ISYE 6644: Blackjack Simulation

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Abstract

Blackjack is a game card game enjoyed by many throughout the century. Though old, some strategy to beat the house/dealer have only recently emerged thanks to invents of computer. Blackjack game methods such as double down and card counting along with statistic are use in combination to determine the best strategy that would maximize the player's profit. Multiples variation of strategies are simulated, and the average of the return is determined for the most profit.

Background

Overview of Blackjack

The history of Blackjack dated back to 17th century with the first rules appear in 1800 under the name of Vingt-Un French for twenty-one [1]. Since then, Blackjack has grown exponentially in popularity and hit the height of its popularity in the '90s. To win a game, the player must have a higher face value than the dealer and remain under 21 [2]. Stand, Hit, Double Down, Split, Surrender, and Card Counting are some main actions that a player can take in blackjack. Stand and Hit are the most common action a player can take where stand refers to cease of car draw and Hit vice versa. Double Down allows a player to take one more hit with the bets doubling. A player can Split when two same value cards are drawn. Surrender lets players give up 50% of their bets to stop further loss. Lastly, Card Counting is the act of keeping a running value of the card in order to adjust the best size in the player's favor.

The sequence of the game follows as such. At the start of the game, the player is dealt two face-up cards with the dealer one face up and one face down. The player is allowed to draw as many cards as needed to reach the face card value of 21. If the player goes over 21, the game is ended and the player's bet is forfeited. However, if the players stop at under 21, the dealer will then reveal the hidden card. The dealer plays draw and continues to draw they reach a face value of over 16. When both player and dealer have stopped, the card's face value is compared and the winner is determined. There are three possible outcomes: Player win, Dealer win, and Push (tie). If the player wins via regular mean (no double), the player is awarded the bet size. If a player wins via 'Blackjack' (ace + 10 value), the player is awarded half of the bet size. Lastly, if a player win via double down, the player is awarded double the

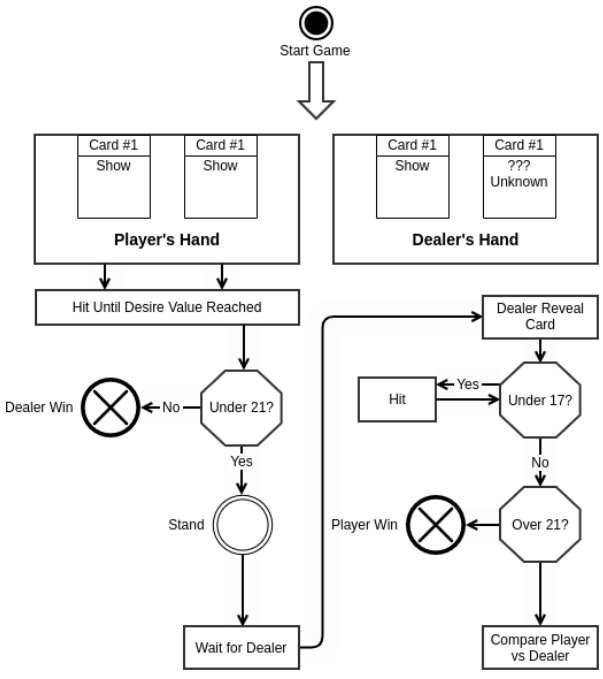

Figure 1: General Blackjack Game Flow

bet size. Otherwise, the player loses the bet if the dealer wins, and no award for a push.

Game Strategy Approaches

A basic strategy published by Roger Balwin is the first scientific and mathematical approach to playing blackjack [3]. This paper will examine the basic strategy and simulate the two fundamental play strategies (Double Down and Card Counting) along with simpler strategies such as mirroring the dealer's rules. The outcome of these various strategies is examined to find the most profitable strategy.

Method

Defining Game Strategies

Strategies # 1: You Hella Basic [Universal Basic Strategy]

This strategy follows a predetermine action per dealer's face-up card and the total of the player's cards. For this strategy, no double down or card counting is considered. The result from this strategy is the baseline to compare with other strategies.

