Let's Taste R Through Cliquet Option Pricing

Dong Up, Kim

전제된 것들

- 기업에서 calculation을 위해 사용되는 excel과 대비하여 R의 도입가능성과 사용성에 대해 고민한 결과를 공유하고자 했습니다.
- 제가 몸 담은 회사의 상사(실장님)로 부터 미리 공개에 대해서 허락을 받았습니다.
- 실제 도입하여 쓰고 있는 상황은 아니라서 taste 라는 표현을 썼습니다.
- R과 Excel 및 기타 프로그래밍에 대한 기본적인 지식을 가진 상태에서 썼습니다.

- **굵은 빨간글씨** 는 중요 포인트입니다.

상황

1. Equity Indexed(Linked) Annuity 판매 - 부채가 발생
2. 부채가격산정(계산)방법 마련이 필요해짐
3. 방안제시
   - 방안1 :판매상품과 동일 구조의 금융상품구매 (계산방법不要)
   - 방안2 : 모회사의 지원을 통한 in-house 개발
4. 다른 지사에서 계산모델제공(excel 파일)
5. 한국지사의 담당자들과 협의

한국의 담당자 = **user ≠ developer(implementer)**

EIA?

장내파생상품 Exchange-Traded Derivative

- 정형 금융상품
- 옵션, 선물

장외파생상품 OTC Derivative

- 비정형 금융상품
- 선도, 스왑, 이색옵션, ELS, DLS, …

1. 주가연계증권(ELS; Equity Linked Security)
2. 주가연계펀드(ELF; Equity Linked Fund)
3. 주가연계예금(ELD; Equity Linked Deposit)

클리켓 옵션 Cliquet Option

최소상환수익률(G_f)을 보장하고, 매 분기마다 기준지수가 조정된 분기 주가상승률을 최대수익률(C)과 최소수익률(F)을 적용하여 결정된 분기수익률의 합으로 Payoff를 결정하는 다음과 같은 European Cliquet Option

$$\boldsymbol{Payoff} = \boldsymbol{e^{-rt} \cdot N} \cdot \boldsymbol{Max}\left[\boldsymbol{G_f}, \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{Max}\{\boldsymbol{Min}\left(\boldsymbol{\alpha} \cdot \frac{\boldsymbol{S_i}}{\boldsymbol{S_{i-1}}} - \boldsymbol{1}, \boldsymbol{C}\right), \boldsymbol{F}\}\right]$$

제공받은 모델

- 엑셀파일로 작성된 파일 (254열 제한)
- Cliquet Option의 가격을 Monte Carlo Simulation을 통해 산출
- 1년만기 기준 : 시나리오가 254 step만 있음
- **주요 함수는 TDFunc2.dll** 안에 들어 있고 VBA Project의 참조를 통해 Excel파일에서 사용 (confidential?)

# Contracts	Iteration
2	

Data from the inforce file for the current contract

	2	6	7	9	10	11	16	17	18	19
Contract	MVLOcode	Mid or End	IssueDate	Monthly Cap	Monthly Floor	Annual Floor	Notional	Hedge Factor	astMonthEndInde	Monthly Sum
2	R03201103G151(	15	2010-9-15	2.40%	-7.00%	0.00%	5,000,000,000	100.000%	254.033	0.00%

INPUT - policy

Spot	254.03	KOSPI index value at the start of the scenarios
Time step	0.003921569	Time step size of the scenarios - approximately 1 working day expressed as fractions of a year.
Valuation Date	2010-10-29	Start date of the interest rate scenarios
Issue Date	2010-9-15	Policy issue date
Contract month	2	Current policy month
Contract year		Current policy year
Last mo index	254.0	KOSPI index value at the end of the last contract month.
Mo reset?		
Ann mo sum ac	0.00%	
Mo mo sum ad	0	
Mo sum	0	
Adj last mo ind	254.033	
Last ann	2010-9-15	Most recent contract anniversary date
Acc*HF	5,000,000,000	Notional amount to be hedged, after adjustment for expected lapses using the hedging factor.
Last con ann m	12	Contract month at end of current contract year.
Disc rate	3.5397%	Risk-free rate to discount from option payoff date to the valuation date.
Disc time	0.8745	2010-10-31 Time in years from valuation date to payoff date.

