

# **Economic Indicators**

**Economic Decision  
Analysis in Engineering  
Course notes**

**Guy Allinson  
2025**

# Economic Indicators

## Contents

1. Introduction
2. Net present value
3. Features of NPV
4. Nominal and real NPVs
5. Internal rate of return
6. Multiple IRRs
7. Comparing investments
8. Effect of delay
9. Nominal and real IRRs
10. Capital productivity index
11. Payback
12. Discounting methods
13. Comparing indicators
14. Summary

## 1 Introduction

In earlier sections, we examined different aspects of constructing cash flow projections of an oil and gas development. The focus of that chapter was the derivation of net after tax cash flow starting from the present day and finishing at some time in the future. These cash flow projections are required to assist us to decide whether or not the project ought to be undertaken or what its value is. However, the derivation of a forecast of net cash flow cannot on its own help us to decide whether the project should be undertaken. We need something more. We need to have some means of deciding whether a particular cash flow gives us an economic/financial return and, if so, how much.

The following sections are concerned with the derivation and meaning of economic indicators. These are devices which reduce a net cash flow projection to single numbers. Therefore, instead of having to deal with a stream of money amounts stretching out from now into the future, we calculate single number indicators of the relative value of the project.

More importantly, however, the indicators are measures of the relative economic attractiveness of the cash flow compared to the cash flows of other projects. In other words, they tell us whether one investment gives a greater economic benefit than other investments which the company could make.

We will examine the meaning and use of four main economic indicators, or measures of economic benefit, which are used currently in the oil and gas industry. These are:-

- (1) Net Present Value
- (2) Internal Rate of Return
- (3) Payback period
- (4) Capital Productivity Index

All of these are single-number indicators of the economic benefit of a stream of future cash flows. Some are more useful than others, depending on the context, and the relative usefulness of the different indicators will also be discussed.



## 2 Net present value

### Time value of money

The equivalent value today of a sum of money received or spent sometime in the future is its present value ("PV"). The present value of net cash flow occurring at some point in the future is referred to as the net present value ("NPV") of that future cash flow.

By its very name, a net present value recognises that time has a critical bearing on the value of money. Money has more or less value depending on when it is received or spent. A sum of money received now is worth more than the same sum of money received several years in the future. It is not a matter of opinion that money has time value. It is a matter of fact. The fact that we can invest money in the bank and earn interest on it demonstrates that the money invested will be worth more later on. The recognition that money has time value is very important in the petroleum industry. Oil and gas projects involve the investment of large sums of money, often over a period of several years before any revenue is obtained. The projects can last for twenty years or more. Therefore the choice of an economic indicator which recognises time and takes time quantitatively into account is critical for investment decision making in the industry.

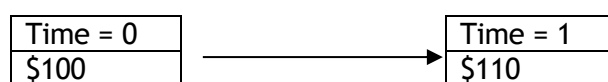
### Future value

The basis for the calculation of present value is the existence of interest - interest which can be earned by putting money into the bank or into some other investment. This is best explained by means of an example.

Suppose that \$100 is placed in a bank account which earns interest at 10% per year. The value of the \$100 after one year will be \$110 and, if required, this \$110 can be withdrawn from the bank. The \$110 is calculated as follows -

Future value of \$100 after 1 year =  $\$100 \times (1 + 10\%) = \$110$

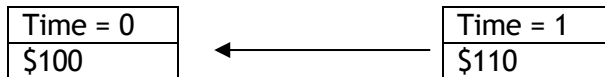
This calculation shows that, after one year, the original \$100 is still there, but, by leaving it in the bank, interest of 10% of \$100 (equals \$10) has been earned. Adding the interest of \$10 to the original amount gives \$110. In this case \$100 is compounded forward at an interest rate of 10%. Diagrammatically, the process of compounding interest can be represented as follows -



Another way of looking at this is that \$100 today is equivalent to \$110 in one year's time. In other words the future value of \$100 is \$110. Therefore future values are obtained by compounding at a given interest rate.

## Present value

The converse of this is that \$110 in one year's time is worth \$100 today. That is, \$110 discounted to the present day is \$100. In other words, the present value of \$110 in one year's time is \$100. Therefore, present values are obtained by discounting at a given discount rate. Present values measure what would have had to be placed in the bank today to get a given future value. Diagrammatically -



Mathematically, the present value of \$110 in one year's time using a discount rate of 10% is derived as follows :-

$$\text{Present value of \$110} = \$100 = \frac{\$110}{(1 + 10\%)}$$

If we consider the equivalence between \$100 today and its value in two years time at an interest rate of 10%, then we would need to compound \$100 over two years as shown in Table1 -

**Table 1 - Future value of \$100 after 2 years**

Value today (time = 0)	= \$100
Value in 1 year (time = 1)	= \$100 * (1 + 10%) = \$110
Value in 2 years (time = 2)	= \$110 * (1 + 10%) = \$100 * (1 + 10%) * (1 + 10%) = \$100 * (1 + 10%) <sup>2</sup> = \$121

Therefore, the future value in two years of \$100 held today is \$121. Conversely, the present value today of \$121 held in two year's time is \$100. Therefore, the effect of time is to reduce the present value. The further away the money is in time, the lower the present value.

In the general case and in algebraic form, we can express the equivalence of present and future values for the nth year in the future as shown in Table 2.

**Table 2 - Equations for present and future value**

$$FV = PV * (1 + r)^n$$

$$PV = \frac{FV}{(1 + r)^n}$$

where

FV = Future value at the end of year n  
 PV = Present value  
 R = Interest rate

## Discount factors

The expression  $(1+r)^n$ , which is shown in Table 2, is called the compound factor for year n of the project. The expression  $1 / (1+r)^n$  is called the discount factor for year n of the project. As an example, if we assume a discount rate of 10% per year, then the discount factors for net cash flow at the end of different years would be as shown in Table 3.

**Table 3 - Discount factors using a discount rate of 10%**

$$\begin{aligned} 1 / (1 + 10\%)^1 &= 0.909 \\ 1 / (1 + 10\%)^2 &= 0.826 \\ 1 / (1 + 10\%)^{10} &= 0.386 \end{aligned}$$

The discount factors clearly become smaller the further into the future they apply. Therefore, cash flows occurring late in a project's life will be discounted heavily, while cash flows occurring early in a project's life will be discounted lightly. Note in the above table that a cash flow occurring at the end of year 1 will be discounted by 0.9091, while a cash flow occurring at the end of year 10 will be discounted by 0.3855. Thus, by its very nature, the NPV calculation takes time into account, putting more weight on early cash flows and less weight on late cash flows.

Table 4 presents discount factors for any given combination of discount rate and period over which a sum of money is discounted. The top row of the table shows discount rates from 0% to 30%. The left hand column shows the number of years in the future that a sum of money is spent or received. For instance, reading down the column labelled "10%" until we reach the 10th year, we see a value of 0.3855. This is the discount rate for a sum of money spent or received in 10 years time when the discount rate is 10%. It is calculated as shown in Table 3.

**Table 4 -Table of discount factors**

Yr	0%	5.0%	10.0%	12.5%	15.0%	17.5%	20.0%	22.5%	25.0%	27.5%	30.0%
1	1	0.952	0.909	0.889	0.870	0.851	0.833	0.816	0.800	0.784	0.769
2	1	0.907	0.826	0.790	0.756	0.724	0.694	0.666	0.640	0.615	0.592
3	1	0.864	0.751	0.702	0.658	0.616	0.579	0.544	0.512	0.483	0.455
4	1	0.823	0.683	0.624	0.572	0.525	0.482	0.444	0.410	0.378	0.350
5	1	0.784	0.621	0.555	0.497	0.447	0.402	0.363	0.328	0.297	0.269
6	1	0.746	0.565	0.493	0.432	0.380	0.335	0.296	0.262	0.233	0.207
7	1	0.711	0.513	0.439	0.376	0.323	0.279	0.242	0.210	0.183	0.159
8	1	0.677	0.467	0.390	0.327	0.275	0.233	0.197	0.168	0.143	0.123
9	1	0.645	0.424	0.346	0.284	0.234	0.194	0.161	0.134	0.112	0.094
10	1	0.614	0.386	0.308	0.247	0.199	0.162	0.131	0.107	0.088	0.073
11	1	0.585	0.351	0.274	0.215	0.170	0.135	0.107	0.086	0.069	0.056
12	1	0.557	0.319	0.243	0.187	0.144	0.112	0.088	0.069	0.054	0.043
13	1	0.530	0.290	0.216	0.163	0.123	0.094	0.072	0.055	0.043	0.033
14	1	0.505	0.263	0.192	0.141	0.105	0.078	0.058	0.044	0.033	0.025
15	1	0.481	0.239	0.171	0.123	0.089	0.065	0.048	0.035	0.026	0.020
16	1	0.458	0.218	0.152	0.107	0.076	0.054	0.039	0.010	0.021	0.015
17	1	0.436	0.198	0.135	0.093	0.065	0.045	0.032	0.023	0.016	0.012
18	1	0.416	0.180	0.120	0.081	0.055	0.038	0.026	0.018	0.013	0.009
19	1	0.396	0.164	0.107	0.070	0.047	0.031	0.021	0.014	0.010	0.007
20	1	0.377	0.149	0.095	0.061	0.040	0.026	0.017	0.012	0.008	0.005
21	1	0.359	0.135	0.084	0.053	0.034	0.022	0.014	0.009	0.006	0.004
22	1	0.342	0.123	0.075	0.046	0.029	0.018	0.012	0.007	0.005	0.003
23	1	0.326	0.112	0.067	0.040	0.025	0.015	0.009	0.006	0.004	0.002
24	1	0.310	0.102	0.059	0.035	0.021	0.013	0.008	0.005	0.003	0.002
25	1	0.295	0.092	0.053	0.030	0.018	0.011	0.006	0.004	0.002	0.001
26	1	0.281	0.084	0.047	0.026	0.015	0.009	0.005	0.003	0.002	0.001
27	1	0.268	0.076	0.042	0.023	0.013	0.007	0.004	0.002	0.001	0.001
28	1	0.255	0.069	0.037	0.020	0.011	0.006	0.003	0.002	0.001	0.001
29	1	0.243	0.063	0.033	0.017	0.009	0.005	0.003	0.002	0.001	0.001
30	1	0.231	0.057	0.029	0.015	0.008	0.004	0.002	0.001	0.001	0.000

### NPV when discount rate is zero

A special case occurs when the discount rate is zero. In this case, the discount factors for all years are 1, because  $1 / (1+0)^n$  is always 1 no matter what the value of n is. Therefore, when the discount rate is zero, the cash flows are effectively undiscounted and the net present value of future net cash flow is the same as the sum of all future net cash flows.



## NPV when time is zero

Another special case occurs when  $n = 0$  (that is, today, when time is zero). In this case, the present value is given as shown in Table 5.

**Table 5 - The present value of money today**

	$PV = \frac{FV_0}{(1 + r)^0} = \frac{FV_0}{1}$
Therefore	$PV = FV_0$

In other words, the present value of a sum of money spent or received today is the same as the sum of money. That is, the sum of money remains undiscounted because it is spent or received now.

## NPV balances money in time

Discounting allows us to discriminate between money amounts at different points in time. The use of present values can therefore help us make a balance between money now and money in the future. This is precisely what is required when making investment decisions.

For instance, how can we assess whether it is worth an investment (that is, an expenditure) of \$100 now in order to receive \$120 (that is, a revenue) in one year's time? In other words, is \$120 revenue in one year's time sufficient to balance and even outweigh \$100 spent now?

Using present value calculations and assuming a discount rate of 10%, we can assess the balance as follows:-

a) Value of \$100 spent now = -\$100

b) Value today of \$120 received in one year's time =  $+\$120 / (1+10\%) = +\$109$

Therefore, \$120 received in one year is worth \$109 today, which is \$9 more than the expenditure of \$100. The implication is that the investment is worthwhile, because in today's terms (that is, in terms of present value), the amount received (\$109) is more than the amount spent (\$100).

## Present value of a stream of cash flows

Table 2 above gives the equation for the present value of an amount  $FV_n$  in a single year (the  $n$ th year). If we want to calculate the net present value of a series, or stream of cash flows from year 1 through to year  $n$ , then we simply add together the present values for individual years is given in Table 6.

**Table 6 - Present value of stream of net cash flows**

$$\text{Total NPV} = \frac{\text{NCF}_1}{(1+r)^1} + \frac{\text{NCF}_2}{(1+r)^2} + \frac{\text{NCF}_3}{(1+r)^3} + \dots + \frac{\text{NCF}_n}{(1+r)^n}$$

where

NPV = Net present value  
 NCF<sub>1</sub> = Net cash flow in year 1  
 NCF<sub>2</sub> = Net cash flow in year 2  
 NCF<sub>3</sub> = Net cash flow in year 3  
 NCF<sub>n</sub> = Net cash flow in year n  
 r = Discount rate

An example of the derivation of the net present value of a stream of net cash flows is shown in Figure 1. In this figure, the net cash flow in each year in the future is "brought back" to today's value by discounting. Once all the separate net cash flows are expressed in terms of today's values in this way, they can then be added together to give a single value for the net present value of the whole cash flow stream.

**Figure 1 - Finding net present value**

	Time 0	1	2	3	4	5	6	7	8
Net cash flow \$MM	-200	+100	+90	+80	+70	+60	+50	+40	-100
Discount rate %		10%	10%	10%	10%	10%	10%	10%	10%
Discount factor		0.909	0.826	0.751	0.683	0.621	0.565	0.513	0.467
Present values \$MM	-200.0								
	+90.9								
	+74.4								
	+60.1								
	+47.8								
	+37.3								
	+28.2								
	+20.5								
	-47.0								
Project NPV \$MM =	+112								

**Finding net present value at 10%**

**Use the discount factor table to find the discount factor**

**Then calculate the net present value**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Discount rate	\$MM		10%	10%	10%	10%	10%	10%	10%
Discount factor	\$MM								
Present values	\$MM								
	\$MM		<-----						
	\$MM		<-----						
	\$MM		<-----						
	\$MM		<-----						
	\$MM		<-----						
	\$MM		<-----						
	\$MM		<-----						
Net present value	\$MM								

**Finding net present value at 10% - answer****Use the discount factor table to find the discount factor****Then calculate the net present value**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Discount rate	\$MM		10%	10%	10%	10%	10%	10%	10%
Discount factor	\$MM	1.000	0.909	0.826	0.751	0.683	0.621	0.564	0.513
Present values	\$MM	-200.0							
	\$MM	-45.5	<-----						
	\$MM	82.6	<-----	-----					
	\$MM	67.6	<-----	-----	-----				
	\$MM	47.8	<-----	-----	-----	-----			
	\$MM	37.3	<-----	-----	-----	-----	-----		
	\$MM	28.2	<-----	-----	-----	-----	-----	-----	
	\$MM	20.5	<-----	-----	-----	-----	-----	-----	-----
Net present value	\$MM	38.6							

**Finding net future value at 10%**

Find the compound factor by compounding forward at the compound rate.

Use the compound factor to derive the future value at the end of year 7.

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Compound rate	\$MM	10%	10%	10%	10%	10%	10%	10%	
Compound factor	\$MM								
Future values	\$MM								
	\$MM							->	
	\$MM							----->	
	\$MM						----->		
	\$MM					----->			
	\$MM				----->				
	\$MM			----->					
	\$MM		----->						
Net future value#	\$MM								

**Finding net future value at 10% - answer**

**Find the compound factor by compounding forward at the compound rate.**

**Use the compound factor to derive the future value at the end of year 7.**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Compound rate	\$MM	10%	10%	10%	10%	10%	10%	10%	
Compound factor	\$MM	1.949	1.772	1.611	1.464	1.331	1.210	1.100	1.000
Future values	\$MM								40.0
	\$MM							->	55.0
	\$MM							----->	72.6
	\$MM						----->		93.2
	\$MM					----->			131.8
	\$MM				----->				161.1
	\$MM			----->					-88.6
	\$MM		----->						-389.7
Net future value#	\$MM								75.3

**Finding net present value at 15% then 10%**

**Calculate the discount factor (be careful about year 3 onwards).**

**Then derive the net present value.**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Discount rate	\$MM		15%	15%	10%	10%	10%	10%	10%
Discount factor	\$MM								
Present values	\$MM								
	\$MM		<-----						
	\$MM		<-----	-----					
	\$MM		<-----	-----	-----				
	\$MM		<-----	-----	-----	-----			
	\$MM		<-----	-----	-----	-----	-----		
	\$MM		<-----	-----	-----	-----	-----	-----	
	\$MM		<-----	-----	-----	-----	-----	-----	-----
Net present value	\$MM								

**Finding net present value at 15% then 10% - answer**

**Calculate the discount factor.**

**Then derive the net present value.**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-200	-50	100	90	70	60	50	40
Discount rate	\$MM		15%	15%	10%	10%	10%	10%	10%
Discount factor	\$MM	1.000	0.870	0.756	0.687	0.625	0.568	0.516	0.470
Present values	\$MM	-200.0							
	\$MM	-43.5	<-----						
	\$MM	75.6	<-----	-----					
	\$MM	61.9	<-----	-----	-----				
	\$MM	43.7	<-----	-----	-----	-----			
	\$MM	34.1	<-----	-----	-----	-----	-----		
	\$MM	25.8	<-----	-----	-----	-----	-----	-----	
	\$MM	18.8	<-----	-----	-----	-----	-----	-----	-----
Net present value	\$MM	16.4							



### 3 Features of NPV

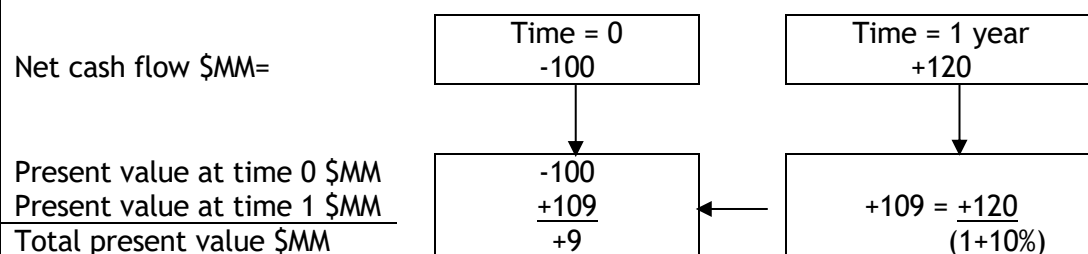
In this section we highlight certain features of the use of net present value (NPV) as it affects investment in oil and gas projects.

