EE092IU
DIGITAL SIGNAL PROCESSING LABORATORY

Lab 5

Z TRANSFORM AND TRANSFER FUNCTION

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Class:……………………………………..

Date:……………………………………...

I. OBJECTIVES

Students know how to write m-file in Matlab to illustrate the properties of Z transform and transfer function of discrete time system.

II. REQUIRED EQUIPMENT

1. Computer
2. Matlab software

III. INTRODUCTION

3.1. Transfer Function of Discrete-time System

The transfer function H(z) of a discrete-time system is defined in a similar way to the transfer function of continuous-time systems. So the transfer function of an LTI discretetime system is defined as the ratio of the z-transform Y(z) of the system's output signal to the z-transform X(z) of the input signal that is applied to the system, supposing zero initial conditions. The mathematical expression is

$$H(z) = \frac{Y(z)}{X(z)} \tag{1}$$

An alternative definition for a discrete-time system transfer function is that the transfer function H(z) of a system is the z-transform of the impulse response h[n] of the system. The mathematical expression is

$$H(z) = Z\{h[n]\}. \tag{2}$$

Example 1

Compute the transfer function of the discrete-time system with impulse response $h[n] = 2^n u[n]$.

Commands	Results	Comments

`syms n z`
`h=2^n;`
`H=ztrans(h,z)`
`H=simplify(H)`
	`H=z/(z-2)`	The transfer function $H(z)$ is computed directly from (11.12)

The transfer function of a discrete-time system can be derived also from the difference equation that describes the system.

Example 2

Compute the transfer function of a system described by the difference equation $y[n]-y[n-1]=x[n]+x[n-1]$ assuming that the initial conditions are zero.

First, z-transform is applied to both sides of the difference equation. The initial conditions are zero; hence, the property $Z\{x[n-1]\}=z^{-1}Z\{x[n]\}=z^{-1}X(z)$ is valid and can be used. Next, the difference equation (which is now transformed into algebraic equation) is solved for Y(z), and the transfer function H(z) is obtained by dividing Y(z) with X(z).

Commands	Results	Comments

`syms n z X Y`
`Y1 = (z^-1)*Y;`
`X1 = (z^-1)*X;`
`G = Y-Y1-X-X1;`
`Y = solve(G,Y);`
`H = Y/X`
	`H = (z+1)/(z-1)`	The transfer function $H(z)$ is derived by first solving the z-transform of the difference equation for $Y(z)$ and then dividing $Y(z)$ by $X(z)$ according to (11.11).

The coefficients of the denominator polynomial of the system transfer function are the same as the coefficients of the output signal y in the difference equation, while the coefficients of the numerator polynomial of the system transfer function are the same as the coefficients of the input signal x in the difference equation. Hence, if a system transfer function is known, we can directly derive the difference equation that specifies the system, and vice versa.

3.2. The Command tf for Discrete-Time Systems

The use and syntax of the *tf* command in the discrete-time case is similar to the one in the continuous-time case except from the fact that we have to indicate one more input argument that specifies the sampling time. The syntax of the *tf* command for discrete-time systems is *H = tf(num,den, Ts)*, where *num* and *den* are the numerator and denominator coefficients of the transfer function and *Ts* is the sampling time.

Commands	Results	Comments

`num = [2 1];`
`den = [1 3 2];`
`Ts = 0.4;`
`H = tf(num,den,Ts)`
	Transfer function: $\frac{2*z+1}{z\text{^}2+3*z+2}$
	Sampling time: 0.4
		If we specify the sampling time argument, the transfer function is written in terms of z.

3.3. Stability of Discrete-Time Systems

The calculation of the zeros and poles of a discrete-time transfer function is very important as the knowledge of the poles provides a criterion for the stability of a system. The command sin traduced for the continuous-time case a real soused for discrete-time systems.

A discrete-time system is stable if all the poles of its transfer function lie inside the unit circle: If one pole lies outside the unit circle the system is unstable.

