1) a) Binomial$(n=2, p=.72)$

b) Poisson(15)

c) Bernoulli $(p=\frac{1}{52})$ or Binomial$(n=1, p=\frac{1}{52})$

d) Geometric $(7/10)$

2) a) $n=15 \quad p=.2$

$E[X] = 15\cdot(.2) = 3$

$SD[X] = \sqrt{Var(X)} = \sqrt{n\cdot p(1-p)} = \sqrt{(15)(.2)(.8)} \approx 1.549$

$P(X \geq 2) = 1 - P(X<2)$

$= 1 - (P(X=0) + P(X=1))$

$= \cancel{.8}\ .60197$

b) $\lambda = 3$

$E[X] = \lambda = 3$

$SD[X] = \sqrt{Var(X)} = \sqrt{\lambda} = \sqrt{3}$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - P(X=0) - P(X=1)$

$= .800$

c) $p = .2$

$E[X] = \frac{1}{p} = \frac{1}{.2} = 5$

$SD[X] = \sqrt{Var(X)} = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{1-.2}{(.2)^2}} = 4.47$

$P(X=2 \text{ or } X \geq 5) = P(X=2) + P(X \geq 5)$ since they are mutually exclusive

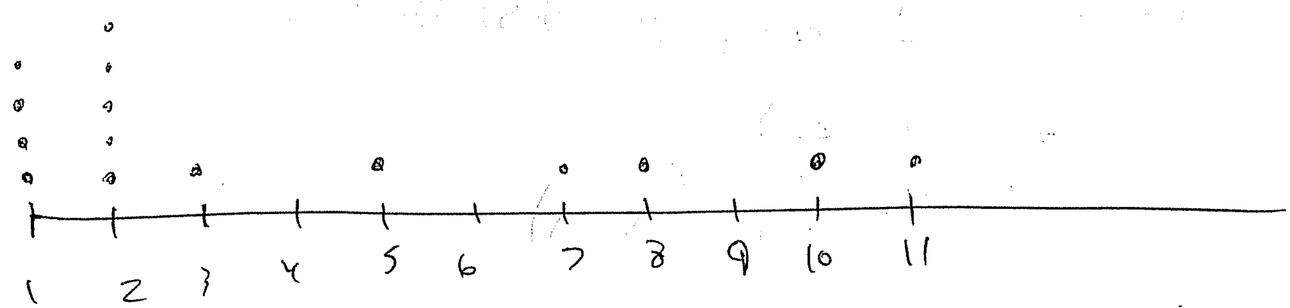
$= P(X=2) + 1 - P(X \leq 4)$

$= .128 + .32768$

$= .45568$

3)

a)



b) $\text{mean} = \bar{x} = \frac{\Sigma x}{n} = 3.866$

$sd = \sqrt{\frac{1}{n-1} \Sigma (x_i - \bar{x})^2} = 3.461$

c) median = 2

$Q_1 = Q(.25) \Rightarrow i' = np + .5 = (15)(.25) + .5 = 4.25$

$\Rightarrow Q(.25) = .75 X_4 + .25 X_5 = (.75)(1) + (.25)(2)$
$= 1.25$

$Q(.75) \Rightarrow i' = (15)(.75) + .5 = 11.75$

$\Rightarrow Q(.75) = .25 X_{11} + .75 X_{12} = (.25)(5) + (.75)(7)$
$= 6.5$

$\boxed{IQR = Q(.75) - Q(.25) = 6.5 - 1.25 = 5.25}$

d) Mode $= 2$ (it occurs 5 times, no other value occurs this often)

