# Sistemi tempo discreto – LT Cap.4

Controllo Digitale

Corso di Laurea in Ingegneria Informatica

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March 22, 2023

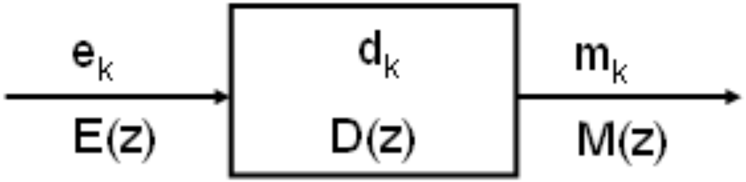
# Indice




- Funzione di trasferimento tempo discreto
- Schemi a blocchi

# Integrale vs sommatoria di convoluzione



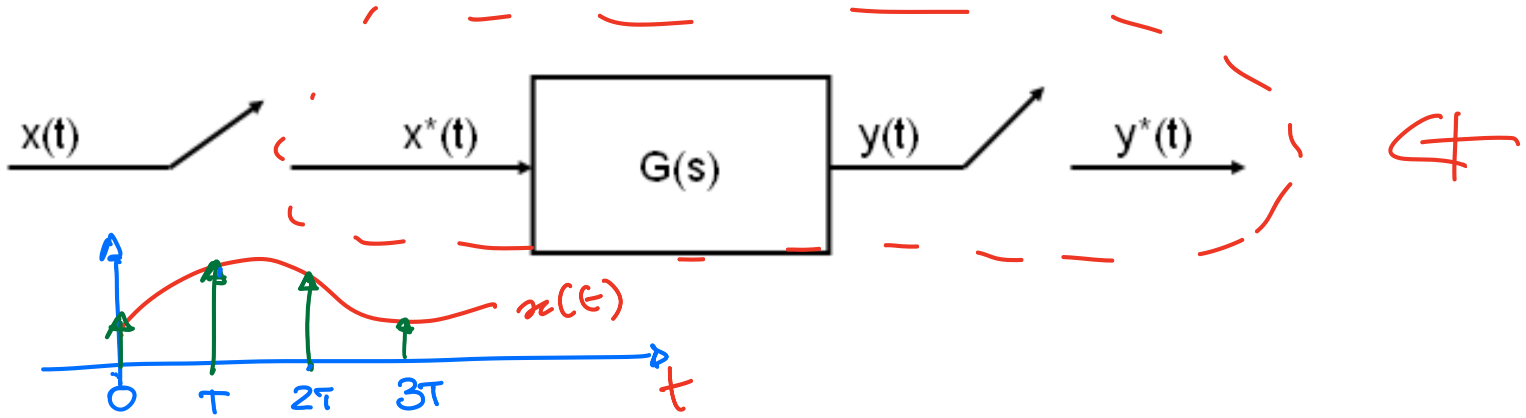




$$
\begin{aligned}
c(t) &= \int_0^t g(\tau)x(t-\tau)d\tau = \\
&= \int_0^t x(\tau)g(t-\tau)d\tau \\
C(s) &= X(s)G(s)
\end{aligned}
$$

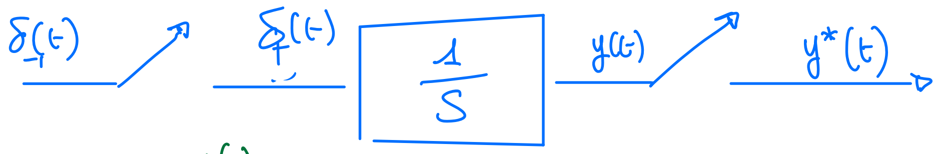
$$
\begin{aligned}
m_k &= \sum_{i=0}^{k} d_i e_{k-i} \\
&= \sum_{i=0}^{k} e_i d_{k-i} \\
M(z) &= D(z)E(z)
\end{aligned}
$$

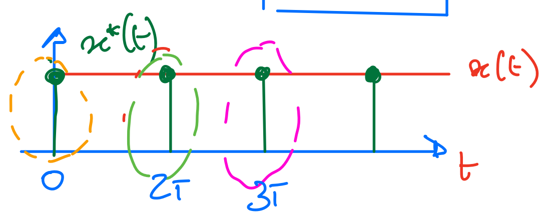
# Dai sistemi tempo continuo ai sistemi tempo discreto (1/2)