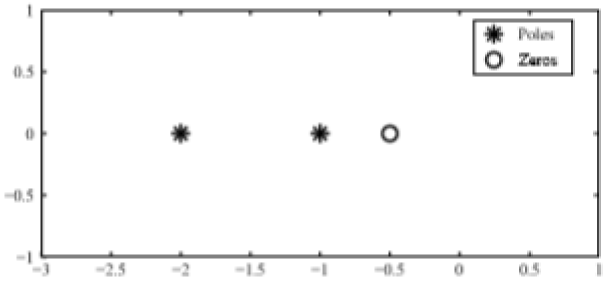
Example 3:

Compute and plot the poles and the zeros of the transfer function

$$H(z)=\frac{2z+1}{z^2+3z+2} \tag{3}$$

First way

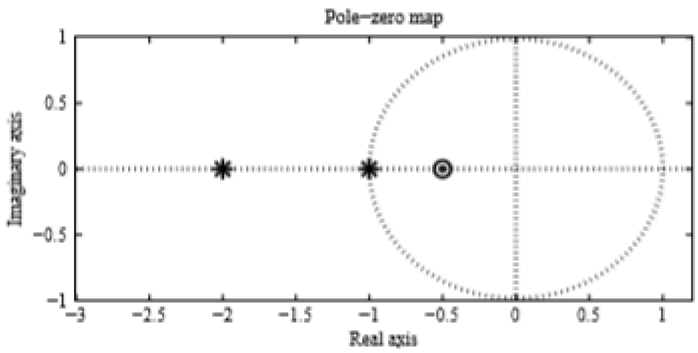
Commands	Results	Comments

`n = [2 1];`
`d = [1 3 2];`
`zer = roots(n);`
`pol = roots(d);`
`plot(real(pol),imag(pol),'*',`
`real(zer),imag(zer),'o')`
`xlim([-3 1]);`
`legend('poles', 'zeros');`
		Computation and graph of the zeros and poles of *H*(z).

Second way

Commands	Results	Comments

`H = tf(n,d,0.1);`
`poles = pole(H)`
`zeros = zero(H)`
	`poles = -2 -1`
	`zeros = -0.5000`
		Calculation of the zeros and poles of *H*(z).

`pzmap(H)` `xlim([-3 1.2])`		The poles and zeros are plotted with use of the command pzmap.

As one can see in the previous figure, the command ***pzmap*** besides the zeros and poles of the transfer function also plots the unit circle in the complex plane. Thus, from the above figure we conclude that the system with transfer function

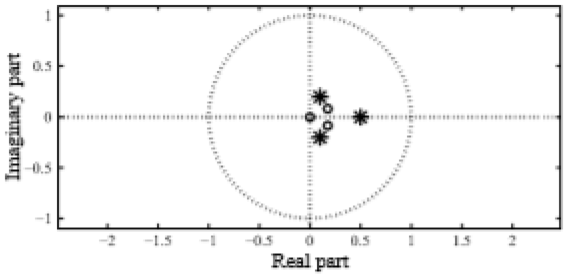
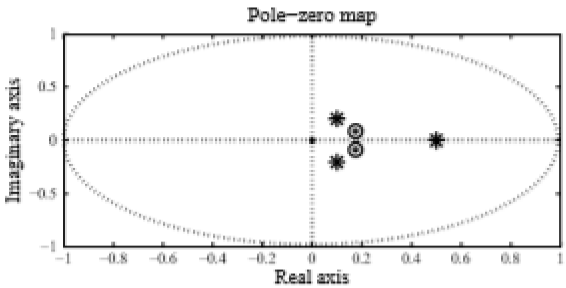
$$H(z)=\frac{2z+1}{z^2+3z+2}$$

is not stable since the pole $p_1=-2$ is not inside the unit circle. Another useful command that plots the poles, the zeros, and the unit circle in the z-plane is the command **zplane**. Its syntax is *zplane(num,den)*, where num and den are the coefficients of the numerator and denominator of the system transfer function, respectively.

Example 4:

Determine if the discrete-time system with transfer function is stable

$$H(z) = \frac{3z^2 - 1.4z + 0.15}{z^3 - 0.7z^2 + 0.15z - 0.025} \tag{4}$$

Commands	Results	Comments
`n=[4 -1.4 .15];` `d=[1 -.7 .15 -.025];` `zplane(n,d)`		Graph of poles and zeros of *H*(*z*) together with the unit circle.
`H=tf(n,d,0.1);` `pzmap(H)`		Equivalent graph obtained with use of the command *pzmap*.