As follow, the programming logic is depicted in Figure 2. After the card is dealt, a while loop keeps the player in total value under 21. From there, the program checks if an ace is in the first two cards. If it is, the hand is considered a soft total because the ace can be 11 or 1. If yes, three conditional statements will be checked. If all three statements do not apply, then the player will stand Any of those conditional statements will if applicable will lead the player to take a hit (draw one card) and return to the start logic check. Once back to the start logic, the player will have three cards, therefore, 'Ace & # card ==2' will return a no. Similar to the top logic, three logic checks are applied to the bottom with a conversion of an ace from 11 to 1 if the total exceeds 21

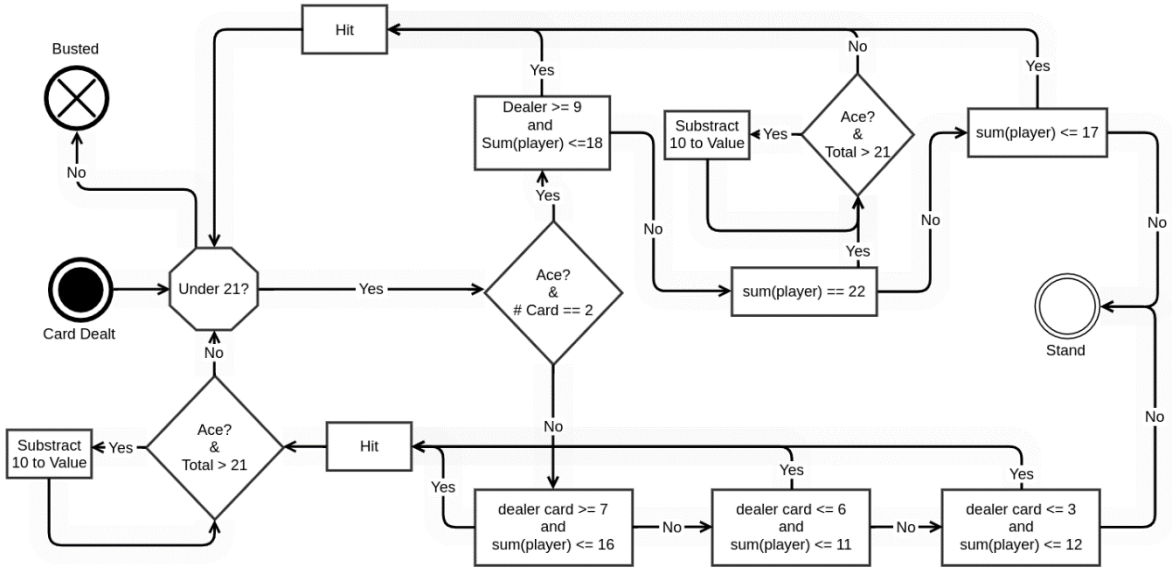

Figure 2: Basic Strategy Diagram, No Double Down, No Card Counting

Strategy #2: Uno Reserve [Mirror Dealer Strategy]

Unlike the Basic Strategy, Uno Reserve mirrored the dealer strategy. As depicted in Figure 3, the program first checks if the total value exceeds 21 and includes an ace. If it doesn't, the player will continue hitting cards under 16 until that logic is satisfied. In theory, this strategy should give the player the same chances as the dealer if the rules are "fair." However, the payment of x1.5 for a blackjack hand (ace + 10) limits the win potential of a player since the expected payout for a winning hand is x2.

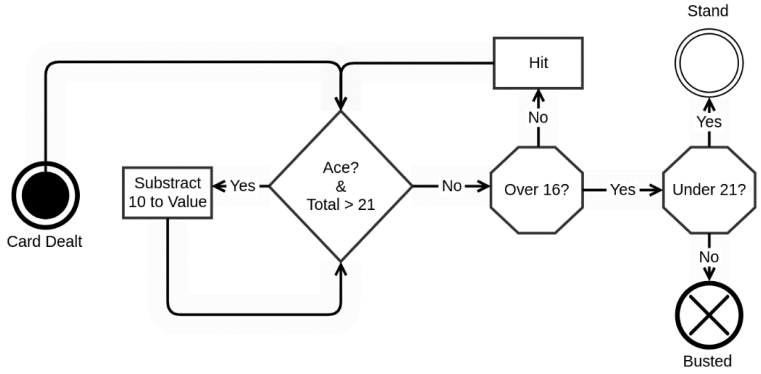

Figure 3: Mirror Dealer Strategy, No Double Down, No Card Counting

Strategy #3: Double Trouble [Basic Strategy with Double Down Strategy]