Calc 1

	11	33	55	77	97	120	140	163	185	206	229	
	10	32	54	76	96	119	139	162	184	205	228	
	1900-01-10	1900-02-01	1900-02-23	1900-03-12	1900-03-31	1900-04-23	1900-05-12	1900-06-02	1900-06-24	1900-07-14	1900-08-04	
	MVLO	126,649,927										
EoMonth	2010-10-29	2010-11-30	2010-12-30	2011-01-31	2011-02-28	2011-03-31	2011-04-29	2011-05-31	2011-06-30	2011-07-29	2011-08-31	2011-09-30
	2010-10-29	2010-11-30	2010-12-31	2011-01-31	2011-02-28	2011-03-31	2011-04-29	2011-05-31	2011-06-30	2011-07-29	2011-08-31	2011-09-30
Month	1	2	3	4	5	6	7	8	9	10	11	12
Month	2	3	4	5	6	7	8	9	10	11	12	13
Fix	2010-11-15	2010-12-15	2011-01-15	2011-02-15	2011-03-15	2011-04-15	2011-05-15	2011-06-15	2011-07-15	2011-08-15	2011-09-15	2011-10-15
Time	0.043651	0.126984	0.206349	0.289683	0.365079	0.456349	0.531746	0.619048	0.702381	0.785714	0.873016	0.9563
Step	11	32	53	74	93	116	136	159	179	200	223	2
MC time	0.043137	0.125490	0.207843	0.290196	0.364706	0.454902	0.533333	0.619608	0.701961	0.784314	0.874510	0.956863
MC rate	3.0529%	3.1457%	3.3069%	3.4054%	3.4531%	3.4840%	3.4991%	3.5102%	3.5190%	3.5283%	3.5397%	3.5501%
Stop	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE
NetWorkD	1900-2-9	1900-3-2	1900-3-23	1900-4-10	1900-4-29	1900-5-22	1900-6-9	1900-6-30	1900-7-22	1900-8-11	1900-8-31	1900-9-20

Calc 2

Path												
1	247.244	258.443	251.489	257.996	234.178	236.074	245.774	250.737	235.519	230.758	236.592	#N/A
2	252.660	245.626	236.278	184.628	186.411	200.327	201.947	201.089	206.022	211.114	216.838	#N/A
3	266.257	280.914	267.498	276.055	300.987	310.797	296.635	283.344	300.667	316.993		
4	253.230	238.822	244.341	292.446	239.022	283.627	290.779	296.758	273.756	228.407		
5	252.908	265.215	281.940	276.092	284.563	297.270	307.354	296.527	263.741	290.12		
6	266.230	259.331	235.840	237.794	255.064	246.323	231.515	241.362	240.197	267.216	258.334	#N/A
7	251.483	237.774	239.523	246.553	256.957	259.697	268.210	280.867	283.419	279.476	279.195	#N/A
8	262.195	273.668	252.296	245.967	266.421	274.793	286.173	294.193	297.201	287.176	267.682	#N/A
9	256.838	228.768	202.333	157.676	157.808	164.943	175.445	172.338	175.091	173.624	186.515	#N/A
10	262.830	265.683	262.039	280.539	288.600	291.382	311.229	314.726	336.578	339.253	323.107	#N/A
11	244.431	249.672	199.755	175.031	199.292	228.272	239.496	239.362	244.677	234.442	240.398	#N/A
12	248.860	240.133	240.095	237.392	239.093	248.557	241.461	233.649	219.131	227.096	223.681	#N/A
13	252.629	269.681	274.739	261.897	275.301	256.934	255.707	269.392	246.191	244.651	239.512	#N/A

