EE092IU
DIGITAL SIGNAL PROCESSING LABORATORY

Lab 3

CONVOLUTION

Full name:………………………………..

Student number:………………………...

Class:……………………………………..

Date:……………………………………...

I. OBJECTIVES

Students know how to write m-file in Matlab to illustrate the concept of linear convolution and how to compute the convolution product of two finite duration sequences.

II. REQUIRED EQUIPMENT

1. Computer
2. Matlab software

III. INTRODUCTION

3.1. Convolution formula

The direct and LTI forms of convolution, which describe the filtering equation of an LTI system in general, is given by:

$$y(n) = \sum_{m} h\,(m)x(n-m) = \sum_{m} x\,(m)h(n-m) \tag{1}$$

3.2. Direct form

Consider a causal FIR filter of order M with impulse response h(n), n = 0, 1, . . . , M. It may be represented as a block:

$$\boldsymbol{h} = [h_o, h_1, \dots, h_M] \tag{2}$$

Its length (i.e., the number of filter coefficients) is one more than its order:

$$L_h = M + 1 \tag{3}$$

The convolution of the length-L input x of which

$$\boldsymbol{x} = [x_o, x_1, \dots, x_{L-1}], \tag{4}$$

with the order-M filter h will result in an output sequence y(n). We must determine: (i) the range of values of the output index n, and (ii) the precise range of summation in m. For direct form, we choose the convolution expression as:

$$y(n) = \sum_{m} h\,(m)x(n-m)$$

The index of $h(m)$ must be within the range of indices in Eq. (2), that is, it must be restricted to the interval:

$$0 \le m \le M \tag{5}$$

Similarly, the index of $x(n-m)$ must lie within the legal range of indices in Eq. (4), that is,

$$0 \le n-m \le L-1 \tag{6}$$

To determine the range of values of the output index n, we rewrite (6), and in addition to the (5), we get:

$$\begin{gathered} 0 \le m \le n \le L-1+m \le L-1+M, or \\ 0 \le n \le L-1+M \end{gathered} \tag{7}$$

This is the index range of the output sequence y(n). Therefore, it is represented by a block:

$$\boldsymbol{y} = [y_o, y_1, \dots, y_{L-1+M}] \tag{8}$$

Its length is:

$$L_y = L + M \tag{9}$$

Thus, y is longer than the input *x* by *M* samples. As we will see later, this property follows from the fact that a filter of order *M* has memory *M* and keeps each input sample inside it for *M* time units. Setting $L_x = L$, and $L_h = M+1$, we can rewrite Eq. (9) in the more familiar form of Eq. (10). The relative block lengths are shown in Fig.1

$$L_y = L_x + L_h - 1 \tag{10}$$

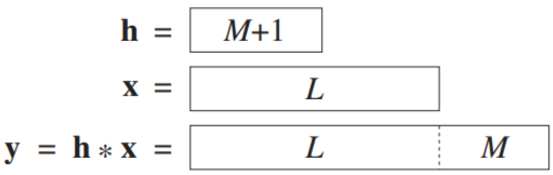

Figure 1: Relative lengths of filter, input, and output blocks

3.3. The relationship between convolution and DFT

If

$$x(n) \xrightarrow{\mathcal{F}} X(e^{jw}) \tag{11}$$

and,

$$h(n) \xrightarrow{\mathcal{F}} H(e^{jw}) \tag{12}$$

And if,

$$y(n) = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n] \tag{13}$$

Then,

$$Y(e^{jw}) = X(e^{jw})H(e^{jw}) \tag{14}$$

Thus, convolution of sequences implies multiplication of the corresponding Fourier transform.

IV. PROCEDURE

Problem 1:

a) In order to be able to perform the convolution procedure, write a script in MATLAB to input the 2 finite sequential values of $x(k)$ and $h(k)$ from the keyboard, then compute the output sequence $y(n)$ using MATLAB function *conv()*. The Input-output relationship for discrete linear time invariant system is described by:

$$y(n) = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

b) Given the following system response signal:

$$h[n] = 2^n u[n], \qquad n = [0:5]$$

and the input signal

$$x[n] = u[n+10] - u[n-5]$$

Plot the signals in MATLAB individually, the range of display is $n = [-15:15]$.

c) Verify the results by computing the Fourier transform of $x(k)$ and $h(k)$, which are $X(e^{jw})$ and $H(e^{jw})$ respectively, taking their multiplication to find $Y(e^{jw})$, and finally compute its inverse Fourier transform to find $y(n)$. Plot both of the results using ***subplot()*** function to show their similarity.

Problem 2: Repeat Problem 1, however, this time write your own function to perform the convolution calculation without using *conv()* function. Test your function with the new signals:

$$h[n] = \begin{cases} e^{-n}, -10 \leq n \leq 10 \\ 0, \text{elsewhere} \end{cases}$$

$$x[n] = \begin{cases} 1, 0 \leq n \leq 5 \\ 0, \text{elsewhere} \end{cases}$$

Problem 3: Write your own function to input the 2 finite sequential values of $x(k)$ and $h(k)$ from Problem 1, then compute the output sequence $y(n)$ by perform the convolution procedure by three different ways according to the theory class, which are the following methods:

- Convolution table
- LTI form
- Overlap – Add block by inputting the number of elements in a single block by keyboard (Ex: 4 elements per block)

$$h[n] = [2,3,0,-5,2,1] \text{ with } n = 0 \text{ at } x = 3$$
$$x[n] = [3,11,7,0,-1,4,2] \text{ with } n = 0 \text{ at } x = 0$$