# STA305/1004-Class 21

March 29, 2017

# Today's Class

- Fractional factorial design

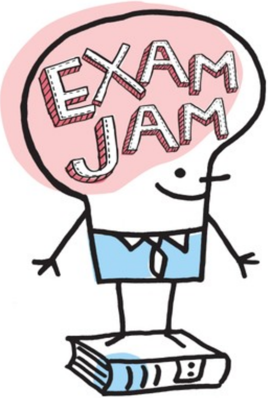
# Exam review session

**Date:** Thursday, April 6th

**Time:** 10 am - 11 am

**Location:** SS 2118

Stop
by the SS lobby to take a few photos in the Photobooth, enjoy some free coffee and snacks and engage in other fun activities (lobby activities run 11-3).



# Fractional factorial designs

- A $2^k$ full factorial requires $2^k$ runs.
- Full factorials are seldom used in practice for large k (k>=7).
- For economic reasons fractional factorial designs, which consist of a fraction of full factorial designs are used.

# Example - Effect of five factors on six properties of film in eight runs

Five factors were studied in 8 runs (Box, Hunter, and Hunter (2005)). The factors were:

1. Catalyst concentration (A)
2. Amount of additive (B)
3. Amounts of three emulsifiers (C, D, E)

Polymer solutions were prepared and spread as a film on a microscope slide. Six different responses were recorded.

| run | A | B | C | D | E | y1 | y2 | y3 | y4 | y5 | y6 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | 1 | -1 | no | no | yes | no | slightly | yes |
| 2 | 1 | -1 | -1 | 1 | 1 | no | yes | yes | yes | slightly | yes |
| 3 | -1 | 1 | -1 | -1 | 1 | no | no | no | yes | no | no |
| 4 | 1 | 1 | -1 | -1 | -1 | no | yes | no | no | no | no |
| 5 | -1 | -1 | 1 | -1 | 1 | yes | no | no | yes | no | slightly |
| 6 | 1 | -1 | 1 | -1 | -1 | yes | yes | no | no | no | no |
| 7 | -1 | 1 | 1 | 1 | -1 | yes | no | yes | no | slightly | yes |
| 8 | 1 | 1 | 1 | 1 | 1 | yes | yes | yes | yes | slightly | yes |

# Example - Effect of five factors on six properties of film in eight runs

- The eight run design was constructed beginning with a standard table of signs for a $2^3$ design in the factors A, B, C.
- The column of signs associated with the BC interaction was used to accommodate factor D, the ABC interaction column was used for factor E.
- A full factorial for the five factors A, B, C, D, E would have needed $2^5 = 32$ runs.
- Only 1/4 were run. This design is called a quarter fraction of the full $2^5$ or a $2^{5-2}$ design (a two to the five minus two design).
- In general a $2^{k-p}$ design is a $\frac{1}{2^p}$ fraction of a $2^k$ design using $2^{k-p}$ runs. This design can study $k$ factors in $\frac{1}{2^p}$ fraction of the runs.

# Effect Aliasing and Design Resolution

- A chemist in an industrial development lab was trying to formulate a household liquid product using a new process.
- The liquid had good properties but was unstable.
- The chemist wanted to synthesize the product in hope of hitting conditions that would give stability, but without success.
- The chemist identified four important influences: A (acid concentration), B (catalyst concentration), C (temperature), D (monomer concentration).

# Effect Aliasing and Design Resolution

- His 8 run fractional factorial design is shown below.

| test | A | B | C | D | y |
|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | -1 | 20 |
| 2 | 1 | -1 | -1 | 1 | 14 |
| 3 | -1 | 1 | -1 | 1 | 17 |
| 4 | 1 | 1 | -1 | -1 | 10 |
| 5 | -1 | -1 | 1 | 1 | 19 |
| 6 | 1 | -1 | 1 | -1 | 13 |
| 7 | -1 | 1 | 1 | -1 | 14 |
| 8 | 1 | 1 | 1 | 1 | 10 |

- The signs of the ABC interaction is used to accommodate factor D. The tests were run in random order. He wanted to achieve a stability value of at least 25.