INPUT - scenario

Calc 3

Return				
-0.02673	0.02400		0.02400	
-0.00540	-0.02784	-0.03806	-0.07000	
0.02400	0.02400	-0.04776	0.02400	
-0.00316	-0.05690	0.02311	0.02400	
-0.00443	0.02400	0.02400	-0.02074	
0.02400	-0.02591	-0.07000	0.00829	
-0.01004	-0.05451	0.00795	0.02400	
0.02400	0.02400	-0.07000	-0.02509	
0.01104	-0.07000	-0.07000	-0.07000	
0.02400	0.01086	-0.01372	0.02400	
-0.03780	0.02144	-0.07000	-0.07000	
-0.02036	-0.03507	-0.00016	-0.01128	
-0.00548	0.02400	0.01876	-0.04674	

제공받은 모델을 수정

- 엑셀파일로 작성된 파일 (254열 제한)
- **주요 함수는 Excel의 기본함수로** 대체

# Contracts	Iteration
2	

Data from the Inforce file for the current contract:

	2	6	7	9	10	11	16	17	18	19
Contract	ID	Mid or End	IssueDate	Monthly Cap	Monthly Floor	Annual Floor	Notional	Hedge Factor	astMonthEndIndex	Monthly Sum
2	222	15	2010-9-15	2.40%	-7.00%	0.00%	5,000,000,000	100.000%	254.033	3.00%

Calculations for the contract:

Spot	254.03	KOSPI index value at the start of the scenarios
Time step	0.003921569	Time step size of the scenarios - approximately 1 working day expressed as fractions of a year.
Valuation Date	2010-10-31	Start date of the interest rate scenarios
IssueDate	2010-9-15	
Current policy month	2	
Current policy year	1	
Monthly Sum	3.00%	
LastMonthEndIndex	254.033	
Last anniversary	2010-9-15	Most recent contract anniversary date
Notional * Hedge Factor	5,000,000,000	Notional amount to be hedged, after adjustment for expected lapses using the hedging factor.
Last ann. contract month	12	Contract month at end of current contract year.
Disc rate	3.5372%	Risk-free rate to discount from option payoff date to the valuation date.
Disc time	0.854902	Time in years from valuation date to payoff date.
MVLO	190,138,280	

Order	1	2	3	4	5	6	7	8	9	10	11	12
Month	2	3	4	5	6	7	8	9	10	11	12	13
Fix	2010-11-15	2010-12-15	2011-01-15	2011-02-15	2011-03-15	2011-04-15	2011-05-15	2011-06-15	2011-07-15	2011-08-15	2011-09-15	2011-10-15
Step	11	33	54	73	92	115	133	155	177	197	218	238
MC time	0.043137	0.129412	0.211765	0.286275	0.360784	0.450980	0.521569	0.607843	0.694118	0.772549	0.854902	0.933333
MC rate	3.0529%	3.1546%	3.3131%	3.4020%	3.4512%	3.4820%	3.4972%	3.5088%	3.5181%	3.5269%	3.5372%	3.5472%
Stop	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	FALSE