#### What NPV means

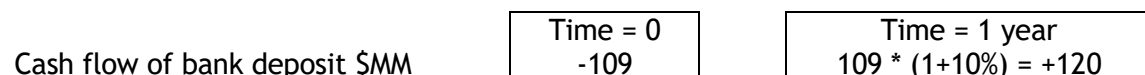
What does the net present value of a projection of net cash flow of an investment tell us? The answer is that it is a measure of how much more we gain by putting our money into the project by comparison with putting it into the bank or some alternative investment. Another way of expressing it is that net present value measures how much more we would have to put in the bank today (or an alternative investment) to give the same net cash flow as the project.

Suppose that we have an investment opportunity which involves investing \$100 today with the promise of \$120 in exactly 1 year's time. Assume that the project is risk-free so that we can legitimately compare it with putting the money in the bank (which we assume is also risk-free). Assume that the bank rate is 10%. The cash flow of the investment and the present value of our cash flow using a discount rate of 10% are shown in Figure 1.

**Figure 1 - Present value of a simple investment**



This tells us that, at a discount rate of 10%, the investment is \$9 better than putting the money in the bank. It means that, we would have to put \$9 more in the bank (that is, \$109 in total) to get the same return as the investment. We can demonstrate this by calculating the cash flow associated with putting \$109 in the bank at an interest rate of 10%. It is as follows -



In other words the output of both the investment and the bank are the same. However, the inputs are different. The bank requires an input of \$109. The investment requires an input of \$100 which is \$9 less. This point is summarised in Figure 2.

**Figure 2 - Input / output for simple project**

	Input (time 0)	Output (time 1)
Investment \$MM	-100	+120
Bank alternative \$MM	-109	+120
NPV = extra in bank \$MM	+ 9	0

Therefore, the value of the investment in today's terms is \$9, because this is how much more we would need to put in the bank now to give the same cash flow as the investment. Clearly, if the present value of the investment had turned out to be negative, then the investment would have been worse than putting the money into the bank

The NPV has the same meaning no matter how many time periods we are considering. Figure 3 shows a similar calculation for a project with cash flows which are projected for a period of 5 years. It demonstrates that the NPV of the cash flow stream is precisely the extra amount of money that would have to be placed in the bank to enable the bank to give exactly the same annual cash flow as the project being considered.

**Figure 3 - The meaning of NPV**

<b>A NPV calculation</b>						
Time	0	1	2	3	4	5
Net cash flow \$MM	-100.0	+50.0	+40.0	+30.0	+20.0	+10.0
Discount rate %		10%	10%	10%	10%	10%
Discount factor		0.909	0.826	0.751	0.683	0.621
Present values \$MM	-100.0					
	+45.5					
	+33.3					
	+22.5					
	+13.7					
	+6.2					
Project NPV \$MM =	+20.9					
<b>B Calculation of bank balances (\$MM)</b>						
Time	0	1	2	3	4	5
Original investment	100.0					
Extra investment required	20.9					
Total investment in bank	120.9					
Interest at 10%		12.1	8.3	5.1	2.6	0.9
Bank balance at start of year	120.9	133.0	91.3	56.4	29.1	10.0
Withdrawal (= net cash flow)		50.0	40.0	30.0	20.0	10.0
Bank balance at end of year		83.0	51.3	26.4	9.1	0

In Figure 3 we have invested \$120.9 in the bank at 10% and withdrawn at the end of each year the same money that we would have earned from the project. At the end of year 5, we have a bank balance of zero. Therefore, to get the same each year from the bank as we do

from the project, we have to invest \$120.9, which is \$20.9 more than is required when we invest in the project. This point is summarised in Figure 4.

**Figure 4 - Input / output for 5 year project**

	Input (time 0)	Output (time 1)
Investment \$MM	-100.0	+50,40,30,20,10
Bank alternative \$MM	-120.9	+50,40,30,20,10
NPV = extra in bank \$MM	+20.9	0

## The alternative investment

In the previous section, we established that the NPV of a project told us how much better (or worse) the project is than investing our money in the bank.

We don't necessarily need to make reference to the bank as an alternative to the project we are evaluating. We can make reference to other kinds of investment. We might make reference, for instance, to investing in oil industry shares on the stock market. In this case, there would be a risk attached to the investment and the discount rate would be correspondingly greater than the bank rate. Alternatively, we might compare the return from the project with the return from the company's own shares, and would therefore discount at that company's "cost of capital". These topics are dealt with in another section. However, the main point is that, when we make an NPV calculation, we are implicitly making a comparison with alternative investments and calculating how much better or worse the project is than those alternatives.

## The discount rate is not the loan rate

It follows from the discussion above that our discount rate is not equal to the interest rate we pay on loans from the bank - that is, on funds we might borrow. It is rather the interest rate we would receive on our money invested in an alternative project - for instance, in a bank deposit. These interest rates would, in general, be different. We might expect the interest we pay on borrowed funds to be higher than the interest we would earn by investing our money in a deposit account in the bank.

## The NPV is a single indicator

The NPV combines into one number all the physical and financial attributes of a project - the oil and gas in place, the production profile, the capital and operating costs, the fiscal terms, the timing of net cash flow and the discount rate. Therefore the NPV fulfils our first requirement of an economic indicator, which is to represent the project by a single number.

## It is not valid to add cash flows

The NPV takes time into account. It weights the net cash flow in each year depending on when the cash flow occurs. Therefore it is not valid to add up the individual net cash flows of a project unless we first discount them to bring them back to the present day. This is

because the individual net cash flows occur at different points in time and therefore do not have the same quality. We can only add cash flows after we have discounted them and translated them into equivalent units.

### **The NPV is value in excess of the capital costs**

The NPV is the value of the asset over and above the capital costs of the project. For instance, if the capital costs of a project are \$250 million and the NPV is \$100 million, then we know that the project returns \$350 million in total. Of this, \$250 million covers the capital costs and the remaining \$100 million (the NPV) is the excess return or value to the owner of the project.

### **The NPV aids decision making**

The NPV is useful in making a decision whether or not to go ahead with a project. A positive NPV tells us that we should go ahead with the project. We would be better off investing in the project than investing in the bank. A negative NPV tells us that we should not go ahead with the project. We would be better off investing in the bank than investing in the project.

Of course, this ignores other considerations that need to be taken into account in the decision making process. These other considerations include project risk and uncertainty, as well as political and strategic factors. Such considerations are discussed in other sections. However, the NPV is an important part of the go/no-go decision.

### **The NPV is a measure of value**

The NPV is useful as a means to determine how much a project is worth to a buyer or seller of the project, or to company shareholders who need to know how much the project contributes to the value of their company. It represents an equivalent cash value of the project as at today. Ignoring other considerations, we are indifferent between having (a) the project and (b) cash equal to the NPV of the project.

### **NPV and asset acquisitions**

Because the NPV is a measure of value, it helps to determine the price the seller would seek in a sale of a petroleum asset. The NPV, or selling price, is based on the seller's view of the asset's attributes (its reserves, production potential, capital and operating costs and so on) and its economic circumstances (future oil prices, escalation rates and so on).

Similarly, the NPV helps to determine the price the buyer would be prepared to pay. However, if the potential buyer of the asset had exactly the same view of the attributes of the asset and its economic circumstances, then there would be no basis for a sale. This is because the buyer would pay \$X million (equal to the NPV) and, in return, would receive an asset which was worth \$X million. The two values would cancel and leave no incentive for the buyer to purchase the asset.

The only basis on which a sale could take place is if the buyer calculated the NPV to be much higher than the NPV calculated by the seller. This is because the buyer might believe the reserves to be higher, the costs to be lower, or any number of attributes to be different.

Because he has a higher NPV than the seller, the buyer would gain value from the transaction and the seller would translate its perceived value into cash.

We know from evidence of past acquisitions that buyers tend to pay only a fraction of their estimation of the NPV of a single asset. For instance, for the purchase of a single oil field, the buyer might be prepared to pay only 75% of its estimate of the NPV based on the field's proven plus probable reserves. However, if there is a portfolio of assets in the sale, some with significant upside potential, it might be prepared to assign a higher price to an individual asset in the portfolio. Much depends on the circumstances of the particular acquisition under consideration.

## Summary of NPV

We are now in a position to summarise what the net present value indicator does for us and what it tells us about the economic attractiveness of an investment:-

- (a) First, the net present value indicator is a single measure of the economic benefit or the value of an investment/project. It is derived from a projection of the future cash flow from that investment/project.
- (b) Second, net present value takes time into account. It does this through the discounting process, by placing more weight on cash flow in the near term than it does on cash flow in the longer term. It therefore helps us to balance cash flows which occur later in time with cash flows which occur nearer in time.
- (c) Third, net present value tells us by how much an investment is better or worse than putting our money into the bank or some alternative investment. It therefore tells us whether the investment should be undertaken or not and also the value of the investment.



**Proof that NPV is the extra to be placed in the bank**

**Prove that you must put in the bank the investment in the project (5.00) plus the NPV (4.12) to allow the bank to give back the same cash flow as the project assuming an interest rate of 20%.**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-5.00	2.00	2.20	2.42	2.66	2.93	3.22	3.54
Discount rate	\$MM		20%	20%	20%	20%	20%	20%	20%
Discount factor	\$MM	1.000	0.833	0.694	0.579	0.482	0.402	0.335	0.279
Present values	\$MM	-5.00							
	\$MM	1.67	<-----						
	\$MM	1.53	<-----	-----					
	\$MM	1.40	<-----	-----	-----				
	\$MM	1.28	<-----	-----	-----	-----			
	\$MM	1.18	<-----	-----	-----	-----	-----		
	\$MM	1.08	<-----	-----	-----	-----	-----	-----	
	\$MM	0.99	<-----	-----	-----	-----	-----	-----	-----
Net present value	\$MM	4.12							

		Units	0	1	2	3	4	5	6	7
Put in bank	\$MM	5.00								
Extra in bank	\$MM	4.12								
Total in bank	\$MM	9.12								
Bank balances with interest at 20%										
Balance brought forward			9.12							
Interest to end of year	\$MM									
Revised balance	\$MM	9.12								
Take out of bank	\$MM	0.00								
Balance carried forward	\$MM	9.12								

**Proof that NPV is the extra to be placed in the bank - answer**

**Prove that you must put in the bank the investment in the project (5.00) plus the NPV (4.12) to allow the bank to give back the same cash flow as the project assuming an interest rate of 20%.**

		Time in years							
	Units	0	1	2	3	4	5	6	7
Net cash flow	\$MM	-5.00	2.00	2.20	2.42	2.66	2.93	3.22	3.54
Discount rate	\$MM		20%	20%	20%	20%	20%	20%	20%
Discount factor	\$MM	1.000	0.833	0.694	0.579	0.482	0.402	0.335	0.279
Present values	\$MM	-5.00							
	\$MM	1.67	<-----						
	\$MM	1.53	<-----	-----					
	\$MM	1.40	<-----	-----	-----				
	\$MM	1.28	<-----	-----	-----	-----			
	\$MM	1.18	<-----	-----	-----	-----	-----		
	\$MM	1.08	<-----	-----	-----	-----	-----	-----	
	\$MM	0.99	<-----	-----	-----	-----	-----	-----	-----
Net present value	\$MM	4.12							

		Units	0	1	2	3	4	5	6	7
Put in bank	\$MM	5.00								
Extra in bank	\$MM	4.12								
Total in bank	\$MM	9.12								
Bank balances with interest at 20%										
Balance brought forward			9.12	8.95	8.53	7.82	6.73	5.14	2.95	
Interest to end of year	\$MM		1.82	1.79	1.71	1.56	1.35	1.03	0.59	
Revised balance	\$MM	9.12	10.95	10.73	10.24	9.39	8.07	6.17	3.54	
Take out of bank	\$MM	0.00	2.00	2.20	2.42	2.66	2.93	3.22	3.54	
Balance carried forward	\$MM	9.12	8.95	8.53	7.82	6.73	5.14	2.95	0.00	

**Therefore 4.12 (= the NPV) is the extra you must put in the bank at time =zero to allow the bank to match the cash flow of the project.**



## 4 Nominal and real NPVs

In this section we discuss the meaning of "nominal" and "real" NPVs and point out some of the problems in their use in evaluating oil and gas projects.

### Discounting and deflation

As a first step in the discussion of nominal and real NPVs, it is important to recognise that compounding has nothing to do with inflation and that discounting has nothing to do with deflation.

Inflation is the way in which the prices of goods and services increase over time as a result of macro-economic influences. By contrast, compounding is the way in which money increases if it is earning interest in the bank, or if it is earning a return by being invested somewhere else. Depending on the circumstances, the interest or return earned may or may not be sufficient to cover inflation.

Deflation is the process of expressing future inflated amounts in terms of their purchasing power, or in their real terms. That is, deflating cash flows expresses them in terms of what goods and services and how many goods and services they will buy.

By contrast, discounting is the way in which future amounts are expressed in today's terms. The process of discounting a future amount of money gives a measure how much more, or less, we would have had to place in a bank today to get that future sum. The present value of the future amount may or may not buy the same goods and services as the future amount. Table 1 summarises the point:-

**Table 1 -Inflation - Compounding, Deflation - Discounting**

Inflation	=	Prices increasing
Compounding	=	Earning interest
Deflation	=	Calculating purchasing power
Discounting	=	Comparing with alternatives

Although the mathematics of the processes of (a) inflating and compounding or (b) deflating and discounting are similar, in fact they are entirely different concepts.

### Defining nominal & real NPVs

Some oil and gas companies use nominal NPVs to evaluate their investments. Some use real NPVs and some use both. Nominal NPVs are nominal net cash flows discounted using nominal discount rates. Real NPVs are real net cash flows discounted using real discount rates. It is not valid to use the same discount rate for both calculations. We discuss in an earlier section the difference between a nominal and a real net cash flow. It remains to define the difference between a nominal discount rate and a real discount rate.

A nominal discount rate is the actual interest we would earn by investing our money in an alternative project (for instance, the bank).

A real discount rate is the purchasing power of the interest we would earn by investing our money in an alternative investment. For instance, suppose we can earn 10% per year on a bank deposit, and inflation was 3% per year. The 10% is our nominal discount rate. However, after one year the 10% we earn on our money would only buy about 7% more goods and services. In other words, if we invested \$100, it would be worth \$110 after one year. However, it would buy only about \$107 of goods and services after one year. This is only an approximation. The accurate calculation yields 6.8% as shown in Table 2 below.

**Table 2 - Nominal and real interest or discount rates**

	Value at time = 0	Value at time = 1
(a) Invest \$1 for 1 year at 10% where 10% is the nominal interest rate  Assume that inflation is 3% per year.	\$1	$\$1 \cdot (1+10\%)$
(b) Real value of investment after 1 year		$\frac{\$1 \cdot (1+10\%)}{(1+3\%)}$
(c) Real interest rate $r(\text{real})$ is given by - $1 + r(\text{real}) =$		$\frac{(1+10\%)}{(1+3\%)}$
(d) Therefore $r(\text{real}) = \frac{(1+10\%)}{(1+3\%)} - 1 = 6.8\%$  Note that 6.8% is approximately 7% which is 10% less 3%		

In other words, in order to work out the real interest/discount rate (based on the interest earned after the effects of inflation), we must divide by an expression which includes the rate of inflation. However, it is often sufficiently accurate to calculate the real interest/discount rate approximately, by deducting the inflation rate from the nominal interest/discount rate.

## Calculating nominal and real NPVs

We are now in a position to calculate and compare nominal and real net present values. The calculations are shown in Table 3 for an investment with a net cash flow over two periods. We assume a nominal discount rate of 10% and an inflation or deflation rate of 3%.

