Tutorial 3

1)

a) $7! = 5040$

b) $2! \times 6! = 720 \times 2$
$= 1440$

c) $4! \times 4! = 24 \times 24$
$= 576$

2) ~~T~~ ~~U~~ ~~E~~ ~~S~~ ~~D~~ ~~A~~ ~~Y~~

a) ${}^{7}C_{4} \times 4! = \binom{7}{4} \times 4 \times 3 \times 2 \times 1$
$= 35 \times 24$
$= 840$

b) $7! = 5040$

c) $\underline{U/E/A}$ ___ ___ ___ ___ ___ ___
$= 3! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 4320$

3) $A-Z = 26$, $0-9 = 10$

a) $= {}^{26}P_{3} \times 10 \times 10 \times 10 \times 10$
$= 15600 \times 10000$
$= 156000000$

b) ${}^{26}P_{3} \times {}^{10}P_{4} = 15600 \times 5040$
$= 78624000$

4) 9b 4g

a) _ _ _ _ _ _ _

$$\underbrace{\_\ \_\ \_}\quad {}^{4}C_3 \times {}^{9}C_4 = 4 \times 126$$

$$= 504$$

b) ${}^{4}C_3 \times {}^{9}C_4 = 4 \times 126 = 504$

${}^{4}C_4 \times {}^{9}C_3 = 1 \times 84 = 84$

$= 504 + 84 = 588$

c) ${}^{4}C_3 \times {}^{9}C_4 = 4 \times 126 = 504$

${}^{4}C_2 \times {}^{9}C_5 = 6 \times 126 = 756$

${}^{4}C_1 \times {}^{9}C_6 = 4 \times 84 = 336$

${}^{4}C_0 \times {}^{9}C_7 = 1 \times 36 = 36$

$504 + 756 + 336 + 36 = 1632$

5) $P = 100$, prize $= 3$

$$\frac{{}^{100-1}C_{3-1}}{{}^{100}C_3} = \frac{{}^{99}C_2}{{}^{100}C_3} = \frac{4950}{161700} = 0.031$$

CamScanner

6)

$G_1 = \text{bag } 1$ $\quad$ $B = \text{black ball}$

$G_2 = \text{bag } 2$

$$P(G_1) = P(G_2) = \frac{1}{2}$$

$$P(B \mid G_1) = \frac{3}{4+3} = \frac{3}{7}$$

$$P(B \mid G_2) = \frac{5}{5+3} = \frac{5}{8}$$

$$P(B) = P(B \mid G_1) \times P(G_1) + P(B \mid G_2) \times P(G_2)$$

$$= \left(\frac{3}{7} \times \frac{1}{2}\right) + \left(\frac{5}{8} \times \frac{1}{2}\right)$$

$$= \frac{59}{112}$$

$$P(G_2 \mid B) = \frac{P(B \mid G_2) \times P(G_2)}{P(B)}$$

$$= \frac{\frac{5}{8} \times \frac{1}{2}}{\frac{59}{112}}$$

$$= \frac{35}{59}$$

7) T = take advance calculus

H = apply pre-health

N = not take advance calculus

$P(T) = 0.6$

$\cancel{P(H) =}$ $P(N) = 1 - P(T) = 0.4$

$P(H|T) = 0.8$

$P(H|N) = 0.3$

$$P(H) = P(H|T) \times P(T) + P(H|N) \times P(N)$$

$$= (0.8 \times 0.6) + (0.3 \times 0.4)$$

$$= 0.6$$

$$P(T|H) = \frac{P(H|T) \times P(T)}{P(H)}$$

$$= \frac{0.8 \times 0.6}{0.6}$$

$$= 0.8$$

$$= 80\%$$

Subject .... Date:

8. a) $P(\text{Over } 60) = 0.15$

$P(\text{Loan} \mid \text{Over } 60) = 0.28$

Over 60 years old and has a loan :

$P(\text{over } 60 \text{ and Loan}) = P(\text{over } 60) \times P(\text{Loan} \mid \text{Over } 60)$

$= 0.15 \times 0.28$

$= 0.042$

∴ Hence, the probability of person over 60 years old and has a loan is 0.042.

b) $P(\text{Loan} \mid \leq 60) = 0.56$

$P(\leq 60) = 1 - P(\text{Over } 60)$

$= 1 - 0.15$

$= 0.85$

The person has no loan.