Path													Return	
1	247.244	261.729	254.818	259.393	229.081	236.909	244.943	246.342	239.816	232.082	234.754	#N/A	-0.02673	0.02400
2	252.660	248.199	232.974	180.860	185.897	201.386	200.520	193.386	203.613	210.398	217.417	#N/A	-0.00540	-0.01765
3	266.257	283.584	272.846	274.910	301.926	313.886	299.777	288.951	297.570	314.239	307.167	#N/A	0.02400	0.02400
4	253.230	232.396	249.787	289.737	245.357	283.677	287.394	298.831	282.662	234.814	216.315	#N/A	-0.00916	-0.07000
5	252.908	263.011	283.559	275.656	289.090	294.378	305.219	301.326	278.811	283.443	306.409	#N/A	-0.00443	0.02400
6	266.230	260.985	239.854	236.676	252.325	245.549	229.770	251.893	232.004	261.672	262.339	#N/A	0.02400	-0.01970
7	251.483	237.051	243.941	253.037	253.213	259.693	265.306	277.110	280.386	283.789	288.146	#N/A	-0.01004	-0.05739
8	262.195	272.644	246.652	244.072	265.658	280.089	279.664	293.660	296.443	287.165	285.596	#N/A	0.02400	0.02400
9	256.838	233.375	191.775	157.031	160.882	162.731	174.820	172.007	174.793	178.086	185.533	#N/A	0.01104	-0.07000
10	262.830	265.440	260.425	280.927	288.367	296.712	309.515	315.186	335.897	338.365	340.844	#N/A	0.02400	0.00993
11	244.431	246.275	207.444	180.959	195.154	229.574	232.127	242.215	241.412	226.973	236.968	#N/A	-0.03780	0.00754
12	248.860	237.609	240.641	238.392	238.087	248.533	246.575	235.678	225.364	226.815	227.538	#N/A	-0.02036	-0.04521
13	252.639	269.706	273.788	268.490	277.551	259.241	258.868	263.217	240.704	239.335	230.673	#N/A	-0.00549	0.02400
14	248.341	258.965	270.446	265.627	260.835	268.460	267.667	282.201	279.089	285.050	263.574	#N/A	-0.02241	0.02400
15	247.195	186.432	207.123	214.607	221.290	219.018	216.518	233.298	236.328	235.474	224.902	#N/A	-0.02692	-0.07000
16	248.859	245.282	248.610	254.161	253.336	273.345	263.140	271.184	268.450	271.299	274.622	#N/A	-0.02037	-0.01437
17	261.288	244.866	240.439	241.421	192.404	203.308	203.809	227.730	228.817	238.482	220.101	#N/A	0.02400	-0.06285
18	265.253	274.274	244.182	230.614	237.113	239.196	216.639	229.432	229.890	226.221	219.074	#N/A	0.02400	0.02400
19	265.531	274.091	271.422	273.159	261.219	268.377	267.078	253.955	278.179	245.179	259.717	#N/A	0.02400	0.02400
20	262.957	286.753	281.031	268.376	259.454	268.171	238.259	208.826	234.773	215.581	235.222	#N/A	0.02400	0.02400
21	227.022	245.455	256.500	247.398	244.235	225.757	233.746	220.663	223.457	226.967	213.990	#N/A	-0.07000	0.02400

R 모델을 쓰게 된 이유

- 주요 논문을 참조
- **개념 수립부터** 차근차근 접근
- 실제 영업일 산출이 core function이 됨
  - 영업일 산출 로직이 cliquet option계산로직보다 더 커지게 되고, 정확도도 떨어짐
  - 결국 lubridate 패키지 사용.
  - lubridate의 모태라고 할 수 있는 java 라이브러리인 **joda-time을 이미 사용한 경험이 있는 상태.**

Dates and Times

Garrett Grolemund
Phd Student / Rice University
Department of Statistics
http://vita.had.co.nz/papers/lubridate.pdf