**Table 3 - Nominal and real NPVs**

	Time = 0	Time = 1
<b>Nominal NPV</b>		
(a) Nominal net cash flow =	$NCF_0$	$NCF_1$
(b) Nominal NPV =	$NCF_0$	$\frac{NCF_1}{(1+10\%)}$
<b>Real NPV</b>		
(c) Real net cash flow =	$NCF_0$	$\frac{NCF_1}{(1+3\%)}$
(d) Real NPV with real discount rate 6.8%=	$NCF_0$	$\frac{NCF_1}{(1+3\%) * (1+6.8\%)}$
(e) However, $(1+6.8\%) = \frac{(1+10\%)}{(1+3\%)}$		
(f) Therefore, the real NPV =	$NCF_0$	$\frac{NCF_1 * (1+3\%)}{(1+3\%) * (1+10\%)}$
We can cancel out the expressions "(1+3%)" in the numerator and the denominator		
(g) Therefore, the real NPV =		$\frac{NCF_1}{(1+10\%)}$
<b>Therefore, the real NPV is the same as the nominal NPV</b>		

This analysis shows that, if the deflator used to obtain the real net cash flow is the same as the deflator used to obtain the real discount rate, then the real NPV is identical to the nominal NPV. Under these circumstances, once we have a nominal NPV, the real NPV is redundant because it is precisely the same number.

### Presenting nominal and real NPVs

Some oil and gas companies present both nominal and real NPVs in tabulations similar to that shown in Table 4. The table shows the nominal and real NPV at different discount rates.

**Table 4 - Presenting nominal and real NPVs - version 1**

<b>Discount rate %</b>	<b>Nominal NPV \$MM</b>	<b>Real NPV \$MM</b>
5%	128	93
6%	116	82
7%	105	72
8%	94	63
9%	84	54
<u>10%</u>	<u>74</u>	<u>45</u>
11%	65	37
etc	etc	etc

The presentation of NPVs as shown in Table 4 can be very misleading. By its very layout, we are invited to read the NPV at a single discount rate. For instance, we might be tempted to read NPVs from the table at a discount rate of 10% (underlined in the table). This would give us \$74MM for the nominal NPV and \$45MM for the real NPV. However, with any non-zero rate of inflation, we cannot choose the same discount rate for the nominal and the real NPV. The nominal and the real discount rates must be different. However, the table does not tell us what rate of inflation is assumed. Therefore, for instance, if we take the first column to represent nominal discount rates, we are not able to determine the real discount rate and the real NPV.

In fact, the rate of inflation assumed in deriving NPVs for the table above is 3% per year. If we pick 10% for our nominal discount rate, then our real discount rate would be approximately 7% (10% less 3%). Therefore, a better real NPV reading from the table would be \$72MM, which is shown alongside the 7% discount rate in the "Real NPV" column. The real NPV is not \$45MM as mentioned above.

As we would expect, the real NPV of \$72MM is very close to the nominal NPV of \$74MM. However, using a more accurate real discount rate of 6.8% instead of 7%, the real NPV is \$74MM, which is precisely the same as the nominal NPV.

A better presentation of NPVs would be as set out in Table 5. In this table, we specify both nominal and real discount rates as well as nominal and real NPVs. The real discount rates are a function of the nominal discount rate and the rate of inflation. The assumed rate of inflation is made explicit at the top of the table. In this example, we assume a rate of 3% per year. With a nominal discount rate of 10%, the nominal NPV is \$74MM and with the corresponding real discount rate of 6.8%, the real NPV is the same numerically. However, the units are different. Strictly speaking, the NPV should be G&S\$74MM. The units are not \$, they are G&S (Goods and Services).

However, because the nominal and real NPVs are identical, in fact the columns showing real discount rates and real NPVs are redundant. All we need is a footnote explaining that the real and nominal NPVs are the same.

**Table 5 - Presenting nominal and real NPVs - version 2**  
Assumes that the rate of inflation is 3% per year.

Nominal discount rate %	Nominal NPV \$MM	Real discount rate %	Real NPV G&SMM
5.0%	128	1.9%	128
6.0%	116	2.9%	116
7.0%	105	3.9%	105
8.0%	94	4.9%	94
9.0%	84	5.8%	84
10.0%	74	6.8%	74
11.0%	65	7.8%	65
etc	etc	etc	etc

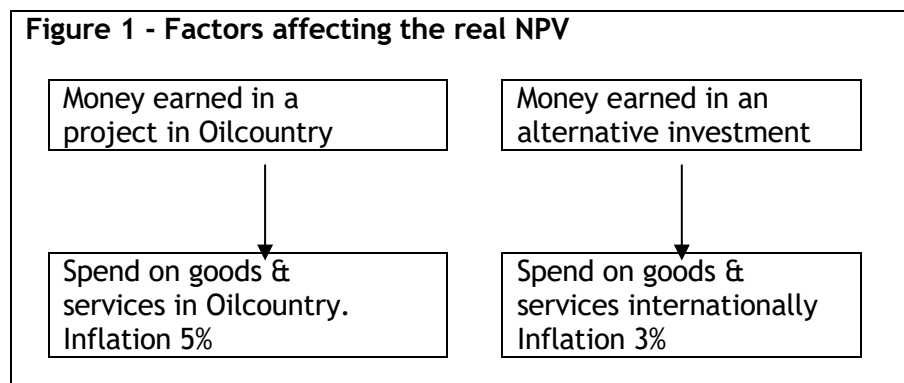
## Nominal and real NPVs can be different

In the discussion of real NPVs so far, we have assumed that the deflator used to obtain the real net cash flow is the same as the deflator used to obtain the real discount rate. Under these circumstances, the nominal and real NPVs are identical. This is usually the case.

However, if the deflators are different, then the nominal and real NPVs will not be the same. In practice, this is unlikely to be the case, but might happen in rare circumstances. The circumstances are discussed in the following.

Consider, for instance, an oil company which is evaluating a potential project in a country called "Oilcountry". The situation is illustrated in Figure 1.

**Figure 1 - Factors affecting the real NPV**



Suppose that a Government condition of undertaking the project is that any money earned from the project in Oilcountry must be spent on goods and services in that country. If the rate of inflation in the country is expected to be 5% per year, then, when calculating real

cash flows from the project, the deflator will be 5% per year. In other words, the future purchasing power of the money earned from the project is calculated using a deflator of 5%. Suppose also that the company's alternative investments are oil and gas projects worldwide and that any money the company earns on these alternative projects would be spent worldwide. Assume that the rate of inflation affecting these alternative projects is 3% per year. Therefore, the company's real discount rate would be calculated using a deflator of 3% per year and not 5% (the deflator used to obtain the project's real cash flows).

The real NPV calculation is as shown in Table 6. In this table, we assume a nominal discount rate of 10%. That is the rate earned on an alternative investment

**Table 6 - NPVs when deflator for the discount rate is not the deflator for the cash flows**

	Time = 0	Time = 1
<b>Nominal NPV</b>		
(a) Nominal net cash flow =	$NCF_0$	$NCF_1$
(b) Nominal NPV =	$NCF_0$	$\frac{NCF_1}{(1+10\%)}$
<b>Real NPV</b>		
(c) Real net cash flow =	$NCF_0$	$\frac{NCF_1}{(1+5\%)}$
(d) Real NPV with real discount rate 6.8%=	$NCF_0$	$\frac{NCF_1}{(1+5\%) * (1+6.8\%)}$
(e) However, $(1+6.8\%) = \frac{(1+10\%)}{(1+3\%)}$		
(f) Therefore, the real NPV =	$NCF_0$	$\frac{NCF_1 * (1+3\%)}{(1+5\%) * (1+10\%)}$
<b>Therefore, the real NPV is not the same as the nominal NPV</b>		

In these circumstances, the deflator for the cash flows is not the same as the deflator for the discount rates and the real NPV is not the same as the nominal NPV.

However, it is rare for there to be such restrictions on where money from an oil project is spent. In practice, it is normal for oil and gas companies to spend the money from a project or from the alternative investment on the same goods and services purchased internationally. Therefore, in most circumstances, the deflator for the discount rate is the same as the deflator for the net cash flows from the project, and then the real NPV is the same as the nominal NPV.

**Abandonment costs - question****1 Escalation / deflation**

Work out the nominal abandonment costs of a field in year 30.

The real abandonment costs at time zero (now) are \$20 million.

This is equivalent to 20 million BOS.

Assume that escalation is 3% per year.

How many million BOS can you buy in year 30 with this money?

In other words, what is its purchasing power in year 30?

	Units	Time = 0		Time = 30	
Real/nominal abandonment costs	\$MM	20.0	= Real ab costs		Nominal = Actual abandonment costs (estimate)
Deflation factor with 3% escalation	%				Real = Purchasing power of actual abandonment costs
Purchasing power	G&SMM				

**2 Discounting**

How much money would you need to put into the bank now to give you a bank balance sufficient to pay the abandonment costs in year 30?

	Units	Time = 0		Time = 30	
Nominal abandonment costs in year 30.	\$MM				Nominal abandonment costs
Discount factor with a 10% discount rate	%				
Bank deposit	\$MM		= Present value of nominal abandonment costs		

**Abandonment costs - answer****1 Escalation / deflation**

Work out the nominal abandonment costs of a field in year 30.  
 The real abandonment costs at time zero (now) are \$20 million.  
 This is equivalent to 20 million BOS.  
 Assume that escalation is 3% per year.  
 How many million BOS can you buy in year 30 with this money?  
 In other words, what is its purchasing power in year 30?

	Units	Time = 0		Time = 30	
Real/nominal abandonment costs	\$MM	20.0	= Real ab costs	48.5	Nominal = Actual abandonment costs (estimate)
Deflation factor with 3% escalation	%			0.412	
Purchasing power	G&SMM			20.0	Real = Purchasing power of actual abandonment costs

**2 Discounting**

How much money would you need to put into the bank now to give you a bank balance sufficient to pay the abandonment costs in year 30?

	Units	Time = 0		Time = 30	
Nominal abandonment costs in year 30.	\$MM			48.5	Nominal abandonment costs
Discount factor with a 10% discount rate	%			0.057	
Bank deposit	\$MM	2.8	= Present value of nominal abandonment costs		



## Nominal and Real NPVs - question

1 The following is the nominal net cash flow (NCF) of a project overseas.

Work out the real NCF assuming that the NCF must be spent overseas and that the deflator is 5%.

	Units	0	1	2	3	4	5	6	
Nominal NCF	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	Actual (=nominal) money from project overseas.
Deflation factor with 5%	%								Deflation factor using the deflation rate overseas = 5%.
Real NCF	G&SMM								Purchasing power of NCF overseas.

2 Your bank is in your home country and offers you a nominal interest rate of 15%.

The rate of inflation in your home country is 3%. Assume that you spend the return from the bank

in your home country and work out the real discount rate with a 3% deflator. Real discount rate =

3 Work out the real NPV using the real NCF and the real discount rate.

	Units	0	1	2	3	4	5	6	
Real NCF from above	G&SMM								Purchasing power of NCF overseas.
Real discount factor	Number								Real discount factors using a real discount rate.
Discounted real NCF	G&SMM								Real NCF discounted using real discount factors
Real NPV	G&SMM								Purchasing power of project today

4 Now assume that you can repatriate the money from the project to the home country and work out the real NPV this time with a 3% deflator for both the real NCF and the real discount rate.

	Units	0	1	2	3	4	5	6	
Nominal NCF from above	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	Actual (=nominal) money from project overseas
Deflation factor with 3%	%								Deflation factor using the deflation rate at home = 3%.
Revised real NCF	G&SMM								Purchasing power at home of project NCF
Real discount factor	Number								Real discount factors using a real discount rate.
Discounted real NCF	G&SMM								Real NCF discounted using real discount factors
Real NPV	G&SMM								Purchasing power of project today

5 Work out the nominal NPV with the nominal NCF and the nominal discount rate.

Note that the nominal NPV = the real NPV in 4 above.

	Units	0	1	2	3	4	5	6	
Nominal NCF from above	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	
Nom disc factor with 15%	Number								Nominal discount factors using a nominal discount rate of
Discounted nominal NCF	\$MM								
Nom NPV	\$MM								Money value of project today

#### 4 Nominal and real NPVs

32

##### Nominal and Real NPVs - answer

1 The following is the nominal net cash flow (NCF) of a project overseas.

Work out the real NCF assuming that the NCF must be spent overseas and that the deflator is 5%.

	Units	0	1	2	3	4	5	6	
Nominal NCF	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	Actual (=nominal) money from project overseas.
Deflation factor with 5%	%	1.000	0.952	0.907	0.864	0.823	0.784	0.746	Deflation factor using the deflation rate overseas = 5%.
Real NCF	G&SMM	-50.0	-100.0	120.0	100.0	80.0	60.0	-100.0	Purchasing power of NCF overseas.

2 Your bank is in your home country and offers you a nominal interest rate of 15%.

The rate of inflation in your home country is 3%. Assume that you spend the return from the bank

in your home country and work out the real discount rate with a 3% deflator.

$$\text{Real discount rate} = (1+15\%)/(1+3\%)-1 = 11.65\%$$

$$\text{Answer} = 11.65\%$$

3 Work out the real NPV using the real NCF and the real discount rate.

	Units	0	1	2	3	4	5	6	
Real NCF from above	G&SMM	-50.0	-100.0	120.0	100.0	80.0	60.0	-100.0	Purchasing power of NCF overseas.
Real discount factor	Number	1.000	0.896	0.802	0.718	0.644	0.576	0.516	Real discount factors using a real discount rate of 11.65%.
Discounted real NCF	G&SMM	-50.0	-89.6	96.3	71.9	51.5	34.6	-51.6	Real NCF discounted using real discount factors
Real NPV	G&SMM	63.0							Purchasing power of project today

4 Now assume that you can repatriate the money from the project to the home country and work out the real NPV this time with a 3% deflator for both the real NCF and the real discount rate.

	Units	0	1	2	3	4	5	6	
Nominal NCF from above	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	Actual (=nominal) money from project overseas
Deflation factor with 3%	%	1.000	0.971	0.943	0.915	0.888	0.863	0.837	Deflation factor using the deflation rate at home = 3%.
Revised real NCF	G&SMM	-50.0	-101.9	124.7	106.0	86.4	66.1	-112.2	Purchasing power at home of project NCF
Real discount factor	Number	1.000	0.896	0.802	0.718	0.644	0.576	0.516	Real discount factors using a real discount rate of 11.65%.
Discounted real NCF	G&SMM	-50.0	-91.3	100.0	76.1	55.6	38.1	-57.9	Real NCF discounted using real discount factors
Real NPV	G&SMM	70.6							Purchasing power of project today

5 Work out the nominal NPV with the nominal NCF and the nominal discount rate.

Note that the nominal NPV = the real NPV in 4 above.

	Units	0	1	2	3	4	5	6	
Nominal NCF from above	\$MM	-50.0	-105.0	132.3	115.8	97.2	76.6	-134.0	
Nom disc factor with 15%	Number	1.000	0.870	0.756	0.658	0.572	0.497	0.432	Nominal discount factors using a nominal discount rate of 15
Discounted nominal NCF	\$MM	-50.0	-91.3	100.0	76.1	55.6	38.1	-57.9	
Nom NPV	\$MM	70.6							Money value of project today

## 5 Internal rate of return

### NPVs and simple project cash flows

In an earlier section, we discussed the nature of the net cash flow profile for a typical petroleum project. It pointed out that oil and gas projects typically have large cash outlays over several years at the beginning of a project. That is, initially cash flows are negative and revenues are not received until production starts. After production starts, the net cash flows are positive.

The net cash flow might become negative at the end of project life when the field is abandoned. However, for the moment, let us ignore this and assume that the net cash flow is simple with an investment and then a return on that investment.

Mathematically, the net cash flow profile and the associated net present value calculation of a typical petroleum project might look like that shown in Table 1.