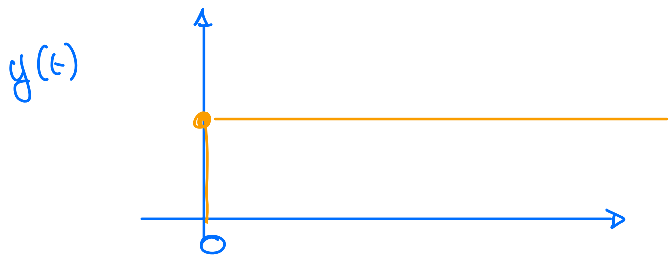




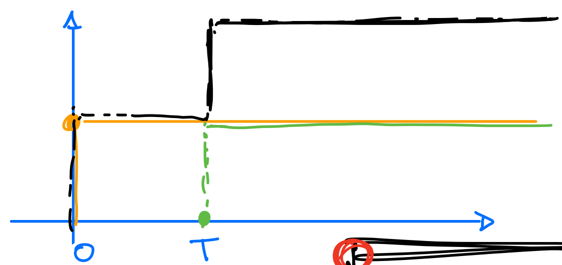

$$
y(t) = \begin{cases}
g(t)x(0) & 0 \le t < T \\
g(t)x(0) + g(t-T)x(T) & T \le t < 2T \\
g(t)x(0) + g(t-T)x(T) + g(t-2T)x(2T) & 2T \le t < 3T \\
\vdots & \vdots \\
g(t)x(0) + g(t-T)x(T) + \cdots + g(t-kT)x(kT) & kT \le t < (k+1)T
\end{cases}
$$



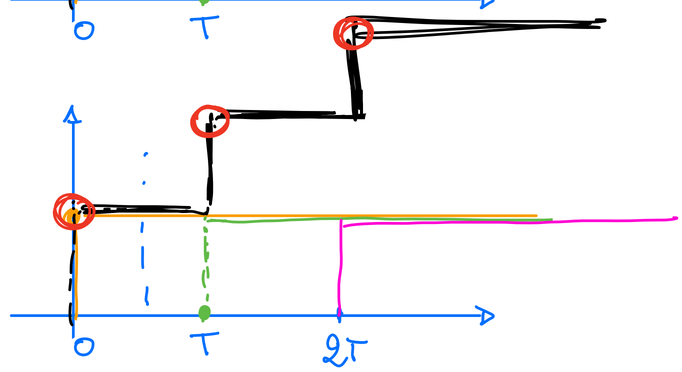




$0 \leq t < T$




$T \leq t < 2T$




$2T \leq t < 3T$

# Dai sistemi tempo continuo ai sistemi tempo discreto (2/2)




In forma compatta

$$
\begin{aligned}
y(t) &= g(t)x(0) + g(t-T)x(T) + \cdots + g(t-kT)x(kT) \quad = \\
&= \sum_{h=0}^{k} g(t-hT)x(hT) \qquad 0 \le t < (k+1)T
\end{aligned}
$$

$t = kT$

campionando la sequenza ottenuta

$$\boxed{y(kT) = \sum_{h=0}^{k} g(kT-hT)x(hT) = \sum_{h=0}^{k} x(kT-hT)g(hT)}$$

SOMMATORIA DI CONVOLUZIONE !!! ...

ricordando che $x(t) = g(t) = 0$, per $t < 0$

$$y(kT) = \sum_{h=0}^{\infty} g(kT-hT)x(hT) = \sum_{h=0}^{\infty} x(kT-hT)g(hT)$$

