T2-2025

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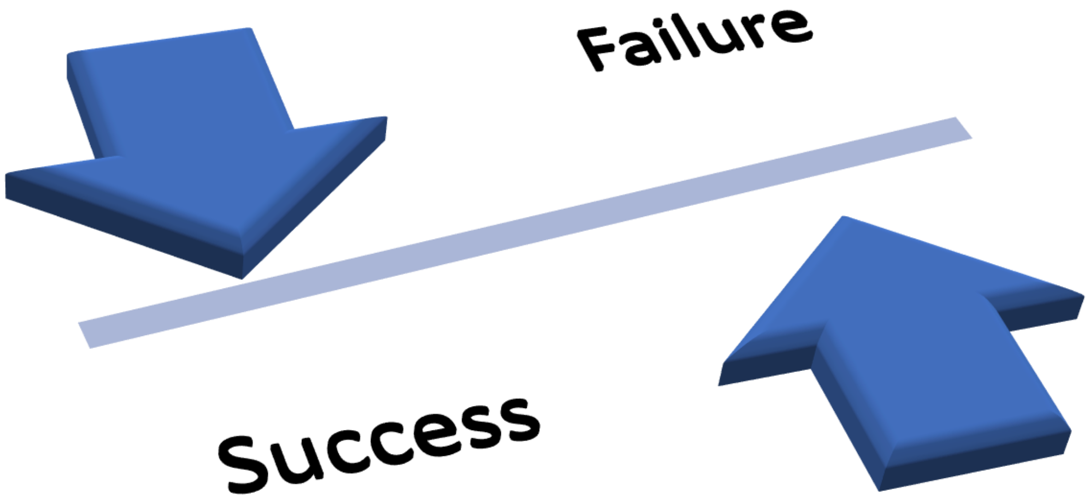
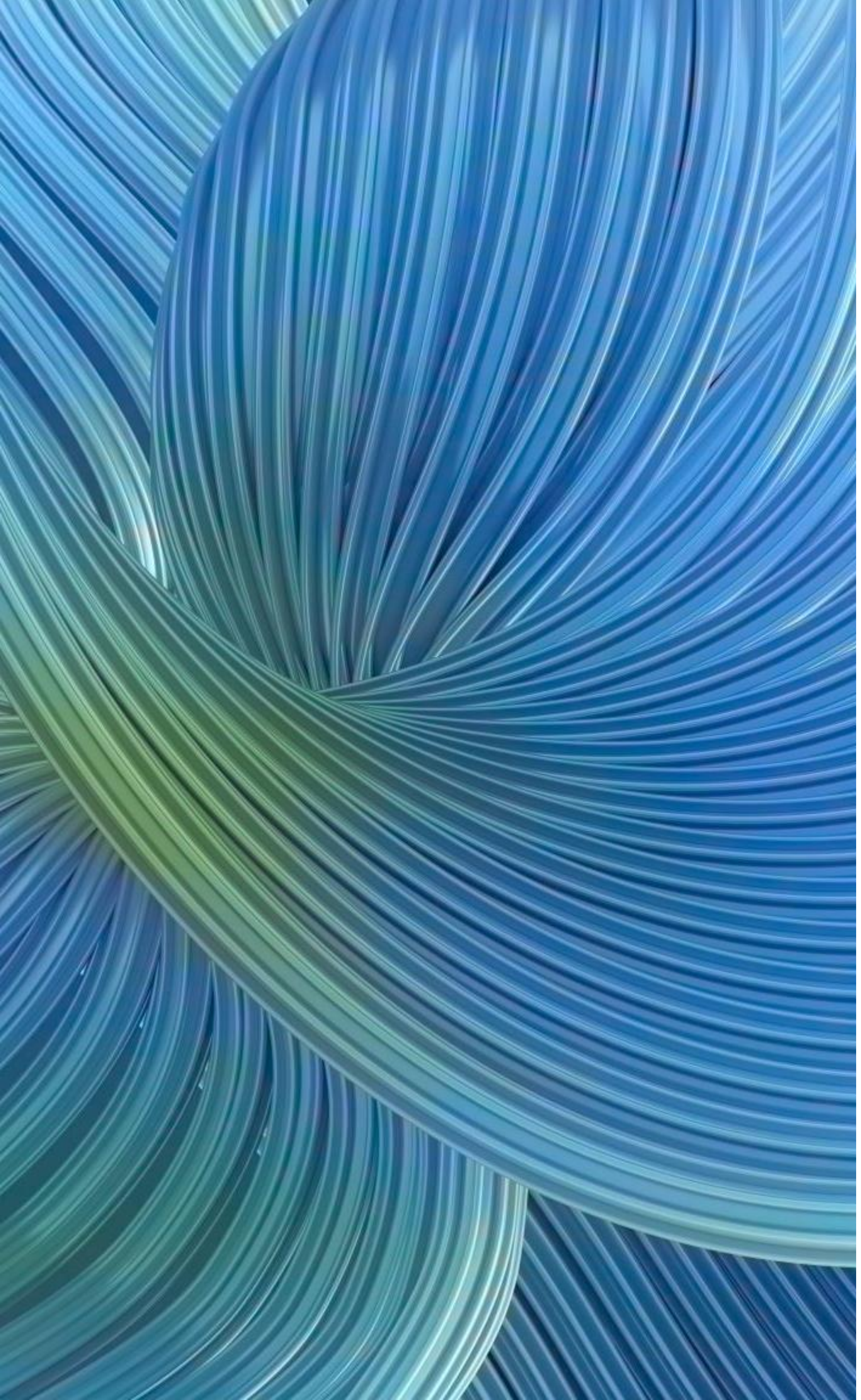
Uncertain decisions and risk sharing