**Table 1 - Present value of stream of net cash flows**

$$\text{Total NPV} = \frac{-\text{NCF}_1}{(1+r)^1} + \frac{\text{NCF}_2}{(1+r)^2} + \frac{\text{NCF}_3}{(1+r)^3} + \dots + \frac{\text{NCF}_n}{(1+r)^n}$$

where

- NPV = Net present value
- NCF<sub>1</sub> = Net cash flow in year 1
- NCF<sub>2</sub> = Net cash flow in year 2
- NCF<sub>3</sub> = Net cash flow in year 3
- NCF<sub>n</sub> = Net cash flow in year n
- r = Discount rate

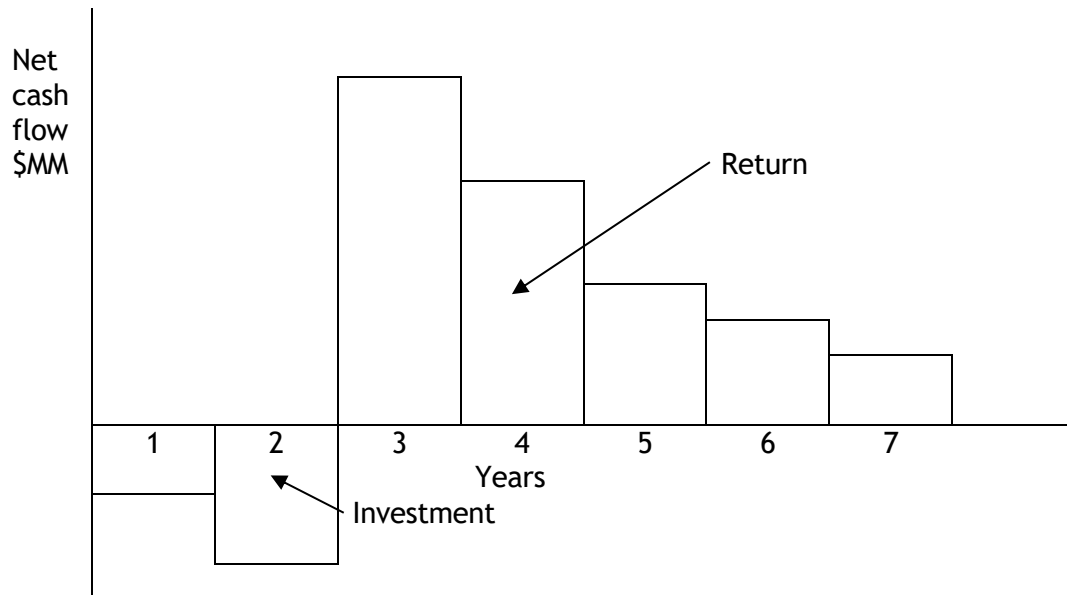
This simple cash flow profile is shown graphically in Figure 1 together with a diagram showing how the NPV of such a cash flow profile changes as the discount rate is varied. The figure shows that the NPV of the net cash flow from a typical oil and gas project decreases as the discount rate increases. The reason for this is as follows -

At a zero discount rate, the NPV of future cash flows is the same as the sum of future cash flows. In this example, the NPV is positive, indicating that the positive net cash flows which occur later in the project are, in total, greater than the negative net cash flows which occur early in the project's life.

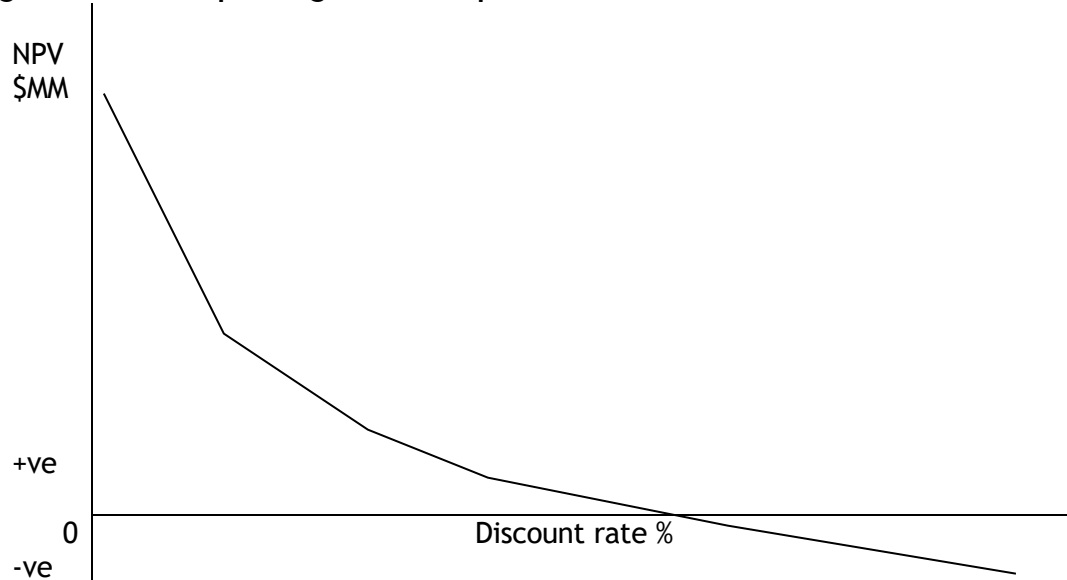
As the discount rate increases above zero, the negative net cash flows which occur early in the project's life are discounted less than the positive cash flows which occur later in the project's life. Therefore as the discount rate increases, the discounted negative net cash flows become more and more prominent in relation to the discounted positive net cash flows. Consequently, the total NPV of the project (that is, the sum of the discounted negative and positive net cash flows) falls.

Eventually, the sum of the discounted negative cash flows becomes greater than the sum of the discounted positive cash flows and at this point the NPV becomes negative. Table 2 demonstrates the point numerically.

**Figure 1A - Net cash flow of a simple investment**



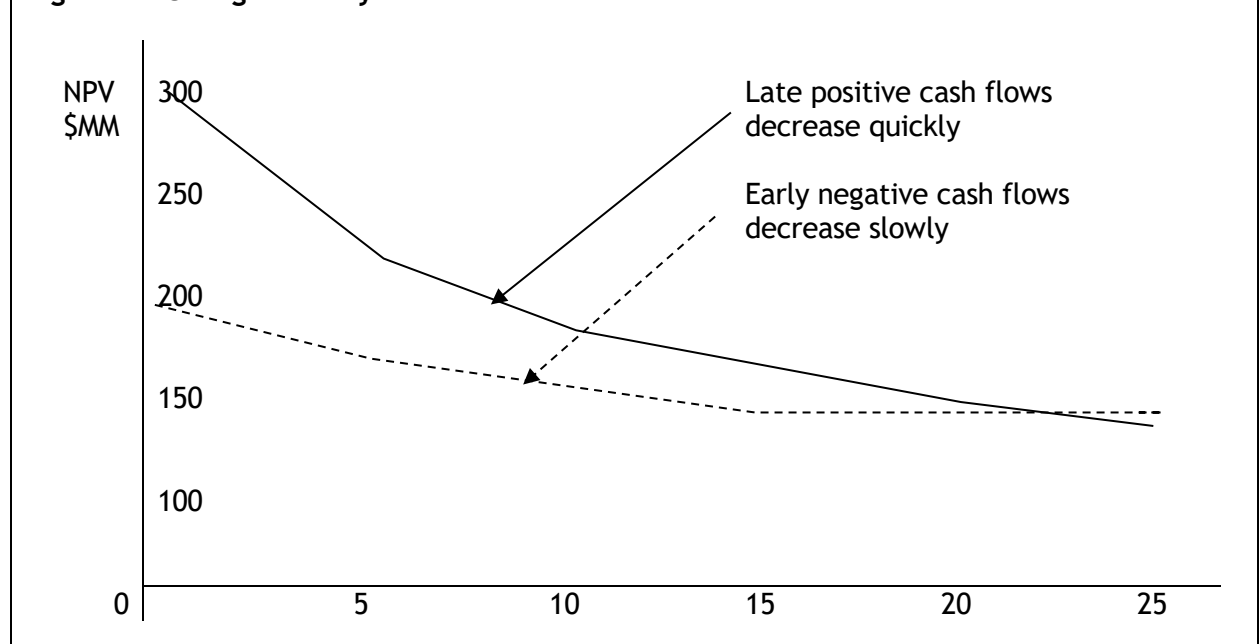
**Figure 1B - Corresponding NPV of simple investment**



**Table 2 - Net present value against discount rate (in \$MM)**

	End year 1	End year 2	Total yr 1&2	End year 3	End year 4	Total yr 3&4	Total all yrs
Net cash flow	-100	-100	-200	+150	+150	+300	+100
Net cash flow discounted at -							
0%	-100	-100	-200	+150	+150	+300	+100
10%	-91	-83	-174	+113	+102	+215	+41
15%	-87	-76	-163	+99	+86	+185	+22
20%	-83	-69	-152	+87	+72	+159	+7
25%	-80	-64	-144	+77	+61	+138	-6

It can be seen from this tabulation that, as the discount rate increases from 0% to 25%, the total discounted negative cash flows decrease more slowly (that is, -\$200 down to -\$144) than the total discounted positive cash flows (+\$300 down to +\$138). The positive cash flows are discounted more because they occur later in time. At a discount rate somewhere between 20% and 25%, the NPV becomes negative, because the total of the discounted negative cash flows is greater than the total of the discounted positive cash flows. The numbers in the table above are illustrated schematically in Figure 2.

**Figure 2 - Change in early and late cash flows as discount rate increases**

## Definition of internal rate of return

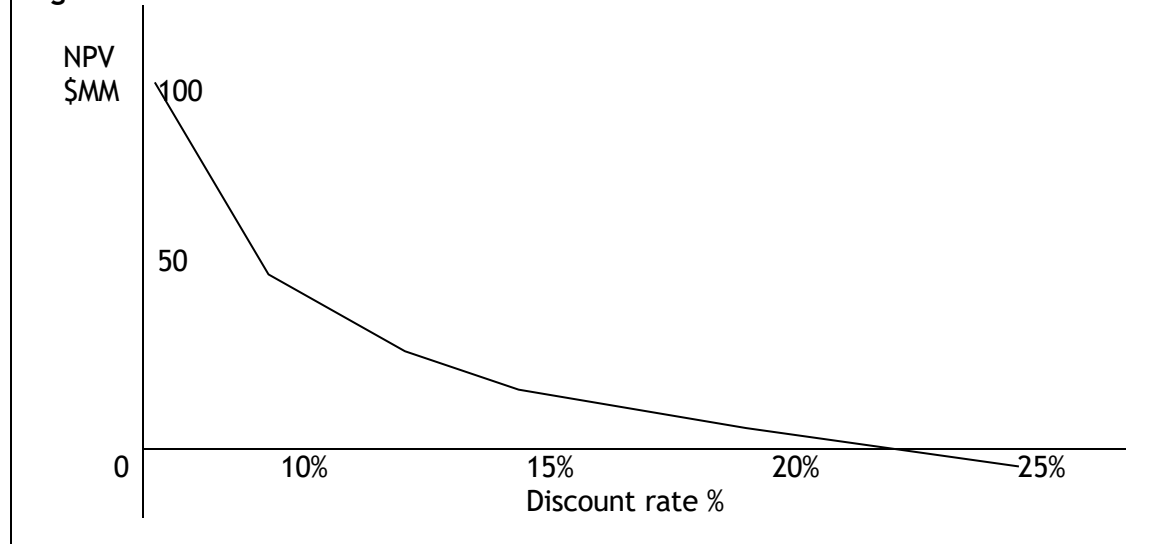
The discount rate for which the NPV becomes zero is defined as the Internal Rate of Return ("IRR") of a net cash flow stream. This is the discount rate for which all discounted negative cash flows are equal to all discounted positive cash flows. For the example cash flow given in the previous section, the Internal Rate of Return is 22.5%. Table 3 demonstrates this

**Table 3 -Internal rate of return = discount rate that makes the NPV = zero**

	End year 1	End year 1	Total yr 1&2	End year 3	End year 4	Total yr 3&4	Total all yrs
Net cash flow	-100	-100	-200	+150	+150	+300	+100
Net cash flow discounted at - 22.5%	-82	-67	-149	+82	+67	+149	0

Figure 3 is a graph of the NPV versus discount rate for this example.

**Figure 3 - Definition of internal rate of return**



The way in which the IRR is used in assessing the economic merit of a net cash flow stream is by comparing it with alternative investments, as was done with the NPV indicator. In other words, if the IRR of a net cash flow stream for a particular project is greater than the rate of return that can be obtained in comparable alternative investments (putting the money in the bank, for instance), then the project should be undertaken. If the rate of return is lower than that which could be obtained in alternative investments, then the project should not be undertaken.

In mathematical form, the definition of the internal rate of return (IRR) is -

$$\text{NPV} = 0 = \frac{\text{NCF}_1}{(1+\text{IRR})^1} + \frac{\text{NCF}_2}{(1+\text{IRR})^2} + \frac{\text{NCF}_3}{(1+\text{IRR})^3} + \dots + \frac{\text{NCF}_n}{(1+\text{IRR})^n}$$

This equation states that the IRR is the discount rate that makes the NPV zero.

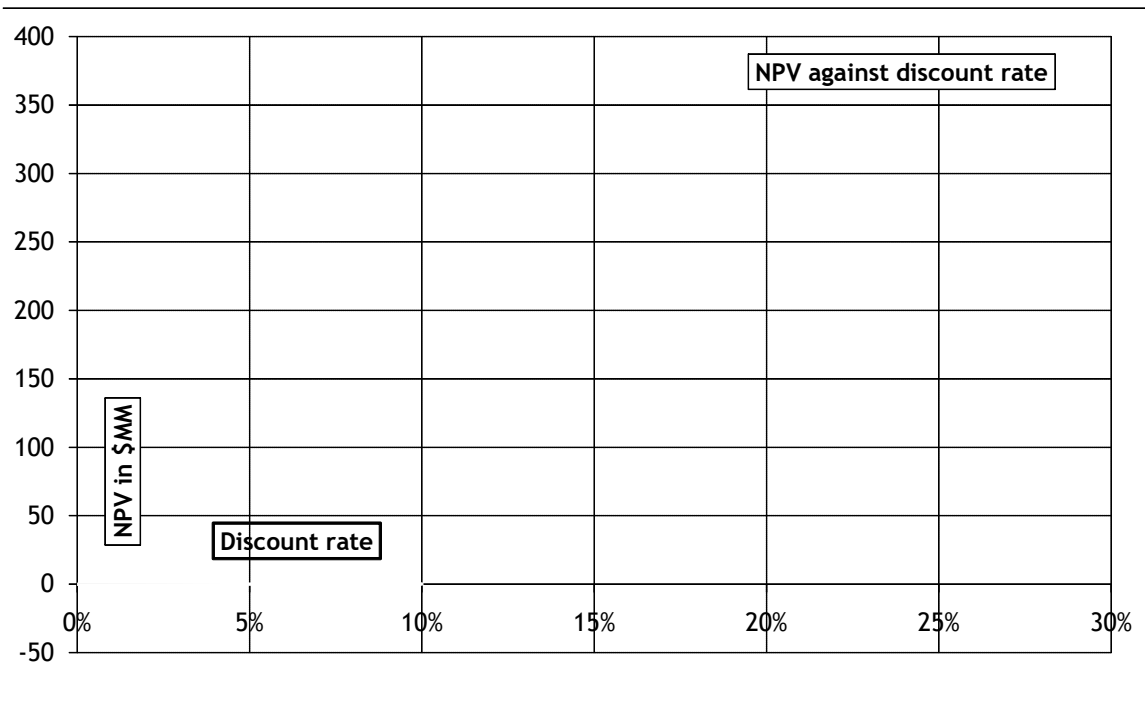




### Finding the Internal Rate of Return

By trial and error find the discount rate which makes the NPV zero. Two NPV calculations have already been done for you. (Hint = choose 30% for your next discount rate and then work backwards. Use a graph to help you.

		Time in years							
		Total	0	1	2	3	4	5	6
Net cash flow	\$MM	393.79	-100	-300	190	185	154	145	120
	Discount rate %	Total present value \$MM	Present values						
Net present values	0.0%	393.79	-100	-300	190	185	154	145	120
	10.0%	186.11	-100	-273	157	139	105	90	68

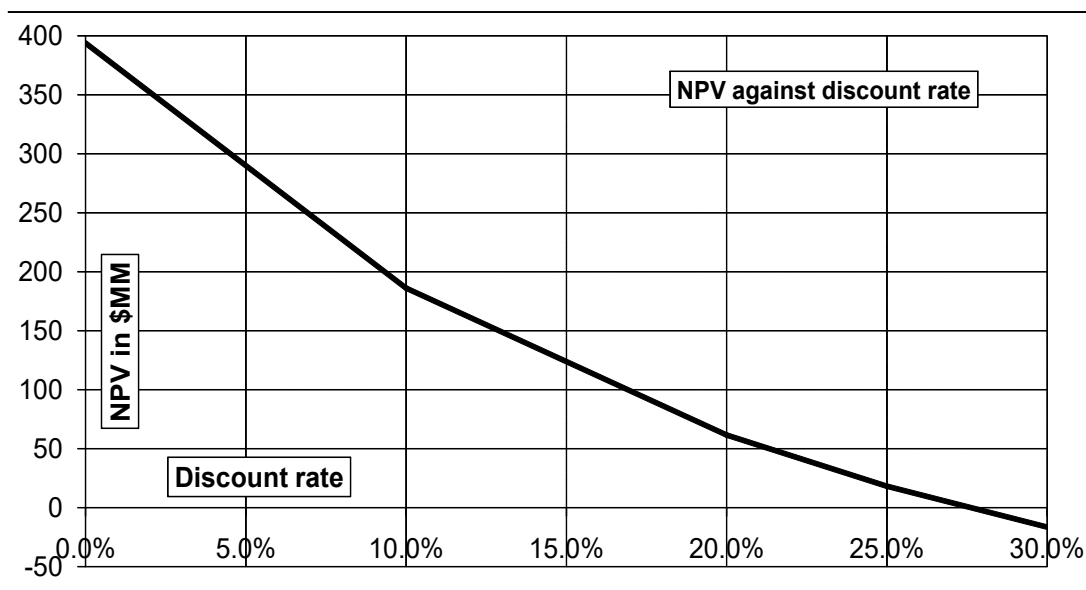


### Finding the Internal Rate of Return

By trial and error find the discount rate which makes the NPV zero. Two NPV calculations have already been done for you. (Hint = choose 30% for your next discount rate and then work backwards. Use a graph to help you.

