Recurrence Relation

1) $a_n = 3a_{n-1} + 2 \quad a_0 = 1$

$a_1 = 3(1) + 2 = 5$
$a_2 = 3(5) + 2 = 17$
$a_3 = 3(17) + 2 = 53$
$a_4 = 3(53) + 2 = 161$
$a_5 = 3(161) + 2 = 485$

2) $b_n = b_{n-1} + n^2 \quad b_1 = 1$

$b_2 = 1 + 2^2 = 5$
$b_3 = 5 + 3^2 = 14$
$b_4 = 14 + 4^2 = 30$

3) $a_n^h = 2a_{n-1}^h$

$a_n^h = \alpha \cdot 2^n$

$c = a_n^h$

$c = 2c + 5$

$c - 2c = 5$

$c = -5$

subs c into $a_n^h = a \cdot 2^n$

$a_n = a \cdot 2^n - 5$

when $a_0 = 3$:

$3 = a \cdot 2^0 - 5$

$3 = a - 5$

$a = 3$

The solution for the recurrence:

$a_n = 3 \cdot 2^n - 5$

4) $T_n = 2T_{n-1} + 1 \quad T_1 = 1$

$T_2 = 2(1) + 1 = 3$
$T_3 = 2(3) + 1 = 7$
$T_4 = 2(7) + 1 = 15$
$T_5 = 2(15) + 1 = 31$
$T_6 = 2(31) + 1 = 63$

$T_n = 1, 3, 7, 15, 31, 63 \ldots n$

$T_n = 2^n - 1$

Proving:

Assume $n = k$

$T_k = 2^{k-1}$

$T_{k+1} = 2T_k + 1$

$T_{k+1} = 2(2^{k-1}) + 1$

$T_{k+1} = 2^{k+1} - 2 + 1$

$T_{k+1} = 2^{k+1} - 1$

$\therefore n = k+1, \quad T_n = 2^n - 1$

DASNEEM BANU
NUREEN FATINI

Subject:
No:
Date:

SECJ1013 : DISCRETE STRUCTURE SEM 1 20242025 (ASSIGNMENT 2)

counting methods & probability

1. a) no digit can be repeated ?
10 9 8 7 6 ⇒ 10 x 9 x 8 x 7 x 6 = 30240 codes

b) First digit must be even, digit cannot be repeated.
5 9 8 7 6 ⇒ 5 x 9 x 8 x 7 x 6 = 15120 codes

even numbers : 0, 2, 4, 6, 8
n even = 5

2. a) ways all 10 people line up in a row
10 9 8 7 6 5 4 3 2 1 ⇒ 10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 3628800 ways

b) two specific people stand next to each other
9 8 7 6 5 4 3 2 1 ⇒ 9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 362880

combined unit = 2!
Total arrangement = 362880 x 2! = 725760 ways.

3. a) P (sum = 7)
n (total) = 6 x 6 = 36
(sum = 7) = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
n (sum = 7) = 6

$$P(\text{sum}=7) = \frac{n(\text{sum}=7)}{n(\text{total})} = \frac{6}{36} = \frac{1}{6}$$

b) at least one number is 6.
5 choices are there if no 6 shows in both dice which is {1, 2, 3, 4, 5}
so, 5 x 5 = 25
n (total) = 36
If at least one 6 shown, then the outcomes is 36 − 25 which is equal to 11.

$$P(\text{at least one 6 appear}) = \frac{11}{36}$$

c) numbers on two dice is equal
n (total) = 36
(equal) = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}
n (equal) = 6

$$P(\text{equal}) = \frac{n(\text{equal})}{n(\text{total})} = \frac{6}{36} = \frac{1}{6}$$

Subject:

Date:

4. a) How many different teams can be formed?

$n = 15$, $r = 4$

$C(n,r) = \dfrac{n!}{r!(n-r)!}$

$C(15,4) = \dfrac{15!}{4!(15-4)!}$

$= 1365$ teams

b) If two specific student must include in team, how many teams formed?

If 2 student must be selected, then remaining will be 13. $(n = 13)$

If 2 student have been selected then the remaining is 2 to be selected. $(r = 2)$

$C(13,2) = \dfrac{13!}{2!(13-2)!}$

$= 78$ teams

5. How many ways the letters of 'STATISTICS' can be arranged?

$n = 10$

For repeated letters : 3 S, 3 T, 2 I

~~Arr~~ Arrangements $= \dfrac{10!}{3! \times 3! \times 2!}$

$= 50400$ ways

a) start with letter 'S'

If start with 'S' then the remaining letters is 9.

$n = 9$

For the repeated letter : 2 S, 3 T, 2 I

Arrangements $= \dfrac{9!}{2! \times 3! \times 2!}$

$= 15120$ ways

b) How many arrangements have all 'T's together?