This strategy builds on top of the Basic Strategy. The inclusion of double down in theory should give the player a better chance of winning more when the position is in the player's favor. The program is more complex, but the logic remains like Basic Strategy. The logic flow for this strategy could be seen in Figure 4: Basic Strategy, Yes Double Down, No Card Counting.

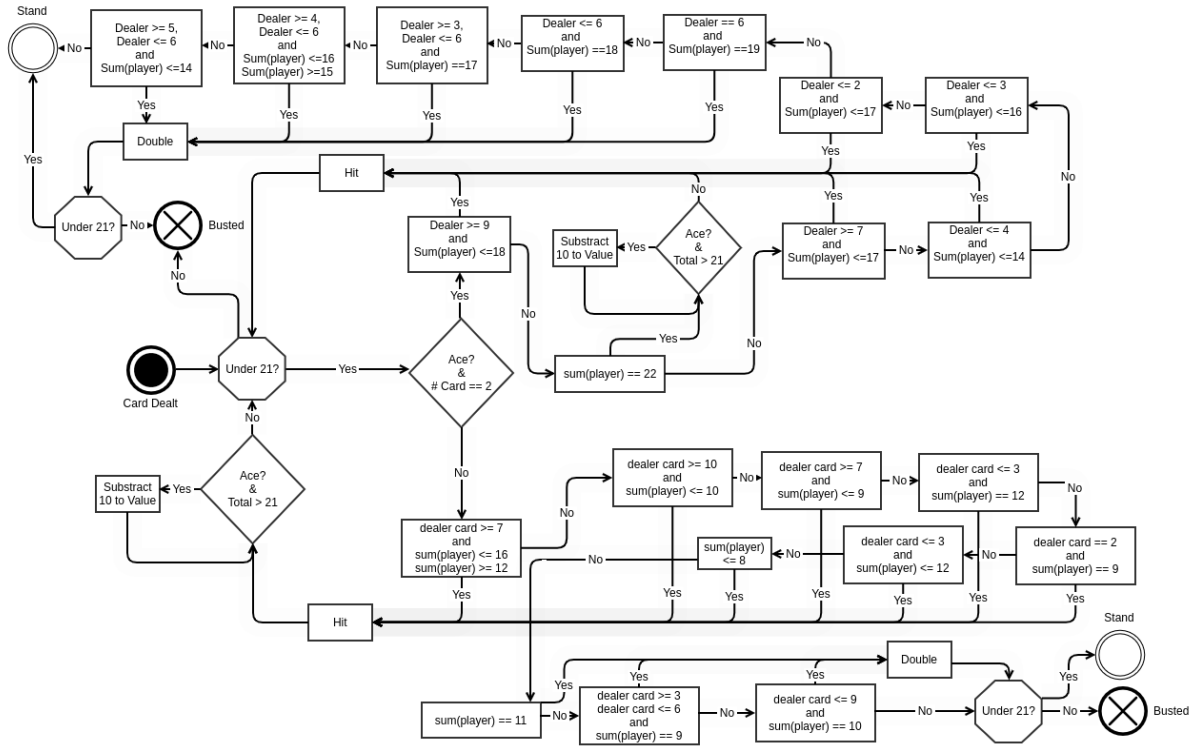

Figure 4: Basic Strategy, Yes Double Down, No Card Counting

Strategy #4: You Move You Lose - [Draw Until => 10 Strategy]

The idea for this strategy plays on the rules that the dealer must follow. Since the dealer must hit if the total value is under 16, the player can wait out the turn for the dealer to bust. Since the player ceases to hit after a total value of 11 is archived, the probability that the player will bust is 0 since the highest card value is 10. The logic for this strategy is depicted in Figure 5.

