Ricostruzione dei segnali
LT Cap.3

Controllo Digitale

Corso di Laurea in Ingegneria Informatica

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March 13, 2023

Indice

Ricostruttori reali

$$a_0 + a_1 t + a_2 t^2 + \dots + a_m t^m$$

Espansione in serie di Taylor

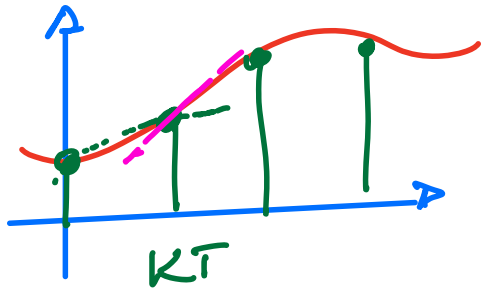
$$x(t) = x(kT) + \left.\frac{dx(t)}{dt}\right|_{t=kT} (t - kT) + \left.\frac{d^2x(t)}{dt^2}\right|_{t=kT} \frac{(t-kT)^2}{2!} + \dots$$

Derivata=rapp. incrementale

$$\left.\frac{dx(t)}{dt}\right|_{t=kT} \simeq \frac{x(kT) - x((k-1)T)}{T}$$

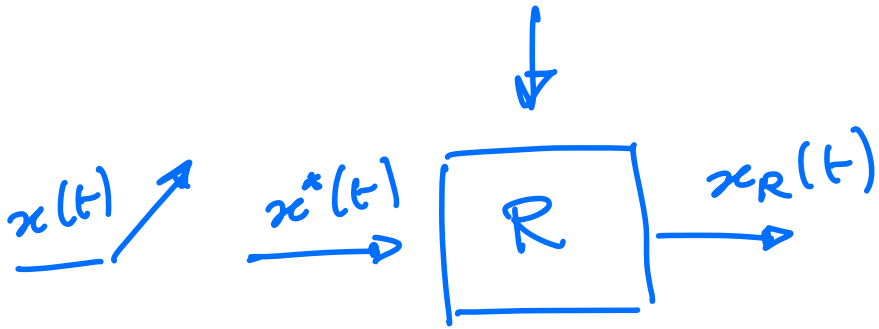
$$\left.\frac{d^2x(t)}{dt^2}\right|_{t=kT} \simeq \frac{\left.\frac{dx(t)}{dt}\right|_{t=kT} - \left.\frac{dx(t)}{dt}\right|_{t=(k-1)T}}{T} \simeq$$

$$\simeq \frac{x(kT) - 2x((k-1)T) + x((k-2)T)}{T^2}$$



Studio dei ricostruttori reali

1. Segnale ricostruito
2. Risposta impulsiva



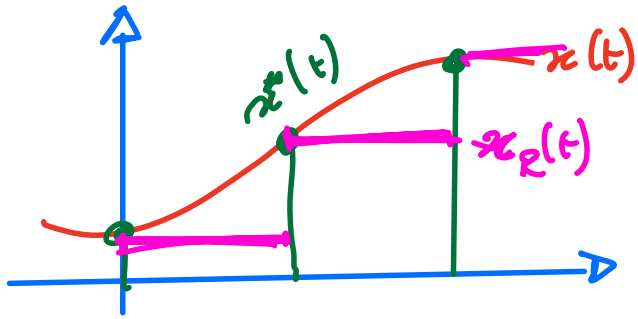
3. $\mathcal{L}$-trasformata (Trasformata di Laplace)
4. Analisi in frequenza
   - Calcolo della funzione in ω
   - Tracciamento della risposta armonica
   - **Considerazioni sul tempo di campionamento**

Indice

Ricostruttore di ordine zero (ZOH)