# Funzione di trasferimento discreta



A partire dalla relazione trovata

$$y(kT) = \sum_{h=0}^{\infty} g(kT - hT)x(hT)$$

si arriva alla relazione di funzione di trasferimento discreta

$$Y(z) = G(z)X(z)$$

$k-h = m$

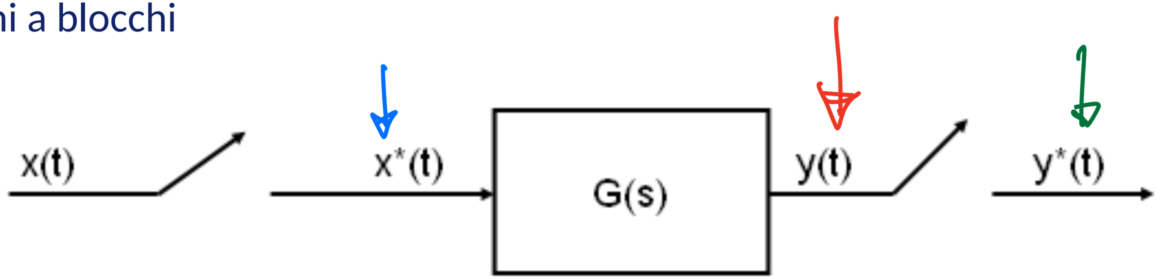
$k = m+h$

$$\begin{aligned} Y(z) &= \sum_{k=0}^{\infty} y(kT)z^{-k} = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} g(kT-hT)x(hT)z^{-k} = \\ Y(z) &= \sum_{m=0}^{\infty}\sum_{h=0}^{\infty} g(mT)x(hT)z^{-m-h} = \sum_{m=0}^{\infty} g(mT)z^{-m}\sum_{h=0}^{\infty} x(hT)z^{-h} = \\ &= G(z)X(z) \end{aligned}$$

# Indice

## 2 Schemi a blocchi


- Funzione di trasferimento tempo discreto
- Schemi a blocchi




Nel caso di ingresso pari a $x^*(t)$, nel dominio di Laplace in uscita si avrà

$$Y(s) = \boxed{G(s)X^*(s)}$$

Campionando l'uscita si ottiene

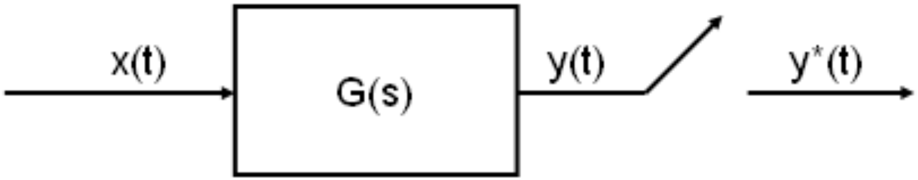
$$Y^*(s) = [G(s)X^*(s)]^* = G^*(s)X^*(s)$$

Passando nel dominio della $\mathcal{Z}$-trasformata

$$\boxed{Y(z) = G(z)X(z)}$$

# Ingresso continuo






Nel caso di ingresso pari a $x(t)$, nel dominio di Laplace in uscita si avrà

$$Y(s) = G(s)X(s)$$

Campionando l'uscita si ottiene

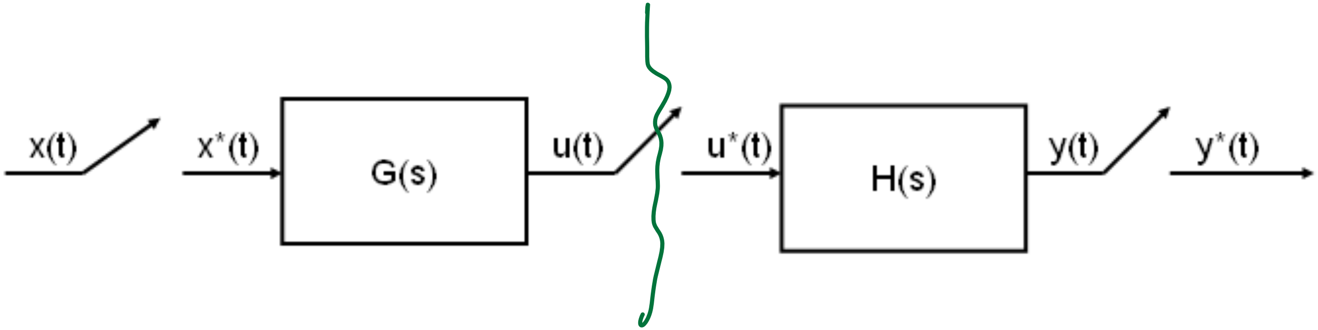
$$Y^*(s) = [G(s)X(s)]^* \boxed{\neq G^*(s)X^*(s)}$$