The strategy is independent of the dealer's face-up card. To maximize the potential of this strategy, the probability of the dealer is utilized to adjust the betting size. However, strategy #4 will not incorporate that adjustment. Therefore, it is expected that this strategy will not maximize a player's profit, but it is still interesting to simulate. A quick explanation of the probability of dealer bust can be found below.

To examine the probability of dealer bust, the face-up card is used.

Assumption: ∞ Deck, 52 cards, 4 copies of each card (2-10, Jack, Queen, King, Ace). 10, Jack, Queen, and King have the same face value, it is assumed that the collection of such is included in a group of 10. 2-9 are face value is equivalent to the corresponding number. Ace can be 1 or 11 so it is considered separately. All in all, there are three separate grouping of values that could be considered.

Probability of drawing from the three groupings and individually are as follows:

$$cdf = \begin{cases} 8/13 = (0.615) & \text{for 2-9} \\ 1/13 = (0.077) & \text{for aces} \\ 4/13 = (0.308) & \text{for 10, J, Q, K} \end{cases}$$

Figure 5: CDF for a full deck

$$pmf = \begin{cases} 1/13 = (0.077) & \text{for value of 2-9} \\ 1/13 = (0.077) & \text{for aces} \\ 4/13 = (0.308) & \text{for vlue of 10} \end{cases}$$

Figure 6: PMF of a full deck

To continue with the example, the possible sums from 2 range between [3,13]. The probability of each sum is the corresponding pmf after 2 is drawn. I.E. P(X=12) = 0.314 and P(X= 5) = 0.078. Where X = Sum of possible values + 2. The pmf of all possible sums are listed in table 1. Since the dealer must hit below 17, the dealer needs to hit one more time. The possibility from table 1 ranges between (4, and 23)[1]. The sum P(X = 17,18,19,20,21) is the probability of dealer safe and P(X > 21) is the probability of dealer bust. Once the calculations are made, the probability of dealer bust based on one face-up card are: 2-35.30%, *3 – 37.56%*, *4 – 40.28%*, *5 – 42.89%*, *6 – 42.08%*, 7 – 25.99%, 8 – 23.86%, 9 – 23.34%, J,Q,K – 21.43%, A – 11.65% [4].

X	P(X)
3	0.077
4	0.077
5	0.077
6	0.077
7	0.077
8	0.077
9	0.077
10	0.077
11	0.077
12	0.308
13	0.077

Table 1: Sum PMF after Drawn 2

[1] Notes: 24 = 13+11 could be excluded because of soft totals

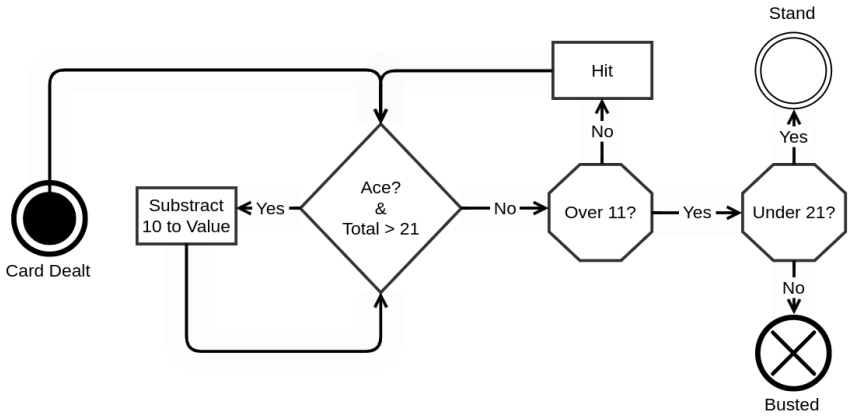

Figure 7: Draw Until => 10 Strategy, No Double Down, No Card Counting

Strategy #5: Go Big or Go Home [Double Down Under 12 otherwise Stay Strategy]

This strategy is similar to Strategy #4: You Move You Lose - [Draw Until => 10 Strategy] with the inclusion of Double Down. The idea is the same. The player plays against the probability that the dealer will bust. A possible problem with this strategy is players cannot draw after doubling down even if the sum is bad. In extreme cases, a 2 + 2 + 2 could be drawn which would result in the sum of 6. Again, this strategy is not expected to produce the best results.