All the poles of H(z) lie inside the unit circle; hence the system is stable. The poles lie inside the unit circle if their magnitude is less than one. To verify this statement, we calculate the poles of H(z), and we compute their magnitudes.

Commands	Results	Comments
`poles=pole(H)`	`poles=0.50  0.10+0.20i  0.10-0.20i`	The poles of the transfer function *H*(*z*).
`mag=abs(poles)`	`mag=0.5000  0.2236  0.2236`	The magnitudes of the poles are less than 1; hence, the system is stable.

Finally, we mention that the transfer function of a discrete-time system written is rational form can be expressed in zero - pole - gain form with use of the command *zpk*.

Example 5:

Find poles, zeros and gain of the transfer function:

$$H(z) = \frac{2z - 1}{z^2 + z - 12} \tag{5}$$

Commands	Results	Comments

`n=[2 0 -1];`
`d=[1 1 -12];`
`H=tf(n,d,0.5);`
`zpk(H)`
	Zero/pole/gain:
	$\frac{2(z-0.7071)(z+0.7071)}{(z+4)(z-3)}$
	Sampling time: 0.5
		Zero/pole/gain form of the transfer function *H*(z).

3.4. Discrete-Time System Response

In this section, we will discuss how to compute the response of a discrete-time system to various input signals when the system transfer function is known.

3.4.1. Step Response

The step response s[n] of a discrete-time system is the response of the system to the unit step sequence

$$u[n]=\begin{cases}1, n\geq 0\\ 0, n<0\end{cases} \tag{6}$$

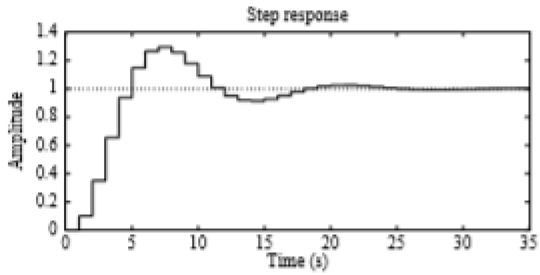
The response of a system to u[n] is computed with the help of the command *stepz*, while the graph of step response is implemented with the command *stairs*. The syntax of *stepz* is *y = stepz(num,den)*, where ***num*** and ***den*** are the coefficients of the numerator and denominator of the system transfer function, respectively.

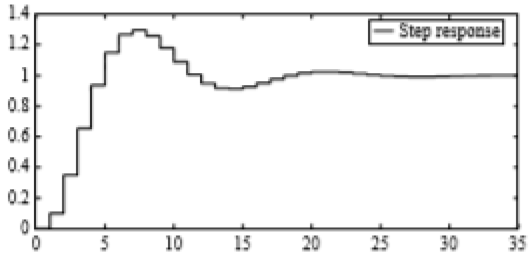
Example 6:

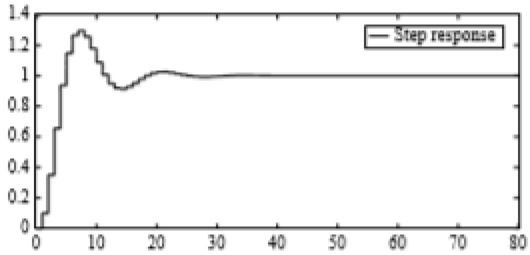
Compute the step response of the discrete-time system with the transfer function:

$$H(z)=\frac{0.1z-0.1}{z^2-1.5z+0.7} \tag{7}$$

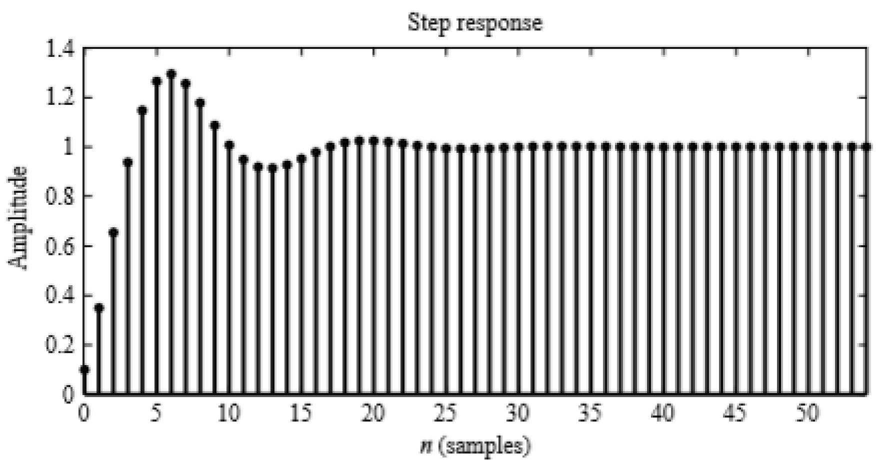
Commands	Results	Comments

`num=[.1 .1];`
`den=[1 -1.5 0.7];`
`dstep(num,den)`
		If no output argument is specified, the command `dstep` plots directly the step response $s[n]$ of the discrete-time system. The output signal is plotted up to the point where it reaches its steady state. The steady state of the system is depicted with the dashed line.

`s=dstep(num,den)`
`stairs(0:length(s)-1,s);`
`legend('Step response')`
		The step response signal $s[n]$ is computed by the command `dstep` and is plotted with use of the command `stairs`.

`n=0:80;`
`s=dstep(num,den,n);`
`stairs(n,s)`
`legend('Step response')`
		Graph of step response for $0 \le n \le 80$.

An alternative way in order to compute and plot the step response of the system is to use the command *stepz*. The syntax is *y = stepz(num,den)*, where num and den are the coefficients of the numerator and denominator of the system transfer function, respectively.

Commands	Results
`stepz(num,den)`	

3.4.2. Impulse Response

The impulse response h[n] of a discrete-time system is the response of the system to the unit impulse sequence (or delta function)

$$\delta(n) = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases} \tag{8}$$

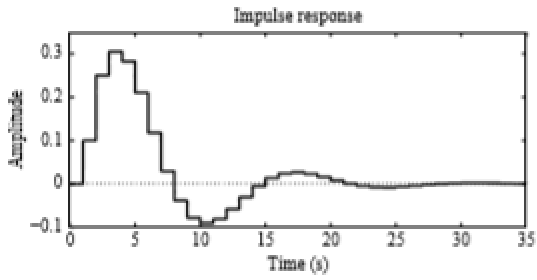
The response h[n] of a system to d[n] is computed with the help of the command *impz*, while the graph of impulse response is also implemented by the command *stairs*. The syntax of *impz* is *y = impz(num,den)*, where *num* and *den* are the coefficients of the numerator and denominator of the system transfer function, respectively.

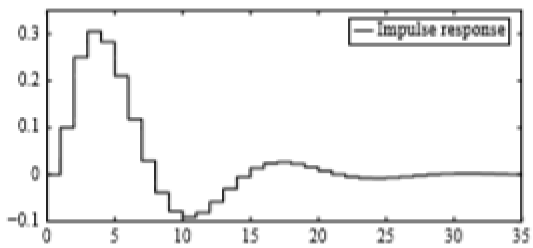
Example 7:

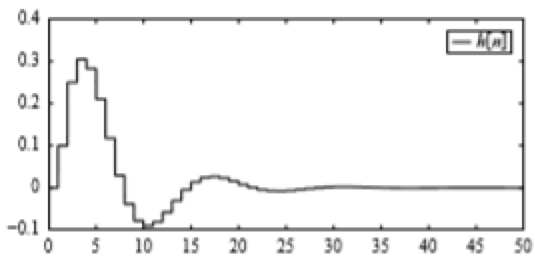
Compute and plot the impulse response of the discrete-time system with transfer function