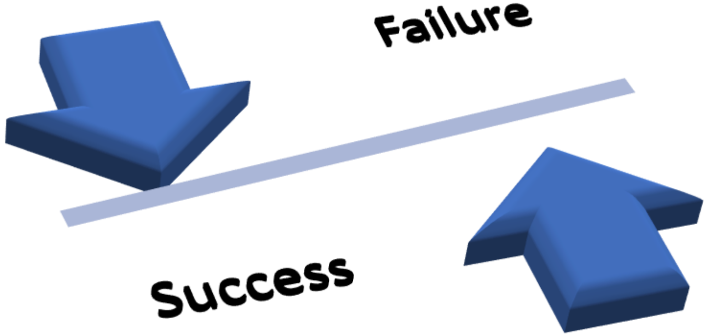
Decisions with risks





Decisions with risks

- Two dimensions to consider
  - Value of success
  - Cost of failure
  - Success probability
  - Failure probability



Investment in Silver mining project is a Risky Decision

- Do we find enough resources?
- Do we find high quality resources?
- What happens if the mind collapse?

Should we invest or not?

Playing a game

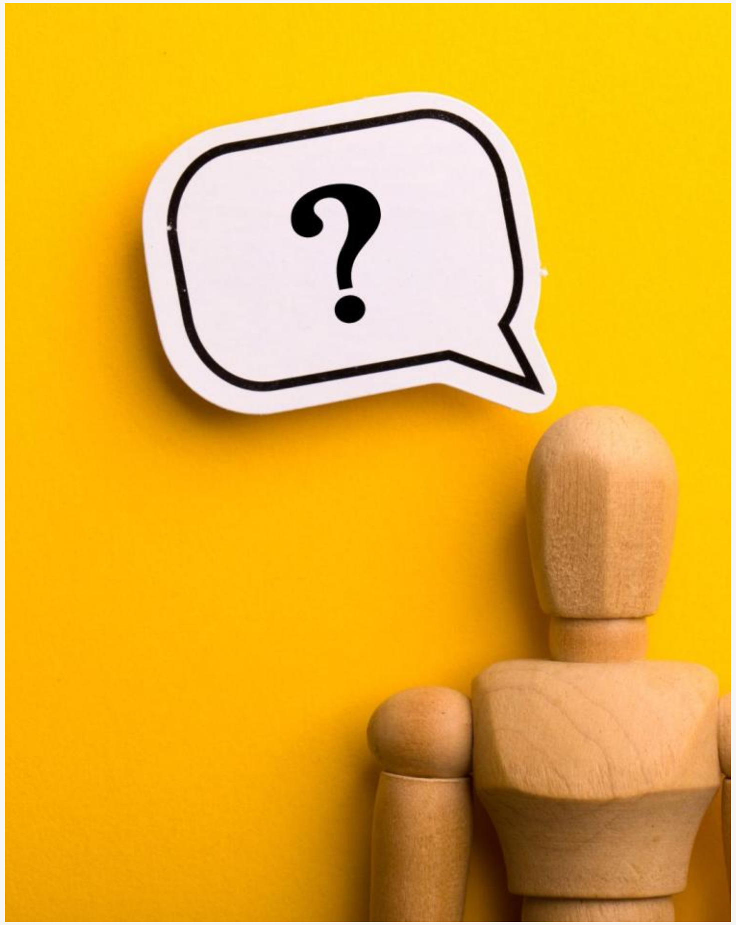
- Heads: $10
- Tails: $0.
- 50% chance of heads and a 50% chance of tails.
- For what price x would you choose to play the game?