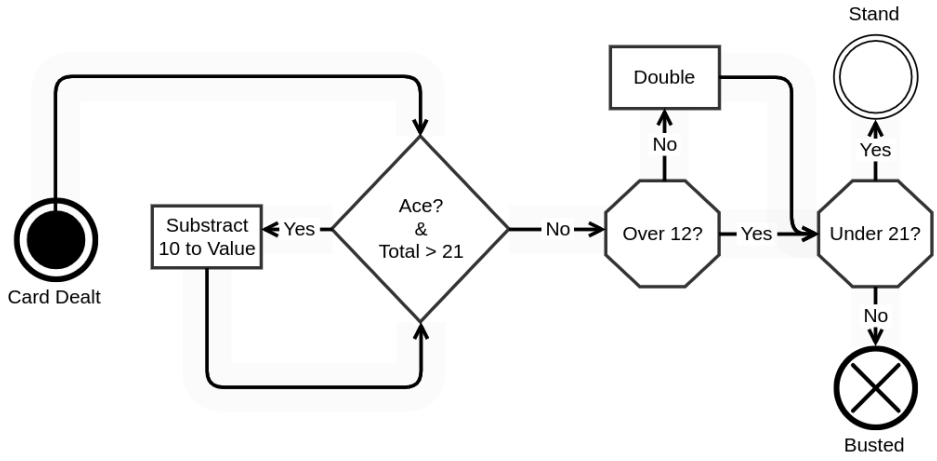

Figure 8: Double Down Under 12 otherwise Stay Strategy, Yes Double Down, No Card Counting

#6: 1+1= 100 [10 Count - Card Counting Strategy]

This strategy is similar to Strategy #3: Double Trouble [Basic Strategy with Double Down Strategy] but with the inclusion of card counting. The act of card counting is to keep track of the running values throughout the game. When the true value (running values/deck) is in the player's favor, the player increases the bet size to capitalize on the advantage [5]. In the simulation, the betting size is multiplied

by (true value – 1) when the value is in the player's favor and betting size/100 for all other bets. There are many card counting methods including Hi-Lo, Omega II, Zen Count, etc… The simulation will use a 10 count which is a +1 running count for 2-9/Ace cards and -2 for 10, J, Q, and K.

For card counting to be effective, the amount of deck must be known and the reshuffled rate must be infrequent. An unknown amount of deck will not give the player an accurate true value and frequent reshuffling will cause the running count to reset every time.

Runs Configuration and Testing Criteria

To test for the best results, the simulation will vary the # run / Turn, Number of decks, Shuffle rate, and starting bet.

The number of runs per turn will test the volatility of a strategy. As more runs increase, it is expected the average values will converge into the expected value due to Central Limit Theorem. The number of decks and shuffle rate will have an impact on card counting. From research, a decreasing number of the deck will have a positive impact on gains, and shuffle rate will negatively impact the gain. Lastly, the starting bet will show whether a strategy works or not by exacerbating the gain and loss. The resulting average ending cash from each strategy will determine which maximizes the player's profit.

Table 2: Program Variables

Variable	Value	Description
Strategy	1-6	The one of 6 strategies that will be examines
Turn	1-1000	Number of independent batch of games that will be run
Run	1-100	Number of blackjack game
Cash_track	Array	Track the current cash count from start_cash for analysis
Start_cash	10000	Amount of money starting
Allow_card_count	True/False	Keep Track of whether card counting is allowed or not
Running_count	+/-	Keep track of running count for card counting
Start_bet	100	Initialize the betting size
Num_deck	1-100	The amount of deck that will be played
Double_down	#	Keep track of the amount of double down
End_cash	#	Average cash of all the runs