2 Ricostruttore di ordine 0 (ZOH)

ORGANO DI TENUTA



Segnale ricostruito

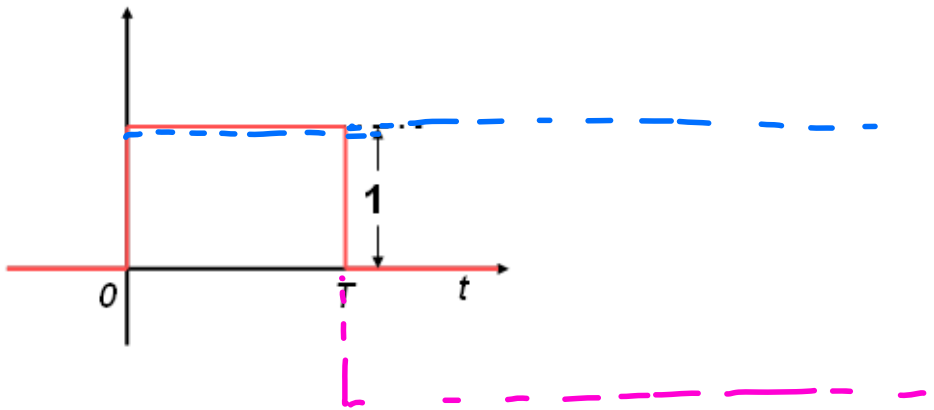
$$x_0(t) = x(kT) \qquad kT \le t < (k+1)T$$

Risposta impulsiva

$$g_0(t) = \delta_{-1}(t) - \delta_{-1}(t-T)$$

$\mathcal{L}$-trasformata

$$H_0(s) = \mathcal{L}[g_0(t)] = \frac{1}{s} - \frac{e^{-sT}}{s} = \frac{1 - e^{-sT}}{s}$$



Analisi in frequenza (1/2)

2 Ricostruttore di ordine 0 (ZOH)

Risposta armonica

$$H_0(j\omega) = \frac{1 - e^{-j\omega T}}{j\omega} = \frac{2e^{-j\omega T/2}}{\omega} \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j} =$$

$$= \frac{2e^{-j\omega T/2}}{\omega} \sin(\omega T/2) = T \frac{\sin(\omega T/2)}{\omega T/2} e^{-j\omega T/2}$$