TTT _ _ _ _ _ _ _

1 2 3 4 5 6 7 8

$n = 8$

For repeated letter : 3 S and 2 I

Arrangements $= \dfrac{8!}{3! \times 2!}$

$= 3360$ ways of arrangements

Permutation & Combination

1) {A, B, C, D, 1, 2, 3, 4, !, @, #, $}

a) $P(12,8) = \frac{12!}{(12-8)!} = \frac{12!}{4!} = 19958400$

b) $\boxed{\underline{A}, \underline{B}, \underline{C}, \underline{D}}$ _ _ _ _ _ _ $\boxed{\underline{1}, 2, 3, 4}$

$= 4 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$

$= 2419200$ ways

2 a) $P(11) = \frac{11!}{3!\,3!\,2!\,2!\,1!} = 277200$

b) $= \frac{9!}{3!\,2!\,2!} = 15120$

3

a) $P(n) = (n-1)!$

$P(8) = (8-1)! = 7! = 5040$

b) $P(n) = \frac{(n-1)!}{2}$

$P(8) = \frac{(8-1)!}{2} = \frac{7!}{2} = \frac{5040}{2} = 2520$

4)

a) woman + man = 10+8 = 18

$C(n,r) = {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$_{18}C_6 = \binom{18}{6} = \frac{18!}{6!(18-6)!} = 18564$

b) when 2 women, 4 men : $^8C_2 \cdot {}^{10}C_4 = 28 \times 210 = 5880$
when 3 women, 3 men : $^8C_3 \cdot {}^{10}C_3 = 56 \times 120 = 6720$
when 4 women, 2 men : $^8C_4 \cdot {}^{10}C_2 = 70 \times 45 = 3150$
when 5 women, 1 man : $^8C_5 \cdot {}^{10}C_1 = 56 \times 10 = 560$
when all women : $^8C_6 \cdot {}^{10}C_0 = 28 \times 1 = 28$
committees can formed : $= 5880 + 6720 + 3150 + 560 + 28$
$= 16338$

5)

a) $C(30,3) = \frac{30!}{10!12!8!} = 3.7848 \times 10^{22}$

b) $C(30,3) = \frac{29!}{9!12!8!} = 1.2616 \times 10^{12}$

6)

a) $C(10,5) = \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} = 252$

b) $C(n+r-1, r) = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$

$C(10+5-1, 5) = \binom{10+5-1}{5} = \frac{(10+5-1)!}{5!(10-1)!}$

$C(14,5) = \binom{14}{5} = \frac{14!}{5!(9!)} = 2002$

Pigeonhole principle

1. - The ~~conta~~ set contains 99 integers from 1 to 99
- All the integers paired up into consecutive pairs of numbers : (1, 2), (3, 4),..., (97, 98), (99)
- Now, the total pairs is 49 and the last integer which 99 does not have any consecutive pairs
- There is 50 pigeonholes which is 50 integers.
~~If we select 50~~
- Since, there are more pigeonholes than the consecutive pairs, it is proven that at least one of these pigeonholes must contain consecutive pair.
- This means at least two of them are consecutive
- Hence, it is totally proved that the ~~co~~ chosen 50 integers from set {1, 2, 3, ..., 99} have atleast two of them are consecutive.

2. - The possible remainders when any integer divided by 8 is 0, 1, 2, 3, 4, 5, 6, 7
- We can identify that integers as pigeonholes and the remainders as pigeons.
- Then $n = 9$ (integers) and $k = 8$. By pigeonholes principle, because $n > k$, at least 2 integers have same remainder.
- For this 2 integers, assume x and y have same remainder, the difference $x - y$ is ~~divisib~~ divisible by 8
- Hence proved that, in any group of 9 integers, there are two integers whose difference is divisible by 8.

3. - Assume that students can identify as pigeons and the days in a week as pigeonholes.
- The days in a week is (Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday)
- Then $n = 30$ (pigeons), $k = 7$ (pigeonholes) and

$$m = \left\lceil \frac{30}{7} \right\rceil = 5$$

- By the generalized pigeonhole principle, at least 5 students must have share the same birthday.
- Hence, it is proved that in a set of 30 students, at least two students born in same day of a week.

Subject: No: Date:

4. - Let socks colour as pigeonholes and drawn socks as pigeons.
- There are 9 drawn socks, $n = 9$ and 3 colours of socks, $k = 3$.
- Assume that we drawn 2 socks of red, 2 socks of blue and 2 socks of green
- Then the last 3 drawn which is 7th, 8th and 9th ~~to~~ must belong to one of red, blue and green ~~making it~~ making it at least three socks ~~mus~~ of same colour.
- Therefore, it is proved that if 9 socks are drawn from a drower, at least three socks must be of same color.

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