Results

Table 3: Testing Configuration

Configuration #	Turn	Run	Num_Deck	Shuffle	Starting Bet	Start_cash
1	1000	100	6	10%	$100	$10,000
2	1000	25	6	10%	$100	$10,000
3	1000	100	3	10%	$100	$10,000
4	1000	25	3	10%	$100	$10,000
5	1000	100	6	10%	$500	$10,000
6	1000	25	6	10%	$500	$10,000
7	1000	100	6	50%	$100	$10,000
8	1000	25	6	50%	$100	$10,000

Table 4: Summary of Various Configurations

	Average Strategy #1	Average Strategy #2	Average Strategy #3	Average Strategy #4	Average Strategy #5	Average Strategy #6
Configuration #1	$ 9,114.50	$ 9,188.05	$ 8,219.65	$ 6,918.80	$ 6,235.20	-$ 1,052.24
Configuration #2	$ 9,038.80	$ 9,213.25	$ 8,164.65	$ 6,897.00	$ 6,235.20	-$ 1,028,88
Configuration #3	$ 9,097.40	$9,269.80	$ 8,187.85	$ 7,008.00	$6,171.80	-$1,996.66
Configuration #4	$9,110.25	$9,240.6	$8,075.05	$6,953.6	$6,150.85	-$412.42
Configuration #5	$ 5,592.00	$ 6,110.75	$876.50	-$5,223.75	- $8,125.50	-$49,464
Configuration #6	$5,413.50	$6,054.00	$944.75	-$5,301,75	$ 8,586.00	-$41,115
Configuration #7	$9,053.55	$9,306.65	$8,201.40	$6,888.25	$7,684.59	$7,466.83
Configuration #8	$9,036.60	$9,227.90	$8,278.15	$ 6,932.20	$6,214.40	$7,193.65

Overall, no strategy yields a positive return. Strategy #2 and Strategy #1 seem to consistently outperform other strategies in most variations. Strategy #6 seem to perform the worst when the shuffle rate is at 10% but outperforms strategy #4 and #5 when rate is at 50%.

Strategies # 1: You Hella Basic [Universal Basic Strategy]

With the universal basic strategy, the first few <20 runs seem to yield better results than long runs. However, the gains are quickly diminished and in the long run fall below starting cash.

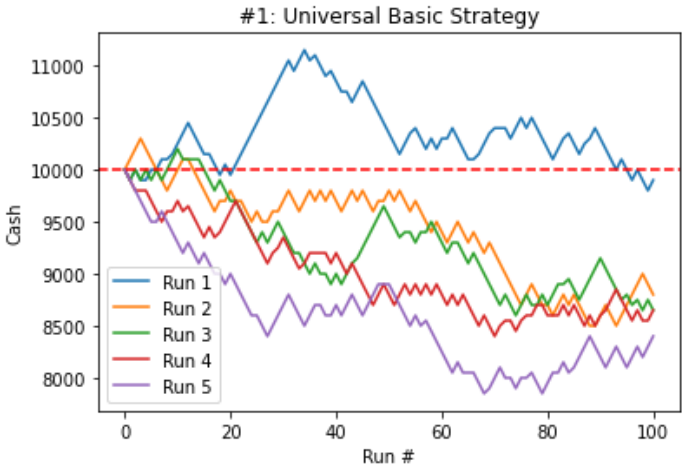

Figure 9: First 5 runs with Strategy #1

#2: Uno Reserve [Mirror Dealer Strategy]

The strategy of copying the dealer worked out better in the long run. From Figure 10: First 5 runs of Strategy 2, the separation of gains and losses happens towards run# 80. This could explain why the dealer uses this strategy and confirms the idea that the casino always comes out on top.

