Project 1

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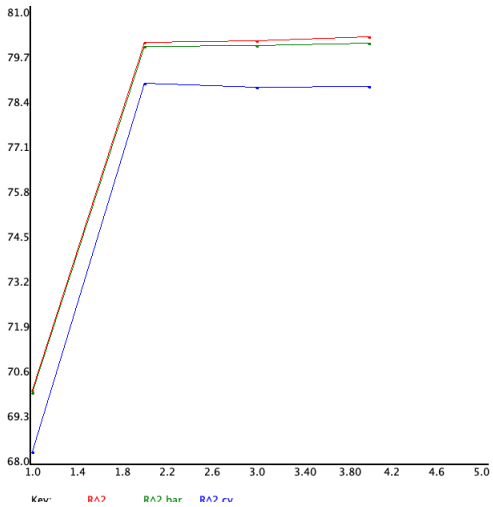
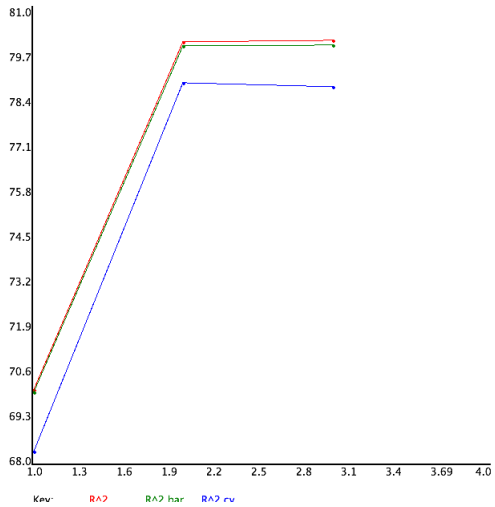
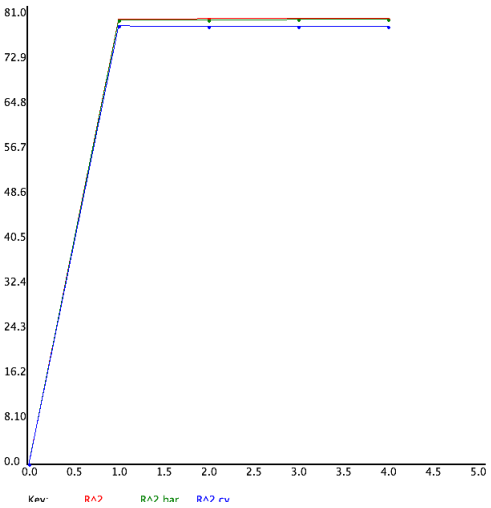
1. Exercise 6 with Forward Selection, Backward Elimination, Stepwise Regression

We called rg.forwardSelAll () and rg.backwardElimAll () for Forward Selec- tion and Backward Elimination, both of which yield the same results for AutoMPG dataset, as this data set is relatively small.