Modulo

$$|H_0(j\omega)| = T \left| \frac{\sin(\omega T/2)}{\omega T/2} \right|$$

Fase

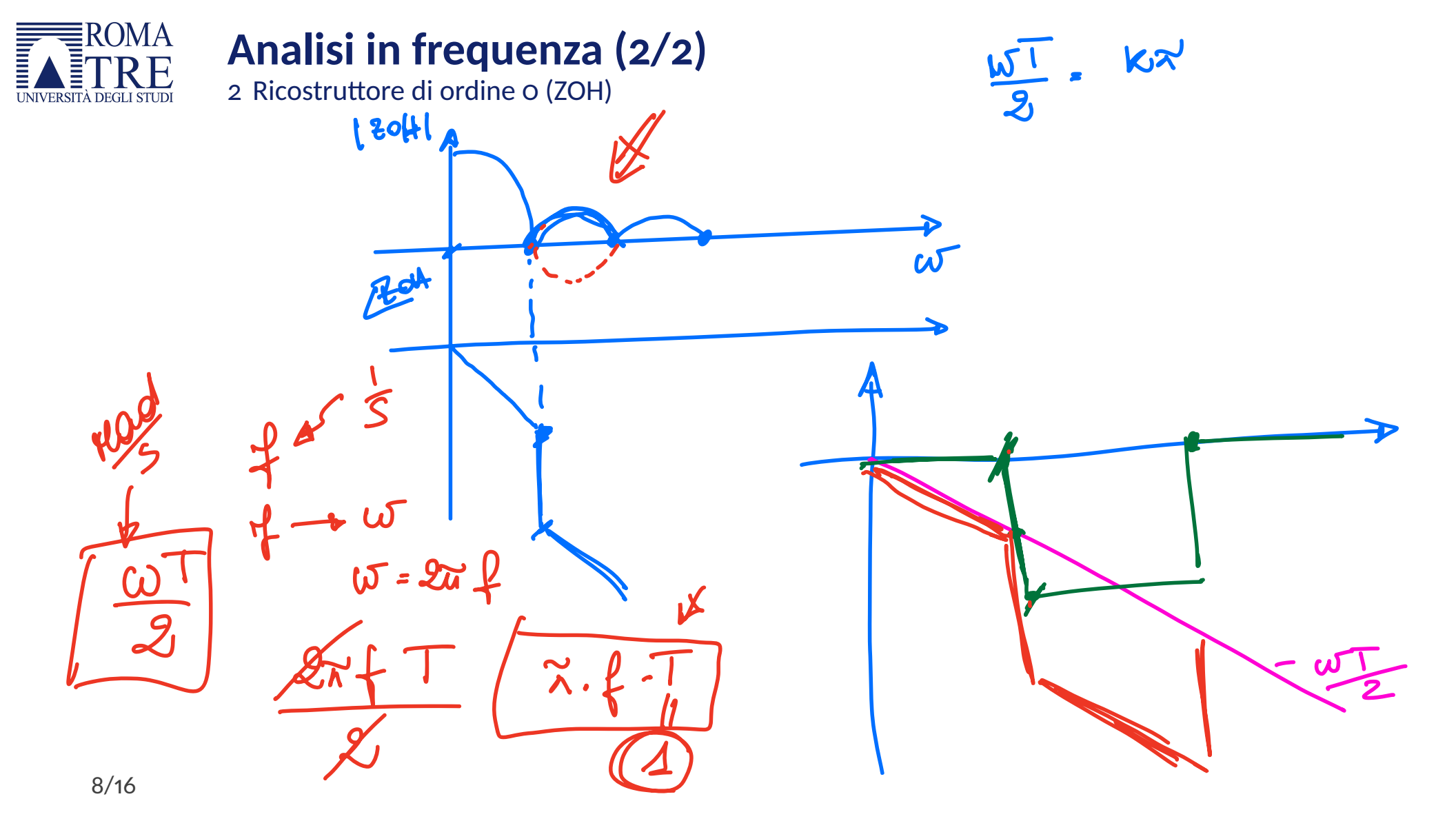
$$\angle H_0(j\omega) = Arg\left[T \frac{\sin(\omega T/2)}{\omega T/2} e^{-j\omega T/2} \right] = Arg\left[\sin \frac{\omega T}{2} \right] - \frac{\omega T}{2}$$

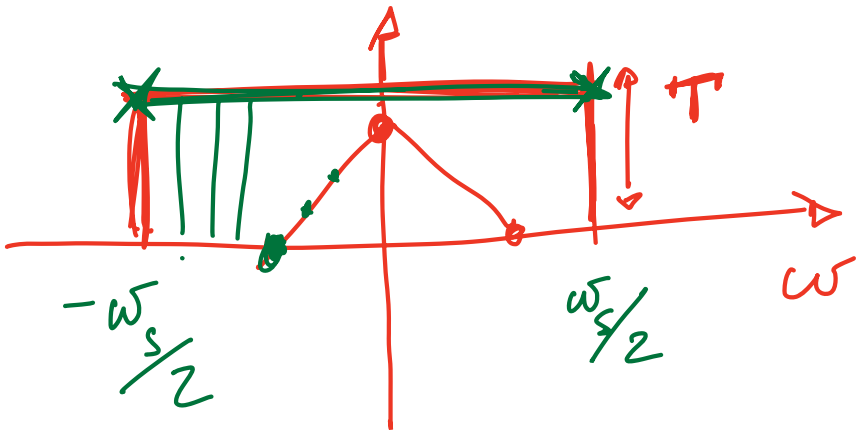
Approssimazione

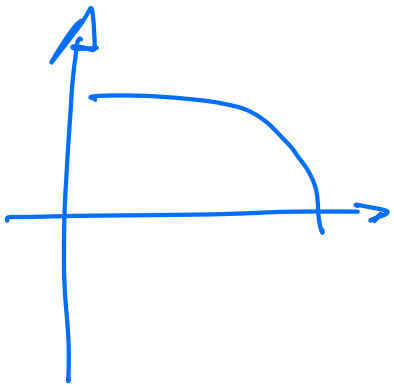
$$H_0(j\omega) \simeq Te^{-j\omega T/2}$$