Passando nel dominio della $\mathcal{Z}$-trasformata

$$Y(z) = \boxed{GX}(z) \neq G(z)X(z)$$

# Blocchi in cascata






Nel dominio di Laplace

$$Y^*(s) = G^*(s)H^*(s)X^*(s)$$

$\mathcal{Z}$-trasformata

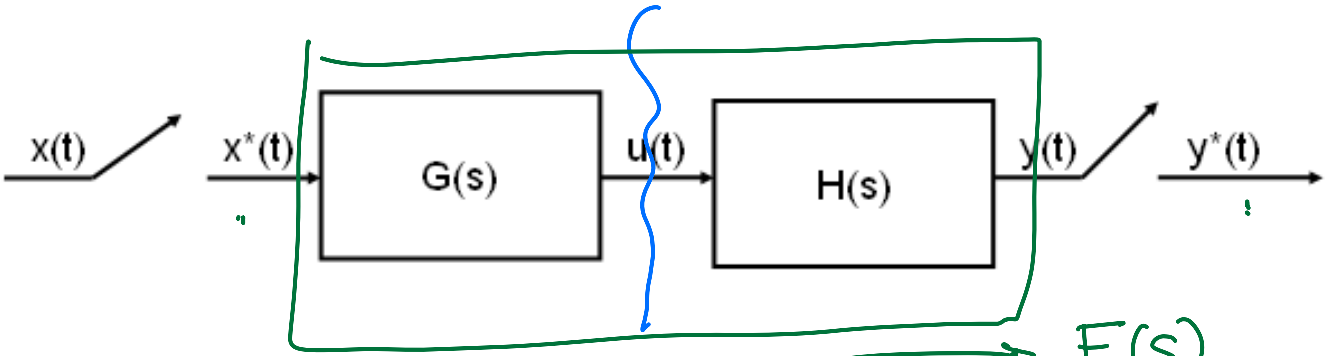
$$Y(z) = G(z)H(z)X(z)$$

FdT

$$\boxed{\frac{Y(z)}{X(z)} = G(z)H(z)}$$

# Blocchi in cascata






F(s)