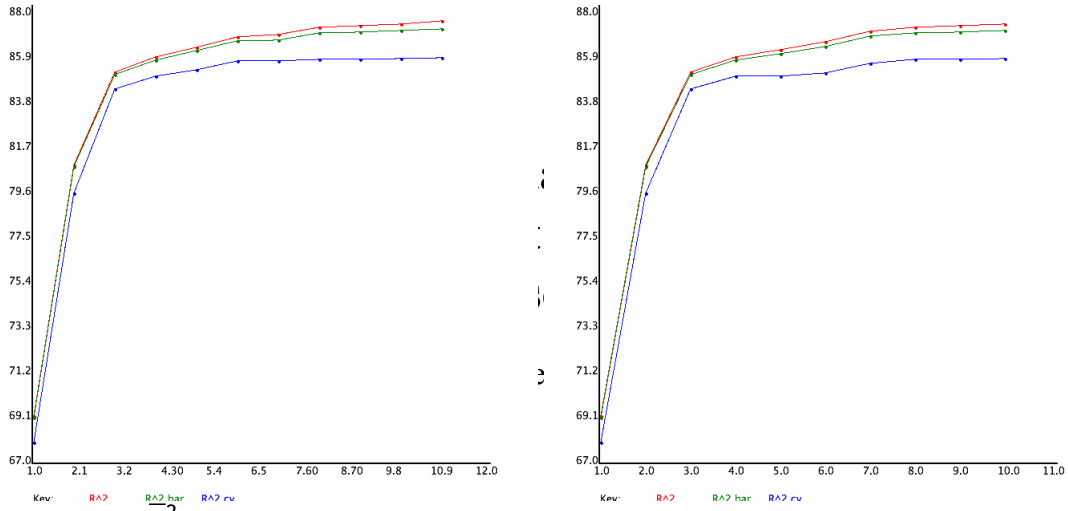




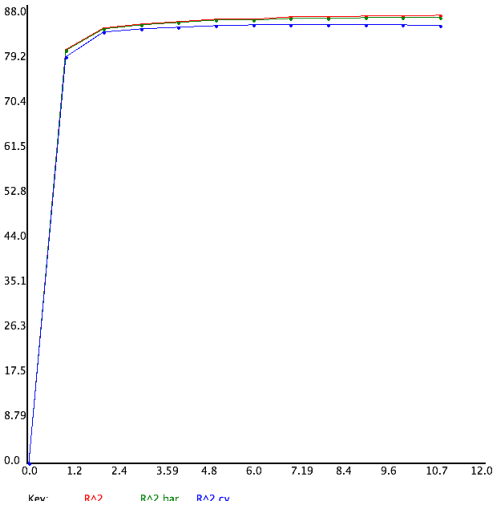


We called rg.stepwiseSelAll () for Stepwise Regression, the result is close to Forward Selection and Backward Elimination, apart from the first row, which corresponds to the first feature included. This is reasonable, as Step- wise Regression starts with no features, then adds the best features, like Forward Selection, but different from Forward Selection, after the second feature is added, we begin to check if we need to eliminate features, namely, do Backward Elimination. So with limited steps, Stepwise Regression should perform better than Forward Selection and Backward Elimination. We note that, even though the result of Stepwise Regression looks similar to Forward Selection and Backward Elimination, this is not always true. With data set of more features, or with Quad Regression, Cubic Regression etc., the differ- ence with show up, as we explain in the following section.

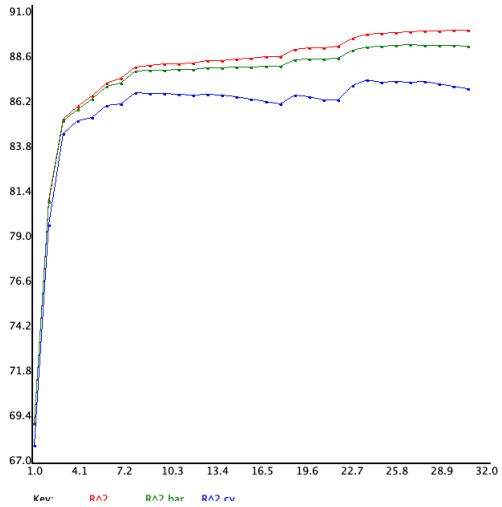
We used R^2 as the selection criterion, we should not use R^2 as R^2 may always encourage us to select more features. However, within this example, the all the three criterion, $\bar{R}^2$, R^2, $\bar{R}^2_{cv}$, are close, $\bar{R}^2$, R^2, $\bar{R}^2_{cv}$ in particular, for data set with more features, and Quad Regression, Cubic Regression etc., this is not true.

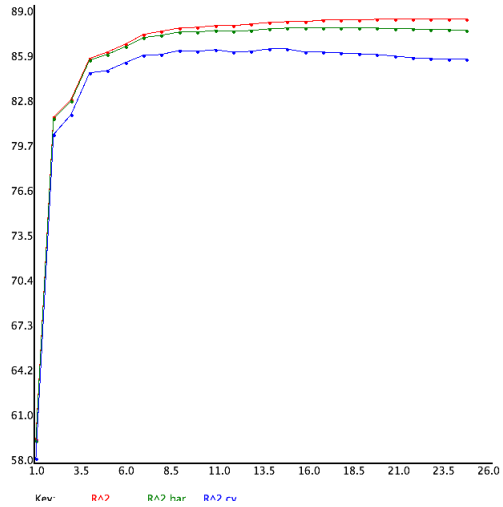


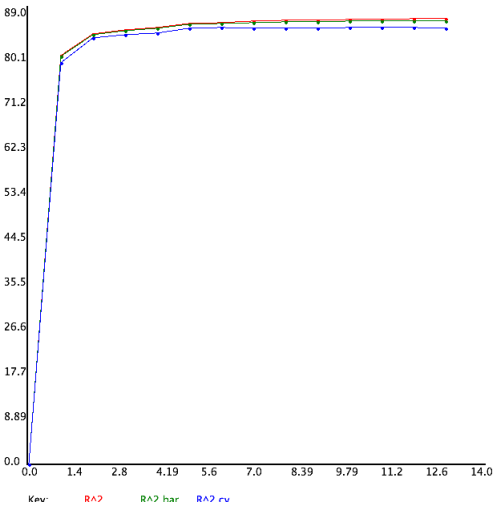
(right), the $\bar{R}^2$ occasionally go downward, meaning that eliminate certain features based on $\bar{R}^2$ hurt the model. Note that there are 13 terms for Quad Regression. Stepwise Regression also differ, and with Quad Regression, Stepwise Regression shows some advantages, it is more stable