$$H(z) = \frac{0.1z - 0.1}{z^2 - 1.5z + 0.7} \tag{9}$$

Commands	Results	Comments

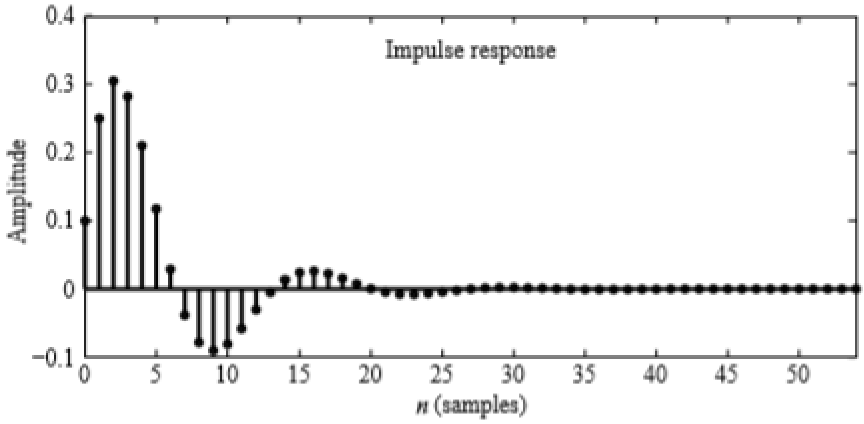
`num=[.1 .1];`
`den=[1 -1.5 0.7];`
`dimpulse(num,den)`
		The impulse response of the discrete-time system is plotted up to the point where it reaches its steady state. The steady state of the system is depicted with the dashed line.

`h=dimpulse(num,den);`
`stairs(0:length(h)-1,h)`
`legend('Impulse response')`
		The impulse response $h[n]$ is computed by the command `dimpulse` and is plotted by the command `stairs`.

`n=0:50;`
`y=dimpulse(num,den,n);`
`stairs(n,y)`
`legend('h[n]')`
		The impulse response $h[n]$ is defined and plotted for $0 \leq n \leq 50$.

Finally, one may use the command *impz* to compute and plot the impulse response of a system.

Commands	Results

`num=[.1 .1];`
`den=[1 -1.5 0.7];`
`impz(num,den)`
	

3.5. Transfer Function and Frequency Response

Example 8:

Compute with the help of z-transform the frequency response $H(\omega)$ of a discrete-time system with impulse response $h[n]=[3,5,2,1], 0 \le n \le 3$

Commands	Results	Comments

`n=0:3;`
`h=[3 5 2 1];`
`syms z w`
`Htf=sum(h.*z.^-n);`
		We define the impulse response $h[n]$, and by applying z-transform to $h[n]$ we compute the transfer function $H(z)$ of the system.

`H=subs(Htf,z,exp(j*w));` `H=simplify(H)`	`H=3+5*exp(-i*w)+2*` `exp(-2*i*w)+exp(-3*i*w)`	We substitute z by $e^{j\omega}$ in the transfer function $H(z)$, and we derive the frequency response $H(\omega)$ of the system.
`Hw=sum(h.*exp(-j*w*n))`	`Hw=3+5*exp(-i*w)+2*` `exp(-2*i*w)+exp(-3*i*w)`	To confirm our result, we compute $H(\omega)$ by applying DTFT to the impulse response $h[n]$.

Example 9:

Compute the frequency response $H(\omega)$ of the discrete-time system with impulse response $h(n)=(2/3)^n u(n)$.

Commands	Results	Comments

`syms n z w`
`h=(2/3)^n*heaviside(n);`
`Hz=ztrans(h,z);`
		Definition of the impulse response $h[n]$ and computation via z-transform of the transfer function $H(z)$.

`H=subs(Hz,z,exp(j*w));` `H=simplify(H)`	`H=3*exp(i*w)/(3*exp(i*w)-2)`	Substituting z by $e^{j\omega}$ in $H(z)$ yields the discrete-time frequency response $H(\omega)$.
`h=(2/3)^n;` `Hw=symsum(h*exp(-j*w*n),n,0,inf)`	`Hw=3*exp(i*w)/(-2+3*exp(i*w))`	Confirmation of our result by directly computing the DTFT of $h[n]$.

Example 10:

Compute with the help of z-transform the frequency response $H(\omega)$ of a system described by the difference equation y[n] = 0.9y[n-1] + x[n] with zero initial conditions.