e) $Q(.3) \Rightarrow i' = (15)(.3) + .5 = 5$

$= X_5$

$\boxed{= 2}$

f) Boxplot

$Q(.25) - 1.5 IQR = 1.25 - (1.5)(5.25) = -6.625$

$Q(.75) + 1.5 IQR = 6.5 + (1.5)(5.25) = 14.375$

So we have no outliers

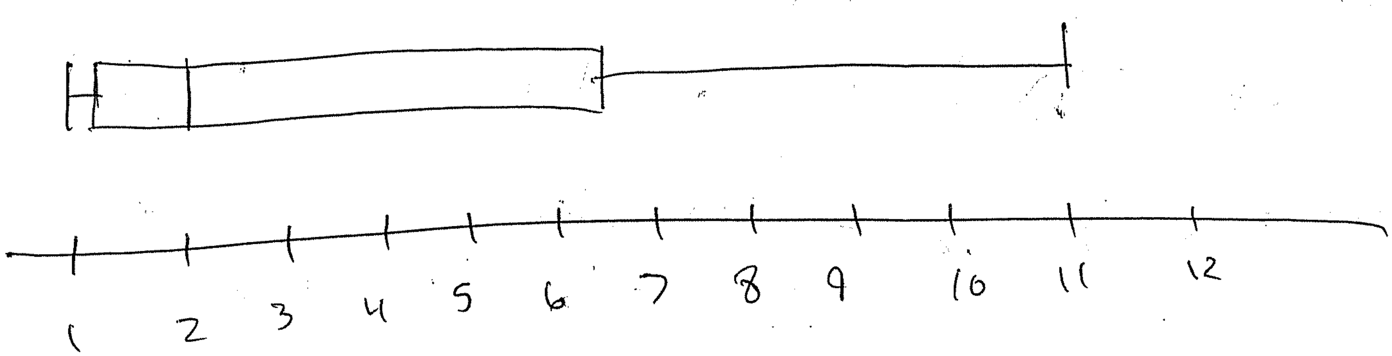
min $= 1$

$Q(.25) = 1.25$

Median $= 2$

$Q(.75) = 6.5$

Max $= 11$



g) This data seems quite skewed so I would prefer to use median and IQR

12

4) a) $P(X \geq 3) = 1 - P(X < 3)$ } since $F(x)$ is constant
$= 1 - P(X \leq 2)$ } on $[2, 3)$
$= 1 - F(2)$
$= 1 - .7 = .3$

b) $P(X < 2) = .3$

Note that $F(t) = \sum_{x \leq t} f(x) = \left(\sum_{x \leq t-1} f(x)\right) + f(t) = F(t-1) + f(t)$

$\Rightarrow f(t) = F(t) - F(t-1)$

c) $P(X = 4) = f(4) = F(4) - F(3) = .95 - .9 = .05$

5) a) All the probabilities are between 0 & 1 ✓
The sum of the probabilities $= .1 + .2 + .3 + .4 + .5 = 1.5$
So this is not a valid probability distribution since the sum of the probabilities exceeds 1.

b) $0 \leq P(X = x) \leq 1 \quad \forall x$ so
and $\sum_x P(X = x) = 1$ so this is a valid pmf

c) $P(X = 2) = -.2$ ← Probabilities can't be negative so this isn't valid.

6) a) $b_1 = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

we are given both the numerator and denominator in the table so we just need to plug in our values

$= \dfrac{2.40}{1.84}$

$= 1.3043$

$b_0 = \bar{y} - b_1\bar{x}$

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{10}{20} = .5$

$\bar{y} = \dfrac{\Sigma y}{n} = \dfrac{30.53}{20} = 1.5265$

$= 1.5265 - (1.3043)(.5)$

$= .87435$

$\boxed{\hat{y} = 0.87435 + 1.3043x}$ is our fitted least squares line

b) $r = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{(\Sigma(x_i - \bar{x})^2)(\Sigma(y_i - \bar{y})^2)}} = \dfrac{2.4}{\sqrt{(1.84)(17.68)}} = .42$

We have a weak, ~~linear~~ positive relationship between x + y.

(without plots we really can't say if its linear or not)

(The residual plot doesn't imply a lack of fit so we would probably be safe saying it was a linear relationship though)

c) $R^2 = r^2 = .42^2 = .1769$

17% of the raw variability in y is accounted for by the fitted regression line.

d) $\hat{y} = .87435 + (1.304348)(.2) = 1.135$

$\hat{y} = .87435 + (1.304348)(1.8) = 2.569$

e) I trust the prediction for $X = .2$ more.

the range of X was from 0 to 1

so $X = .2$ is at least within our data range.