			Time in years							
			Total	0	1	2	3	4	5	6
Net cash flow	\$MM	393.79	-100	-300	190	185	154	145	120	
			Total present value \$MM	Present values						
			Discount rate %							
Net present values	0.0%	393.79	-100	-300	190	185	154	145	120	
	10.0%	186.11	-100	-273	157	139	105	90	68	
	30.0%	-16.37	-100	-231	112	84	54	39	25	
	25.0%	18.29	-100	-240	122	95	63	47	31	
	27.5%	0.00	-100	-235	117	89	58	43	28	

Conclusion = the IRR is approximately 27.5%, because this makes the NPV zero.



## 6 Multiple internal rates of return

In the example net cash flow projection shown in the previous section, a single rate of return of 22.5% was derived. With that net cash flow profile, only that particular IRR gives an NPV of zero. However, some net cash flow profiles can produce several IRRs. In general, the number of IRRs will be the same as the number of times that the sign of the net cash flow changes. With the example used in the two previous sections, there is only one change of sign. That is between year 2 (when the net cash flow is negative) and year 3 (when the net cash flow is positive). Therefore, there is only one IRR.

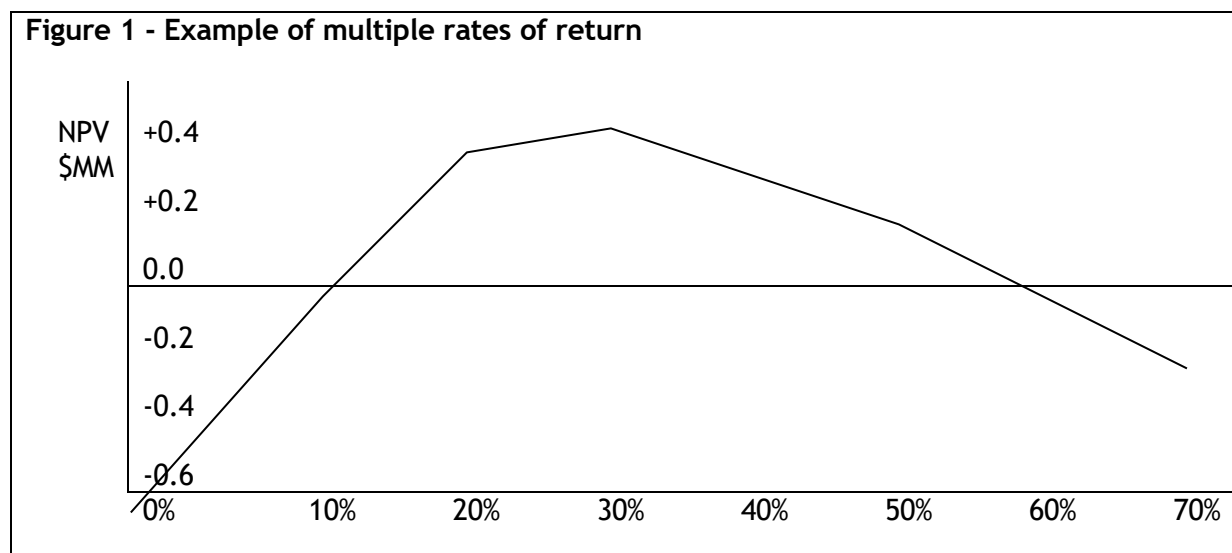
### Example of multiple IRRs

Table 1 demonstrates a situation in which a net cash flow profile can produce two rates of return. In this example, the net cash flow profile has two changes of sign. Here, the petroleum project has an initial capital investment of \$10 million in year 1, gives a net cash flow of \$27 million in year 2, but is uneconomic in year 3 and must be abandoned at a cost of \$17.6 million. The NPV of this project at different discount rates is shown in Table 1 and in graphical form in Figure 1.

**Table 1 - Example of multiple rates of return**

	Time = 0		End yr 1		End yr 2			
Net cash flow \$MM	-10.0		+27.0		-17.6			
Discount rates %	0%	10%	20%	30%	40%	50%	60%	70%
NPVs \$MM	-0.60	0.00	+0.28	+0.36	+0.31	+0.18	0.00	-0.21

**Figure 1 - Example of multiple rates of return**



We can see that the two internal rates of return for this project are 10% and 60% (at these points the NPVs are zero). In this case, relying solely on the IRR indicator might lead to a

false rejection or acceptance of the project - particularly if only one IRR was found. In this case, it would be important to know the complete NPV versus discount rate relationship before a decision is taken. The final decision might then be based on the NPV at a selected discount rate.

Very often we might obtain IRRs by using a spreadsheet function or a financial calculator. Either way, the IRR obtained in this way depends on the software routine used to make the estimate and particularly the seed, or first estimate, assumed as part of the routine. Different IRRs can result from different first estimates. For instance, the routine used in a spreadsheet IRR function might begin with 10% and search for the IRR from that point. In the case of the investment shown in Figure 1, such a routine would yield an IRR of 10%. However, if the starting point for the routine was more than that represented by the highest point on the NPV curve (about 30%) in Figure 1, then the routine would yield a result of 60%. This means that, depending on the first estimate, one spreadsheet might show the IRR to be 10% and another spreadsheet might show the result to be 60%.

Therefore, because there might be multiple rates of return, the IRR can often be unreliable as an indicator of the attractiveness of an investment. Many, if not most, of the oil and gas investments we encounter are likely to have at least two IRRs, because their net cash flows are likely to have at least two changes of sign. There will be an initial investment, then a positive return, then abandonment. In addition, many of the investments we examine will be incremental projects (discussed elsewhere in these notes). Often incremental projects have several IRRs.

Whenever there are two changes of sign in a net cash flow projection - one at the beginning and one at the end, then the shape of the NPV versus discount rate curve will always be as shown in Figure 1 (that is, an inverted bowl shape) and there will always be two IRRs. The lower IRR will be associated with the change in sign at the end of the project and the higher IRR will be associated with the change in sign at the beginning of the project. In some projects the two IRRs might be both positive (as above). In other projects the lower IRR might be negative and the higher IRR positive, or there might be two negative IRRs. However, whatever the case, the IRR criterion always gives ambiguous answers in cases where there is more than one change in sign.

## The meaning of NPV in the example

It is important to note that the NPV measure still has a meaning as a measure of the value of the project. In an earlier section, we defined the meaning of NPV as the extra amount we would have to deposit in the bank to enable us to withdraw the same cash flow from the bank as we would from the project.

Taking the example shown in Table 1 and Figure 1, assume that the bank rate (and therefore the discount rate) was 30%. Then the NPV of the example project above would be \$0.36 million as shown in the table. Therefore, for this example, we would have to invest an extra \$0.36 million in the bank to be able to withdraw the same amounts each year from the bank as we can from the project. That is, we would have to invest in the bank \$10 million (the initial investment in the project) plus \$0.36 million (the NPV of the project) equals \$10.36 million in total. This is illustrated in Table 2.

**Table 2 - Investment in bank alternative (in \$MM)**

	Time = 0	End year 1	End year 2
Original investment	10.00		
Extra investment (NPV at 30%)	0.36		
Total investment	10.36		
Interest at 30%		+3.11	-4.06
Balance 1		+13.46	-17.60
Withdrawal (-) or deposit (+)		-27.00	+17.60
Balance 2		-13.53	0.00

Note that in Table 2 the amount we "withdraw" at the end of year 2 is negative \$17.60 million. In other words, we have to deposit another \$17.60 million to clear the account with the bank. This is equivalent to what happens with the project when, at the end of year 2, we have to spend another \$17.6 million.

Finally, in this example, in which the cash flow changes sign twice, the behaviour of the NPV as the discount rate increases can be described as follows.

At zero or low discount rates, the NPV is negative. As discount rates increase, however, the discounted rate negative cash flow at end Year 2 becomes less and less important. Therefore, the NPVs increase and become positive. At high discount rates, however, even the discounted positive cash flow at the end of year 1 is not large enough to offset the discounted negative cash flow at Time = 0, and so the NPVs become negative again.



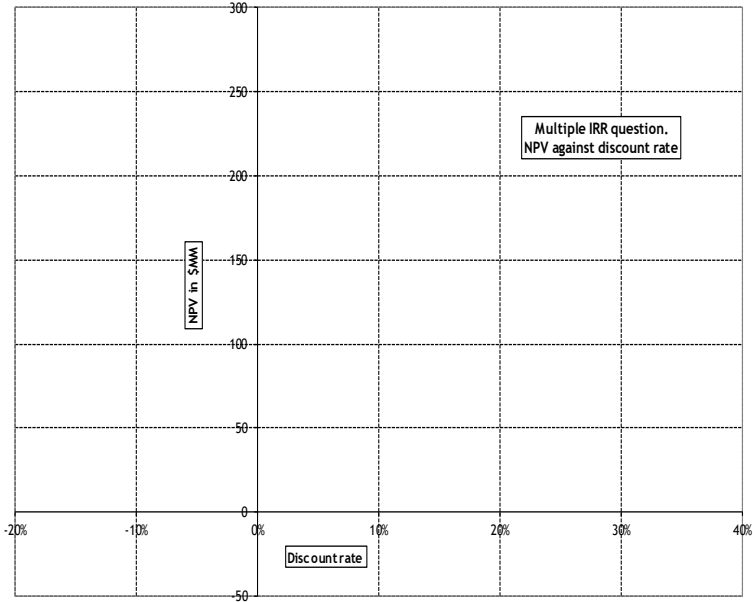
Multiple IRRs - question

Work out the nominal NPVs of the nominal oil field net cash flows (NCFs) below at the discount rates set out below. Draw a graph of NPV against discount rate. Find out their internal rates of return.

What do you conclude from the analysis?

		Time in years										
Units		0	1	2	3	4	5	6	7	8	9	10
Net cash flow 1	\$MM	-250.0	150.0	120.0	90.0	60.0	30.0	10.0	7.0	5.0	3.0	0.0
Net cash flow 2	\$MM	-250.0	150.0	120.0	90.0	60.0	30.0	10.0	7.0	5.0	3.0	-250.0

Discount rate	NPV1 \$MM	NPV2 \$MM
-30%		
-25%		
-20%		
-15%		
-10%		
-5%		
0%		
5%		
10%		
15%		
20%		
25%		
30%		
35%		
40%		
First IRR		
Second IRR		



## Multiple IRRs - answer

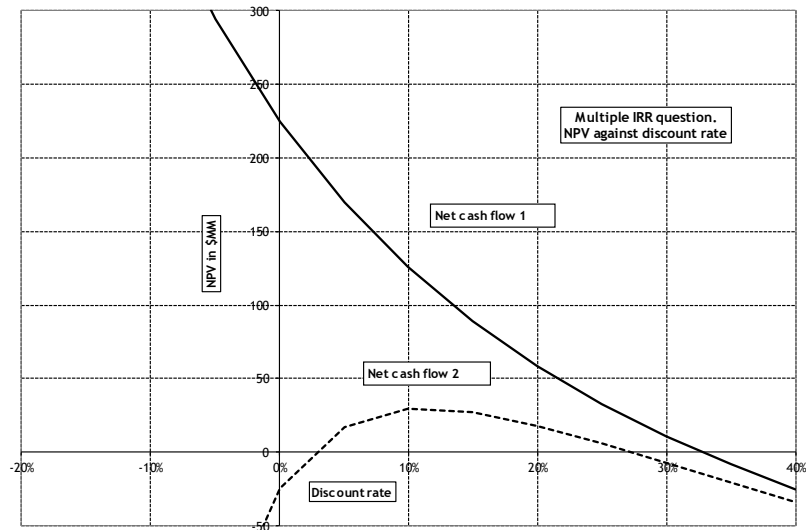
Work out the nominal NPVs of the nominal oil field net cash flows (NCFs) below at the discount rates set out below. Draw a graph of NPV against discount rate. Find out their internal rates of return.

What do you conclude from the analysis?

		Time in years										
Units		0	1	2	3	4	5	6	7	8	9	10
Net cash flow 1	\$MM	-250.0	150.0	120.0	90.0	60.0	30.0	10.0	7.0	5.0	3.0	0.0
Net cash flow 2	\$MM	-250.0	150.0	120.0	90.0	60.0	30.0	10.0	7.0	5.0	3.0	-250.0

Discount rate	NPV1 \$MM	NPV2 \$MM
-30%	1,231	-7,619
-25%	891	-3,548
-20%	662	-1,666
-15%	501	-769
-10%	383	-334
-5%	294	-123
0%	225	-25
5%	170	17
10%	126	29
15%	89	27
20%	58	18
25%	33	6
30%	11	-8
35%	-9	-21
40%	-25	-34
First IRR	32.7%	27.2%
Second IRR		or 2.4%

The first NCF has no field abandonment and has a single IRR. The second NCF includes abandonment and has two IRRs. The effect of introducing a negative NCF at the end for abandonment is (a) to reduce the IRR dominated by the NCF in the early years and (b) to introduce a second IRR, lower than the first. This lower IRR is dominated by the NCF at the end of the project. The higher IRR is dominated by the NCF at the start of the project. The spreadsheet IRR search routine could easily display the lower value for the IRR, which could be very misleading.





## 7 Comparing investments

In earlier sections, we discussed how the decision to invest funds in a project can be made using two economic indicators - NPV and IRR. In practice, the company might have several projects in which to invest. Some or all of these might be mutually exclusive, either because of the very nature of the projects (for instance, they might be different ways of doing the same thing), or because the company might have sufficient funds for only one or a limited number of projects. It can be the case when comparing mutually exclusive projects that the NPV and IRR indicators give contradictory results. This is demonstrated by means of an example in the following. The intention of examining this example is to highlight the differences between the NPV and the IRR as indicators of the attractiveness of different investments.

### Example of mutually exclusive investments

Suppose that a company has the option to invest in one of two petroleum developments and has a required discount rate of 10% for assessing their relative merits. The two projects are mutually exclusive. In this case they are mutually exclusive because the required investment in each is \$100MM and the company cannot afford to invest in both. The cash flows and economic indicators are shown in Table 1. The NPV against discount rate plot for both of these projects is shown in Figure 1

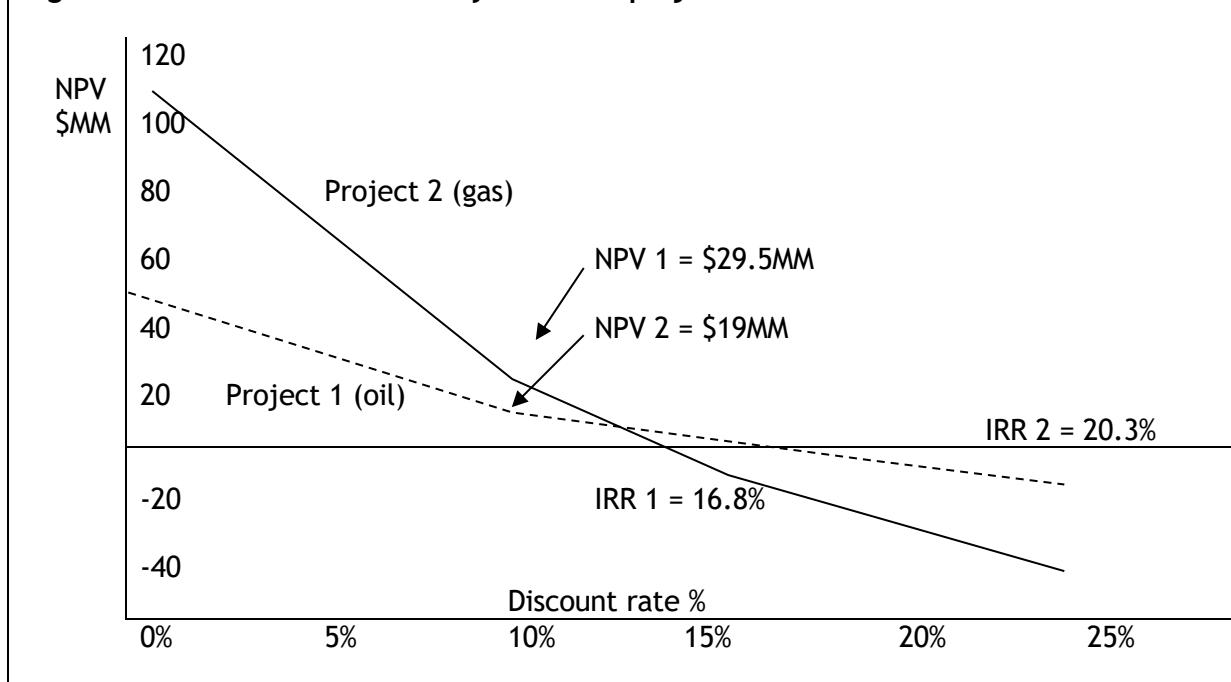
**Table 1 - Comparing mutually exclusive projects**

	End yr 1	End yr 2	End yr 3	End yr 4	End yr 5	End yr 6
Project 1 NCF	-100	50	40	30	20	10
Project 2 NCF	-100	1	2	10	20	175
Project 1 NPV at 10%	\$19.0MM					
Project 1 IRR	20.3%					
Project 2 NPV at 10%	\$29.5MM					
Project 2 IRR	16.8%					

For the purposes of illustration, suppose that Project 1 is an oil discovery which brings early net cash flow. Project 2 is a gas discovery and we must wait several years for the gas market to develop. Therefore, the cash flows from selling gas occur later in project life.