With QuadX Regression, the three Feature Selection methods vary more. There are 28 terms, some of them should correlate with each other. Neither Forward Selection nor Backward Elimination truly performs well, Stepwise Regression may be the best choice. This is evident if we compare the last plot (Stepwise Regression) with the previous two.



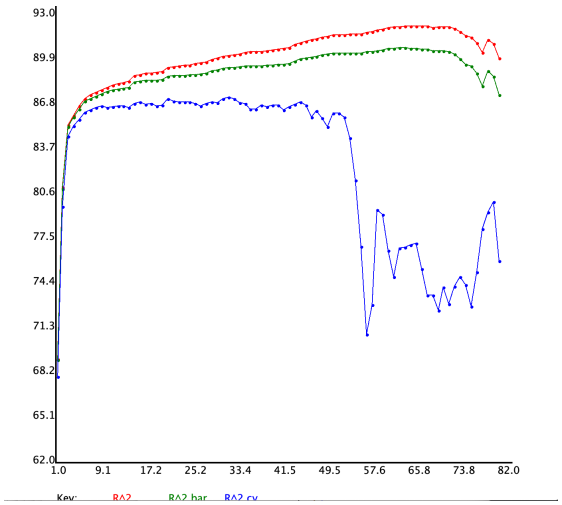




3. Cubic Regression, CubicX Regression with Forward Selection, Backward Elimination, Stepwise Regression

There are 34 terms for Cubic Regression, based on similar reasons, Stepwise Regression is a better choice than Forward Selection or Backward Elimina- tion, this is even more clearly demonstrated in the following 3 plot (the last plot is Stepwise Regression)

There are 84 terms for CubicX Regression, Forward Selection and Backward Elimination become even more unstable, while Stepwise Regression behaves well.


93.0
89.9
86.8
83.7
80.6
77.5
74.4
71.3
68.2
65.1
62.0
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17.2
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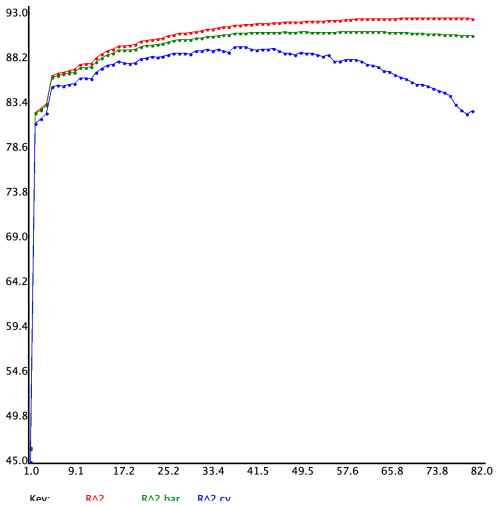
33.4
41.5
49.5
57.6
65.8
73.8
82.0
Key:
RA2
RA2 bar
RA2 cv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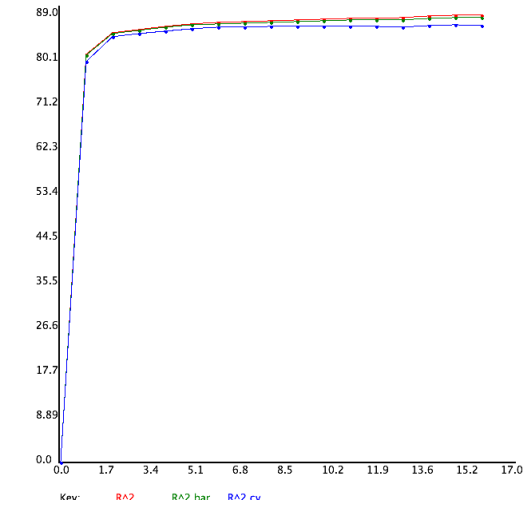
73.8
82.0
Key:
RA2
RA2 bar
RA2 cv