Nel dominio di Laplace

$$Y^*(s) = \boxed{[G(s)H(s)]}^* X^*(s)$$

$\mathcal{Z}$-trasformata

$$Y(z) = \underline{GH(z)}X(z)$$

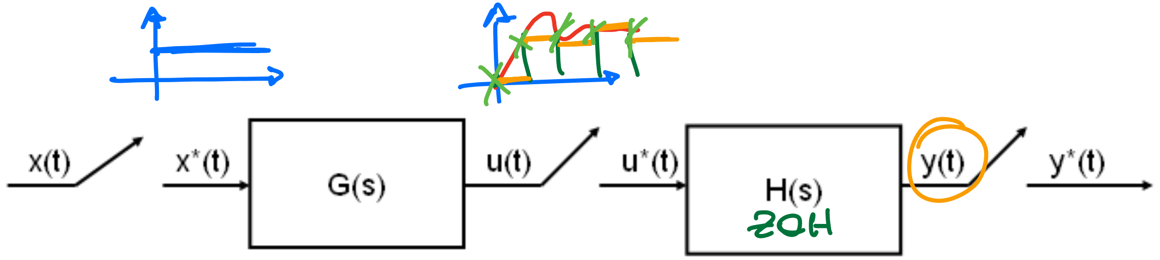
$$Y(z) = F(z)\,X(z)$$

FdT

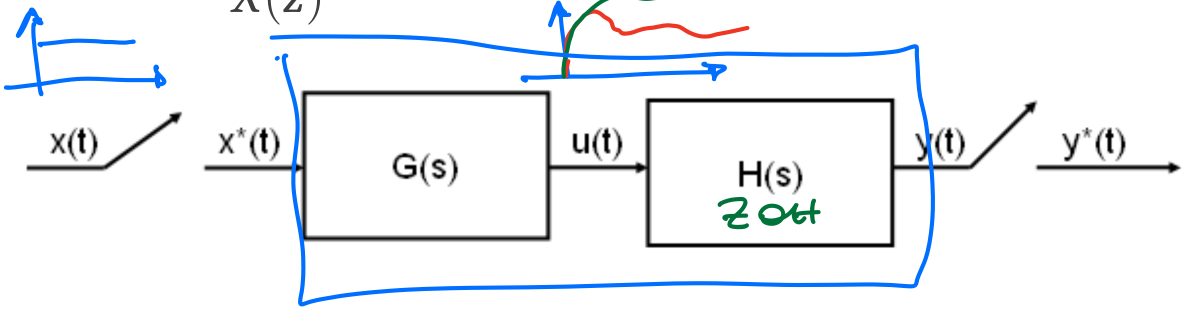
$$\frac{Y(z)}{X(z)} = GH(z)$$

# Esempio: confronto blocchi in cascata






$$\frac{Y(z)}{X(z)} = H_0(z)G(z) = G(z) = \mathcal{Z}[G(s)]$$

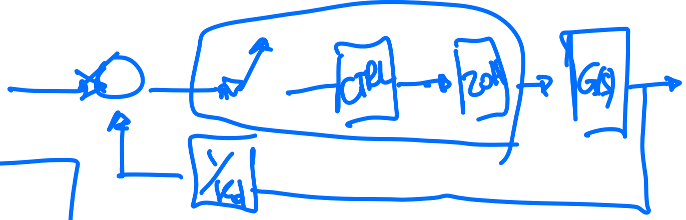


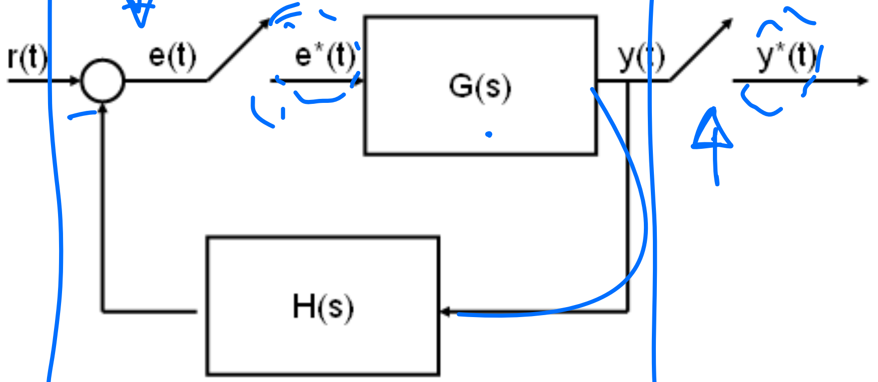

$$\frac{Y(z)}{X(z)} = H_0G(z) = \mathcal{Z}[H_0(s)G(s)] = (1 - z^{-1})\mathcal{Z}\left[\frac{G(s)}{s}\right]$$

$$\mathcal{Z}\left\{\frac{1-e^{-sT}}{s}G(s)\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

# Controllo a retroazione (1/2)

## 2 Schemi a blocchi






$$
\begin{aligned}
1 \quad E(s) &= R(s) - H(s)Y(s) \\
2 \quad Y(s) &= G(s)E^*(s)
\end{aligned}
$$

sostituendo si ottiene

$$E(s) = R(s) - H(s)\overbrace{\boxed{G(s)E^*(s)}}^{y(s)}$$

# Controllo a retroazione (2/2)



campionando le relazioni precedenti si ottiene

$$
\begin{aligned}
(1)\quad E^*(s) &= R^*(s) - GH^*(s)E^*(s)\\
(2)\quad Y^*(s) &= G^*(s)E^*(s)
\end{aligned}
$$

Dalla (1)

$$[1+GH^*(s)]E^*(s) = R^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1+GH^*(s)}$$

Sostituisco nella (2) e ottengo

La FdT del sistema campionato nel dominio di Laplace risulta

$$Y^*(s) = \frac{G^*(s)R^*(s)}{1+GH^*(s)}$$

nel dominio della $\mathcal{Z}$-trasformata

$$Y(z) = \frac{G(z)R(z)}{1+GH(z)}$$

# Sistemi tempo discreto – LT Cap.4

*Thanks for sharing your thoughts*

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