Expected value of Playing the game

- The average payoff is: $0.5(10 - x) + 0.5(- x) = 5 - x$
- Thus the value of the game is \$5 - x.

Decision making under risk

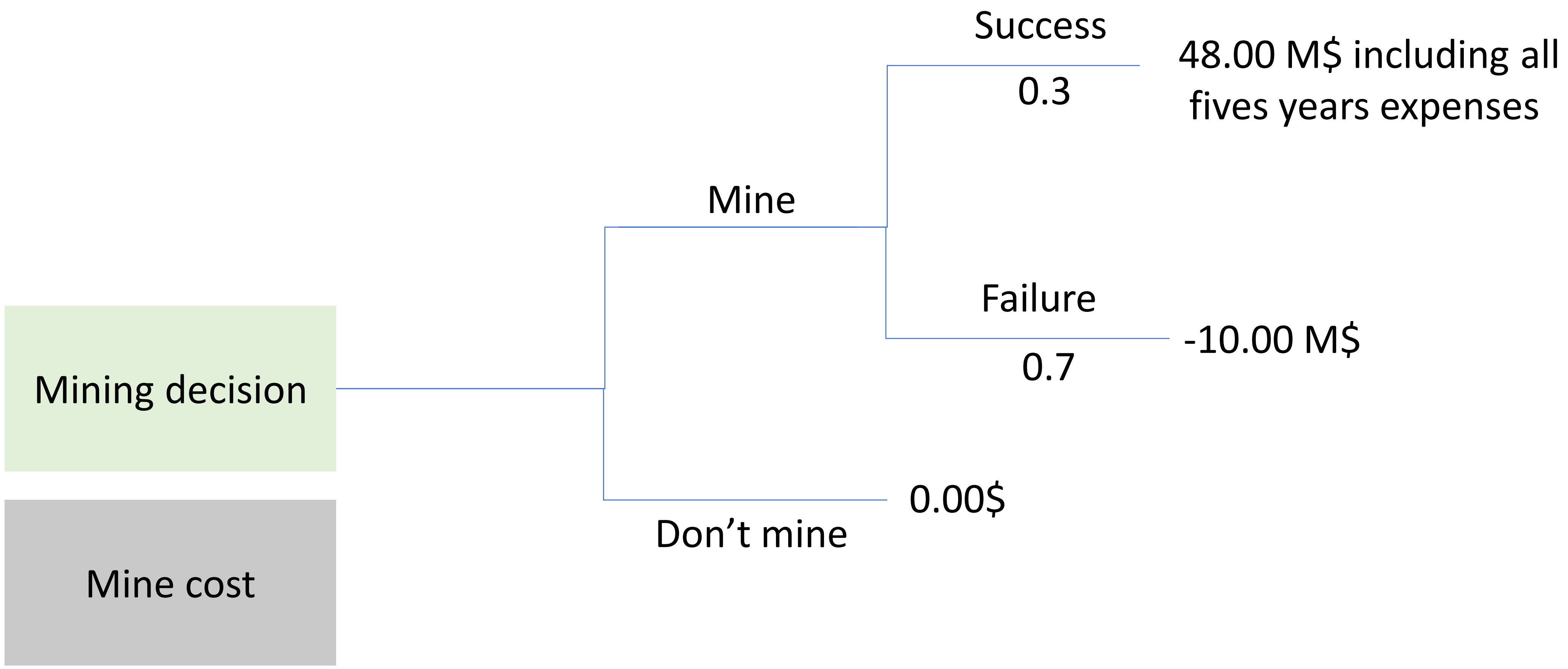
- A process of choosing between different lotteries.
- A lottery (or prospect) consists of a number of possible outcomes with their associated probability It can be described as:
- $\boldsymbol{q}$ = $(x_1 , p_1 ; x_i , p_i ; \ldots x_n , p_n)$
- x_i represents the i th outcome
- p_i is its associated probability
- $\sum p_i = 1$



Expected Value Calculation

- **1. Define Outcomes and Probabilities**
- List all possible outcomes of the game.
- Assign a probability to each outcome. Make sure the total probabilities sum up to 1.
- **2. Determine Payoffs**
- Identify the payoff associated with each outcome (this could be a win amount or a loss).
- **3. Calculate Expected Value**
- Use the formula:
- $EV = \sum(\text{Probability} \times \text{Payoff})$
- This means you multiply each outcome's probability by its payoff and then sum these values.