Analisi in frequenza (2/2)

2 Ricostruttore di ordine 0 (ZOH)







$$-\frac{\omega^* T}{2} = -\frac{\pi}{10} \qquad \omega^* = \frac{2\pi}{T/10} = \frac{\omega_s}{10}$$

$$\omega_{-3} = \frac{\omega_s}{10}$$

$$\boxed{\omega_s \geq 10\,\omega_{-3}}$$

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3 Ricostruttore di ordine 1 (FOH)

Ricostruttore di ordine uno (FOH)

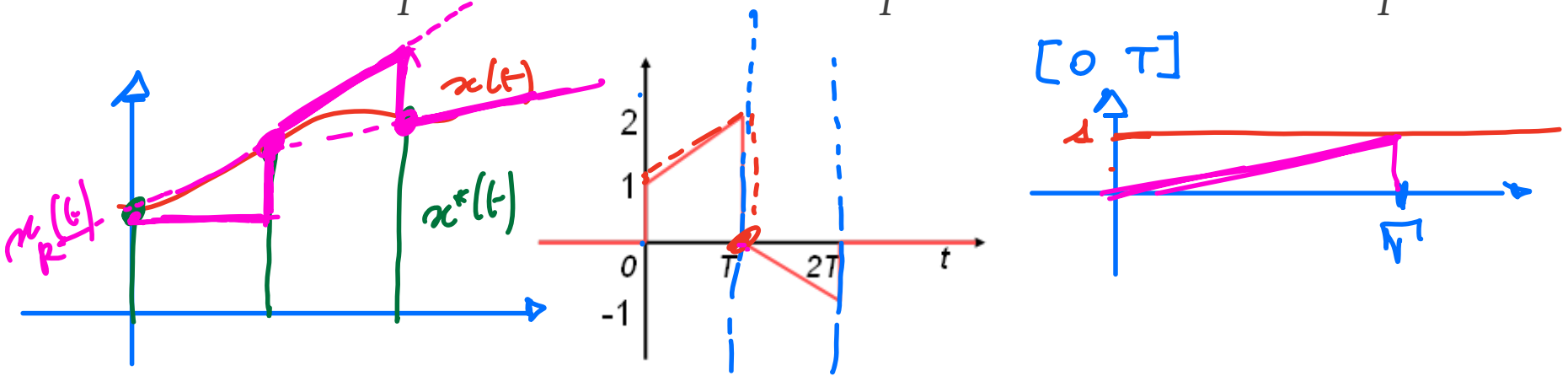
3 Ricostruttore di ordine 1 (FOH)