# Effect Aliasing and Design Resolution

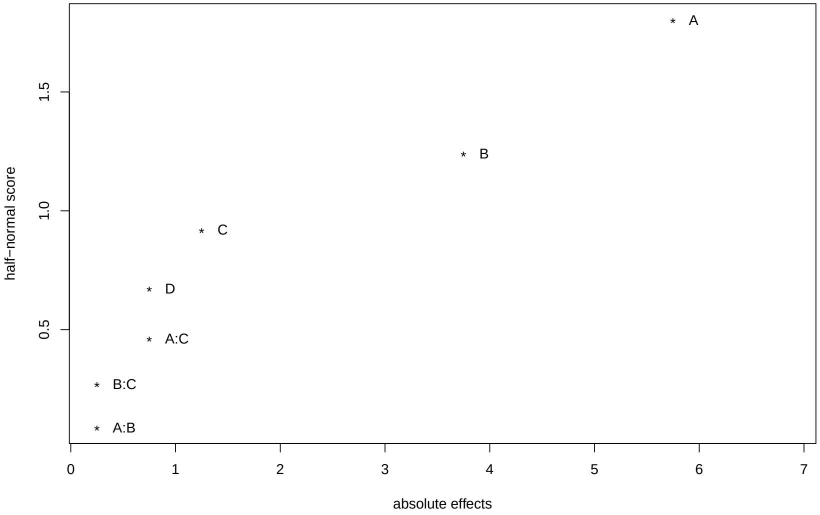
```
fact.prod <- lm(y~A*B*C*D,data=tab0602)
fact.prod1 <- aov(y~A*B*C*D,data=tab0602)
round(2*fact.prod$coefficients,2)
```

```
(Intercept)           A           B           C           D         A:B
      29.25       -5.75       -3.75       -1.25        0.75        0.25
        A:C         B:C         A:D         B:D         C:D       A:B:C
       0.75       -0.25          NA          NA          NA          NA
      A:B:D       A:C:D       B:C:D     A:B:C:D
         NA          NA          NA          NA
```

Even though the stability never reached the desired level of 25, two important factors, A and B, were identified.

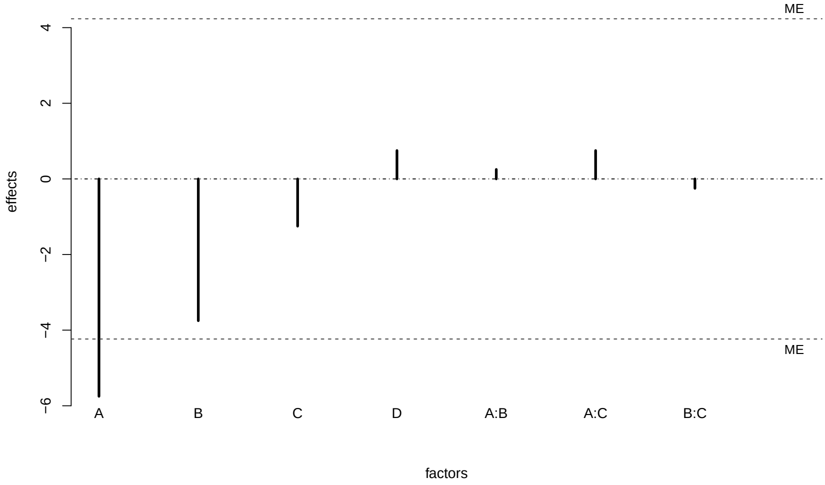
# Effect Aliasing and Design Resolution

```r
BsMD::DanielPlot(fact.prod,half = T)
```



# Effect Aliasing and Design Resolution

```r
BsMD::LenthPlot(fact.prod1)
```




```
##     alpha       PSE        ME       SME
##  0.050000  1.125000  4.234638 10.134346
```

# Poll Question

**A factorial design to assess the effects of seven factors (each has two levels) in eight runs is an example of a**

Respond at **PollEv.com/nathantaback**

Text **NATHANTABACK** to **37607** once to join, then **A, B, C, or D**

| | |
|---|---|
| $2^7$ factorial design | **A** |
| $2^3$ factorial design | **B** |
| $2^{7-4}$ factorial design | **C** |
| $2^{8-5}$ factorial design | **D** |

Figure 1:

# Effect Aliasing and Design Resolution

What information could have been obtained if a full $2^5$ design had been used?

| Factors | Number of effects |
|---|---|
| Main | 5 |
| 2-factor | 10 |
| 3-factor | 10 |
| 4-factor | 5 |
| 5-factor | 1 |

- 31 degrees of freedom in a 32 run design.
- 16 used for estimating three factor interactions or higher.
- Is it practical to commit half the degrees of freedom to estimate such effects?
- According to effect hierarchy principle three-factor and higher not usually important.
- Thus, using full factorial wasteful. It's more economical to use a fraction of full factorial design that allows lower order effects to be estimated.

