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Assignment 1

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1. A = {1, 3, 5, 7}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8}

a) $A \cap B = \{3, 5\}$
$B \cap C = \{5, 6\}$
$(A \cap B) \cup (B \cap C) = \{3, 5, 6\}$

$\subseteq B = \{\}, \{3, 4, 5, 6\}, \{3\}$
$\subseteq B = \{\}\ \{3\}\ \{4\}\ \{5\}\ \{6\}\ \{3, 4\}\ \{3, 5\}, \{3, 6\}, \{4, 5\}, \{4, 6\}$
$\{5, 6\}, \{3, 4, 5\}, \{3, 4, 6\}, \{3, 5, 6\}, \{4, 5, 6\}, \{3, 4, 5, 6\}$

$(A \cap B) \cup (B \cap C) \subseteq B$ is true

b) $(A \cup C) - B = \{1, 7, 8\}$
$A - B = \{1, 7\}$
$C - B = \{7, 8\}$
$(A - B) \cup (C - B) = \{1, 7, 8\}$

$\{1, 7, 8\} = \{1, 7, 8\}$
$(A \cup C) - B = (A - B) \cup (C - B)$ proven true

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c) $A = \{1, 3, 5, 7\}$

$B \cap C = \{5, 6\}$

$A \oplus (B \cap C) = \{1, 3, 6, 7\}$

$A \oplus B = (A - B) \cup (B - A)$

$A - B = \{1, 7\}$

$B - A = \{4, 6\}$

$(A - B) \cup (B - A) = \{1, 4, 6, 7\}$

$A \oplus B = \{1, 4, 6, 7\}$

$C = \{5, 6, 7, 8\}$

$(A \oplus B) \oplus C = \{1, 4, 5, 8\}$

$\{1, 3, 6, 7\} \neq \{1, 4, 5, 8\}$

$\therefore A \oplus (B \cap C) \neq (A \oplus B) \oplus C$

2. $P = \{\cancel{2}, \cancel{3}, 5, 7, 11, 13, 17, 19\}$

$E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$D = \{1, 6, \cancel{3}, 12, \cancel{2}, 18\}$

a) $\forall x\ (P(x) - D(x)) : \exists x\ E(x)$

b) $P(x) - D(x) = \{5, 7, 11, 13, 17, 19\}$

$E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$\forall x \{5, 7, 11, 13, 17, 19\} \neq \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$\therefore$ statement in (a) is disprove

2 c) $P \cap E = \{2\}$

$D = \{1, 2, 3, 6, 18\}$

$(P \cap E) \subseteq D$

~~$\emptyset$ inter~~

$\therefore$ since P intersect with E is the subset of D, then ~~subset~~ subset of D union with D will get set D back

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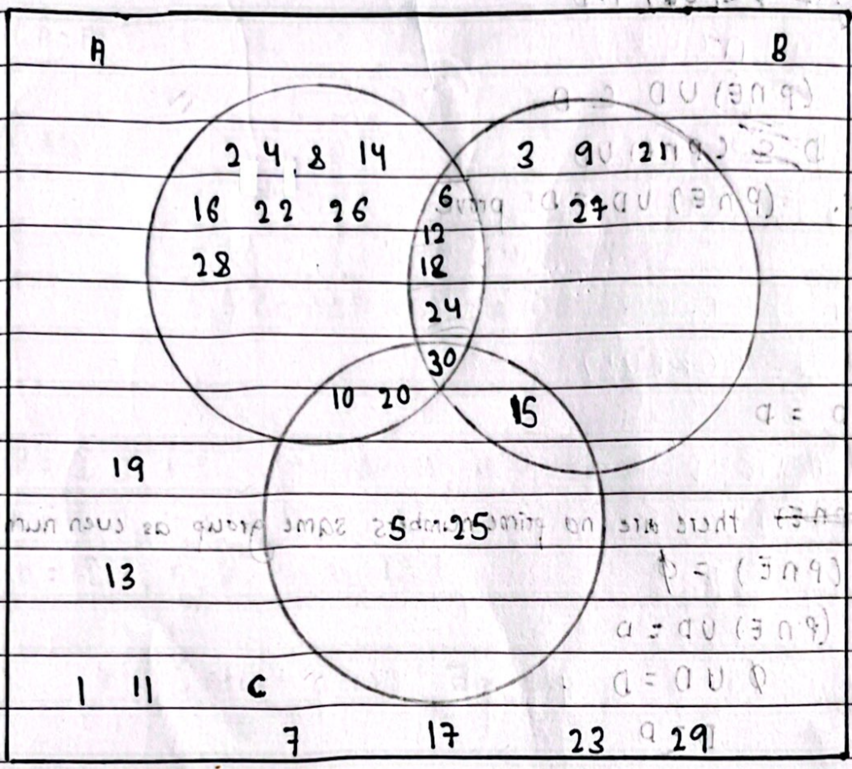
3. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$

$B = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$

$C = \{5, 10, 15, 20, 25, 30\}$

a)



b) U = 30 elements

A = 15 elements

B = 10 elements

C = 6 elements

$A \cap B$ = 7 elements

$B \cap C$ = 2 elements

$A \cap C$ = 3 elements

$A \cap B \cap C$ = 1 elements

$A \cup B \cup C$ = 22 elements

$(A \cup B \cup C)'$ = 8 elements

$(A - B) - C$ = 8 elements

$(B - A) - C$ = 4 elements

$(C - B) - A$ = 2 elements

$(A \cap B) - C$ = 4 elements

$(B \cap C) - A$ = 1 elements

$(A \cap C) - B$ = 2 elements

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ASSIGNMENT 1 (DISCRETE STRUCTURE)

4 $X = \{0, 1, 2\}$ $Y = \{0, b\}$

a) Cartesian product $X \times Y$

$X \times Y = \{(0,a), (0,b), (1,0), (1,b), (2,0), (2,b)\}$

b) $R \subseteq X \times X$ such that $(x,y) \in R$ if and only if $x+y$ is even.