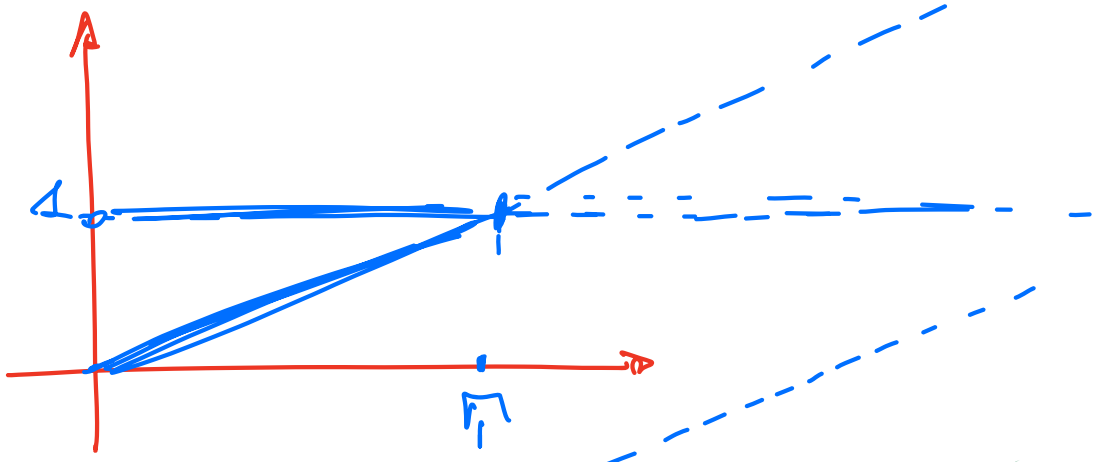
Segnale ricostruito

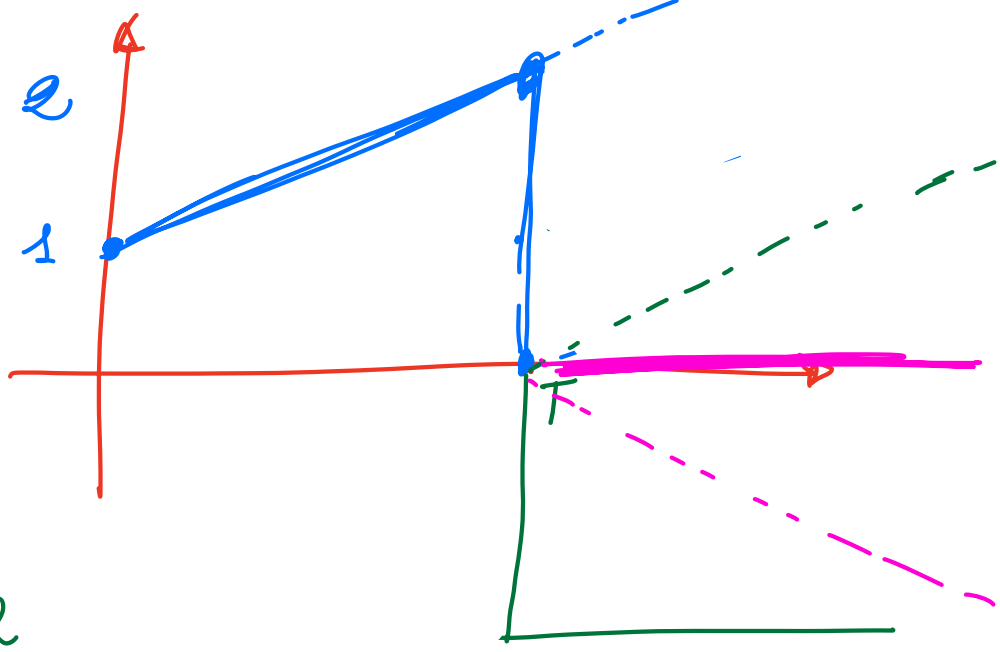
$$x_1(t) = x(kT) + \frac{x(kT) - x((k-1)T)}{T}(t - KT) \qquad kT \le t < (k+1)T$$

Risposta impulsiva

$$g_1(t) = \delta_{-1}(t) + \frac{\delta_{-2}(t)}{T} - 2\delta_{-1}(t-T) + -\frac{2\delta_{-2}(t-T)}{T} + \delta_{-1}(t-2T) + \frac{\delta_{-2}(t-2T)}{T}$$







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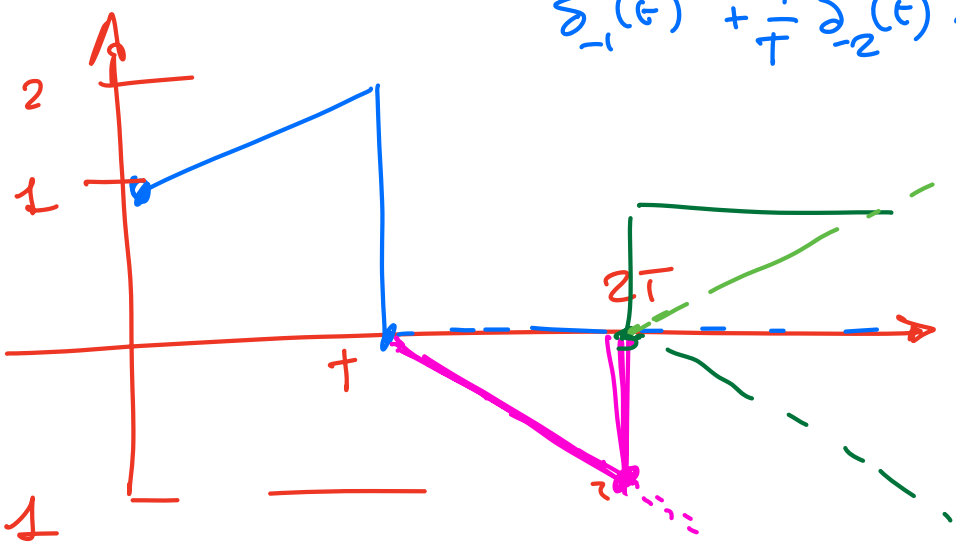