R code snippet

```
scentimes<-(as.vector(t(read.table(scenfile,head=FALSE,skip=5,nrows=1))))[1:366]
scenrates<-(as.vector(t(read.table(scenfile,head=FALSE,skip=7,nrows=1))))[1:366]
scendvyld<-(as.vector(t(read.table(scenfile,head=FALSE,skip=12,nrows=1))))[1:366]
scenarios<-(as.matrix(t(read.table(scenfile,head=FALSE,skip=14))))[1:366,]
matplot(scentimes, scenarios, type="l"); title(main="1000 KOSPI Scenario for 300 Trading Days")
```

```
span <- valuationDate-issueDate
elapsed <-valuationDate-(issueDate+years(1))
disc_time = as.duration(elapsed) %/% days(1);
disc_rate = exp(-1*disc_time/255 * scenrates[disc_time+1])
Fix= dueDates(issueDate,valuationDate)
Step = dueDurations(valuationDate,Fix,holidays) ;Step
```

```
networkdays <- function(start, end, holidays) {
intervals = 0
if(missing(holidays)) intervals = 0 else {
for(i in 1:length(holidays)){
if((holidays[i] >= start) && (holidays[i]<=end) && (wday(holidays[i])!=7)
&& (wday(holidays[i])!=1)) intervals<-intervals+1}
}
dates <- seq(as.Date(start), as.Date(end), by="day")
return( sum(as.numeric(format(dates, "%w") > 1)) - intervals)
}
```

R code snippet – cont.

```
z<-rbind(V0=1, scenarios[Step,])
b <- tail(z, stop) / head(z, stop)
c <- apply(b,1:2, function(x) max(min(x-1,mcap),mfloor))
d<-apply(c,2,sum)
e<-sapply(d,function(x) max(x,afloor))
notional*hedgefactor*sum(e)/length(e)*disc_rate
```

1. apply

Description: "Returns a vector or array or list of values obtained by applying a function to margins of an array or matrix."

OK – we know about vectors/arrays and functions, but what are these "margins"? Simple: either the rows (1), the columns (2) or both (1:2). By "both", we mean "apply the function to each individual value." An example:

```
# create a matrix of 10 rows x 2 columns
m <- matrix(c(1:10, 11:20), nrow = 10, ncol = 2)
# mean of the rows
apply(m, 1, mean)
 [1]  6  7  8  9 10 11 12 13 14 15
# mean of the columns
apply(m, 2, mean)
[1]  5.5 15.5
# divide all values by 2
apply(m, 1:2, function(x) x/2)
      [,1] [,2]
 [1,]  0.5  5.5
 [2,]  1.0  6.0
 [3,]  1.5  6.5
 [4,]  2.0  7.0
 [5,]  2.5  7.5
 [6,]  3.0  8.0
 [7,]  3.5  8.5
 [8,]  4.0  9.0
 [9,]  4.5  9.5
[10,]  5.0 10.0
```

R 모델의 경험

- **개념에서 코드**로 넘어가기 좋음

excerpted from Mats Kjaer, On the Pricing of Cliquet Options with Global Floor and Cap, Goeteborg, Sweden 2004, Chapter2 2.1Floored cliquet options

Let T be a future point in time, and divide the interval $[0, T]$ into N subintervals called *reset periods* of length $\Delta T_n = T_n - T_{n-1}$, where $\{T_n\}_{n=0}^{N}, T_0 = 0, T_N = T$ are called the *reset days*. The return of an asset with price process S_t over a reset period $[T_{n-1}, T_n)$ is then defined as

$$R_n = \frac{S_{T_n}}{S_{T_{n-1}}} - 1.$$

Truncated returns, $\overline{R}_n = \max(\min(R_n, C), F)$ are returns truncated at some floor and cap levels F and C respectively with $F < C$ as illustrated in Figure 2.1 below. Absence of floor and/or cap corresponds to $F = -1$ and $C = +\infty$. A general cliquet option has a payoff Y at time T of