# Effect Aliasing and Design Resolution

Consider a design that studies five factors in 16 run. A half fraction of a $2^5$ or $2^{5-1}$.

| Run | B | C | D | E | Q |
|---|---|---|---|---|---|
| 1 | -1 | 1 | 1 | -1 | -1 |
| 2 | 1 | 1 | 1 | 1 | -1 |
| 3 | -1 | -1 | 1 | 1 | -1 |
| 4 | 1 | -1 | 1 | -1 | -1 |
| 5 | -1 | 1 | -1 | 1 | -1 |
| 6 | 1 | 1 | -1 | -1 | -1 |
| 7 | -1 | -1 | -1 | -1 | -1 |
| 8 | 1 | -1 | -1 | 1 | -1 |
| 9 | -1 | 1 | 1 | -1 | 1 |
| 10 | 1 | 1 | 1 | 1 | 1 |
| 11 | -1 | -1 | 1 | 1 | 1 |
| 12 | 1 | -1 | 1 | -1 | 1 |
| 13 | -1 | 1 | -1 | 1 | 1 |
| 14 | 1 | 1 | -1 | -1 | 1 |
| 15 | -1 | -1 | -1 | -1 | 1 |
| 16 | 1 | -1 | -1 | 1 | 1 |

- The factor E is assigned to the column BCD.
- The column for E is used to estimate the main effect of E and also for BCD.
- The main factor E is said to be **aliased** with the BCD interaction.

# Effect Aliasing and Design Resolution

- This aliasing relation is denoted by

$$E = BCD \text{ or } I = BCDE,$$

  where $I$ denotes the column of all $+$'s.
- This uses same mathematical definition as the confounding of a block effect with a factorial effect.
- Aliasing of the effects is a price one must pay for choosing a smaller design.
- The $2^{5-1}$ design has only 15 degrees of freedom for estimating factorial effects, it cannot estimate all 31 factorial effects among the factors B, C, D, E, Q.

# Effect Aliasing and Design Resolution

- The equation $I = BCDE$ is called the **defining relation** of the $2^{5-1}$ design.
- The design is said to have resolution IV because the defining relation consists of the "word" BCDE, which has "length" 4.
- Multiplying both sides of $I = BCDE$ by column B

$$B = B \times I = B \times BCDE = CDE,$$

  the relation $B = CDE$ is obtained.
- B is aliased with the CDE interaction. Following the same method all 15 aliasing relations can be obtained.

# Effect Aliasing and Design Resolution

- To get the most desirable alias patterns, fractional factorial designs of highest resolution would usually be employed.
- There are important exceptions to this rule that we will not cover in the course.

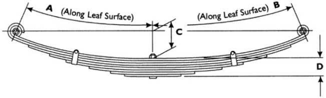
# Class Question

Consider a $2^{5-1}$ fractional factorial design.

1. How many factors does this design have?
2. How many runs are involved in this design?
3. How many levels for each factor?
4. The factor $E$ is assigned to the four-way interaction ($ABCD$). What is the defining relation? What is the design resolution? What are the aliasing relations?

# Example - Leaf spring experiment

An experiment to improve a heat treatment process on truck leaf springs (Wu and Hamada (2009)). The height of the unloaded spring is an important quality characteristic.



# Example - Leaf spring experiment

Five factors that might affect height were studied in this $2^{5-1}$ design.

| Factor | Level |
|---|---|
| B. Temperature | 1840 (-), 1880 (+) |
| C. Heating time | 23 (-), 25 (+) |
| D. Transfer time | 10 (-), 12 (+) |
| E. Hold down time | 2 (-), 3 (+) |
| Q. Quench oil temperature | 130-150 (-), 150-170 (+) |

# Example - Leaf spring experiment

| B | C | D | E | Q | y |
|---|---|---|---|---|---|
| -1 | 1 | 1 | -1 | -1 | 7.7900 |
| 1 | 1 | 1 | 1 | -1 | 8.0700 |
| -1 | -1 | 1 | 1 | -1 | 7.5200 |
| 1 | -1 | 1 | -1 | -1 | 7.6333 |
| -1 | 1 | -1 | 1 | -1 | 7.9400 |
| 1 | 1 | -1 | -1 | -1 | 7.9467 |
| -1 | -1 | -1 | -1 | -1 | 7.5400 |
| 1 | -1 | -1 | 1 | -1 | 7.6867 |
| -1 | 1 | 1 | -1 | 1 | 7.2900 |
| 1 | 1 | 1 | 1 | 1 | 7.7333 |
| -1 | -1 | 1 | 1 | 1 | 7.5200 |
| 1 | -1 | 1 | -1 | 1 | 7.6467 |
| -1 | 1 | -1 | 1 | 1 | 7.4000 |
| 1 | 1 | -1 | -1 | 1 | 7.6233 |
| -1 | -1 | -1 | -1 | 1 | 7.2033 |
| 1 | -1 | -1 | 1 | 1 | 7.6333 |