In order to compute the transfer function H(z) of the system we will apply z-transform to both sides of the difference equation taking into account that $Z\{y[n-1]\} = z^{-1}Z\{y[n]\} = z^{-1}Y(z)$.

Next, we will substitute z by $e^{j\omega}$ in order to derive the frequency response $H(\omega)$ of the system.

Commands	Results	Comments

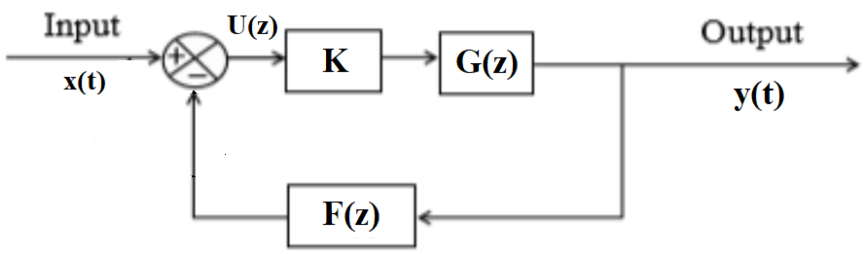
`syms z n w Yz X`
`Y1z=z*(-1)*Yz;`
`G=Yz-0.9*Y1z-X;`
`Yz=solve(G,Yz);`
		We apply z-transform to both sides of the difference equation and solve for $Y(z)$.

`Hz=Yz/X`	`Hz=10/(10+9*z)`	The transfer function is computed as $H(z)=\frac{Y(z)}{H(z)}$.
`Hw=subs(Hz,z,exp(j*w))`	`Hw=10/(10+9*exp(i*w))`	Substituting z by $e^{j\omega}$ in $H(z)$ yields the discrete-time frequency response $H(\omega)$.

PROCEDURE

Problem 1:

Compute the closed-loop transfer function $\boldsymbol{H(z)}$ of the system that is depicted in the figure below if $\boldsymbol{G(z) = \frac{0.5}{z-0.1}, F(z) = \frac{0.2}{z-0.4}, and\ K = 2}$. Then, find the inverse z – transform of the obtained transfer function.



Problem 2:

Consider the discrete-time system described by the transfer function

$$H(z) = \frac{8z^2 + 10z - 6}{z^3 + 2z^2 - z - 2}$$

a. Find out if the system is stable.
b. Express the transfer function in zero - pole - gain form.
c. Express the transfer function in partial fraction form.

Problem 3:

A linear transfer function is given as

$$H(z) = \frac{z^2 + 1}{z^2 - 1.39z + 1.21}$$

Write a Matlab code to make two subplots. On the left, plot the poles and zeros of H(z). On the right, plot the first 80 samples of the impulse response.

Problem 4:

Consider a discrete-time system with transfer function

$$H_1(z) = \frac{z^2}{z^2 + 0.2z + 0.01}$$

a. Find the transfer function $H_2(z)$ of a system with the same behavior that causes a delay of 2 units to an applied input signal. Also, plot for $0 \le n \le 8$.
b. The impulse response of the system with transfer function $H_1(z)$.
c. The impulse response of the system with transfer function $H_2(z)$.

Problem 5:

Consider the system described by the difference equation

$y[n] = 1.6y[n-1] - 0.8y[n-2] + 0.01x[n] + 0.03x[n-1] + 0.015x[n-2]$ with zero initial conditions. Compute and plot the response y[n] of the system to the input signal x[n] when

a. x[n] is the unit impulse sequence $\delta[n]$.

b. x[n] is the unit step sequence u[n].

Problem 6:

Consider a discrete-time system with frequency response

$$H(\omega) = \frac{0.1 + 0.1e^{-j\omega} + 0.18e^{-2j\omega} + 0.18e^{-3j\omega} + 0.09e^{-4j\omega} + 0.09e^{-5j\omega}}{1 - 1.5e^{-j\omega} + 2.2e^{-2j\omega} - 1.5e^{-3j\omega} + 0.8e^{-4j\omega} + 0.18e^{-5j\omega}}$$

a. Plot the frequency response of the system.

b. Compute and plot the impulse response and the step response of the system.