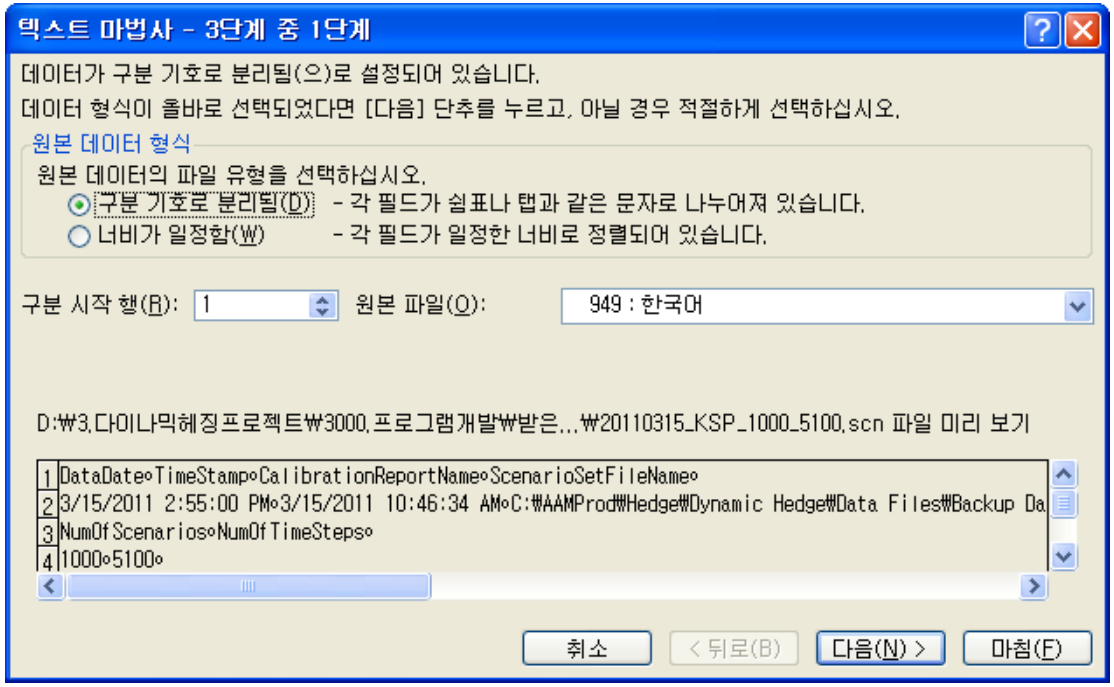
$$Y = B \times \min(\max(\sum_{n=1}^{N} \overline{R}_n, F_g), C_g)$$

where the *global floor* F_g and *global cap* C_g are minimum and maximum returns respectively and B is a notional amount which is set to one for the remainder of this thesis. For F_g and C_g to be of interest, they must satisfy $NF < F_g < C_g < NC$.

```
z<-rbind(V0=1, scenarios[Step,])
b <- tail(z, stop) / head(z, stop)
c <- apply(b,1:2, function(x) max(min(x-1,mcap),mfloor))
d<-apply(c,2,sum)
e<-sapply(d,function(x) max(x,afloor))
notional*hedgefactor*sum(e)/length(e)*disc_rate
```

R 모델의 경험 – cont.

- Front end 기능은 excel에 비해 약함.
  - **시나리오를 읽는데 몇 분이 소요됨**
  - Excel은 SAS(PC)와 같이 text import wizard 존재



R 모델의 경험 – cont.

- core function에 집중하게 됨.
  - 대신 visual intuition을 얻기에는 excel에 비해 어려움.
- functional language의 간결함 (lambda함수 “apply”, “sapply”)
- 재사용성이 높음
  - 현재 엑셀2003의 254제한은 엑셀2010으로 업그레이드 하여 해결하였으나 frame을 고치기 어려움.
  - R은 language
- Enterprise IT 환경에 embed가 가능함.
  - **Personal level vs. Corporate level**

R 모델의 경험 – cont.

- lubridate 패키지로 보는 **open source** 문제
  - Commercial Use에 있어서의 한계
    - 어디까지 테스트해야 하는가?
    - 모두 분석해서 재코딩해야 하나?
    - 오픈소스정책과 충돌이 일어날 수 있나?
  - Excel의 기본함수에 대해서는 신뢰하게 됨.
    - 오류의 가능성에 대해 신경쓰지 않음
    - 있다면 즉각 고침.
    - 안정성이 매우 높음