89.0
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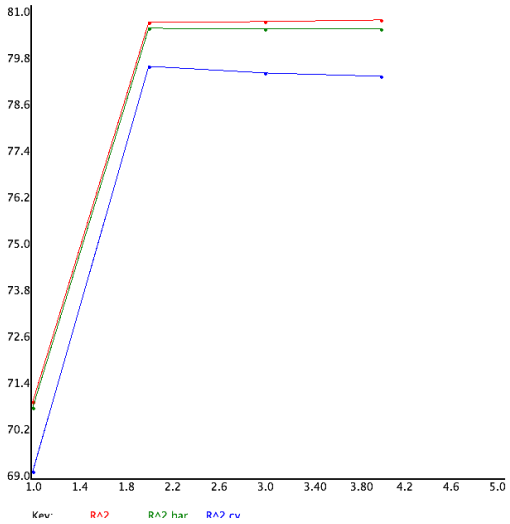
Key:
RA2
RA2 bar
RA2 cv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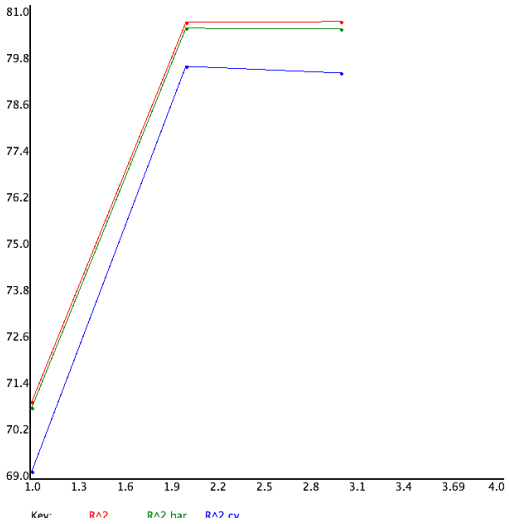
79.8
78.6
77.4
76.2
75.0
73.8
72.6
71.4
70.2
69.0
1.0
1.4
1.8
2.2
2.6
3.0
3.40
3.80
4.2
4.6
5.0
Key:
RA2
RA2 bar
RA2 cv


81.0
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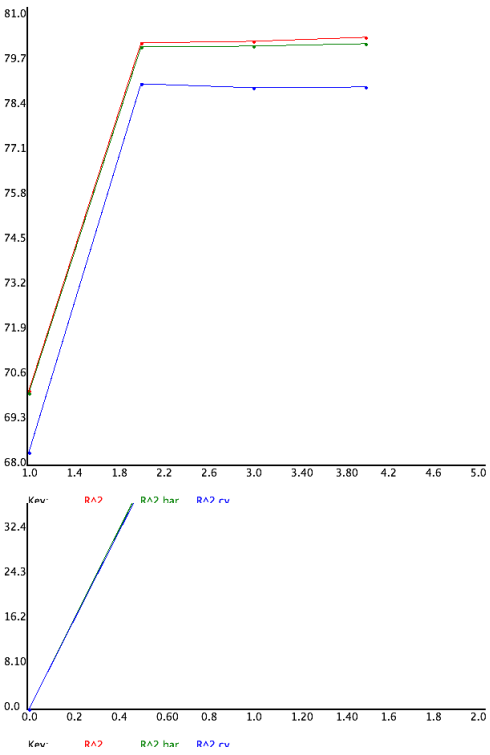
1.3
1.6
1.9
2.2
2.5
2.8
3.1
3.4
3.69
4.0
Key:
RA2
RA2 bar
RA2 cv

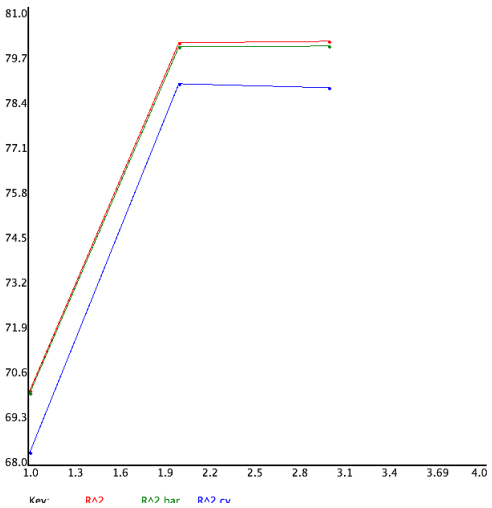
4. Ridge Regression, Lasso Regression with Forward Selection, Backward Elimination, Stepwise Regression

Though the Rubric asks for QR, QXR, CR, CXR, with Ridge Regression and Lasso Regression, based on 7.6 and 7.7 of our textbook, (the two equa- tions) Ridge Regression and Lasso Regression do not involve Quad nor cubic terms, so I did Ridge Regression and Lasso Regression with Forward Selection, Backward Elimination, Stepwise Regression.

