STA305/1004-Class 16

March 13, 2017

Today's Class

- Sample size for ANOVA
- Factorial designs at two levels
- Cube plots
- Calculation of factorial effects

Sample size for ANOVA - Designing a study to compare more than two treatments

- Consider the hypothesis that k means are equal vs. the alternative that at least two differ.
- What is the probability that the test rejects if at least two means differ?
- Power $= 1 - P(\text{Type II error})$ is this probability.

Sample size for ANOVA - Designing a study to compare more than two treatments

The null and alternative hypotheses are:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k \text{ vs. } H_1 : \mu_i \neq \mu_j.$$

The test rejects at level α if

$$MS_{Treat}/MS_E \geq F_{k-1,N-K,\alpha}.$$

The power of the test is

$$1 - \beta = P\left(MS_{Treat}/MS_E \geq F_{k-1,N-K,\alpha}\right),$$

when H_0 is false.

Sample size for ANOVA - Designing a study to compare more than two treatments

When H_0 is false it can be shown that:

- MS_{Treat}/σ^2 has a non-central Chi-square distribution with $k-1$ degrees of freedom and non-centrality parameter δ.
- MS_{Treat}/MS_E has a non-central F distribution with the numerator and denominator degrees of freedom $k-1$ and $N-k$ respectively, and non-centrality parameter

$$\delta = \frac{\sum_{i=1}^{k} n_i \left(\mu_i - \bar{\mu}\right)^2}{\sigma^2},$$

where n_i is the number of observations in group i, $\bar{\mu} = \sum_{i=1}^{k} \mu_i / k$, and σ^2 is the within group error variance .

This is dentoted by $F_{k-1,N-k}(\delta)$.

Direct calculation of Power

- The power of the test is

$$P\left(F_{k-1,N-k}(\delta) > F_{k-1,N-K,\alpha}\right).$$

- The power is an increasing function δ
- The power depends on the true values of the treatment means μ_i, the error variance σ^2, and sample size n_i.
- If the experimentor has some prior idea about the treament means and error variance the sample size (number of replications) that will guaruntee a pre-assigned power of the test.

Blood coagulation example - sample size

Suppose that an investigator would like to replcate the blood coagulation study with only 3 animals per diet. In this case $k = 4, n_i = 3$. The treatment means from the initial study are:

Diet	A	B	C	D
Average	61	66	68	61

```r
anova(lm.diets)
```

```
## Analysis of Variance Table
##
## Response: y
##           Df Sum Sq Mean Sq F value    Pr(>F)
## diets      3    228    76.0  13.571 4.658e-05 ***
## Residuals 20    112     5.6
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Blood coagulation example - sample size

- $\mu_1 = 61$, $\mu_2 = 66$, $\mu_3 = 68$, $\mu_4 = 61$.
- The numerator df = 4-1=3, and the denominator df = 12-4=8.
- The error variance σ^2 was estimated as $MS_E = 5.6$.
- Assuming that the estimated values are the true values of the parameters, the non-centrality parameter of the F distribution is

$$\delta = 3 \times \left((61 - 64)^2 + (66 - 64)^2 + (68 - 64)^2 + (61 - 64)^2\right) /5.6 = 20.35714$$

Blood coagulation example - sample size

If we choose $\alpha = 0.05$ as the significance level then $F_{3,20,0.05} = 4.0661806$. The power of the test is then

$$P\left(F_{3,8}(20.35714) > 4.066181\right) = 0.8499.$$

This was calculated using the CDF for the F distribution in R `pf()`.

```r
1-pf(q = 4.066181,df1 = 3,df2 = 8,ncp = 20.35714)
```

```
## [1] 0.8499
```

Calculating power and sample size using the pwr library

- There are several libraries in R which can calculate power and sample size for statistical tests. The library `pwr()` has a function for ANOVA.
- `pwr.anova.test(k = NULL, n = NULL, f = NULL, sig.level = 0.05, power = NULL)`

for computing power and sample size.

- `k` Number of groups
- `n` Number of observations (per group)
- `f` Effect size

The effect size f

$$f = \sqrt{\frac{\sum_{i=1}^{k} \left(\mu_i - \bar{\mu}\right)^2 / k}{\sigma^2}},$$

is related to the non-centrality parameter δ via $\delta = k \cdot n_i \cdot f^2$.

- n_i is the number of observations in group i, $\bar{\mu} = \sum_{i=1}^{k} \mu_i / k$, and σ^2 is the within group error variance.

Calculating power and sample size using the pwr library

In the previous example $\delta = 20.35714$ so $f = \sqrt{\frac{\delta}{k \cdot n_i}} = \sqrt{20.35714/4 \cdot 3} = 1.3024701$.

```
library(pwr)
pwr.anova.test(k = 4,n = 3,f = 1.30247)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##               k = 4
##               n = 3
##               f = 1.30247
##       sig.level = 0.05
##           power = 0.8499
##
## NOTE: n is number in each group
```

Calculating power and sample size using the pwr library