Multiple random variable analysis and Monte Carlo simulation

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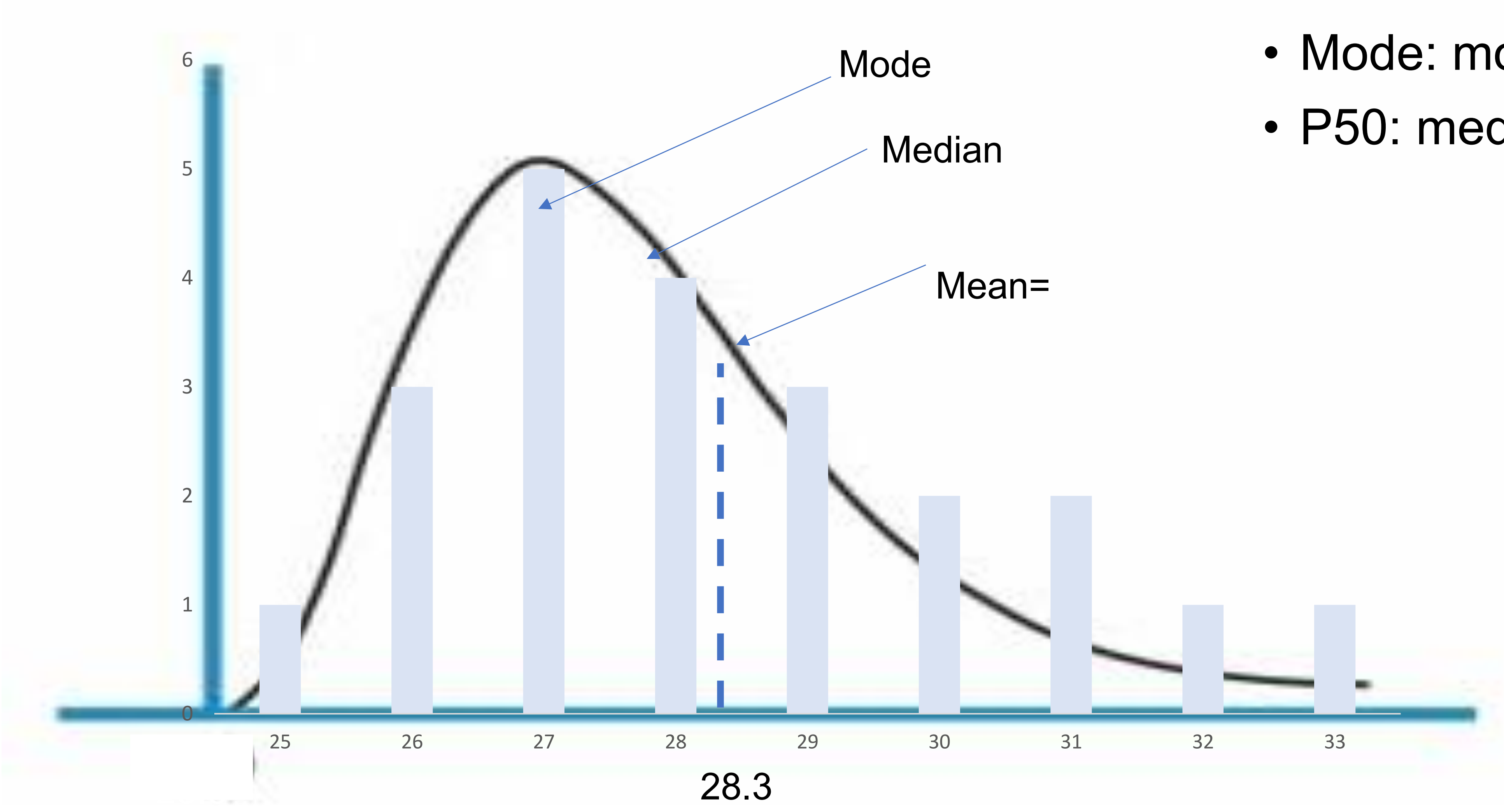
Probability concepts in economic analysis

