Unicité et existence d'un nœud à chaque position

Existence d'au moins un nœud à la position i

$$\Phi_{\text{existence\_of\_u}}(i) = \bigvee_{u=0}^{N-1} \bigvee_{h=0}^{H} P(u, i, h)$$

Unicité d'un nœud à la position i

$$\Phi_{\text{unicity\_of\_node}}(i) = \bigwedge_{u=0}^{N-1} \bigwedge_{\substack{v=0 \\ v \neq u}}^{N-1} \bigwedge_{h_u=0}^{H} \bigwedge_{h_v=0}^{H} \Big(\neg P(u, i, h_u) \vee \neg P(v, i, h_v) \Big)$$

Combinaison existence + unicité pour la position i

$$\Phi_{\text{existence\_and\_unicity}}(i) = \Phi_{\text{existence\_of\_u}}(i) \wedge \Phi_{\text{unicity\_of\_node}}(i)$$

3. Unicity of Stack Height for Each Node

Each node occupies at most one height per position:

$$\Phi_{\text{unicity\_of\_height}}(u) = \bigwedge_{i=0}^{L} \bigwedge_{0 \leq k < k' \leq H} (\neg P(u, i, k) \vee \neg P(u, i, k'))$$

4. Simple Path Constraint for Each Node

Each node appears at most once along the path (regardless of height):

$$\Phi_{\text{simple\_path}}(u) = \bigwedge_{0 \leq i < i' \leq L} \bigwedge_{0 \leq k, k' \leq H} (\neg P(u, i, k) \vee \neg P(u, i', k'))$$

5. Combined Node Constraints

For all nodes, combining stack height unicity and simple path:

$$\Phi_{\text{height\_and\_simple\_path}} = \bigwedge_{u=0}^{N-1} \Big(\Phi_{\text{unicity\_of\_height}}(u) \ \wedge \ \Phi_{\text{simple\_path}}(u) \Big)$$