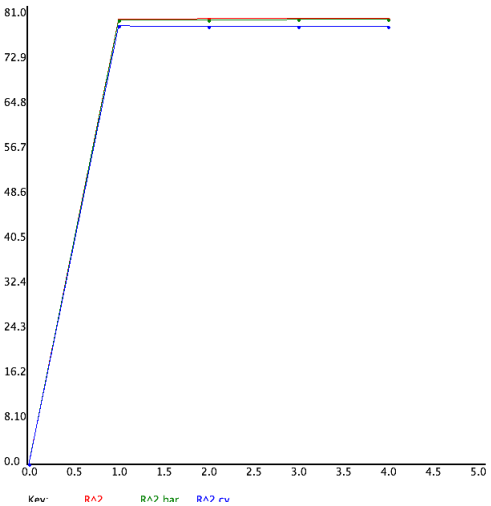
The AutoMPG dataset is relatively small, Ridge Regression and Lasso Regression do not truly show their advantages, however, they should be better for feature selection. The advantage is showed in RidgeRegressionstepwise, for this data set.

Results for Ridge Regression (in order of Forward Selection, Backward Elimination, Stepwise Regression)





Results for Ridge Regression (in order of Forward Selection, Backward Elimination, Stepwise Regression)



AIC Values for Auto MPG Data Set

AIC					
	Forward	Backward	Stepwise	Lasso	Ridge
MLR	-1068	-1070	-1032	-1068	-1066
QR	-980	-982	-970	-	-
QXR	-902	-904	-886	-	-
CR	-958	N/A	-940	-	-
CXR	-897	N/A	-884	-	-

AIC(Akaike Information Criteron) is used to check the best fit for the data to compare different possible models. AIC depends on the predictors in the model and the maximum likelihood estimate of the model . The best-fit model explains the greatest amount of variation(with largest R squared) using least number of predictors. AIC should be lower for better fit model. If two models have same R squared value, then the one with lower AIC will be a better fit model.
We consider the **absolute** value of AIC.

Here, for Auto MPG dataset, AIC Values are computed for different models :MLR,

QR, QXR, CR and CXR for Forward, Backward , Stepwise, Lasso and Ridge Regression. We can analyze from the above table that Forward CubicX model has lower AIC than Forward MLR model. Likewise, we can analyze AIC value for all the models and compare.

Since, AIC values for Stepwise Regression model is lower as compared to forward and backward Regression models, Stepwise Regression is a better fit model.