Probability distribution of a random variable

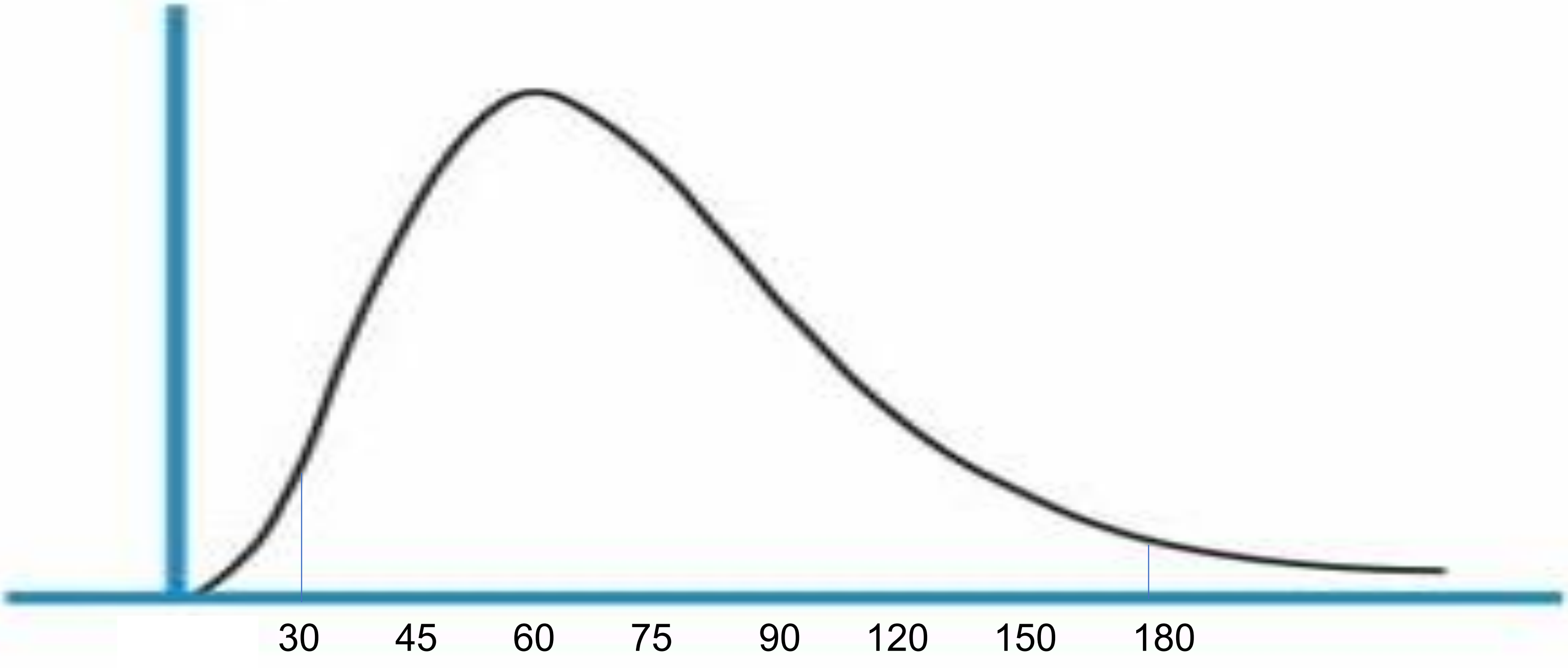


- Mode: most likely
- P50: median

Using extreme values to describe the uncertainty

Cumulative density function (CDF): $F(x)=P\,[X \le x]=\alpha$

Percentile :$P_{x\%}$: the value of x such that the probability of getting a value less than or equal to x is α/100 and the probability of getting value greater than or equal to x is (100-α)/100

