A scientist would like to test a new drug that will inhibit a mouse's ability to run through a maze. Two mice are randomly chosen to receive the drug and another two mice don't receive the drug (control group). The time each mouse takes to go through the maze is measured in seconds. The results of the experiment are as follows:

Drug		Control	
30	25	18	21

The average time for the drug group is 27.5s and the average time for the control group is 19.5s. The mean difference in times is 27.5-19.5=8.0s. (*Adapted from Mathematical Statistics with Resampling and R, Chihara and Hesterberg*)

Answer the following questions:

1. If the drug does not really influence times then the split of the observations into two groups was essentially random. Give an example of how the outcomes of the experiment could have been distributed into the two groups? Is the mean difference in your example the same or different than the observed mean difference?

Drug		Control	
18	25	30	21

2. If the drug does not really influence times then how many possible ways could the outcomes be distributed into the two groups?

$$\binom{4}{2} = 6.$$

3. Explicitly write out all possible distributions of the observations in the table below.

Drug		Control		$\bar{X}_D$	$\overline{X_C}$	$d_i = \bar{X}_D - \bar{X}_C$
30	**25**	**18**	**21**	**27.5**	**19.5**	**8**
30	18	25	21	24	23	1
30	21	25	18	25.5	21.5	4
25	18	30	21	21.5	25.5	-4
25	21	30	18	23	24	-1
18	21	30	25	19.5	27.5	-8

4. Let μ_D and μ_C be the mean time in the drug and control groups respectively. The hypothesis is that mean time in the drug group is shorter compared the control group. Therefore, calculate the P-value of the test $H_0 : \mu_D = \mu_C$ versus $H_0 : \mu_D < \mu_C$.

   The P-value is $\dfrac{\sum_{i=1}^{\binom{4}{2}} I\left(d_i \le d^{obs}\right)}{\binom{4}{2}}$. Where d_i is the value of the test statistic $\bar{X}_D - \bar{X}_C$ for the ith randomization, $I\left(d_i \ge d^{obs}\right) = \begin{cases} 1, & \text{if } d_i \le d^{obs} \\ 0, & \text{if } d_i > d^{obs} \end{cases}$, and $d^{obs} = 8$ is the observed value of the test statistics. Therefore the P-value is $6/6 = 1$.

5. What has been assumed in calculating the P-value?

   - Randomization of the subjects to the treatments.
   - The null hypothesis is true. This means that the labels of the mice are free to be exchanged.

6. What can you conclude about the effectiveness of the drug on time to complete the maze?

   In this sample of mice there is no evidence that mice in the drug group are faster compared to mice in the control group. Therefore there is no evidence that the drug is effective in inhibiting (slowing) the time to go through the maze.