If we use the NPVs to compare the two projects, then we would select project 2 because it has a higher value (\$29.5 million compared to \$19.0 million). In contrast, when comparing the IRRs of the two projects, we should select project 1 because it has an IRR of 20.3% as opposed to 16.8%. The two investment criteria give contradictory results.

We might choose Project 1, the oil project, because it gives early cash flow and a faster return. This is reflected in the fact that the IRR is higher for Project 1. The IRR is effectively a measure of speed by which we obtain our money back (hence the word "rate" of return).

**Figure 1 - NPV curves for mutually exclusive projects**

### Does speed generate wealth?

Does the fact that Project 1 gives our money back quickly enable us to invest that money elsewhere and make us wealthier than Project 2 at the end of the 6 years? In answering this question, suppose that the only other investment is to put our money in the bank. We know the bank interest rate is 10%, because that is our discount rate. We also know that, at a 10% discount rate, the NPV of Project 2 is more than the NPV of Project 1. Therefore, by the very meaning of NPV, we also know that investing the cash flow from Project 2 in the bank will make us wealthier at the end of 6 years. In other words, in 6 years from now the future value of Project 2 will be more than the future value of Project 1. Table 2 demonstrates this point.

If there were potential investments other than the bank, then the (presumably different) return on those would determine our discount rate rather than the bank rate. Then the new NPVs of Project 1 and Project 2 would determine which gave us more wealth at the end of 6 years.

### Does speed reduce risk?

Does the fact that we receive our money back quickly with Project 1 make this project less risky? The speed with which we obtain a return is often used as a measure of safety. However, this is a crude way of taking risk into account. It ignores the specific risks associated with each project. For instance, in this example, because Project 1 is an oil project it is therefore subject to the volatility of oil prices. In fact, taking oil price risk into account, the cash flow might be significantly lower if we have a long period of low oil prices. By contrast, because Project 2 is a gas project, we might have a legally binding contract with firm sales volumes and prices. Under these circumstances, taking specific risk into account, Project 2 might be significantly less risky than Project 1. In sum, a fast return does not necessarily indicate lower risk.

**Table 2 - Proof that Project 2 gives more wealth than Project 1**

	End yr 1	End yr 2	End yr 3	End yr 4	End yr 5	End yr 6
Net cash flow \$MM for Project 1	-100.0	50.0	40.0	30.0	20.0	10.0
Future values at 10%						10.0
						22.0
						36.3
						53.2
						73.2
						-161.1
Total	PV=19.0					FV=33.6
	End yr 1	End yr 2	End yr 3	End yr 4	End yr 5	End yr 6
Net cash flow \$MM for Project 2	-100.0	1.0	2.0	10.0	20.0	175.0
Future values at 10%						175.0
						22.0
						12.1
						2.7
						1.5
						-161.1
Total	PV=29.5					FV=52.3

### NPV is money, IRR is a %

In this example, Project 2 is clearly the one which adds more to the value of the company. The problem with the IRR measure is that it is just a percentage. Although Project 2 gives a lower rate of return than Project 1, the return is based on a larger total cash flow in money terms. This illustrates the problem with any percentage figure, namely that it tells us nothing about the absolute magnitudes involved. In contrast, the NPV measure tells us the magnitude of value of the project in today's terms. The NPV tells us by how much the value of our company increases today by investing in a project.



### Comparing investments

The nominal net cash flows below are for two mutually exclusive oil developments. Derive the NPVs for the table below and draw a graph of NPV against discount rate. What is the NPV at 10% (10% is the bank rate and our discount rate)? What are their internal rates of return?

Which project would you choose and why?

If we invested the net cash flow from the projects in the bank at 10% per year, which project would give us the most wealth at the end of year 7? Prove this by calculating the future value of each project at the end of year 6.

	Units	Time 0	1	2	3	4	5	6
Net cash flow 1	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0
Net cash flow 2	\$MM	-100.0	10.0	10.0	20.0	70.0	100.0	100.0

Discount rate	NPV1 \$MM	NPV2 \$MM
0%		
10%		
20%		
30%		
40%		
50%		
IRRs		

	Units	Time 0	1	2	3	4	5	6
Net cash flow 1	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0
Future values 1	\$MM							
Total future value	\$MM							

	Units	Time 0	1	2	3	4	5	6
Net cash flow 2	\$MM	-100.0	10.0	10.0	20.0	70.0	100.0	100.0
Future values 2	\$MM							
Total future value	\$MM							

## Comparing investments - answer

The nominal net cash flows below are for two mutually exclusive oil developments. Derive the NPVs for the table below and draw a graph of NPV against discount rate. What is the NPV at 10% (10% is the bank rate and our discount rate)? What are their internal rates of return?

Which project would you choose and why?

If we invested the net cash flow from the projects in the bank at 10% per year, which project would give us the most wealth at the end of year 7? Prove this by calculating the future value of each project at the end of year 6.

		Time	End years -					
	Units	0	1	2	3	4	5	6
Net cash flow 1	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0
Net cash flow 2	\$MM	-100.0	10.0	10.0	20.0	70.0	100.0	100.0

	NPV1	NPV2
Discount rate	\$MM	\$MM
0%	120	210
10%	74	99
20%	42	34
30%	20	-5
40%	3	-30
50%	-10	-47
IRRs	42.1%	28.4%

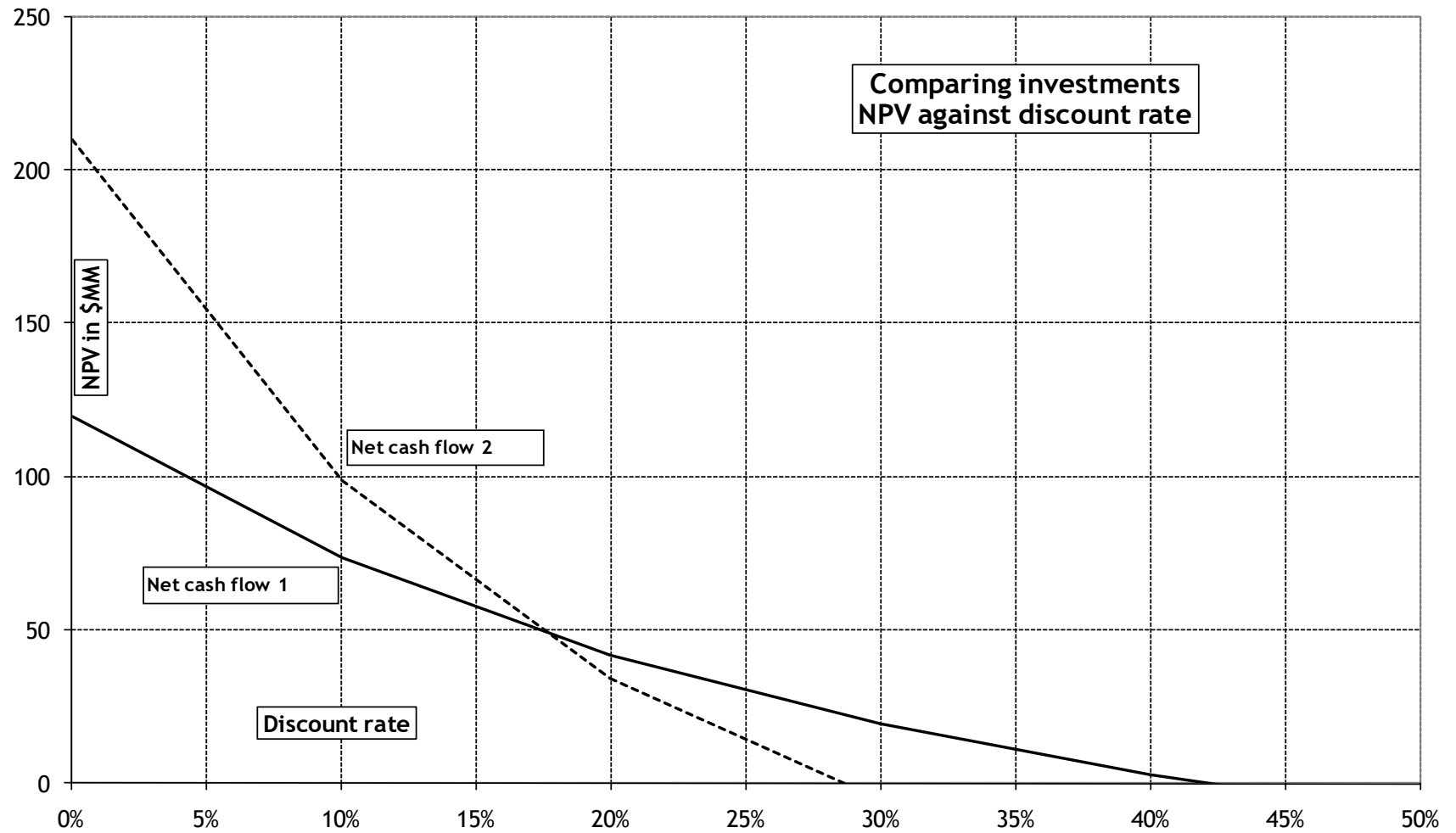
See also the chart on separate sheet.

Choose project 2, because it has the highest NPV at our discount rate, even though the rate of return is lower. Our company will increase in value by more if we choose Project 2.

		Time	End years -					
	Units	0	1	2	3	4	5	6
Net cash flow 1	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0
Future values 1	\$MM	-177.2	112.7	73.2	53.2	36.3	22.0	10.0
Total future value	\$MM							130.3

		Time	End years -					
	Units	0	1	2	3	4	5	6
Net cash flow 2	\$MM	-100.0	10.0	10.0	20.0	70.0	100.0	100.0
Future values 2	\$MM	-177.2	16.1	14.6	26.6	84.7	110.0	100.0
Total future value	\$MM							174.9

Project 2 makes us more wealthy because it has the highest future value.







## 8 The effect of delay on IRR

In our review of the different economic indicators, we have implicitly assumed that project start is the beginning of the current year. In practice, we might examine projects that begin several years from now. For example, we might examine the effects of delay in project start.

If the project start is several years in the future, the net present value of the project will be reduced because its annual net cash flows must be discounted more. If, for instance, the NPV of a development is \$100 million, a two year delay in project start would mean a reduction in the NPV to  $\$100 / (1+r)^2$ .

### Example of effect of delay on IRR

In contrast, a delay in project start leaves the internal rate of return of the project unchanged. We demonstrate this in Table 1 for a project which has a rate of return of 22.5%. This is the same project as is used in the section which shows how IRRs are derived.

Table 1 - Example of effect of delay on IRR

	End yr 1	End yr 2	End yr 3	End yr 4	End yr 5	End yr 6	Total
Project net cash Flow (NCF)	-100	-100	+150	+150			
Project NCF discounted at 22.5%	$\frac{-100}{1.225^1}$	$\frac{-100}{1.225^2}$	$\frac{+150}{1.225^3}$	$\frac{+150}{1.225^4}$			= 0
NCF delayed 2 yrs			-100	-100	+150	+150	
Delayed NCF discounted at 22.5%			$\frac{-100}{1.225^3}$	$\frac{-100}{1.225^4}$	$\frac{+150}{1.225^5}$	$\frac{+150}{1.225^6}$	= 0

In this table the project is delayed for 2 years. However, the discount rate that makes the NPV zero (that is, the internal rate of return) remains the same at 22.5%.

In the general case, by definition the NPV of any cash flow is 0 if we discount by the IRR. The NPV of the delayed cash flow is simply the NPV of the original cash flow divided by one plus the IRR raised to the power of the number of years of delay. However, because the original NPV is zero, this division will leave the NPV unchanged at zero. Therefore, the IRR is unchanged by shifting the project in time. This is shown in the following for the example in Table 1.

NPV for delayed project = (NPV for non-delayed project) /  $1.225^2$

Therefore, if the NPV for the non-delayed project = 0, then the NPV for the delayed project must be zero. Therefore the IRR of Project 2 must be the same as the IRR of Project 1.

This, then, is a disadvantage of the IRR as a sole measure of the economics of a project.

It cannot help us distinguish between projects which start early and projects which start late. In contrast, the NPV measure indicates that projects which start later have less value than those which start now, all other things being equal. This is clearly important. A profitable gas project which can begin now, for instance, is preferred by comparison with a similar project which cannot begin for several years because of market constraints or some other reason.

It should be noted that this feature of the IRR only applies if the cash flows of the delayed project are identical to those of the original project. If inflation applies and changes the cash flows, then, in general, the IRR will change if the cash flows are delayed.

## 9 Nominal and real IRR

In another section, we discussed the use of nominal and real NPVs. In some cases, oil and gas companies also use nominal and real internal rates of return (IRRs) to evaluate investments. We discuss these in this section.

### Definitions

Based on the discussion in the section on defining IRR, the IRR is the discount rate that yields a net present value (NPV) of zero.

It follows that -

- (a) A nominal IRR is the discount rate that makes zero the NPV of a nominal net cash flow and
- (b) A real IRR is the discount rate that makes zero the NPV of a real net cash flow.

However, the nominal and the real IRR of a project are related by a simple formula as shown in the following.

$$\text{Real IRR} = \frac{(1 + \text{Nominal IRR})}{(1 + \text{Deflator})} - 1$$

### Example of nominal and real IRR

The derivation of nominal and real IRRs is illustrated in Table 1.

	Total	End yr 1	End yr 2	End yr3	End yr 4
<b>Nominal analysis</b>					
Nominal NCF	77.17	-100	20	50	107.17
Discounted at 26.5%	0	-79.05	12.50	24.70	41.85
Nominal IRR = 26.5%					
<b>Real analysis</b>					
Real NCF - 10% deflator	36.38	-90.91	16.53	37.57	73.20
Discounted at 15.0%	0	-79.05	12.50	24.70	41.85
Real IRR = 15%					
<b>Alternatively -</b>					
1 + nominal IRR =	1.265				
1 + deflator =	1.100				
1+Real IRR =	1.265/1.100				
Real IRR =	15%				



### Nominal and real IRRs

Find the nominal IRR of the nominal net cash flow below.

Deflate the nominal net cash flow by 3% per year to obtain a real net cash flow.

Find the real IRR of the real net cash flow below.

Show how you can calculate the real IRR from a formula once you know the nominal IRR.

	Units	Time		End years -					
		0	1	2	3	4	5	6	
Nominal net cash flow	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0	
Nominal IRR =									
Deflator (3% per year)									
Real net cash flow	\$MM								
Real IRR =									

Real IRR =

### Nominal and real IRRs - answer

Find the nominal IRR of the nominal net cash flow below.

Deflate the nominal net cash flow by 3% per year to obtain a real net cash flow.

Find the real IRR of the real net cash flow below.

Show how you can calculate the real IRR from a formula once you know the nominal IRR.

	Units	Time 0	1	2	3	4	5	6
Nominal net cash flow	\$MM	-100.0	70.0	50.0	40.0	30.0	20.0	10.0
Nominal IRR =	42.1%							
Deflator (3% per year)		1.00	0.97	0.94	0.92	0.89	0.86	0.84
Real net cash flow	\$MM	-100.0	67.96	47.13	36.61	26.65	17.25	8.37
Real IRR =	38.0%							

$$\text{Real IRR} = \frac{(1 + \text{nominal IRR})}{(1 + \text{deflator})} - 1$$

$$\text{Real IRR} = \frac{(1 + 42.1\%)}{(1 + 3\%)} - 1$$

$$\text{Real IRR} = 38.0\%$$

## 10 Capital productivity index

An economic indicator frequently used in the oil and gas industry is the "Capital Productivity Index" (CPI). Other labels used for the same indicator are "Profit Investment Ratio" (PIR), "Value Investment Ratio" (VIR) and "Present Worth Index" (PWI). Essentially, these ratios or indices are the net monetary output of a project divided by the monetary input. The input is the investment, or the upfront capital expenditure. The output is the net cash flow or, more appropriately, the NPV of the project. Therefore, the ratios measure the output per unit of input. In other words, they measure capital productivity. This is why we use the term "capital productivity index" in these notes.

### Definition

The definition of the CPI of a project used in the notes is -.

$\text{Capital productivity index (CPI)} = \frac{\text{Project NPV}}{\text{Present value of capital investment}}$
---

We established in the section on NPV that, by definition, acceptable investments are those with NPVs greater than zero. It follows mathematically that acceptable investments are also those with CPIs greater than zero. Any project with a positive NPV will also have a positive CPI.

Oil and gas companies often use the CPI as an initial screening device to select those projects that warrant further examination. Such companies often require the CPI to be significantly more than zero. For instance, companies might require projects to have CPIs of more than 0.2 or 0.3. In doing this, implicitly the companies are incorporating an element of risk in their selection. The idea is that, by selecting projects with CPIs more than 0.2 or 0.3 (for instance), they are filtering out more risky projects.

Such practices are arbitrary and rather crude methods of dealing with risk. Some projects with low CPIs might have very low levels of risk and be very attractive. For instance, gas discoveries having low CPIs, but with high NPVs, established gas markets and firm gas contracts (that is low risk high value projects) would be highly attractive. However, they might be filtered out and discarded using CPI cut offs greater than zero.