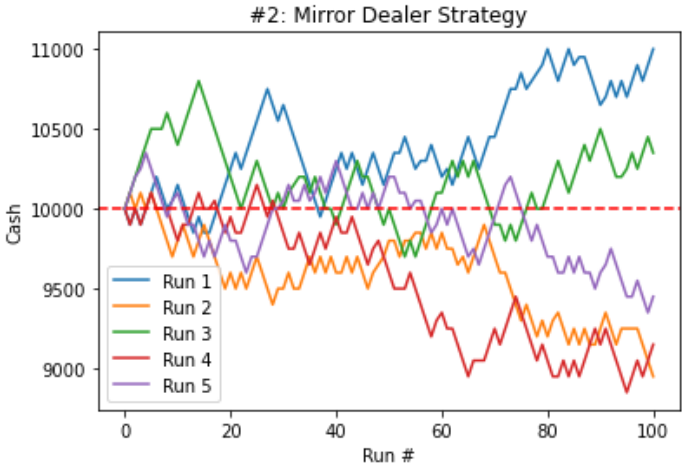

Figure 10: First 5 runs of Strategy 2

#3: Double Trouble [Basic Strategy with Double Down Strategy]

As expected, this strategy does not work to maximize profit. From Figure 11: First 5 runs of Strategy 3, 1/5 runs ended up above the starting cash. However, this could be explained by variations in the deck or a "lucky" run.

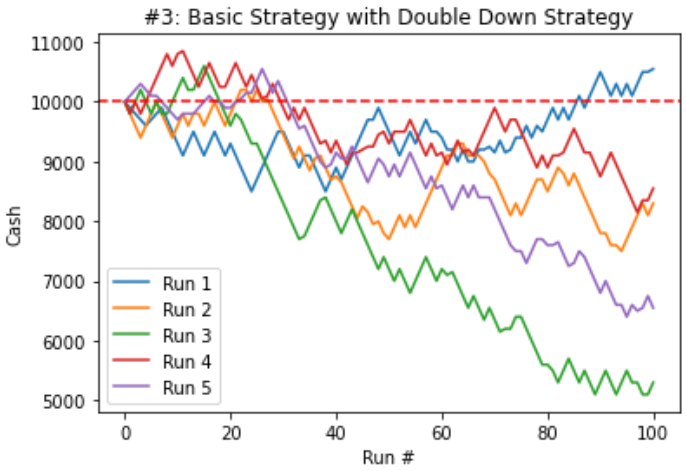

Figure 11: First 5 runs of Strategy 3

#4: You Move You Lose - [Draw Until => 10 Strategy]

This is another strategy that falls within expectations. From Figure 12: First 5 runs of Strategy 4, constant decrease as # run increase. Don't use this strategy.

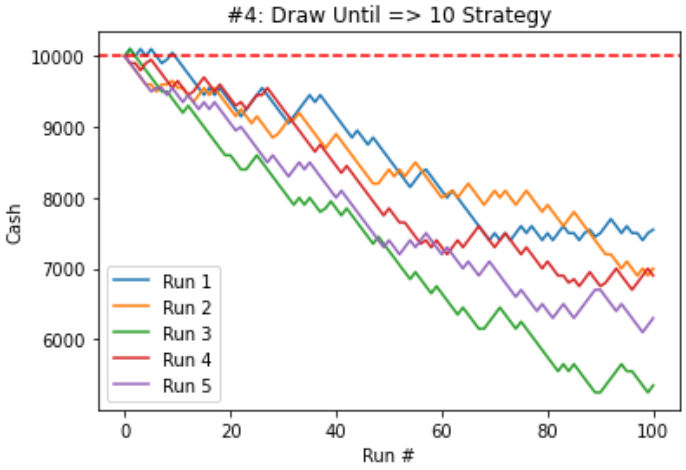

Figure 12: First 5 runs of Strategy 4

#5: Go Big or Go Home [Double Down Under 12 otherwise Stay Strategy]

Similar to Strategy 4, this strategy holds consistence toward <20 runs but began to push toward loss afterward.