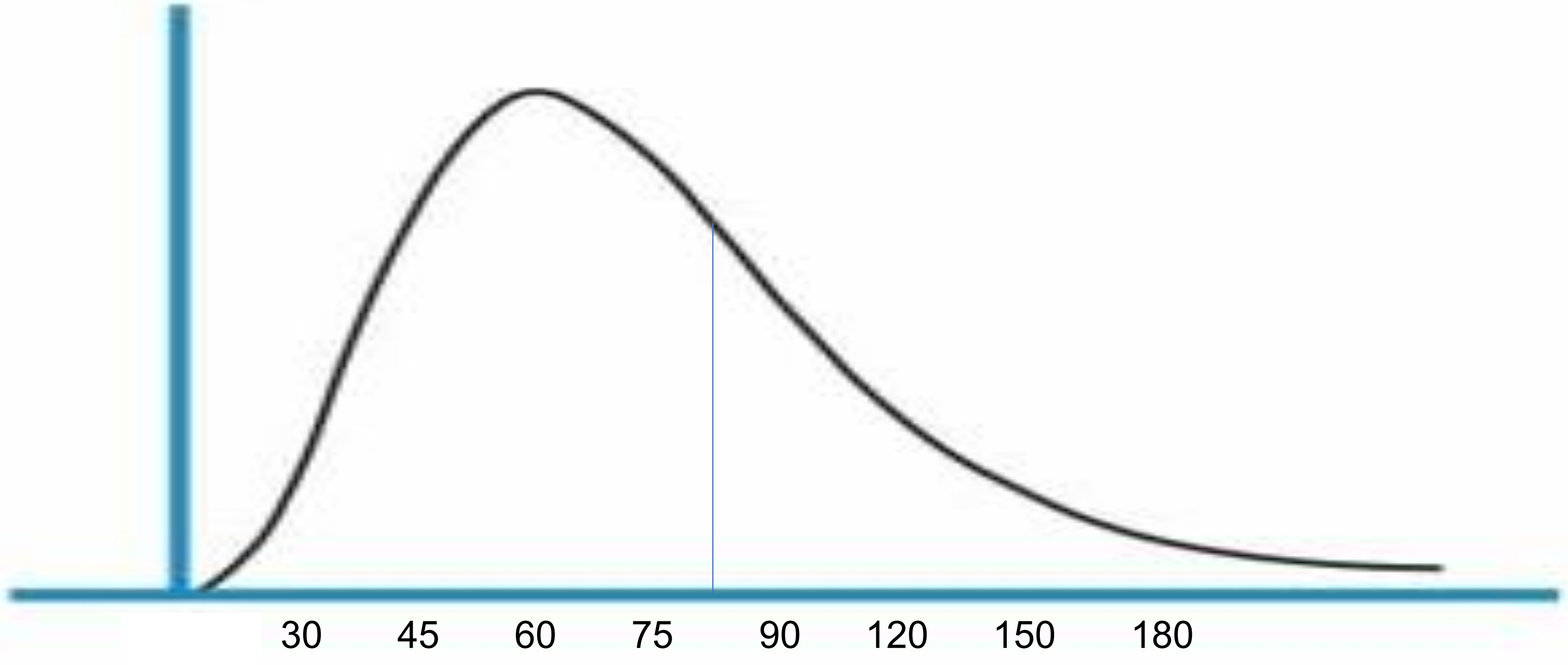


P_{90} F(30)=0.1

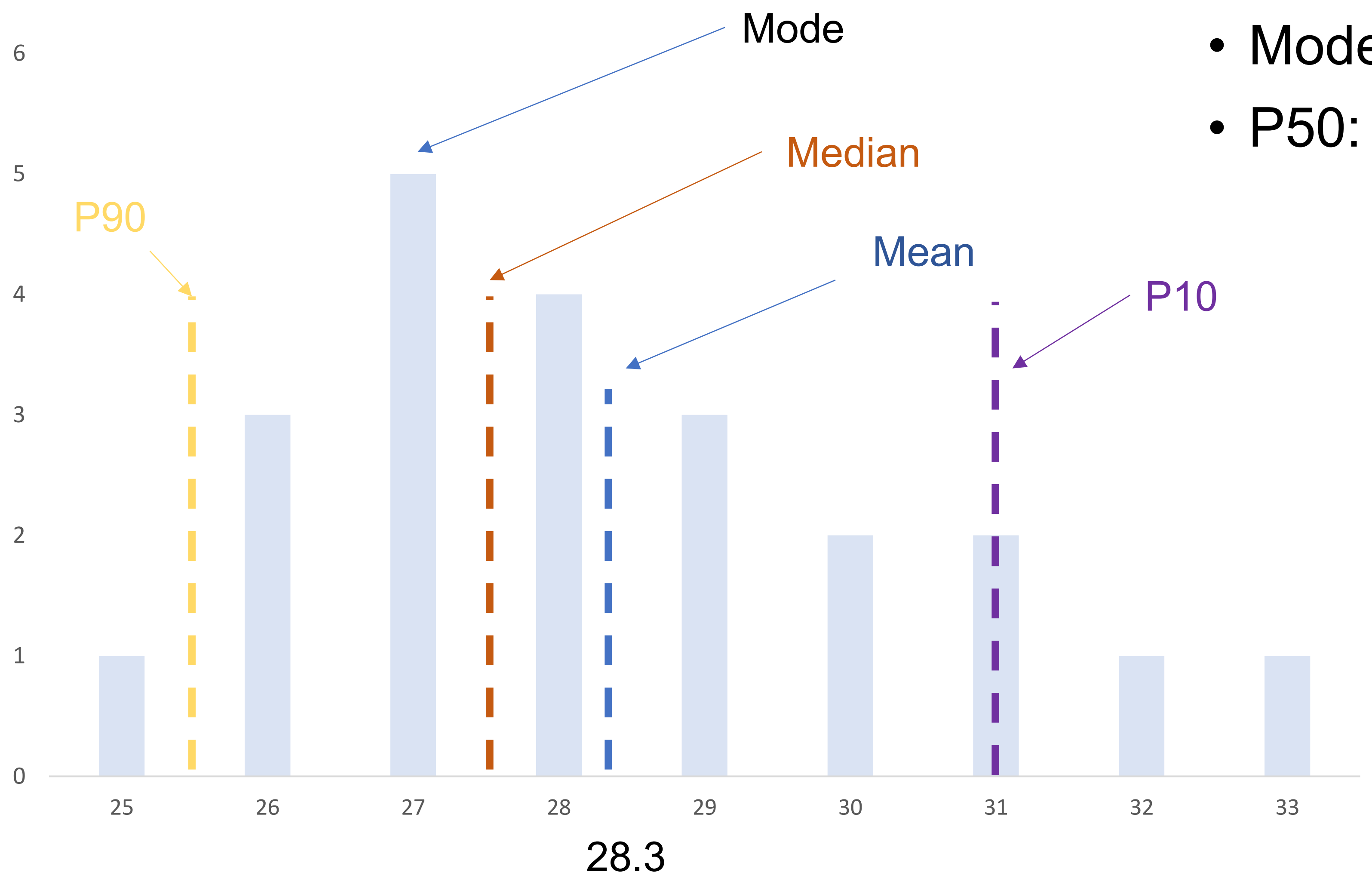
P_{10} F(180)=0.9

Mean, Expected value

- Takes uncertainty into account by considering the probability of each possible outcome and using this information to calculate an expected value.
- The information is reduced to a single number resulting in easier decisions.