### CPI example

The derivation of the CPI of a simple example investment is illustrated in Table 1. In this example, the capital expenditure occurs at the end of year 1. We must discount this to derive its present value. Then it is in today's terms and can be properly compared with the NPV.

**Table 1 - Example CPI calculation**

	Total	End yr 1	End yr2	End yr 3
Nominal net cash flow \$MM	40.6	-100.0	62.5	78.1
NPV at 10% \$MM	19.4			
Capital investment \$MM	100.0	100.0		
Discounted capital investment \$MM (assuming a 10% discount rate)	90.9			
CPI = $\frac{19.4}{90.9}$				
Therefore, CPI = 0.213				

In this example, the CPI is more than zero and therefore theoretically would be an acceptable project.

### Alternative definition

One alternative definition of the CPI of a project is -

Capital productivity index (CPI) = $\frac{\text{Undiscounted net operating cash flow}}{\text{Undiscounted total capital investment}}$
---

In this formulation, net operating cash flow is simply net cash flow excluding the capital expenditure. It is the pure output of the project.

This alternative definition of CPI - or some variant of it - is sometimes used in production sharing contracts (PSCs) as a means of establishing the profitability of the project for profit sharing. Such PSCs are sometimes called "R over C" contracts, where "R over C" stands for "revenues over costs". Another name used is "Investment Multiple" contracts. A project with a high CPI, or R over C, or Investment Multiple would mean a high Government share of profit. In such PSCs, the CPI would be calculated each year based on cumulative net cash flow and cumulative investment up to that year. The CPI would then determine profit sharing in the following year.

A disadvantage in using an undiscounted CPI is that, by definition, it does not take time into account and does not make a comparison with an alternative investment. We discuss R/C or investment multiple PSCs in another section.



## Investment multiple example

The derivation of the R over C ratio or investment multiple in a simple example investment is illustrated in Tables 2A and 2B.

**Table 2A - Investment multiple example**

	End yr 1	End yr 2	End yr 3	End yr 4	End yr 5
A Operating net cash flow \$MM	0	63	78	90	80
B Investment \$MM	100	25			
C Cum op net cash flow \$MM	0	63	141	231	311
D Cum investment \$MM	100	125	125	125	125
E Investment multiple (=C/D)	0.00	0.50	1.13	1.85	2.49

In this example, the operating net cash flow is the gross revenue less operating costs and taxes. This is accumulated over the life of the project. The capital costs, or the investment, are accumulated in a similar way. The ratio between the cumulative operating net cash flow and the cumulative investment is the R over C ratio or the investment multiple. In some PSCs, this ratio determines the State's share of profit oil or gas in the following year. An example is in Table 2B. For instance, an investment multiple of less than 1.50 this year might mean that the State's share of profit oil is zero next year. An investment multiple of more than 1.50 this year might mean that the State's share of profit oil is, say, 10% in the following year.

**Table 2B - Investment Multiple (IM) shares**

IM Range	Contractor	Government
Below 1.5	100%	0%
1.5 to 2.0	90%	10%
2.0 to 2.5	80%	20%
2.5 to 3.0	70%	30%
3.0 to 3.5	60%	40%
Over 3.5	50%	50%

## Disadvantages of CPI

Using the CPI as the only means of making investment decisions has a number of disadvantages. These are -

(1) The CPI is simply a ratio. It does not on its own tell us the size of the project. A project with a CPI of 2.5 might be (a) a project with an NPV of \$2.5MM and a discounted capital

expenditure of \$1MM or (b) a project with an NPV of \$250MM and a discounted capital expenditure of \$100MM.

(2) It also follows from (1) above that the CPI does not tell us how much money we are making.

(3) The CPI does not take into account the effect of project delay. A project that begins now will have the same CPI as the same project delayed by several years (assuming that the cash flows remain the same). This is demonstrated in Table 3 below.

(4) We cannot add and subtract CPIs of different projects. If a company has a project in Africa with a CPI of 2.5 and a project in Asia with a CPI of 1.5, the combined CPI of both projects is not 2.5 plus 1.5 equals 4.0.

**Table 3 - CPI with a 2 year delay**

<b>Project starting now</b>	
NPV of project in Table 1	\$19.4 MM
Discounted capex from Table 1	\$90.9
CPI	0.213
<b>Same project delayed 2 years</b>	
NPV of project delayed 2 years	$\frac{\$19.4 \text{ MM}}{(1 + 10\%)^2}$
Discounted capex with 2 year delay	$\frac{\$90.9 \text{ MM}}{(1 + 10\%)^2}$
CPI	0.213

## The use of CPI in capital rationing

The CPI is useful in capital rationing situations where many investments are possible, but there are limited investment funds available. Therefore not all profitable investments can be undertaken.

When there is a limited investment budget, we have to select those investments which in total give us the highest return by whatever measure we adopt. For instance, we could do this by choosing those projects which give the highest NPV, but making careful note of the capital expenditure involved in each one and ensuring that we do not exceed our budget.

The CPI can help us make this selection automatically because it measures the NPV per unit of capital expended. By ranking the projects in order of the highest to the lowest CPI, we could go down the ranking and pick only those with the highest values until our budget is exhausted.

An example of this process is given in Table 4. In this table there are eight possible projects in which we could invest our capital. Each of them involves an initial expenditure of either \$1MM or \$0.5MM at time = 0. Thereafter, each produces positive cash flows of varying

amounts in each year of project life (10 years in each case). In this example our task is to select the best projects with a budget limited to \$3MM.

Table 4 indicates that if we select the projects with the highest CPI we would therefore choose projects 5, 4, 8 and 2. This would maximise the NPV per unit of capital invested and exhaust our available funds of \$3MM.

Table 4 shows that with selections made using a Payback indicator, we would select projects 6, 4 and 3. This is a different group of projects compared with that obtained using the IRR (4, 1 and 2). Although both Payback and IRR are effectively a measure of speed or rate of return, they yield different choices.

**Table 4 - Choosing investments with \$3MM to spend**

	Project number								
Year	1	2	3	4	5	6	7	8	Choice
Invest- ment (\$MM)	-1	-1	-1	-1	-0.5	-1	-1	-0.5	
CPI10%	0.288	0.294	0.101	0.317	0.334	0.139	0.290	0.308	4,2,5,8
Payback	4.35	4.45	4.02	4.00	5.80	3.83	4.46	5.03	6,4,3
IRR	17.1%	17.0%	13.8%	18.4%	16.5%	14.4%	16.9%	16.9%	4,1,2
NPV10% (\$MM)	0.288	0.294	0.101	0.317	0.167	0.139	0.290	0.154	4,2,5,8

The use of the Net Present Value (NPV) criteria needs some discussion. Initially we might make the selection using the NPV as shown in Table 5, which ranks the projects starting with the highest NPV.

**Table 5 - Initial selection using NPV**

Rank	Project	NPV	Capex
1	4	\$0.317MM	\$1MM
2	2	\$0.294MM	\$1MM
3	7	\$0.290MM	\$1MM
	Totals	\$0.901MM	\$3MM

However, if we remember that NPVs can be added and we look carefully at the table, we can see that, instead of choosing project 7, we can choose projects 5 and 8. The reason is that, taken together, these two projects give a total NPV of \$0.321MM (compared to \$0.290MM for project 7), yet they involve the same capital expenditure of \$1MM. This is made up of \$0.5MM for project 5 and \$0.5MM for project 8. The revised selection is summarised in Table 6.

Therefore, by choosing projects 4,2,5,8 we achieve a greater total NPV (that is, \$0.932MM) than by choosing projects 4, 2, 7 (which give a total NPV of \$0.901MM).

**Table 6 - Revised selection using NPV**

Rank	Project	NPV	Capex
1	4	\$0.317MM	\$1MM
2	2	\$0.294MM	\$1MM
3	5	\$0.167MM	\$0.5MM
3	8	\$0.154MM	\$0.5MM
Totals		\$0.932MM	\$3MM

Now we can see the usefulness of the CPI indicator because this leads to the same project selection (4, 2, 5, 8) as does the NPV indicator. However, the use of the CPI indicator takes into account both the NPV and the initial capex and automatically makes the selections which maximise the total NPV given the amount of capital we have available. In other words, using the CPI makes the selection process easier.

## Lumpy investments

One problem with the use of the CPI (or the NPV for that matter) when selecting projects with a limited budget, is that it does not explicitly show the size of the investment, and cannot take into account the “lumpiness” of investments. “Lumpiness” refers to the fact that the investment required for a particular project might not generally be divisible into smaller investments for smaller projects. It might not be easy to get a comfortable match between the budget available and the investment requirements of each project. For instance, if each investment in Table 4 involved an initial capital expenditure of \$2MM and our budget remained \$3MM, then it would be impossible to choose more than one project. We would spend only \$2MM on the first and have \$1MM left over. This might not be useable on another project if for some reason it is not possible to undertake only a fraction of a project.

### Investment Multiple and Capital productivity index

Calculate the undiscounted investment multiple each year.

Calculate also the nominal capital productivity index assuming a nominal discount rate of 10%.

	Units	Time						
		0	1	2	3	4	5	6
Gross revenue	\$MM			90.0	80.0	70.0	60.0	50.0
Operating costs	\$MM			10.0	10.0	10.0	10.0	10.0
Tax	\$MM			20.0	16.0	12.0	8.0	4.0
Operating net cash flow (NCF)	\$MM							
Capital investment	\$MM	50.0	100.0					
Cumulative operating NCF	\$MM							
Cumulative investment	\$MM							
Undiscounted investment multiple	No							
After tax cash flow (ATNCF)	\$MM							
NPV of ATNCF	\$MM							
NPV of capital investment	\$MM							
Capital productivity index	No							

**Investment Multiple and Capital productivity index - answer**

Calculate the undiscounted investment multiple each year.

Calculate also the nominal capital productivity index assuming a nominal discount rate of 10%.

	Units	Time						
		0	1	2	3	4	5	6
Gross revenue	\$MM			90.0	80.0	70.0	60.0	50.0
Operating costs	\$MM			10.0	10.0	10.0	10.0	10.0
Tax	\$MM			20.0	16.0	12.0	8.0	4.0
Operating net cash flow (NCF)	\$MM			60.0	54.0	48.0	42.0	36.0
Capital investment	\$MM	50.0	100.0					
Cumulative operating NCF	\$MM	0.0	0.0	60.0	114.0	162.0	204.0	240.0
Cumulative investment	\$MM	50.0	150.0	150.0	150.0	150.0	150.0	150.0
Undiscounted investment multiple	No	0.0	0.0	0.4	0.8	1.1	1.4	1.6
After tax cash flow (ATNCF)	\$MM	-50.00	-100.00	60.00	54.00	48.00	42.00	36.00
NPV of ATNCF	\$MM	28.4						
NPV of capital investment	\$MM	140.9						
Capital productivity index	No	0.20						

## 11 Payback

The "payback" period of a project is frequently used as an indicator of the project's economic merit. This is simply the time taken for the project's positive net cash flow to recoup the initial capital outlay.

### Undiscounted payback

The simplest payback indicator is undiscounted payback, which is based on the undiscounted net cash flow. The example in Table 1 illustrates the calculation.

**Table 1 - Example undiscounted payback period calculation**

	Year 1	Year 2	Year 3	Year 4
Net cash flow (NCF) \$MM	-50	-150	+100	+175
Cumulative NCF \$MM	-50	-200	-100	+75
Payback period (years)	3.57	(=3+100/175)		

In this example, it is 3.57 years before the capital investment of \$200 million is fully recovered from net cash flow. This assumes that the annual net cash flows are spread evenly over each year. Therefore in Year 4, the fraction of the year to payout is given by -

$$\text{In year 4, the fraction of the year until payback} = \frac{\$100 \text{ MM}}{\$175 \text{ MM}} = 0.57 \text{ years}$$

In this calculation, \$100 MM is the outstanding cumulative net cash flow at the end of year 3 and \$175 MM is the net cash flow in year 4. The \$175 is assumed to be spread evenly over year 4. If the net cash flow in year 4 had been \$200 MM, then it would take half a year to recoup the outstanding \$100 MM. However, the net cash flow is less than \$200 MM. Therefore, it takes longer to recoup the outstanding amount.

### Discounted or compounded payback

We can also calculate a payback using the discounted or compounded net cash flow of a project. Although not so often used, these measures are more useful than the undiscounted payback because it takes into account the return we could obtain on our alternative investment. The measures tell us how long it takes to recover our money with interest. They therefore tell us if the project is better or worse than the alternative investment.

The example in Table 2 illustrates the discounted and compounded payback calculations. Table 2 uses the same net cash flow data as Table 1 and is based on a discount or compound rate of 10%.

**Table 2 - Example discounted and compounded payback period calculations**

	Year 1	Year 2	Year 3	Year 4
Net cash flow (NCF) \$MM	-50	-150	+100	+175
Discounted NCF \$MM	-45.45	-123.97	+75.13	+119.53
Cum discounted NCF \$MM	-45.45	-169.42	-94.29	+25.24
Discounted payback (years)	3.79 = $3 + 94.29 / 119.53$			
Net cash flow (NCF) \$MM	-50	-150	+100	+175
Cum compounded NCF \$MM	-50.00	-205.00	-125.50	+36.95
Compounded payback (years)	3.79 = $3 + 125.50 * (1 + 10\%) / 175$			

In this example, because we are discounting or compounding the net cash flows, it takes longer to achieve payback than if we use undiscounted or uncompounded net cash flows. However, we obtain the same answer whether we use discounted payback or compounded payback. Mathematically, they must be the same.

In this example, we have to make an assumption about exactly when the cash flows occur during the year. As an illustration, we assume that they occur at the end of each year. This makes calculating the fraction of the year rather meaningless. It would have more meaning if we split up the annual cash flows into daily amounts and discounted or compounded each day. Our example is too simple to illustrate the calculations properly. It is intended to convey only the method and concept that discounted and compounded payback give the same result. In Table 2, to illustrate that the discounted payback and the compounded payback give the same result, we assume artificially that the annual discounted or annual cumulative compounded amounts apply to the whole of each year.

Some petroleum fiscal regimes rely on compounded payback to determine profit sharing or levels of tax. For instance, in some PSCs in India and the Resource Rent Tax system in Australia, we are required to compound forward our net cash flows at rates specified in the PSC or the tax legislation, determine when compounded payback occurs and then base the tax we pay on the net cash flow remaining after payback. For instance, in the example shown in Table 2, we might pay tax on the \$36.95 MM remaining after compounded net cash flow first becomes positive.

## Disadvantages of payback

Payback is a useful initial indicator of the merits of a project. However, there are a number of drawbacks to the use of the payback period as a sole measure of the project's relative economic merits.

- Measures of payback do not indicate by how much a project increases the value of a company. It is not a measure expressed in money terms.
- Payback tells us nothing about the profitability of the project beyond the payback period. Significant cash flows might occur after payback which we might prefer to be reflected in the measure of the project's profitability.



- c) Payback does not on its own take into account when a project starts since it is usually measured in relation to project start.
- d) Payback indicators cannot be meaningfully added, subtracted or otherwise arithmetically manipulated to assess investment portfolios, incremental economics or to assess risk-weighted returns.



### Payback, discounted payback and compounded payback

Calculate the payback, the discounted and compounded payback periods for the nominal after tax net cash flow shown below.

Assume a nominal discount rate or compound rate of 10%.

		Years -					
Units		1	2	3	4	5	6
<b>Payback period</b>							
Nominal after tax net cash flow	\$MM	-50.0	-100.0	90.0	80.0	70.0	60.0
Cumulative ATNCF	\$MM						
Payback period	Years						
<b>Discounted payback period</b>							
Discounted net cash flow	\$MM						
Cumulative discounted ATNCF	\$MM						
Discounted payback period	\$MM						
<b>Compounded payback period</b>							
Nominal after tax net cash flow	\$MM						
Cumulative compounded ATNCF	\$MM						
Compounded payback period	Years						

**Payback, discounted payback and compounded payback - answer**

Calculate the payback, the discounted and compounded payback periods for the nominal after tax net cash flow shown below.

Assume a nominal discount rate or compound rate of 10%.

		Years -					
Units		1	2	3	4	5	6
<b>Payback period</b>							
Nominal after tax net cash flow	\$MM	-50.0	-100.0	90.0	80.0	70.0	60.0
Cumulative ATNCF	\$MM	-50.0	-150.0	-60.0	20.0	90.0	150.0
Payback period	Years				3.75		
<b>Discounted payback period</b>							
Discounted net cash flow	\$MM	-45.5	-82.6	67.6	54.6	43.5	33.9
Cumulative discounted ATNCF	\$MM	-45.5	-128.1	-60.5	-5.8	37.6	71.5
Discounted payback period	\$MM					4.1	
<b>Compounded payback period</b>							
Nominal after tax net cash flow	\$MM	-50.0	-100.0	90.0	80.0	70.0	60.0
Cumulative compounded ATNCF	\$MM	-50.0	-155.0	-80.5	-8.6	60.6	126.7
Compounded payback period	Years					4.1	

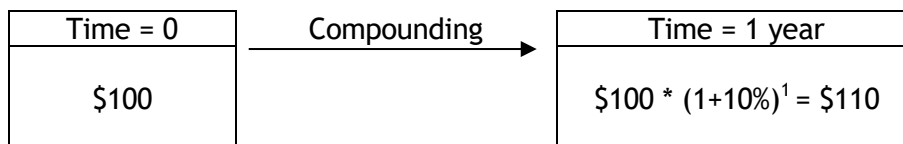
## 12 Discounting methods

In earlier sections, we discussed discounting future net cash flows assuming that they always occur at the end of a year. This is "end year discounting" and is based on an approximate representation of reality. In reality, of course, the annual net cash flows will almost never occur exactly at the end of a year. Costs and revenues will occur at irregular times during any particular year. At the early stages of a large project it may be difficult or impossible to predict when exactly they will occur.