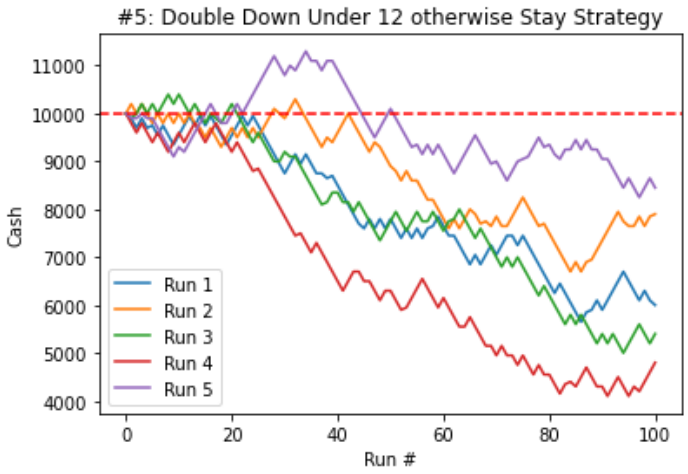

Figure 13: First 5 runs with strategy 5

#6: 1+1=-100 [10 Count - Card Counting Strategy]

This is a wild card strategy. For the most part, the player will lose money on average (some even go into debt). However, further checking suggests that the variance is $489,724,508, max ending cash $80,022, and min cash ending of -$113,931. Therefore, this strategy would win big but also lose big.

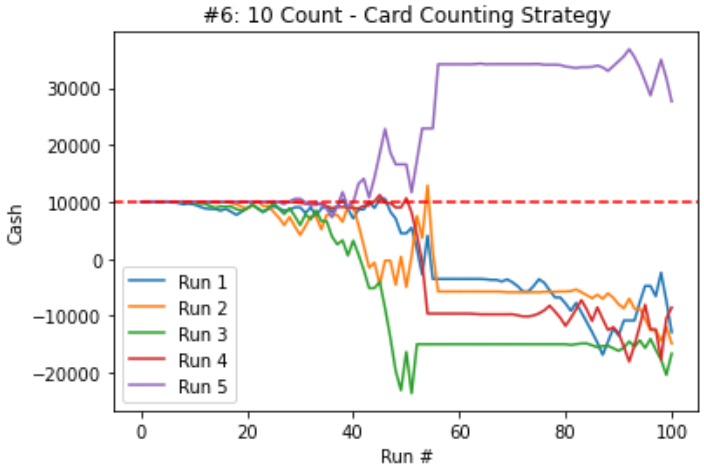

Figure 14: First 5 runs strategy 6

Conclusions

Summary

There were a few main takeaways from the project. 1) You will not beat the house/dealer at Blackjack in the long game, much like other casino games. The rule of the game is designed to give the dealer the advantage and even copying the dealer's strategy does not stop you from losing money. 2) Use Strategy #2: Uno Reserve [Mirror Dealer Strategy] if you must play and want to mitigate your loss. However, use #6: 1+1= 100 [10 Count - Card Counting Strategy] to make the most money (or lose the most). 3) Double down is either going to make you rich or going to bankrupt you.

Challenges

The logic behind each strategy proved to be the hardest to determines and translate into code. The complexity of the logic make it hard to generalize the program often resulting in a branching if statement just for one criteria.

The use of card counting into code was relatively simple to translate but the result was hard to determine. From literatures, card counting should have given the player a higher advantage than the results showed. An explanation for such a difference could be attributed to the large gap between biggest lost and biggest gain. If in the long run player tend to lose, the loses are included more in the average than the gains will be.

Future

The unknown impact of split and surrender is not explored in this paper. It would be expected that a split can be used to increase profit and surrender and minimize loses.

Works Cited

[1] “The History of Blackjack.” Crescent.edu, crescent.edu/post/the-history-of-blackjack.

[2] “Blackjack Rules | Learn How to Play 21 [Tips & Best Practices].” Blackjack.org, www.blackjack.org/blackjack/how-to-play/.

[3] Baldwin, Roger. Playing Blackjack to Win. Las Vegas, Nv, Cardoza Pub, 2008.

[4] Meltzer, Marc. “Here Are Blackjack Dealer Bust Probabilities.” Blackjack Online, 16 Feb. 2018, www.blackjackonline.com/1086/blackjack-dealer-bust-probabilities/. Accessed 25 July 2022.

[5] “Blackjack Card Counting - How to Count Cards with Our FREE Trainer.” Www.casino.org, www.casino.org/blackjack/card-counting/. Accessed 25 July 2022.