Probability distribution of a random variable



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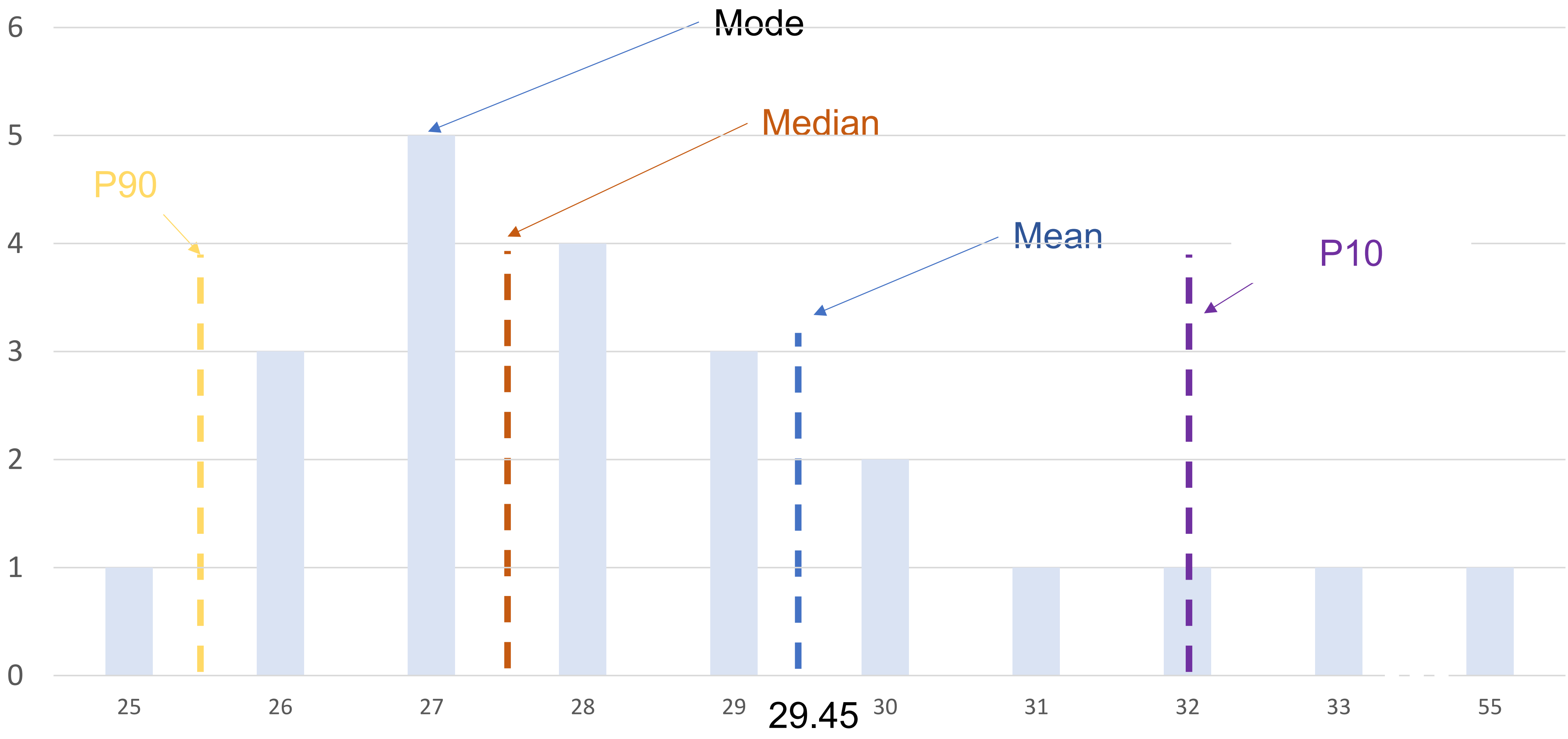
Probability distribution of a random variable



Exercise

A company used to sell its products in two different markets A and B

Each product was offered in both markets for an identical time period.

The historical sales in two markets are given in Excel file: Market

Calculate the statistics to compare the two markets.

Changing the Value of More Than One Factor at a Time

01

To this point we have only looked at changes in one factor at a time.

02

Each factor considered can change, so it is useful to look at the effect of simultaneous changes in factors of interest.

03

One way to accomplish this is to use the Optimistic-Most Likely-Pessimistic (O-M L-P) technique.

Estimating a value as a function of multiple variables

Estimated Profit:Estimated revenue –Estimated cost

Optimistic-Most Likely-Pessimistic

- Establish optimistic (the most favorable), most likely, and pessimistic (the least favorable) estimates for each factor.
- Perform analysis under each condition for insight into the sensitivity of the solution.
- The results can be seen on a spider plot for further insight.