Investment in Silver mining project is a Risky Decision



Expected value of a decision

EV of Outcome 1
Probability × Value

Expected value of success

EV of Outcome 2
Probability × Value

Expected value of failure

Expected value of a decision

EV of Outcome 1+EV of Outcome 2

Expected value of mining decision

Outcome 1:
0.3 × (48=58-10)=14.4

Expected value of success

+

Outcome 2: 0.7 × (-10)=-7

Expected value of failure

Expected value of mining decision

14.4-7=7.4 M

Expected value of an investment decision

Probability × NPV

Reward

Exchange rate

Grade

Silver Market price

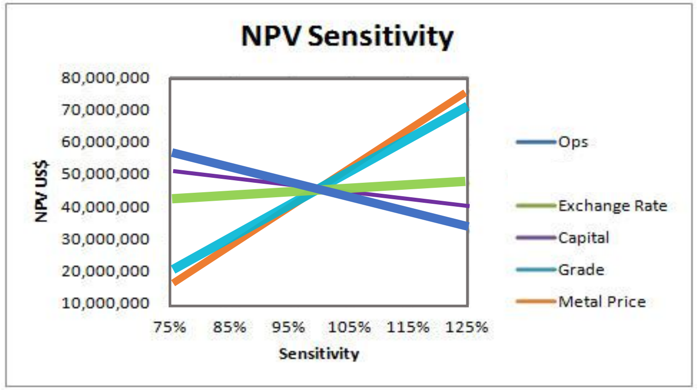
Cost of development

Geology

Risk
Probability × NPV

Expected value of failure

Expected value of a decision



EV of Outcome 1+EV of Outcome 2

Exercise

- 1-Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)

Expected value of Playing the game

- The average payoff is: p_1(10-x) + (1- p_1)(- x) =
- $10p_1$- xp_1-x+ $xp_1 = 10p_1$-x
- Expected value=$10p_1$-x

Expected value of an investment decision

- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times p_{Failure})$
-
- $V_{Success}$: Conditional value of success
- $V_{Failure}$: Conditional value of Failure
- $P_{Success}$: Probability of success
- $P_{Failure}$: Probability of success

- $P_{Success} + P_{Failure} = 1$
- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1 - p_{Success}))$

Expected value of mining decision

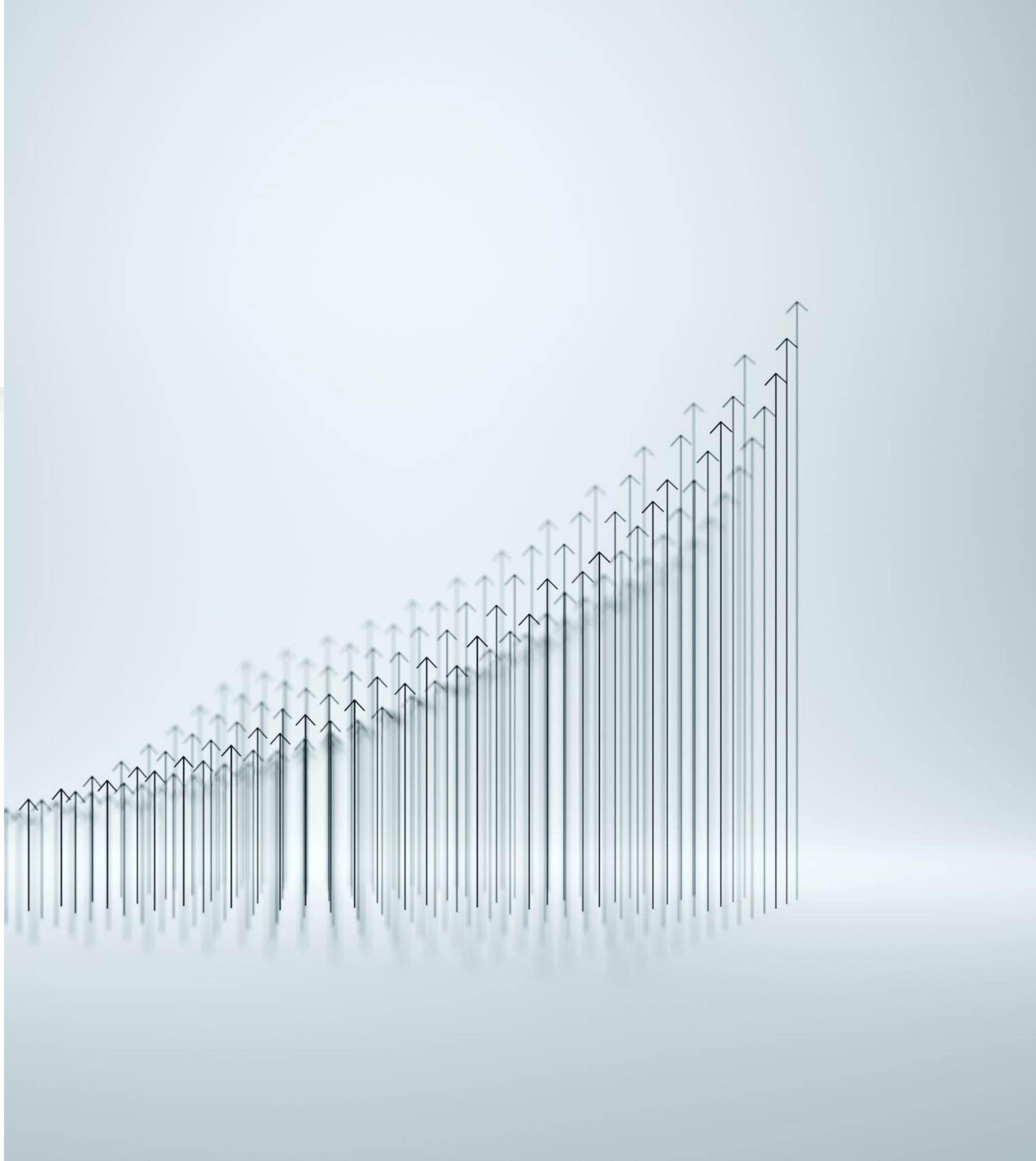
- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1\text{-}p_{Success}))$
- EV=(48 $\times$ $p_{Success}$)+(-10$\times$ (1-$p_{Success}$))
- EV = 58 $\times$ $p_{Success}$-10