$$\delta_{-1}(t) + \frac{1}{T}\delta_{-2}(t) - 2\delta_{-1}(t-T) - \frac{\delta_{-2}(t-T)}{T}$$

$$-\frac{\delta_{-2}(t-T)}{T} + \delta_{-1}(t-2T) + \frac{\delta_{-2}(t-2T)}{T}$$

Ricostruttore di ordine 1 (FOH)

$\mathcal{L}$-trasformata

$$
\begin{aligned}
H_1(s) &= \frac{1}{s} + \frac{1}{Ts^2} - 2\frac{e^{-sT}}{s} - 2\frac{e^{-sT}}{Ts^2} + \frac{e^{-2sT}}{s} + \frac{e^{-2sT}}{Ts^2} = \\
&= \left(\frac{1}{s} + \frac{1}{Ts^2}\right)(1 - 2e^{-sT} + e^{-2sT}) = \\
&= \frac{1+Ts}{T}\left(\frac{1-e^{-sT}}{s}\right)^2
\end{aligned}
$$

ZOH !!!

Analisi frequenziale

3 Ricostruttore di ordine 1 (FOH)

Risposta armonica

$$H_1(j\omega) = \frac{1+j\omega T}{T}\left(\frac{1-e^{-j\omega T}}{j\omega}\right)^2 =$$

$$= T\left(\frac{\sin(\omega T/2)}{\omega T/2}\right)^2 (1+j\omega T)e^{-j\omega T}$$

Modulo

$$|H_1(j\omega)| = T\left|\frac{\sin(\omega T/2)}{\omega T/2}\right|^2 \sqrt{1+\omega^2 T^2}$$

Fase

$$\angle H_1(j\omega) = Arg\left[T\left(\frac{\sin(\omega T/2)}{\omega T/2}\right)^2 (1+j\omega T)e^{-j\omega T}\right] =$$

$$= \arctan(\omega T) - \omega T$$

Analisi frequenziale

3 Ricostruttore di ordine 1 (FOH)

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4 Continua

Ricostruttore a uscita continua

4 Continua

Segnale ricostruito

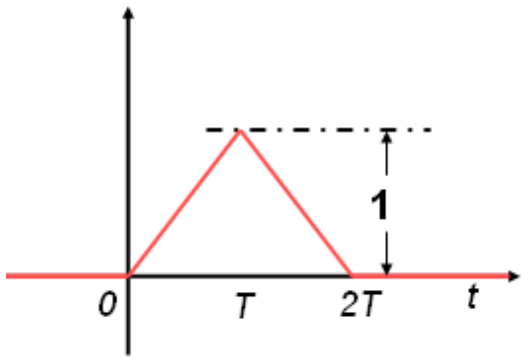
$$x_1(t) = x((k-1)T) + \frac{x(kT)-x((k-1)T)}{T})(t-KT) \quad kT \leq t < (k+1)T$$

Risposta impulsiva

$$g_c(t) = \frac{\delta_{-2}(t)}{T} - 2\frac{\delta_{-2}(t-T)}{T} + \frac{\delta_{-2}(t-2T)}{T}$$

$\mathcal{L}$-trasformata

$$H_c(s) = \frac{1-2e^{-sT}+e^{-2sT}}{Ts^2} = \frac{1}{T}\left(\frac{1-e^{-sT}}{s}\right)^2$$



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Thanks for sharing your thoughts

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