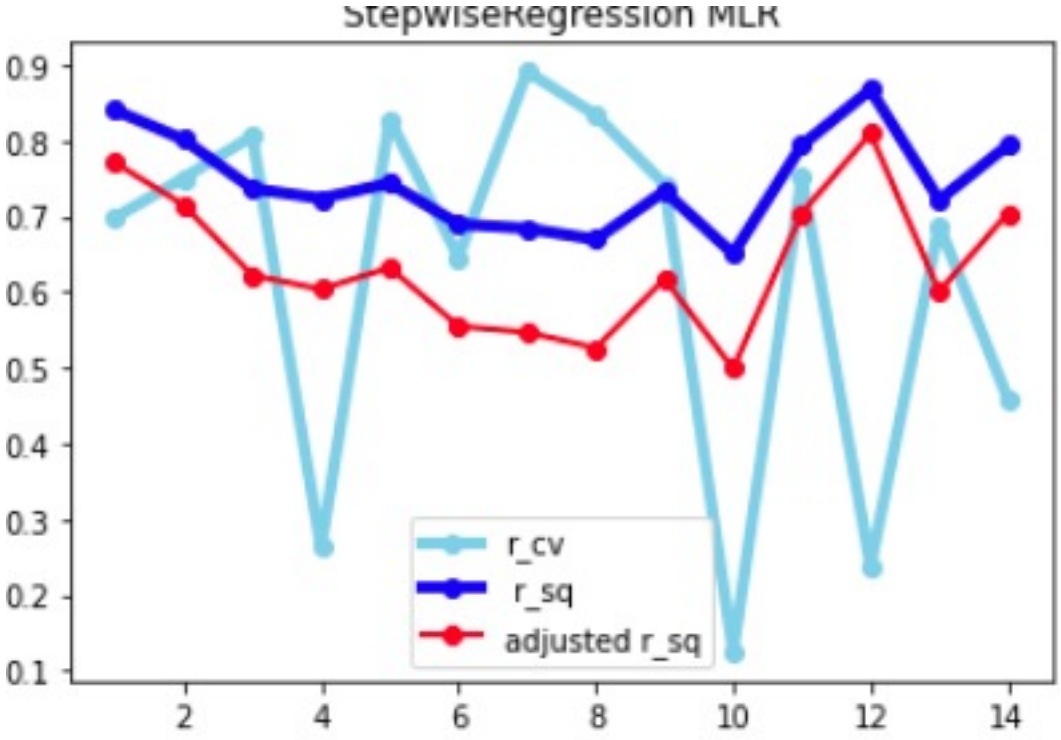
5. Other data sets

We also use Concrete, diabetes, winequality-red, BPressure, basketball, Beijing PM2 , Real Estate and Computer Hardware and several other files from UCI Machine Learning Repository. For those data sets that are already included in scalation, we directly import that; for the rest, we add the csv files under the corresponding folder and import with *scalation.columnar_db.Relation*.
For small data sets (those with fewer features), the three feature selection methods do not differ that much. Forward Selection and Backward Elimina- tion are alike. However, with more complicated features, Stepwise Regres- sion has clear advantages Forward Selection or Backward Elimination. With whatever data set, Stepwise Regression also performs better at the beginning.

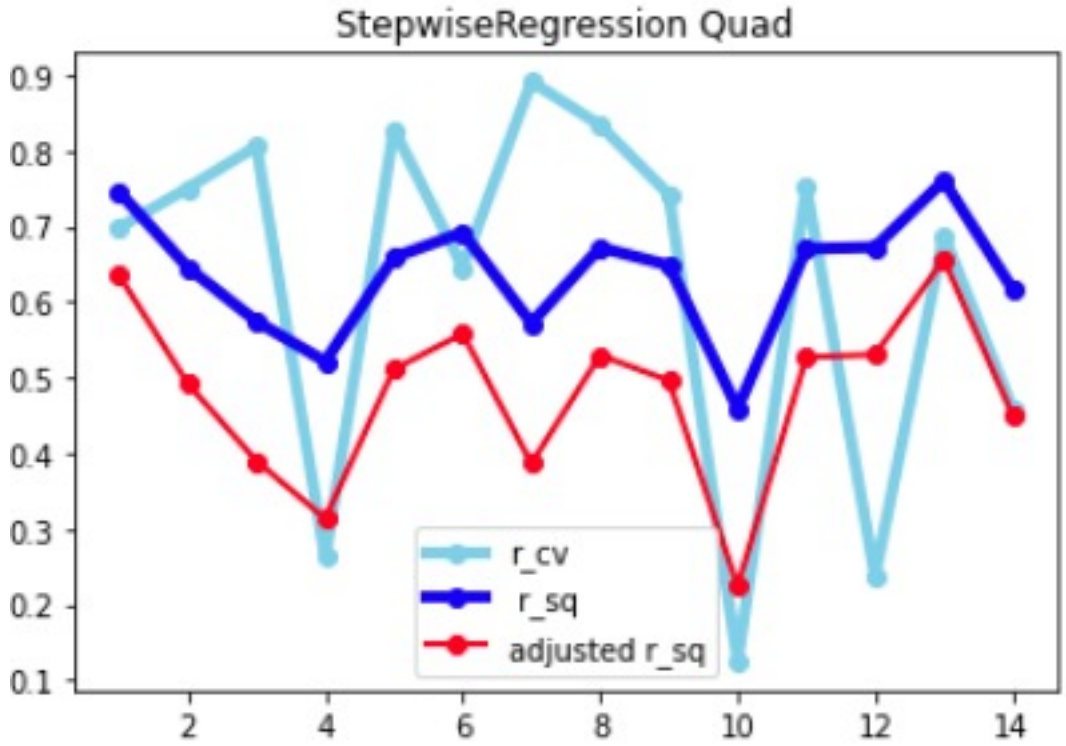
6. Python

In Python, feature selection can be done with *mlxtend.feature_selection* (importing *SequentialFeatureSelector*), we could also write functions with- out *SequentialFeatureSelector* for all three feature selection method. By adjusting $P - value$, we could get results agree with scalation.