Minimum probability of success for indifference

- The minimum probability of success for indifference is a key concept in decision-making under uncertainty, particularly in scenarios involving risk and investment choices. It represents the probability at which a decision-maker is indifferent between two options: one with a risky payoff and another with a guaranteed (certain) payoff.

Minimum probability of success for indifference

- The minimum probability of success is the threshold probability at which the expected value of the risky option equals the value of a sure (certain) outcome.
- The minimum probability of success, where the expected value of the risky option equals the guaranteed value of no investment :
- $p_{Success} \times V_{Success} + (1 - p_{Success}) \times V_{Failure} = 0$



Minimum probability of success for indifference

- EV=0
- $0 = 58 \times p_{Success}$-10
- $p_{Success}$=$\frac{10}{58}$=0.17

Expected value line

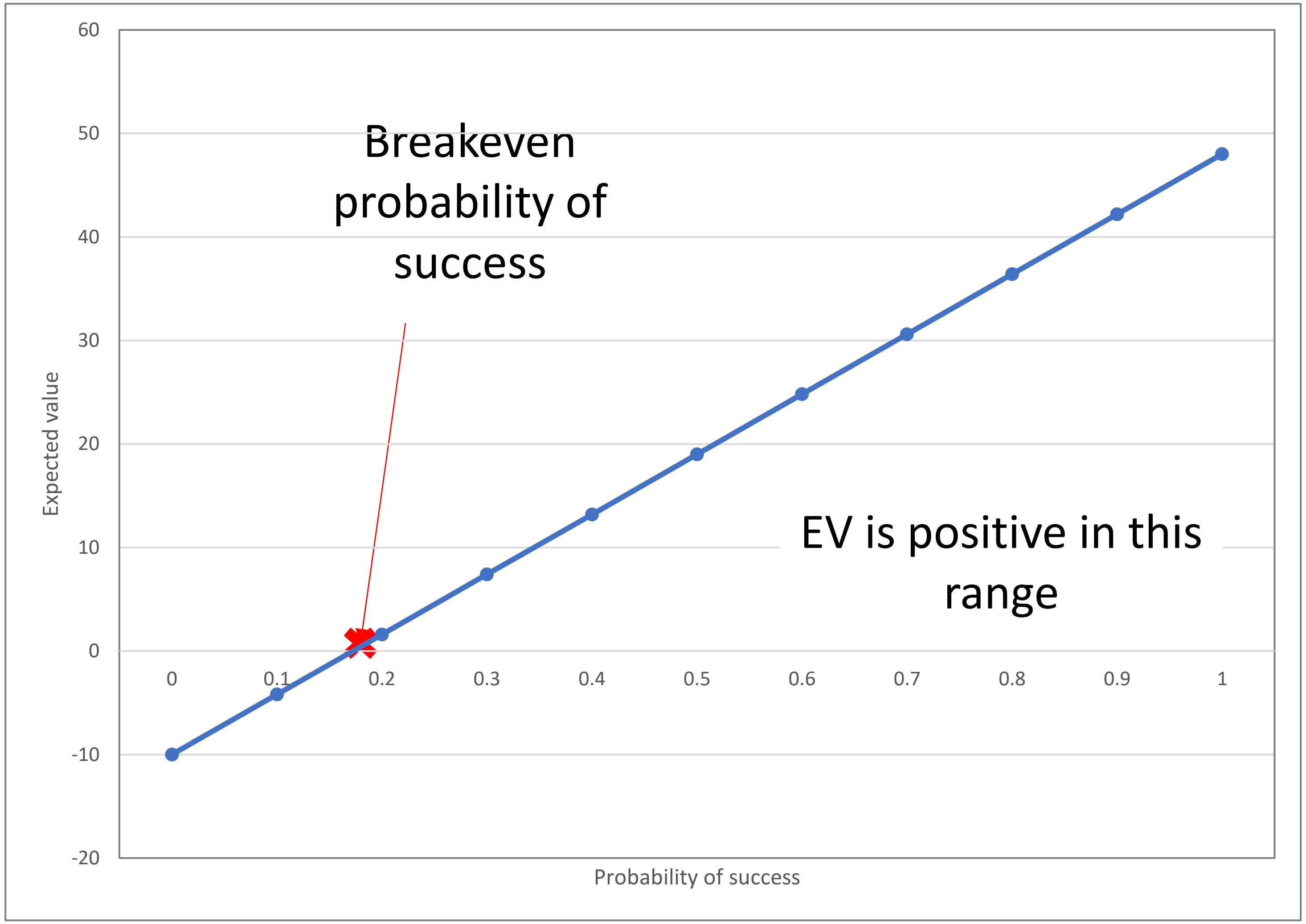
- **Creating an Expected Value Line**
- **1. Define**
- **Outcomes**: Different possible results of an investment.
- **Probabilities**: The likelihood of each outcome occurring.
- **2. Calculate the Expected Value**