$P(\text{loan}) = [P(\text{Loan} \mid \text{Over } 60) \times P(\text{over } 60)] + [P(\text{Loan} \mid \leq 60) \times P(\leq 60)]$

$= [(0.28) \times 0.15] + [0.56 \times 0.85]$

$= 0.518$

$P(\text{no loan}) = 1 - P(\text{loan})$

$= 1 - 0.518$

$= 0.482$

∴ Hence, probability for person with no loan is 0.482.

c) $P(\text{loan} \mid \leq 60) = 0.56$

d) $P(\text{over } 60 \mid \text{Loan}) = \dfrac{P(\text{Over } 60 \text{ and Loan})}{P(\text{loan})}$

$= \dfrac{0.042}{0.518}$

$= 0.0811$

9. i) Vertices, $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

Edges, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

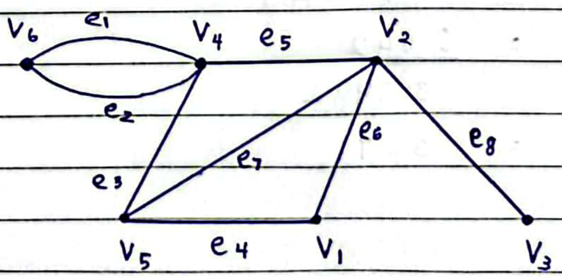
The edge-endpoint function :

Edge	Endpoints
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_3, v_4\}$
e_4	$\{v_4\}$
e_5	$\{v_4\}$
e_6	$\{v_4, v_5\}$
e_7	$\{v_5, v_2\}$
e_8	$\{v_1, v_5\}$
e_9	$\{v_1, v_5\}$
e_{10}	$\{v_1, v_6\}$

ii) Incidence matrix of the graph :

$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

10. Draw graph

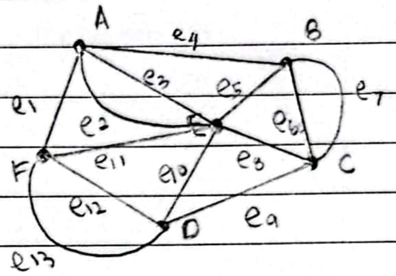


11. Find adjacency matrix for G2 and G3

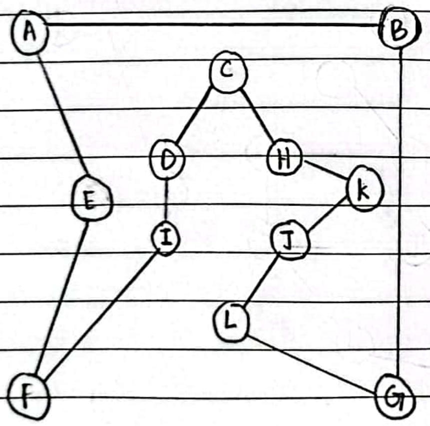
$$A_{G_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_{G_3} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

12. - The graph is Euler circuit.
- ($A, e_4, B, e_7, C, e_6, B, e_5, E, e_8, C, e_9, D, e_{10}, E, e_{11}, F, e_{13}, D, e_{12}, F, e_1, A, e_2, E, e_3, A$)
- The graph is also connected graph
- Every vertex in the graph has even degree.
- Hence, the graph is Euler circuit.



13.



- It is proved that the graph has Hamiltonian cycle.

14. - Both G1 and G2 has 4 vertices.
- Both G1 and G2 has 6 edges.
- Both graph has same number of loop and parallel edges.
- Both have 2 vertices with 4 degree, 1 vertice with 3 degree and 1 vertice with one degree.
- $f(A_{G1}) = Y_{G2}$ $\quad f(C_{G1}) = W_{G2}$
- $f(B_{G1}) = X_{G2}$ $\quad f(D_{G1}) = Z_{G2}$
- Hence it is proved ~~to~~ that both graph is isomorphic.