Optimistic-Most Likely-Pessimistic

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Revenue	Optimistic	Most likely	Pessimistic
	20000	15000	10000

Cost	Optimistic	Most likely	Pessimistic
	4000	8000	15000

Profit	Optimistic	Most likely	Pessimistic
	16000	7000	-5000

Optimistic-Most Likely-Pessimistic

- Establish optimistic (the most favorable), most likely, and pessimistic (the least favorable) estimates for each factor.
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Calculate the revenue range with the Optimistic-Most Likely- Pessimistic approach

Sales volume: (Market share) × (market size)

Market size	Max	Mid	Min
	30000	25000	20000

Revenue	Optimistic	Most likely	Pessimistic
	20000	15000	10000

Cost	Optimistic	Most likely	Pessimistic
	4000	8000	15000

Does not tell anything about uncertainty!

Profit	Optimistic	Most likely	Pessimistic
	16000	7000	-5000

Exercise

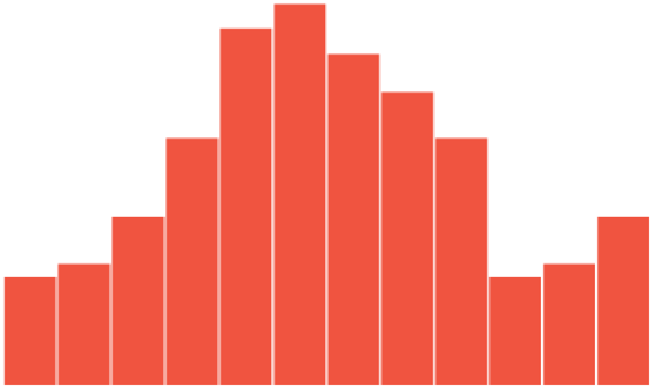
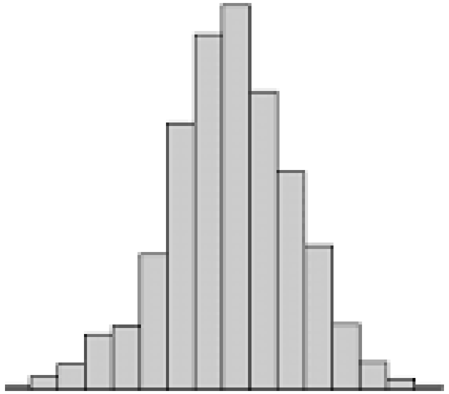
- **Calculate the revenue range with the Optimistic-Most Likely-Pessimistic approach**
- **Sales volume: (Market share) × (market size)**

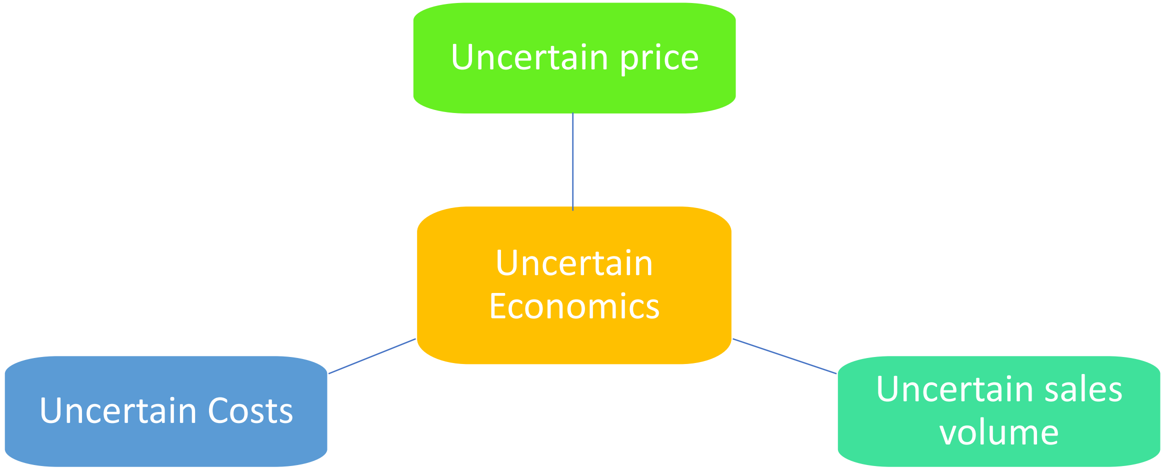
Market share	Max	Mid	Min
	20%	15%	10%

Market size	Max	Mid	Min
	30000	25000	20000

Estimating a value as a function of multiple variables

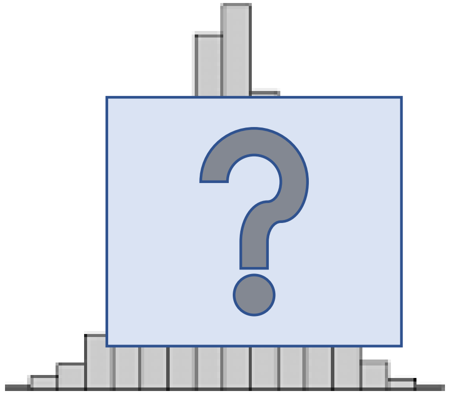
- Estimated Profit: Estimated revenue –Estimated cost

