Master Thesis-GoSafe Controller Parameterization

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Controller Parameterization

Our model:

$$\Lambda_e(q, \dot{q})\frac{dv_e}{dt} + \mu(q, \dot{q}) + p(q, \dot{q}) = F_e$$

Assume exact feedback linearization

$$\frac{dx}{dt} = Ax + Bu$$

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ I \end{bmatrix},\ x = \begin{bmatrix} r_{e,T} \\ v_e \end{bmatrix}, u = F$$

Controller Parameterization

Using A, B model Q, R and apply LQR control. Objective:

- Performance:
  $r_t = -(||r_{e,T}(t)||_2/||r_{e,T}(0)||_2)^2 - ||\tanh(v_e)/5||_2^2 - -||\tanh(u)/5||_2^2$
  Why tanh?
  $\tanh(x) \in [-1, 1]$ and for small x, $\tanh(x) \approx x$
- $g_1(x) = \frac{1}{4}(1 + \exp(-||r_{e,T}||_2/||r_{e,T}(0)||_2))(1 + \exp(-\tanh(||v_e||_2)))$
- $g_2(x) = \min(0, ||r_{e,T}(t)|| - ||r_{e,T}(0)||_2)$

Constraints: $g_1(x_0, a) = \max_{t \geq 0} \bar{g}_1(x(t)) > \alpha$, $g_2(x_0, a) = \min_{t \geq 0} \bar{g}_2(x(t)) \leq \epsilon$

Example parameterization for controller:

$$Q = \begin{bmatrix} Q_r & 0 \\ 0 & Q_d \end{bmatrix}$$

with $Q_r = 10^{q_r} I$ and $Q_d = q_d \sqrt{Q_r}$ and R fixed

Controller Parameterization

In general, we may not know the system matrices exactly (e.g. Λ_e)

$$\Lambda_e(q, \dot{q})\frac{dv_e}{dt} + \mu(q, \dot{q}) + p(q, \dot{q}) = F_e$$

Define $F_e = \mu(q, \dot{q}) + p(q, \dot{q}) + F$

We get: $\Lambda_e(q, \dot{q})\frac{dv_e}{dt} = F$

Assume we know an approximation of Λ_e: $\tilde{\Lambda}_e$ and define $F = \tilde{\Lambda}_e u$.

Then our equation becomes:

$$\frac{dv_e}{dt} = u + (\Lambda_e^{-1}\tilde{\Lambda}_e - I)u$$

Final control law: $F_e = \mu(q, \dot{q}) + p(q, \dot{q}) - \tilde{\Lambda}_e K x$

Thus our linear system ends up neglecting $(\Lambda_e^{-1}\tilde{\Lambda}_e - I)u$.

Controller Parameterization

Objective: Find Q, R such that $u = -Kx$ fulfills all constraints and optimizes our performance given that we have an imperfect model.

Types of controllers:

- Exact feedback lineraization: $\tilde{\Lambda}_e = \Lambda_e$
- Approximate feedback linearization $\tilde{\Lambda}_e = \mathrm{diag}(\Lambda_e)$ (we only know the diagonal terms of the mass matrix)
- Impedance control: $\tilde{\Lambda}_e = I$

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