$X = \{0,1,2\}$

$X \times X = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}$

check:

$(0,0)$ = even
$(0,1)$ = odd
$(0,2)$ = even
$(1,0)$ = odd
$(1,1)$ = even
$(1,2)$ = odd
$(2,0)$ = even
$(2,1)$ = odd
$(2,2)$ = even

$\therefore$ Hence, $R = \{(0,0), (0,2), (1,1), (2,0), (2,2)\}$

c) $R = \{(0,0), (0,2), (1,1), (2,0), (2,2)\}$

Reflective checking

$$M_R = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

$\therefore$ This relationship is reflective because all diagonal elements are 1.

Symmetric checking

$R = \{(0,0), (0,2), (1,1), (2,0), (2,2)\}$

$\therefore$ The relation R is symmetric because all $(x,y) \in R$ and $(y,x) \in R$:

$(0,0)$, $(0,0)$ $\in R$
$(0,2)$, $(2,0)$ $\in R$
$(1,1)$, $(1,1)$ $\in R$
$(2,0)$, $(0,2)$ $\in R$
$(2,2)$, $(2,2)$ $\in R$

transitive checking

∴ The relation R is transitive because $(x,y), (y,z) \in R$ and $(x,z) \in R$.

Proof :

$(0,0), (0,2) \in R$ and $(0,2) \in R$

$(0,2), (2,0) \in R$ and $(0,0) \in R$

$(1,1), (1,1) \in R$ and $(1,1) \in R$

$(2,0), (2,2) \in R$ and $(2,2) \in R$

$(2,2), (2,2) \in R$ and $(2,2) \in R$

⇒ Hence, Relation R is reflexive, symmetric and transitive.

5 a) Statement : If ($x \in Z$ x even and x^2 is also even), then (x is a multiple of 4).

contrapositive : If x is not multiple of 4, then $x \in Z$ is not even or x^2 is not even.

Proof :

① Assume that x is not multiple of 4 but x is even.

② x^2 is still even, but x itself cannot fulfill the condition which is multiple of 4.

③ For example :

- Assume $x = 2$, in this case x is not multiple of 4 but x is even.
- x^2 is 4 and it is even too, but x still does not fulfill the condition which is multiple by 4.

∴ Hence the contrapositive of the following statement is proven.

b) Statement : If mn is even, the m is even or n is even.

∴ This statement is true because if product of mn is even, either m is even or n is even. If both m is odd and n is odd, the product of mn will result is odd.

Hence it is proved that if mn is even, m or n is even.

c) Assume $\sqrt{3} = \frac{x}{y}$ where $y \neq x$, $y \neq 0$ and y and x does not have same common factors.

$\sqrt{3} = \frac{x}{y}$

$3 = \frac{x^2}{y^2}$

$3y^2 = x^2$ (∴ Since $3y^2 = x^2$, x^2 is divisible by 3. so, x is also divisible by 3)

For example : Let $x = 3f$

$(3f)^2 = 3y^2$

$9f^2 = 3y^2$

$3f^2 = y^2$

This shows that $\sqrt{3}$ is irrational because y^2 is divisible by 3 which means y is also divisible by 3.

Hence proved that our initial assumption is false and $\sqrt{3}$ is actually irrational.

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6 a) $f(x) = 3x + 2$

$f(x_1) = 3x_1 + 2$ $\quad$ $f(x_2) = 3x_2 + 2$

$f(x_1) = f(x_2)$

$3x_1 + 2 = 3x_2 + 2$

$3x_1 = 3x_2 + 2 - 2$

$x_1 = \frac{3x_2}{3}$

$x_1 = x_2$

$\therefore$ f is one-to-one function

b) $f(1) = 5$

$f(-1) = -1$

$f(2) = 8$

$f(-2) = -4$

$f = \{(1,5), (-1,-1), (2,8), (-2,-4) \ldots\}$

$\therefore$ f is not onto function

c) f(x) must be one-to-one and onto function to have $f^{-1}(y)$ but f(x) is not onto function

$f(x) = 3x + 1$

$y = 3x + 1$

$y - 1 = 3x$

$x = \frac{y-1}{3}$

$f^{-1}(y) = \frac{x-1}{3}$

f has an inverse function if and only if the codomain is restricted to the set $\{y \in \mathbb{Z} : y \equiv 2 \pmod 3\}$

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7 a) $g(x) = x^2$ $h(x) = x+1$

$g \circ h(x) = g(h(x))$
$= \cancel{g}\ (x+1)^2$

$h \circ g(x) = h(g(x))$
$= x^2 + 1$

b) $g \circ hg = g(hg(x))$
$= (x^2+1)^2$
$= x^4 + 2x^2 + 1$
$=$

~~$g \circ hg = \{(1,4),(2,25),(3,100),(4,289)\ldots\}$~~

$g \circ hg = \{(1,4),(-1,4),(2,25),(-2,25)\ldots\}$

$g \circ hg$ is onto function

$h \circ gh = h(gh(x)) = (x+1)^2 + 1$
$= x^2 + 2x + 1 + 1$
$= x^2 + 2x + 2$

$h \circ gh = \{(1,25),(-1,1),(2,10),(-2,2)\ldots\}$

$h \circ gh$ is one-to-one function

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c) $(h \circ g)(x) = x^2 + 1$

$y = x^2 + 1$

$y - 1 = x^2$

$x = \pm \sqrt{y-1}$

$(h \circ g)^{-1}(x) = \pm \sqrt{x-1}$