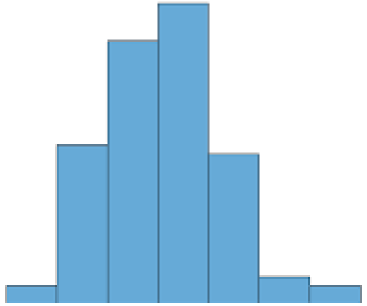



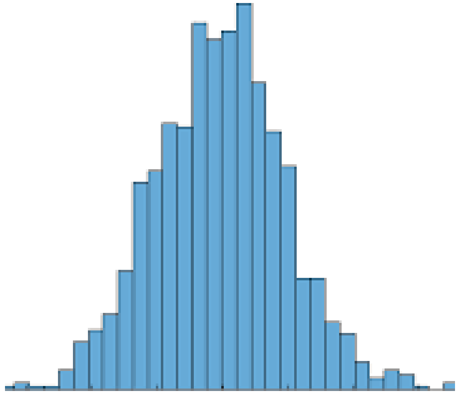



Estimating a value: function of multiple variables

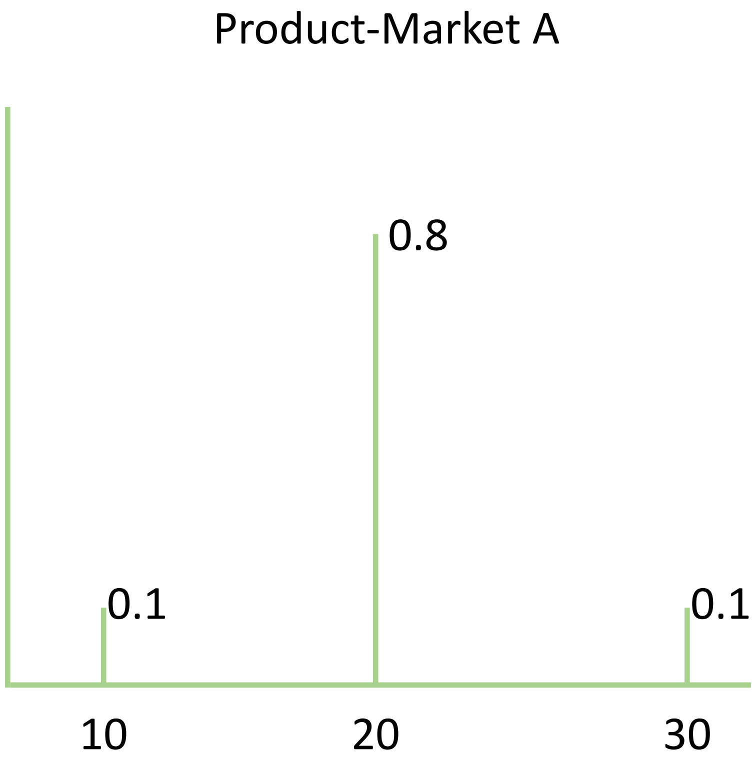
- Estimated Revenue: Product A revenue + Product B revenue

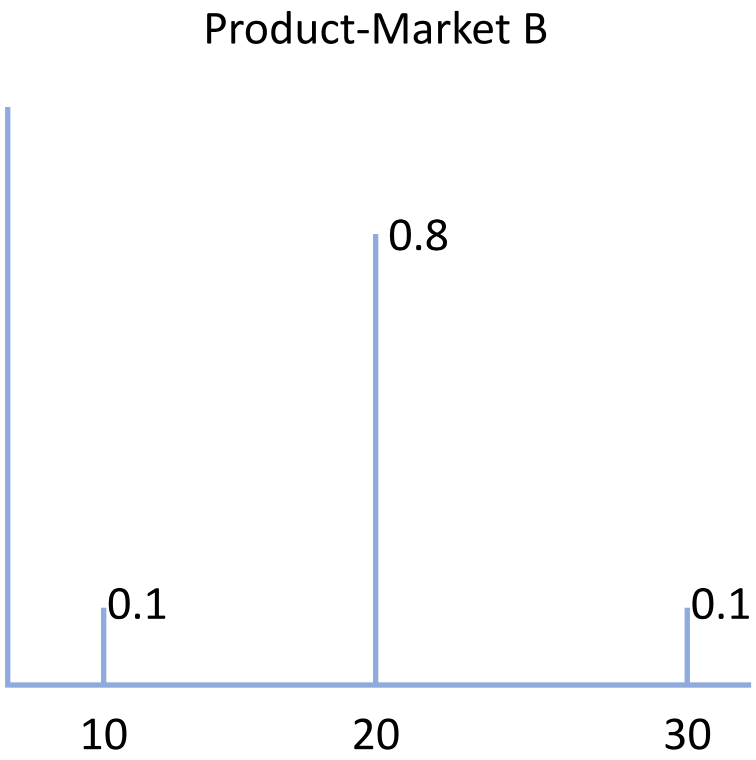






Risk of a product-market portfolio





Assuming Perfect Positive correlation

A perfect positive correlation occurs when two variables move in the same direction at a constant rate.

