y_2|x_3) \\ p(x_1) \cdot p(y_3|x_1) + p(x_2) \cdot p(y_3|x_2) + p(x_3) \cdot p(y_3|x_3) \end{bmatrix}$$

$$p(Y|X) = \begin{bmatrix} 0.95 & 0.025 & 0.025 \\ 0.025 & 0.95 & 0.025 \\ 0.025 & 0.025 & 0.95 \end{bmatrix}$$

$$\begin{bmatrix} 0.4875 \\ 0.25625 \\ 0.25625 \end{bmatrix} = \begin{bmatrix} 0.5 \cdot 0.95 + 0.25 \cdot 0.025 + 0.25 \cdot 0.025 \\ 0.5 \cdot 0.025 + 0.25 \cdot 0.95 + 0.25 \cdot 0.025 \\ 0.5 \cdot 0.025 + 0.25 \cdot 0.025 + 0.25 \cdot 0.95 \end{bmatrix}$$

Conditional Entropy -> Entropy of Y given X
$$H(Y|X) = \Sigma_{k=1}^N \Sigma_{i=1}^N P(x_k, y_i) * (-log_2(\frac{P(x_k, y_i)}{P(x_k)}))$$
Chain Rule
$$H(Y|X) = H(X, Y) - H(X) \ || \ H(Y \backslash X)$$
Bayes Rule
$$H(Y|X) = H(X|Y) - H(X) + H(Y) \ || \ H(Y \backslash X)$$



Transinformation
likelyhood of information being correct at arrival.
$$T = H(X) - H(X|Y) \ || \ H(Y) - H(Y|X)$$
or: $|(X; Y)$

Hamming distance / distance to next valid codeword
$$h = Min_{i,j}(d(x_i, x_j))$$
error detection distance
the amount of bits that differ from input to output
$$e^* = h - 1$$
error correction distance for h even
$$h = 2e + 2 \text{->} e = \frac{h-2}{2}$$
error correction distance for h uneven
$$h = 2e + 1 \text{->} e = \frac{h-1}{2}$$
Consider the valid input either 111 or 000.
The Hamming distance **h** is therefore 3 bits.
The detection distance e^* is 3 - 1
Due to h being uneven, the correction distance **e** is $\frac{h-1}{2}$
which results in 1.

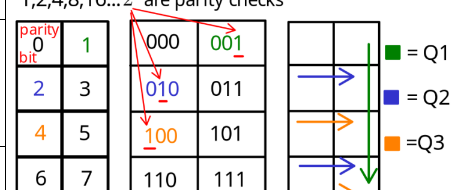
tighly packed coderoom
n = dimension of code
m = dimension of messages $2^m * \Sigma_{w=0}^e \binom{n}{w} \le 2^n$
k = dimension of control -> n = m + k
The code is considered to be tightly packed
if the equation has the result 2. aka == not smaller.

m=2 | k=1

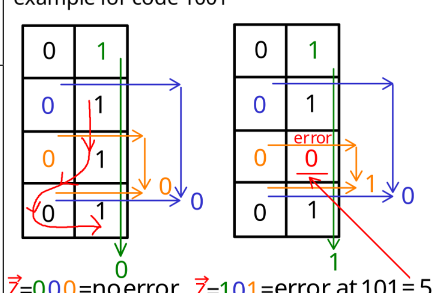
x_1	x_2	x_3
0	0	0
0	1	1
1	0	1
1	1	0
0	0	1
0	1	0
1	0	0
1	1	1

```
0 = (0 + 0) mod 2 = 0 OK
1 = (0 + 1) mod 2 = 1 OK
1 = (1 + 0) mod 2 = 1 OK
0 = (1 + 1) mod 2 = 0 OK
```
$$x_3 = (x_1 + x_2) \bmod 2$$
```
1 = (0 + 0) mod 2 = 0 NOT OK
0 = (0 + 1) mod 2 = 1 NOT OK
0 = (1 + 0) mod 2 = 1 NOT OK
1 = (1 + 1) mod 2 = 0 NOT OK
```

Hamming Codes
The hamming code is very easy to implement
$$\Sigma_i x_i * \vec{P_i} \equiv \vec{0} \, mod2$$
The syndrome $\vec{Z} = \Sigma_i x_i * \vec{P_i} mod2$
1,2,4,8,16...2^x are parity checks



example for code 1001



$\vec{Z}$=000=noerror $\vec{Z}$=101=error at 101 = 5

note that the 001 010 100 of the parity checks are
simply the unit vector $\vec{0}$!!!

parity checks needed:

$par = log_2(\text{bit amount of code})$

1101 = 4 bits -> 3 parity checks
as 4 can be displayed by 3 bits -> 100