The expected value line effectively communicates how various outcomes and their probabilities impact the decision-making process. By visualizing this information, decision-makers can better understand the risk-reward trade-off

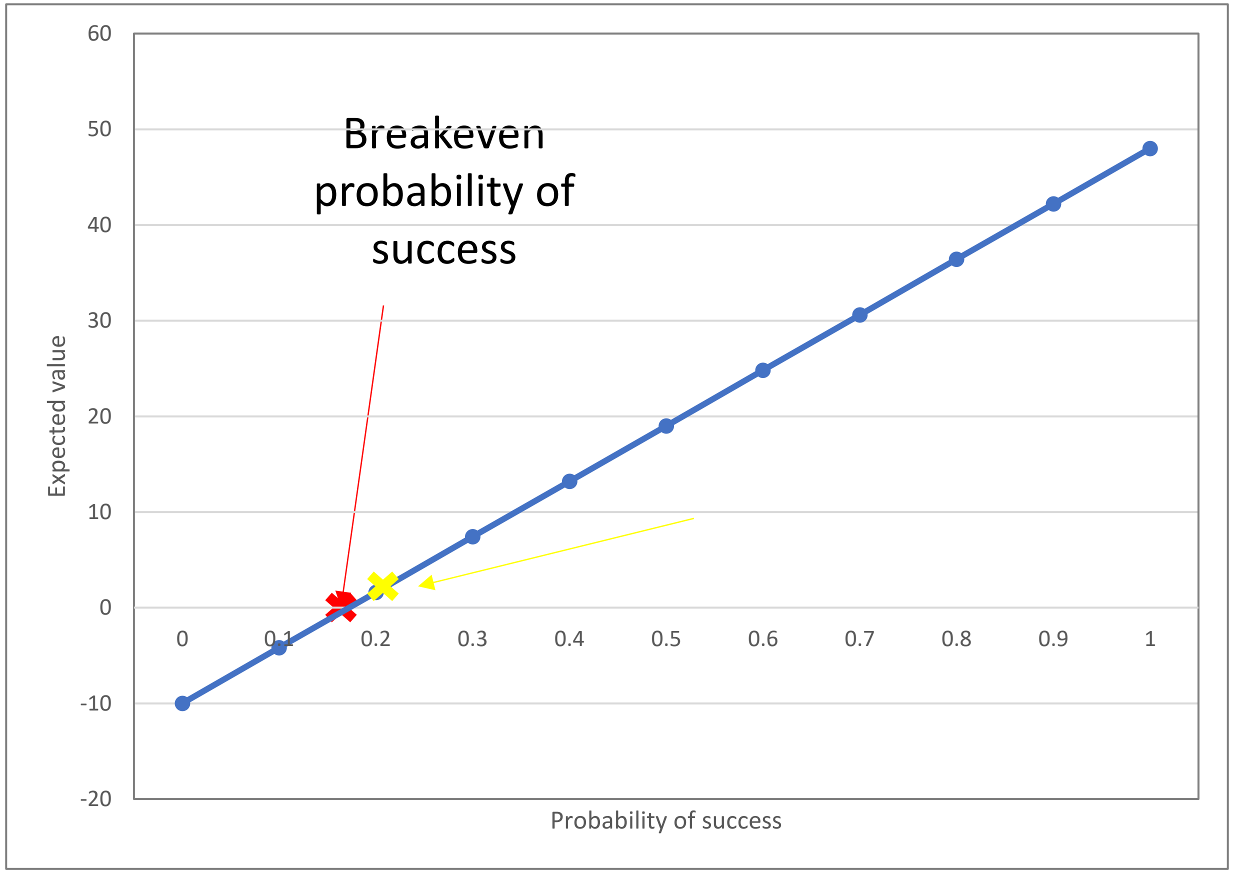
Expected value line

- EV = 58 $\times$ $p_{Success}$-10

The **break-even probability of success** is the probability at which the expected value of a risky investment or decision equals zero. In other words, it's the point where the potential gains from the investment are just enough to offset the potential losses, making the decision neutral in terms of expected value.