This relationship is represented by a correlation coefficient (r) of +1.

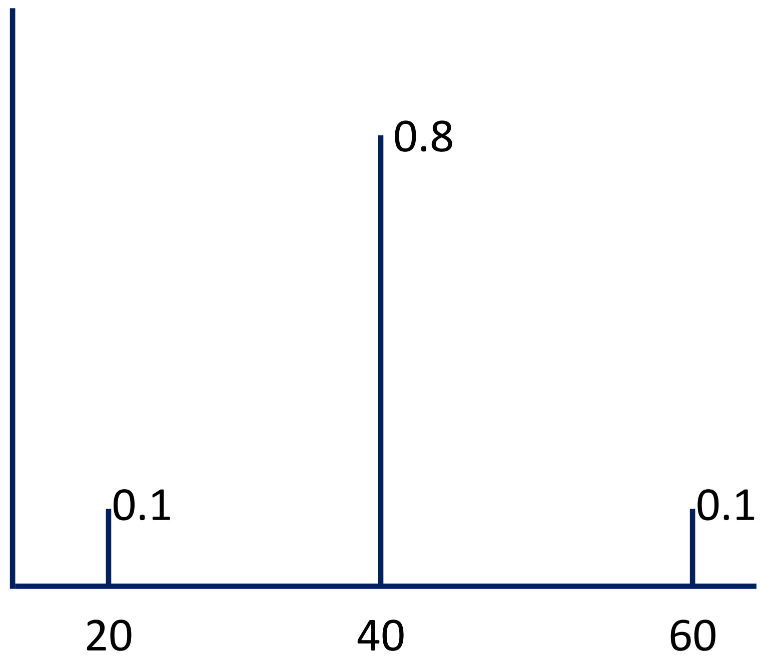
Linear Relationship: Every increase in one variable results in a proportional increase in the other.

Assuming Perfect Positive correlation

Market A- Sales volume	Probability	Market B- Sales volume	Probability	Market A + Market B Sales volume	P(A+B)
10	0.1	10	$1 \times 0.1 = 0.1$	20	0.1
20	0.8	20	$1 \times 0.8 = 0.8$	40	0.8
30	0.1	30	$1 \times 0.1 = 0.1$	60	0.1

Assuming Perfect Positive correlation

Product-Market A+B



Assuming Perfect Negative correlation

A perfect negative correlation occurs when two variables move in opposite directions at a constant rate.

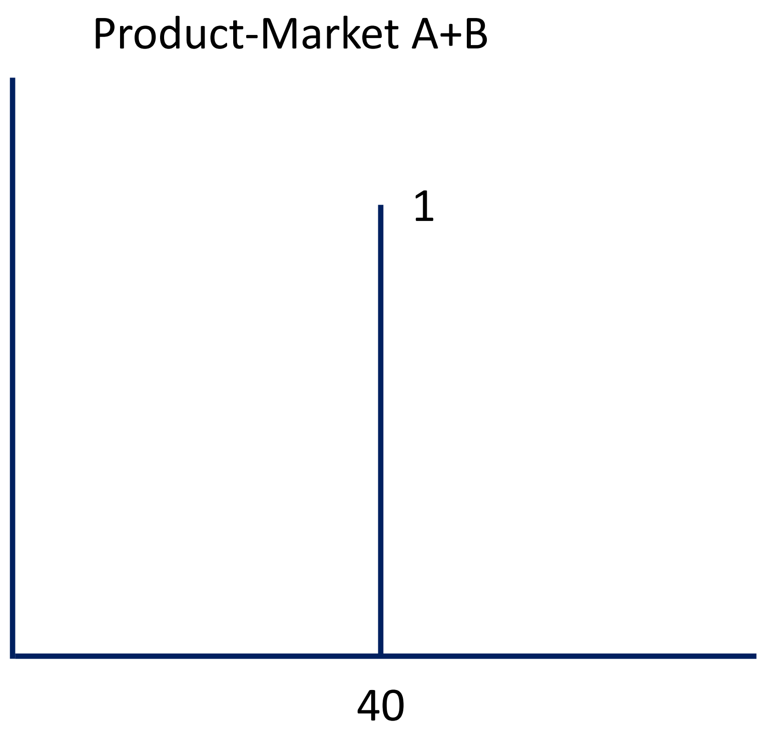
This relationship is represented by a correlation coefficient (r) of -1.

Linear Relationship: Every increase in one variable results in a proportional decrease in the other.

Assuming Perfect Negative correlation

Market A-Sales volume	Probability	Market B-Sales volume	Probability	Market A + Market B Sales volume	P(A+B)
10	0.1	30	1×0.1=0.1	40	0.1
20	0.8	20	1×0.8=0.8	40	0.8
30	0.1	10	1×0.1=0.1	40	0.1

Assuming Perfect Negative correlation



Assuming independence

Independence between two variables means that the occurrence or value of one variable does not affect the occurrence or value of the other.