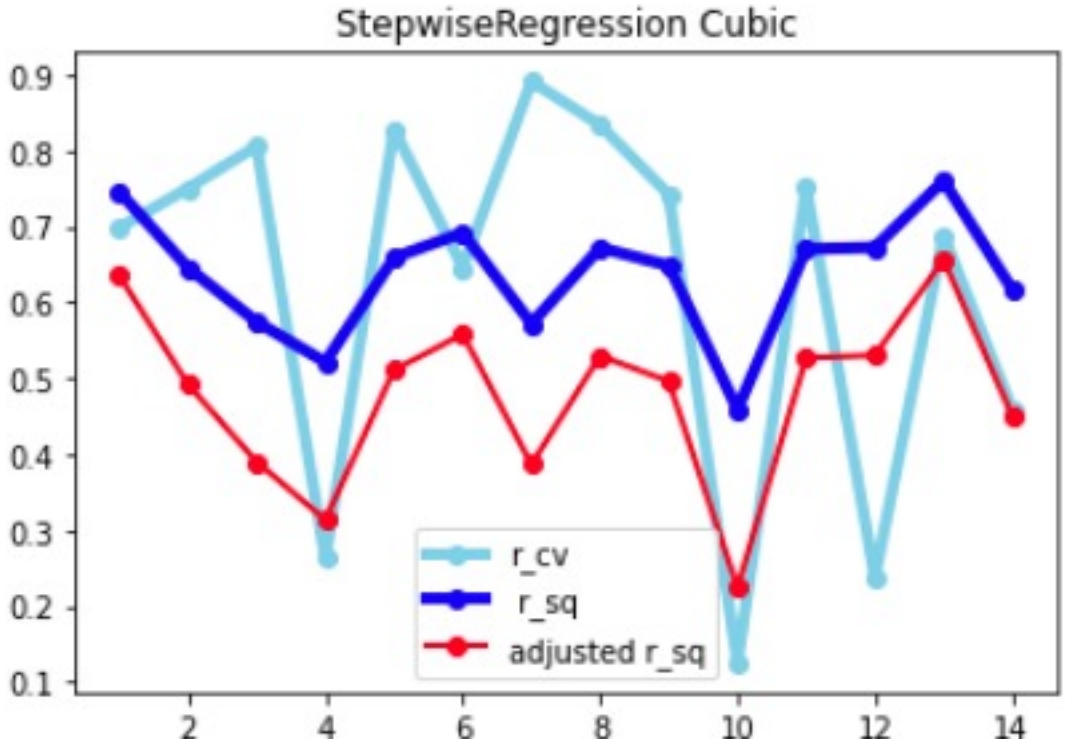
Stepwise Regression: MLR



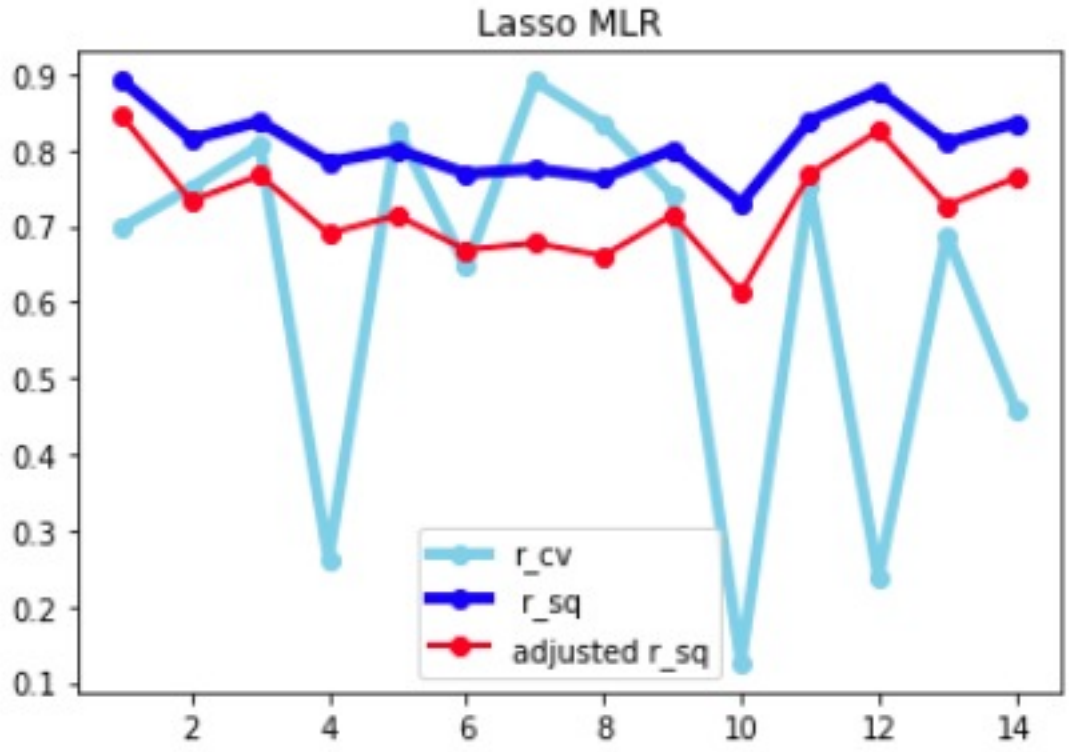
Stepwise Regression - Quad

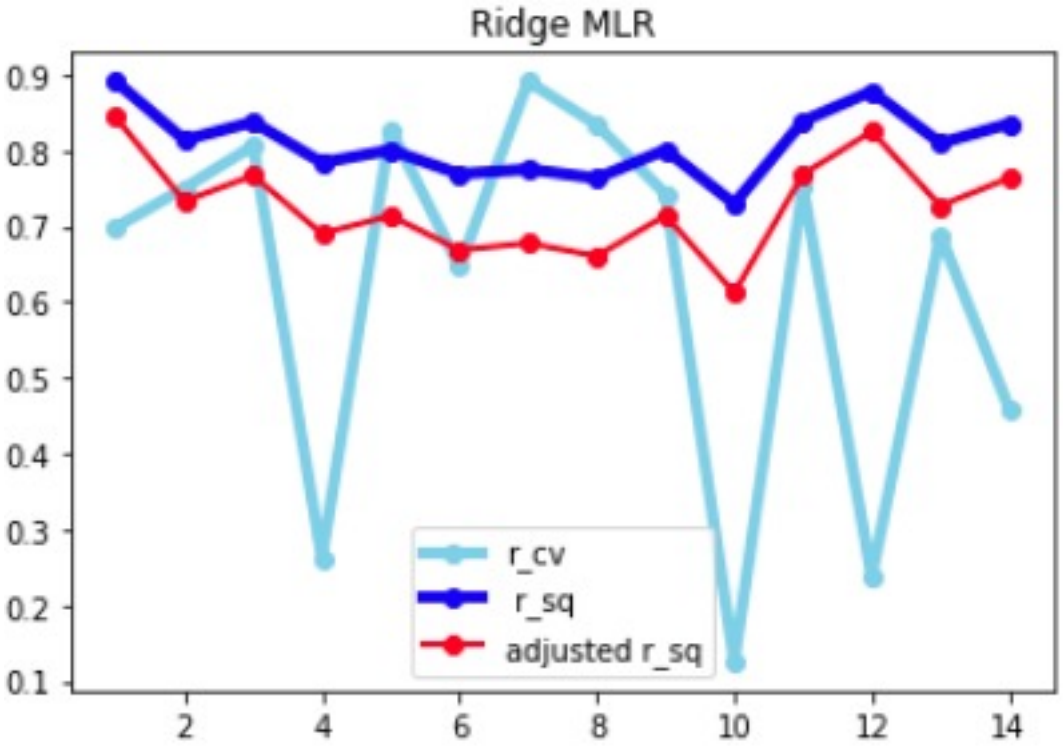


Stepwise Regression- Cubic



Lasso MLR





From the above graphs, we can analyze Stepwise Regression, Ridge and lasso Regression applied on different models in Python.

Stepwise Regression is a better alternative to Forward Selection and Backward Elimination Subset selection.

We also observe from the above plots that Stepwise Quad and Stepwise Cubic are better fit models than Stepwise MLR.

With data set of more features, or with Quad Regression, Cubic Regression etc., the difference will show up, as we explain in the following section using different criterion R, R squared, R_CV . Adjusted R square is always better than R-Square as R-Square produces large test error because Adjusted R-square depends on the decreased RSS(Residual Squared Sum) as well as the variables when the predictors are added to the model. R^2 may always encourage us to select more features.

Ridge and Lasso Regression reduce the variance of the model. It is a better alternative to Best Subset Selection as it regularizes the coefficient estimates and shrink them towards 0.

c

7. Conclusion

We performed Forward Selection, Backward Elimination, Stepwise Regression, Stepwise Regression should be the most stable one, it also performs better at the beginning. The different criterion $\bar{R}^2$, R^2, $\bar{R}^2$ and AIC do behave differently.

Though we use $\bar{R}^2$ for this project, other criteria may yield different results, especially for Forward Selection and Backward Elimination.