Expected value line



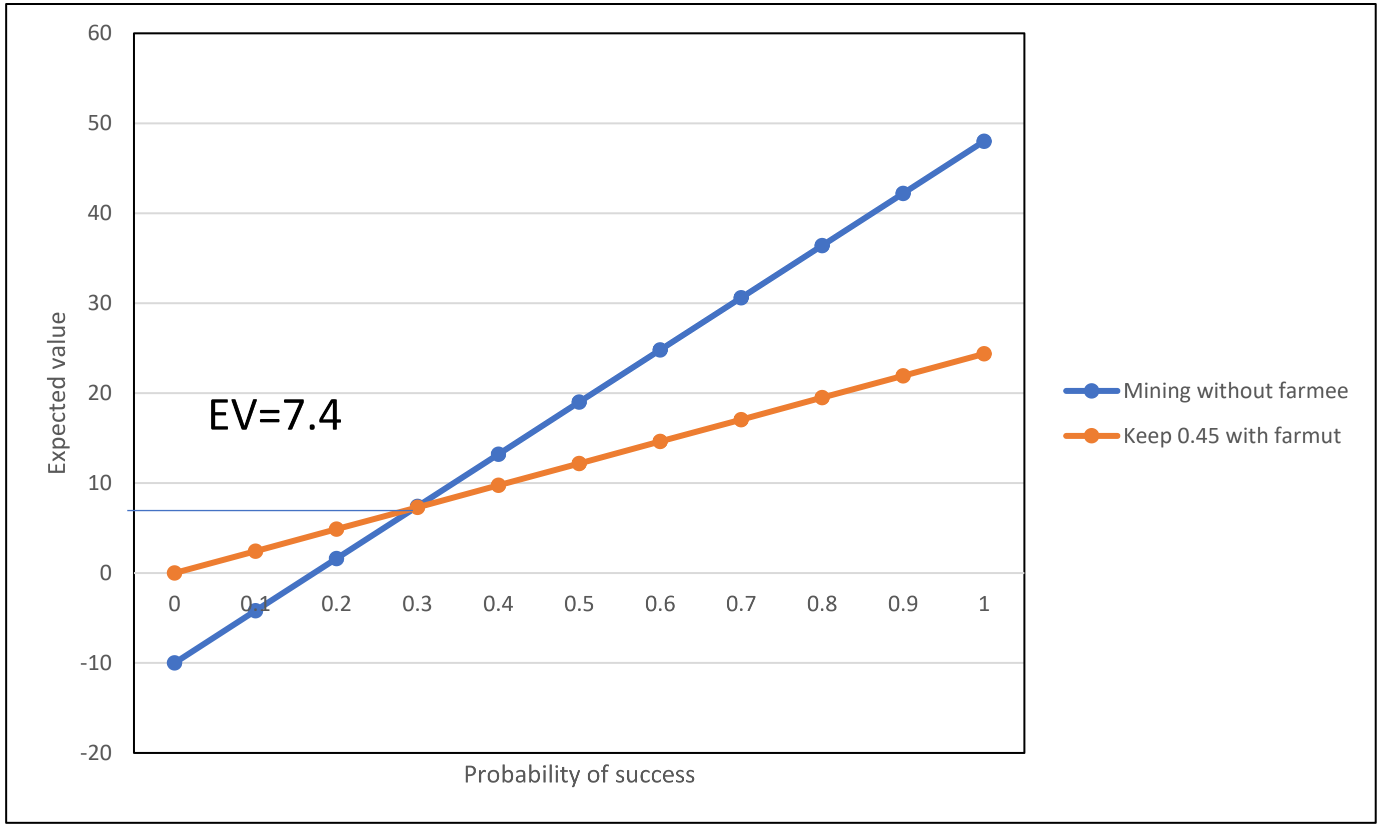
Risk sharing

- **farm-out agreement:**
  An entity (the farmor) agrees to provide a working interest in a mining property to a third party (the farmee), provided that the farmee makes a cash payment to the farmor and/or incurs certain expenditures on the property to earn that interest.

Farmout the mining

- $EV\ of\ decision\ to\ mine = (48 \times 0.3)$ + (-10×0.7)=7.4
- $EV\ of\ decision\ to\ farmout\ = (58 \times 0.3)$ ×(1-x) + (0×0.7)
- Maximize working interest to farmout
- 7.4=(58×0.3) ×(1-x) + (0×0.7)
- X=0.57

Risk sharing in mining decision



Farm-out agreement:

- Key Components of a Farm-Out Agreement:
  - Farmor: The party that owns the rights to the property and is looking to reduce its financial exposure or share the risk.
  - Farmee: The party that takes on the interest and associated responsibilities, usually in exchange for covering some costs.
  - Interest Transfer:Specifies the percentage of interest being transferred from the farmor to the farmee.
  - Cost Sharing: Details on how costs will be allocated between the parties, e.g., all upfront payments or some of it being paid by Farmee.

Exercise

- 1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)
- 2-Find the break even probability point
- 3-Find the farmout value that provides same EV for the original company?

Decision making under risk: more than two outcomes

- A process of choosing between different lotteries.
- A lottery (or prospect) consists of a number of possible outcomes with their associated probability It can be described as:
- $\boldsymbol{q} = (x_1 , p_1 ; x_i , p_i ; \ldots x_n , p_n)$
- x_i represents the ith outcome
- p_i is its associated probability
- $\sum p_i = 1$

Risky decisions:more than two outcomes

- Outcomes
- x_1 : Success:
  - x_{11} Low success