$X = 1.3$ is outside of our data so we

would be extrapolating and predictions based

on extrapolating aren't as trust worthy.

f) The residual plot looks quite good.

there is no discernable pattern left over

so there are no noticeable problems.

7) a)

x	2	4	6	8	10
$f(x)$	$\frac{c}{2}$	$\frac{c}{4}$	$\frac{c}{6}$	$\frac{c}{8}$	$\frac{c}{10}$

we need $\frac{c}{2}+\frac{c}{4}+\frac{c}{6}+\frac{c}{8}+\frac{c}{10}=1 \Rightarrow \boxed{c=.8759}$

b)

$$F(x) = \begin{cases} 0 & x<2 \\ .4379 & 2\le x<4 \\ .6569 & 4\le x<6 \\ .8029 & 6\le x<8 \\ .9124 & 8\le x<10 \\ 1 & 10\le x \end{cases}$$

c) $P(X=4)=\frac{c}{4}=.2189$

d) $P(X \text{ is odd})$ there are no odd values of X with positive probability so

$P(\emptyset)$

$=$ ~~$P(\emptyset)$~~ $=0$

e) $E[x] = \sum_x x\,f(x) = 2(\frac{c}{2})+4(\frac{c}{4})+6(\frac{c}{6})+8(\frac{c}{8})+10(\frac{c}{10})$

$= 5c$

$= 4.379$

f) $E[3x+2] = 3E[x]+2 = 3(4.379)+2 = 15.138$

g) $SD[x] = \sqrt{Var(x)} = \sqrt{E[x^2]-E[x]^2} = \sqrt{26.277-4.379^2} = \boxed{2.66}$

$E[x^2] = \sum_x x^2 f(x) = 2c+4c+6c+8c+10c = 26.277$

4

8) a) we need $c\int_0^{\pi} \sin(x)\,dx = 1$

$$\int_0^{\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi} = -(-1)-(-1) = 2$$

$\Rightarrow 2c = 1$

$c = \frac{1}{2}$

b) $$\int_0^{t} \tfrac{1}{2}\sin(x)\,dx = -\tfrac{1}{2}\cos(x)\Big|_0^{t} = -\tfrac{1}{2}\cos(t) + \tfrac{1}{2}\cos(0)$$

$$= \tfrac{1}{2} - \tfrac{1}{2}\cos(t)$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} - \frac{1}{2}\cos(x) & 0 \le x \le \pi \\ 1 & x \geq \pi \end{cases}$$

c) $$P(2 \le X \le 5) = P(2 \le x) = 1 - P(x \le 2)$$

$$= 1 - F(2)$$

$$= 1 - \left(\tfrac{1}{2} - \tfrac{1}{2}\cos(2)\right)$$

$$= \tfrac{1}{2} + \tfrac{1}{2}\cos(2)$$

$$= .2919$$

d) $P(X = \pi/2) = 0$ since this is continuous

e) $$E[X] = \int_0^{\pi} \tfrac{1}{2}x\sin(x)\,dx = -\tfrac{1}{2}x\cos(x)\Big|_0^{\pi} - \int_0^{\pi} -\tfrac{1}{2}\cos(x)\,dx$$

$$= \left(-\tfrac{1}{2}\pi\cos(\pi) + \tfrac{1}{2}0\cos(0)\right) + \underbrace{\int_0^{\pi}\tfrac{1}{2}\cos(x)\,dx}_{=0}$$

$$\boxed{= \pi/2}$$

f) $E[3x+2] = 3E[x]+2 = 3(\frac{\pi}{2})+2 = \boxed{6.712}$

g) $Q(.1)$:

we want $F(Q(.1)) = .1$

$\frac{1}{2} - \frac{1}{2}\cos(x) = .1$ (solve for x)

$-\frac{1}{2}\cos(x) = .1 - \frac{1}{2}$

$\cos(x) = \frac{.1-\frac{1}{2}}{(-\frac{1}{2})} = \frac{.5-.1}{.5} = \frac{.4}{.5}$

$\cos^{-1}(4/5) = x$

$\boxed{Q(.1) = .6435}$