In principle, we can carry out the discounting process under any assumption that we feel is the best approximate representation of reality. We can assume that the cash flows occur at the mid-point of each year, quarterly, monthly, or even that they are spread evenly over the year. The mathematics of discounting will be different depending on the method, but any method can be assumed. All methods are based on an approximation to the real situation.

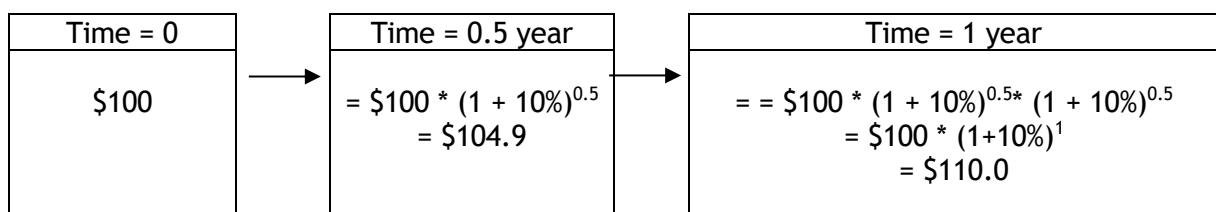
To illustrate the techniques of deriving other methods of discounting, we need to look back at the basic principles of the discounting process. In an earlier section, we derived the present value of a sum of money spent or received at the end of one year assuming a discount rate of 10%. This was derived by reversing the compound interest process which occurs, for example, when a sum of money is deposited in an interest-bearing bank account.

As shown at the beginning of this section, \$100 invested in a bank account at a 10% interest rate for a period of one year becomes \$110 at the end of the year. Diagrammatically -



### Mid-year discounting

As an alternative, we can think of this process occurring in two stages, namely \$100 invested for 6 months, and then the compounded amount being left in the bank for a further 6 months to give the same \$110 after one year. This time the diagram looks like this -



The above calculations indicate that taking the value of \$100 after 6 months as being  $\$100 * (1+10\%)^{0.5}$  is consistent with a value after one year of  $\$100 * (1+10\%)^1$ .

We can deduce that the value of a sum of money after 6 months is obtained by multiplying the sum of money by  $(1+r)^{0.5}$ , where  $r$  is the discount rate expressed as an annual rate.

Similarly, the compounding factor required to give the value of a sum of money after six monthly periods of time is as shown in Table 1 -

**Table 1 - Compounding factors for 6 month time periods**

Value of \$1 after half a year	=	$(1 + r)^{0.5}$
Value of \$1 after 1 year	=	$(1 + r)^{1.0}$
Value of \$1 after 1.5 years	=	$(1 + r)^{1.5}$
Value of \$1 after 2 years	=	$(1 + r)^{2.0}$
Value of \$1 after 2.5 years	=	$(1 + r)^{2.5}$
etc		etc

Conversely, we would use the equation in Table 2 to derive the net present value of net cash flows which occur in the middle of each year instead of the end of each year.

**Table 2 - Formula for mid-year discounting**

	NPV	=	$\frac{NCF_1}{(1 + r)^{0.5}}$	+	$\frac{NCF_2}{(1 + r)^{1.5}}$	+	.....	+	$\frac{NCF_n}{(1 + r)^{n-0.5}}$
where	NPV	=	Net present value						
	$NCF_1$	=	Net cash flow in year 1						
	$NCF_2$	=	Net cash flow in year 2						
	$NCF_n$	=	Net cash flow in year n						
	r	=	The discount rate						

## Comparing end and mid-year discounting

The formula for mid-year discounting in Table 2 makes sense because the net cash flows are in the middle of each year, instead of being at the end of each year,. In other words they are all 0.5 years closer to the present day. This means that we can restate the NPV using end year discounting as follows -

$$\text{NPV with mid-year discounting} = \text{NPV with end year discounting} * (1 + r)^{0.5}$$

We can compare end year discounting and mid-year discounting pictorially as shown in Figures 1 and 2.

Figure 1 assumes that the cash flows occur at the end of each year.

Figure 1 - End year discounting								
	Time							
	0	0.5	1	1.5	2	2.5	3	
NCF at end year	NCF <sub>0</sub>		NCF <sub>1</sub>		NCF <sub>2</sub>		NCF <sub>3</sub>	
End year discounting	$\frac{NCF_0}{(1+r)^0}$							
	$\frac{NCF_1}{(1+r)^1}$							
	$\frac{NCF_2}{(1+r)^2}$							
	$\frac{NCF_3}{(1+r)^3}$							

Figure 2 shows mid-year discounting and assumes that the cash flows occur in the middle of each year.

Figure 2 - Mid-year discounting								
	Time							
	0	0.5	1	1.5	2	2.5	3	
NCF in middle of year	NCF <sub>0</sub>	NCF <sub>1</sub>		NCF <sub>2</sub>		NCF <sub>3</sub>		
End year discounting	$\frac{NCF_0}{(1+r)^0}$							
	$\frac{NCF_1}{(1+r)^{0.5}}$							
	$\frac{NCF_2}{(1+r)^{1.5}}$							
	$\frac{NCF_3}{(1+r)^{2.5}}$							

By using similar logic, we can adopt other forms of discounting. For instance, quarterly net cash flows discounted quarterly from the end of each quarter would be derived as set out in Table 3.

**Table 3 - Formula for quarterly discounting**

$$NPV = \frac{NCF_{q1}}{(1+r)^{0.25}} + \frac{NCF_{q2}}{(1+r)^{0.50}} + \dots + \frac{NCF_{qn}}{(1+r)^{n*0.25}}$$

where

NPV = Net present value  
 $NCF_{q1}$  = Net cash flow in quarter 1  
 $NCF_{q2}$  = Net cash flow in quarter 2  
 $NCF_{qn}$  = Net cash flow in quarter n  
 $r$  = The annual discount rate

If we adopted mid quarter discounting, the discounted value of net cash flow in the nth quarter (" $NCF_{qn}$ ") would be  $NCF_{qn} / (1+r)^{n*0.25}$ .

## Discounting to different dates

Sometimes we need to discount to dates (decision points) other than the present day. This is the case if, for instance, we need to establish the value of a project at different points in time (discount dates).

In order to calculate the value of a project at some point in the future, - that is, if the discount date is in the future - we need first to eliminate all net cash flows before the start date. This is because any net cash flows occurring before the discount date are sunk costs and are therefore irrelevant to the decision (except in so far as they are needed to ensure that the fiscal calculations are correct). If the discount date is in the past, we need to include net cash flows that occurred in the past after the discount date. Once we have done this, one way to derive the discounted net cash flow at different dates is to -

- Calculate the NPV to the present day
- Compound the NPV forward to the future date specified, or
- Discount the NPV back to the past date specified.

For instance, if the NPV of a project is \$100, then the value of the project discounted to the middle of next year is  $\$100 * (1 + 10\%)^{0.5}$  equals \$104.9.

The value of the project discounted to the middle of last year is  $\$100 / (1 + 10\%)^{0.5}$  equals \$95.4 (assuming that the additional net cash flow from last year is zero).

## IRR and discounting methods

In an earlier section on the internal rate of return (IRR), we established that the IRR is the same whatever the start year for the net cash flow. By similar logic to that described above, provided the net cash flow remains the same, the IRR is the same whatever the discount date and whatever the method of discounting.



## Using spreadsheet functions

The net present value function used in some spreadsheets assumes end year discounting and that the first net cash flow in the calculation occurs at the end of year 1. If so, then it is easy to convert the calculation to mid-year or any other form of discounting by multiplying the result by an appropriate conversion factor. For instance, if we want mid-year discounting and the discount rate is 10%, we might enter in the cell in the spreadsheet the following expression -

"=NPV(10%,Range of cells containing the net cash flows)\*(1+10%)^0.5"

The first part of this expression will give the NPV using end year discounting. The remaining part after the multiplication sign will translate the NPV into an NPV with mid-year discounting.



## Discounting methods

Calculate the NPVs of the net cash flow below using different methods of discounting.  
Assume a nominal discount rate of 15%.

	Basis	Units	Total	Mid or end years -					
				1	2	3	4	5	6
Nominal after tax net cash flow		\$MM	150.0	-50.0	-100.0	90.0	80.0	70.0	60.0
Discounting each year's net cash flow									
NPV1 = NPV at time zero with end year discounting		\$MM							
NPV2 = NPV at time zero with mid year discounting		\$MM							
NPV3 = NPV at mid year 1 with end year discounting		\$MM							
NPV4 = NPV at mid year 1 with mid year discounting		\$MM							
NPV5 = NPV at start quarter 2 in year 1 with end year discounting		\$MM							
NPV6 = NPV at start quarter 2 in year 1 with mid year discounting		\$MM							
NPV7 = NPV at start year 3 with end year discounting		\$MM							
NPV8 = NPV at start year 3 with mid year discounting		\$MM							
Manipulating NPVs - draw the timelines and the NPV dates									
NPV1 = NPV at time zero with end year discounting		\$MM							
NPV2 = NPV at time zero with mid year discounting		\$MM							
NPV3 = NPV at mid year 1 with end year discounting		\$MM							
NPV4 = NPV at mid year 1 with mid year discounting		\$MM							
NPV5 = NPV at start quarter 2 in year 1 with end year discounting		\$MM							
NPV6 = NPV at start quarter 2 in year 1 with mid year discounting		\$MM							
NPV7 = NPV at start year 3 with end year discounting		\$MM							
NPV8 = NPV at start year 3 with mid year discounting		\$MM							

Not needed  
Not needed  
Not needed  
Not needed  
Not needed  
Not needed  
Not needed  
Not needed

## Discounting methods - answer

Calculate the NPVs of the net cash flow below using different methods of discounting.

Assume a nominal discount rate of 15%.

	Basis	Units	Total	Mid or end years -					
				1	2	3	4	5	6
Nominal after tax net cash flow		\$MM	150.0	-50.0	-100.0	90.0	80.0	70.0	60.0
Discounting each year's net cash flow									
NPV1 = NPV at time zero with end year discounting	$NCF_n / (1 + 15\%)^n$	\$MM	46.6	-43.5	-75.6	59.2	45.7	34.8	25.9
NPV2 = NPV at time zero with mid year discounting	$NCF_n / (1 + 15\%)^{(n - 0.5)}$	\$MM	49.9	-46.6	-81.1	63.5	49.1	37.3	27.8
NPV3 = NPV at mid year 1 with end year discounting	$NCF_n / (1 + 15\%)^{(n - 0.5)}$	\$MM	49.9	-46.6	-81.1	63.5	49.1	37.3	27.8
NPV4 = NPV at mid year 1 with mid year discounting	$NCF_n / (1 + 15\%)^{(n - 1)}$	\$MM	53.6	-50.0	-87.0	68.1	52.6	40.0	29.8
NPV5 = NPV at start quarter 2 in year 1 with end year discounting	$NCF_n / (1 + 15\%)^{(n - 0.25)}$	\$MM	48.2	-45.0	-78.3	61.3	47.4	36.0	26.9
NPV6 = NPV at start quarter 2 in year 1 with mid year discounting	$NCF_n / (1 + 15\%)^{(n - 0.75)}$	\$MM	51.7	-48.3	-84.0	65.7	50.8	38.6	28.8
NPV7 = NPV at start year 3 with end year discounting	$NCF_n / 1.15^n$ exclude years 1 and 2	\$MM	219.1			78.3	60.5	46.0	34.3
NPV8 = NPV at start year 3 with mid year discounting	$NCF_n / 1.15^{(n - 0.5)}$ exclude years 1 and 2	\$MM	234.9			83.9	64.9	49.4	36.8
Manipulating NPVs - draw the timelines and the NPV dates									
NPV1 = NPV at time zero with end year discounting	= NPV1 from above	\$MM	46.6					Not needed	
NPV2 = NPV at time zero with mid year discounting	= NPV1 * $(1 + 15\%)^{0.5}$	\$MM	49.9					Not needed	
NPV3 = NPV at mid year 1 with end year discounting	= NPV1 * $(1 + 15\%)^{0.5}$	\$MM	49.9					Not needed	
NPV4 = NPV at mid year 1 with mid year discounting	= NPV1 * $(1 + 15\%)^1$	\$MM	53.6					Not needed	
NPV5 = NPV at start quarter 2 in year 1 with end year discounting	= NPV1 * $(1 + 15\%)^{0.25}$	\$MM	48.2					Not needed	
NPV6 = NPV at start quarter 2 in year 1 with mid year discounting	= NPV5 * $(1 + 15\%)^{0.5}$	\$MM	51.7					Not needed	
NPV7 = NPV at start year 3 with end year discounting	= NPV7 from above	\$MM	219.1					Not needed	
NPV8 = NPV at start year 3 with mid year discounting	= NPV7 * $(1 + 15\%)^{0.5}$	\$MM	234.9					Not needed	

## 13 Comparing economic indicators

In earlier sections we discussed the meaning and use of the following economic indicators:-

Net Present Value  
Internal Rate of Return  
Payback  
Capital Productivity Index

We are now in a position to assess the relative merits of these indicators for evaluating the economic/financial merits of investments in the oil and gas industry. Table 1 summarises the relative merits of the different indicators by different criteria. It is intended to show the strengths of each indicator in isolation from the rest.

**Table 1 - Comparison of economic indicators**

Desirable criteria	Net present value	Internal rate of return	Payback	Capital productivity index
Takes time into account	Yes	Yes	Yes	Yes
Gives a unique value	Yes	No	Yes	Yes
Can add and subtract	Yes	No	No	No
Measures money value	Yes	No	No	No
Depends on project start date	Yes	No	No	No
Shows the size of the capex	No	No	No	No

The summary in this table shows that while each of the economic indicators satisfies some of the desirable criteria, only the NPV indicator satisfies all but one. In practice, the complete economic assessment of a project would involve calculating all the economic indicators shown. The IRR and Payback criteria are frequently used as an initial screening device. Company management may, for instance, filter projects by accepting initially all those with an internal rate of return in excess of 15%. However, the NPV is the single indicator which is consistently the most reliable and the one most frequently used in practice.

One problem with the NPV indicator is that it is a net figure. That is, it doesn't indicate the size of the investment (the same NPV could be obtained from the difference between revenues and costs for a large or a small project). This is particularly important in the context of capital budgeting when there is a constraint on the capital available for investment.

However, it should be borne in mind that, in practice, the economic indicators of a project are usually not viewed in isolation. Other, non-quantitative factors (corporate strategy, economic and political risk, etc) are also considered alongside the quantifiable aspects (including economic indicators) when field development decisions or exploration decisions are taken. In the final analysis, it is the task of company management to weigh-up all the

different aspects of the investment decision. Economic indicators should be used merely as an aid to this process.

## 14 Economic indicators summary

The following is a summary of the main points of this section.

### Net present value (NPV)

The NPV is a single measure.

The NPV shows value in excess of capex.

The NPV takes time into account.

The NPV balances cash flows over time.

The NPV compares the project against alternatives.

The NPV is the extra you would need to invest in the alternative investment to match your project.

The discount rate is the return on your alternative investment.

The discount rate is not the loan rate. It is the deposit rate.

NPV is used to make go, no go decisions.

NPV is used to value projects and helps determine sale or purchase price of oil and gas properties.

### Internal rate of return (IRR)

The IRR is a discount rate that makes the NPV zero.

There are as many IRRs as there are changes of sign in the net cash flow.

There are at least 2 changes of sign in the net cash flow of many oil and gas projects (because of abandonment). Therefore they have at least 2 IRRs.

The IRR is just a percentage. It doesn't tell us how much money we make.

The IRR tells us how fast we make money (the rate of return)

The IRR stays the same when we delay the project (provided the net cash flow does not change)

### Capital productivity index (CPI)

The Capital Productivity Index (CPI) is the NPV divided by the PV of the capex.

The CPI should be more than zero for a project to be economically viable.

Some companies reject projects with CPIs less than 0.2 or 0.3.

This is a crude attempt to take risk into account. Such practices can lead to the rejection of very attractive projects.

The CPI does not tell us how much money we make. It is simply a ratio.

The CPI is useful for choosing projects with a limited budget.

## **Payback**

Payback is the time taken to recover the investment.

Payback occurs when the cumulative after tax net cash flow first becomes positive.

Pay back does not tell us how much money we make.

There is no benchmark to tell us what a good payback is and what a bad payback is.

Long payback is not necessarily bad (eg LNG projects). Short payback is not necessarily good.