  - x_{12}: Medium success
  - x_{13} High success
- x_2 : Failure
- 1=$p_1 + p_2$
- $p_{11} + p_{12} + p_{12}$=1

Expected value of mining

Conditional probability

VL:41.65
PL=0.3
EVL=12.495

Low NPV

VH=65
PH=0.2
EVH=13

High NPV

VM=45
PM=0.5
EVM=22.5

Med NPV

Reward

(22.5+13+12.495=48)*0.3=14.4

Expected value of a decision

14.4-7=7.4 M

Expected value of failure

Outcome 2: 0.7 × (-10)=-7

Expected value of mining

Un-Conditional probability

VL:41.65
PL=0.09

Low NPV

+

VH=65
PH=0.06

High NPV

+

VM=45
PM=0.15

Med NPV

3.9+3.7485+6.75=14.4

+

Outcome 2: 0.7 × (-10)=-7

Expected value of failure

Expected value of a decision

14.4-7=7.4 M

Applying fiscal relief

Outcome 1:
0.3 × (48=58-10)=14.4

Expected value of success

Outcome 2: 0.7 × (-10)=-7

Expected value of failure

Expected value of mining decision

Fiscal relief

- Overall rate of tax: 60%
- The cost after tax with full fiscal relief: 10 M
- Cost with no fiscal relief:
- $\frac{10}{1-60\%}$ =25M

Fiscal relief

Outcome 1:
0.3 × (58-25)=9.9

Expected value of success

+

Outcome 2: 0.7 × (-25)=-17.5

Expected value of failure

Expected value of mining decision

9.9-17.5=-7.6

What we had earlier
7.4 M

Fiscal relief

$EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1\text{-}p_{Success}))$

$\text{EV}=((58\text{- }25) \times p_{Success})+(-25\times (1\text{-}p_{Success}))$

$\text{EV} = 58 \times p_{Success}\text{-}25$

EV=0

P=25/58

P=43.1% *Compared to 0.17 without applying the effect fiscal relief*

Exercise

- 1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)
- 2-Find the break even probability point
- 3-Find the farmout value that provides same EV for the original company?
- 4- Recalculate the values of question 1-3 without fiscal relief when the overall rate of tax is 50%. Compare the results with calculated values in questions 1-3.