# Example - Leaf spring experiment

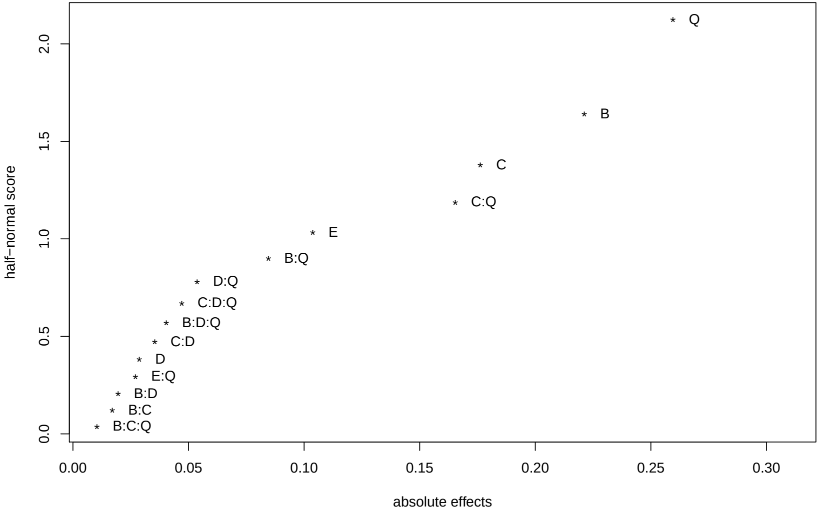
The factorial effects are estimated as before.

```
fact.leaf <- lm(y~B*C*D*E*Q,data=leafspring)
fact.leaf2 <- aov(y~B*C*D*E*Q,data=leafspring)
round(2*fact.leaf$coefficients,2)
```

```
(Intercept)           B           C           D           E           Q
      15.27        0.22        0.18        0.03        0.10       -0.26
        B:C         B:D         C:D         B:E         C:E         D:E
       0.02        0.02       -0.04          NA          NA          NA
        B:Q         C:Q         D:Q         E:Q       B:C:D       B:C:E
       0.08       -0.17        0.05        0.03          NA          NA
      B:D:E       C:D:E       B:C:Q       B:D:Q       C:D:Q       B:E:Q
         NA          NA        0.01       -0.04       -0.05          NA
      C:E:Q       D:E:Q     B:C:D:E     B:C:D:Q     B:C:E:Q     B:D:E:Q
         NA          NA          NA          NA          NA          NA
    C:D:E:Q   B:C:D:E:Q
         NA          NA
```

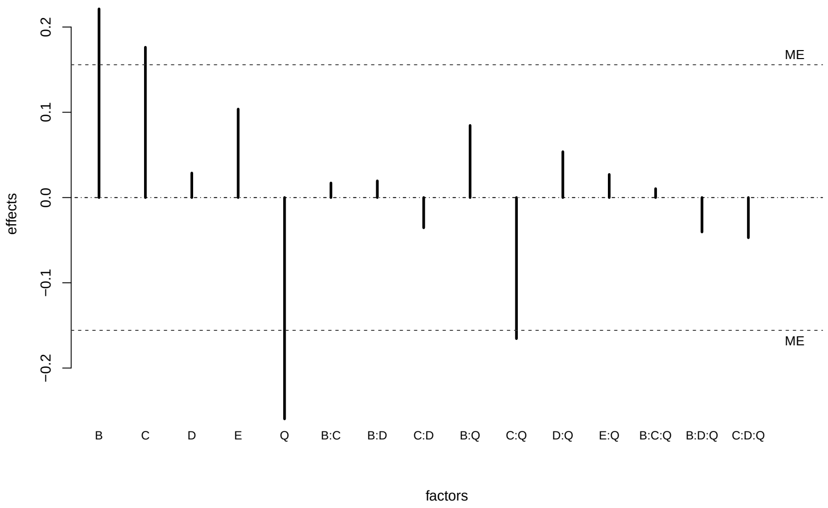
# Example - Leaf spring experiment

```
BsMD::DanielPlot(fact.leaf,half = T)
```



# Example - Leaf spring experiment

```
BsMD::LenthPlot(fact.leaf2,cex.fac = 0.8)
```




```
    alpha       PSE        ME       SME
0.0500000 0.0606000 0.1557773 0.3162503
```