No Relationship: Changes in one variable do not influence the other.

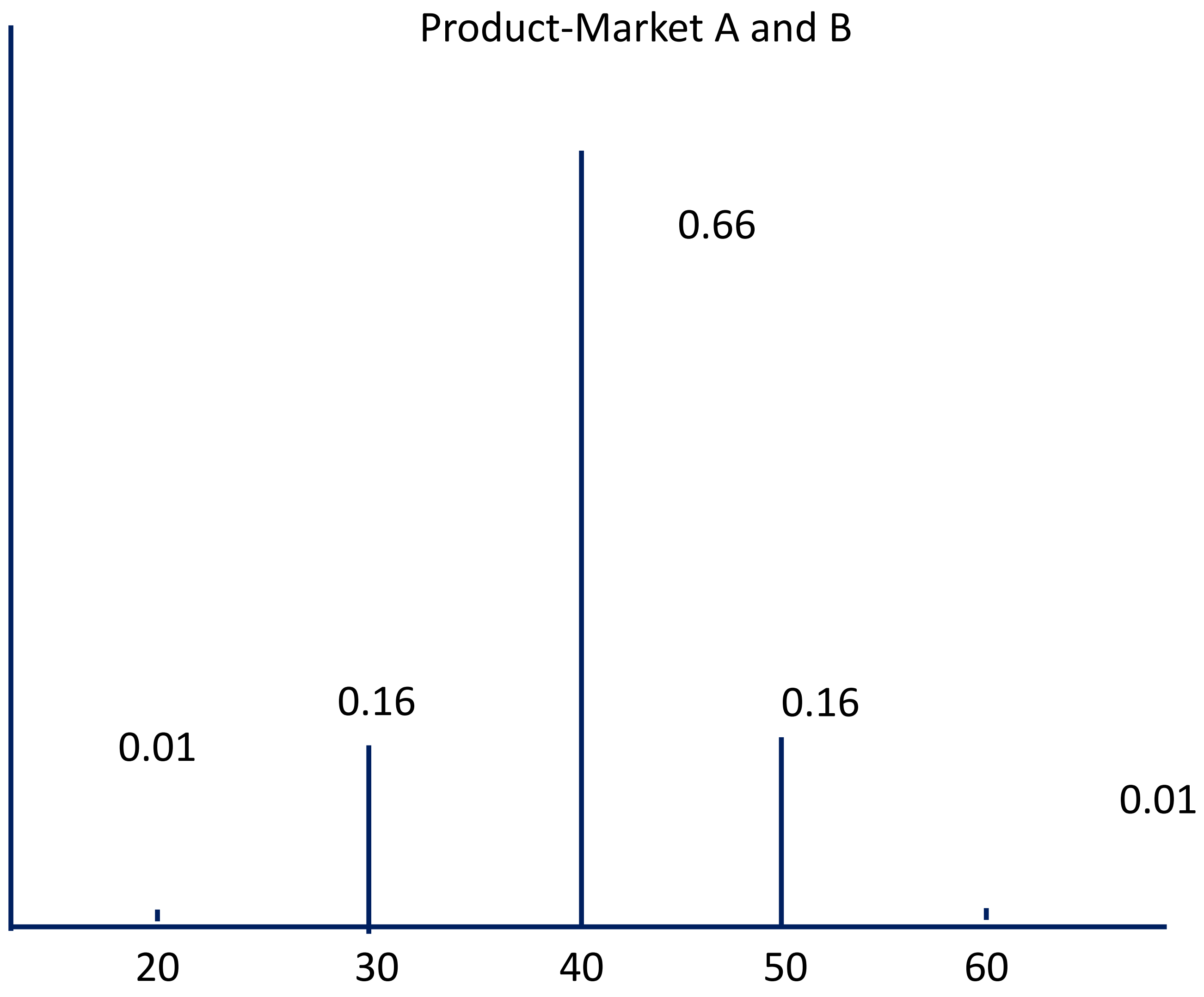
Assuming independence

Market A- Sales volume	Probability	Market B- Sales volume	Probability	Market A + Market B Sales volume	P(A+B)
10	0.1	10	0.1	20	0.1×0.1=0.01
10	0.1	20	0.8	30	0.08
10	0.1	30	0.1	40	0.01
20	0.8	10	0.1	30	0.08
20	0.8	20	0.8	40	0.64
20	0.8	30	0.1	50	0.01
30	0.1	10	0.1	40	0.01
30	0.1	20	0.8	50	0.08
30	0.1	30	0.1	60	0.01

Product-Market portfolio: No independence

Market A- Sales volume	Probability	Market B- Sales volume	Probability	Market A + Market B Sales volume	P(A+B)
10	0.1	10	0.1	20	0.1×0.1=0.01
10	0.1	20	0.8	30	0.08
10	0.1	30	0.1	40	0.01
20	0.8	10	0.1	30	0.08
20	0.8	20	0.8	40	0.64
20	0.8	30	0.1	50	0.01
30	0.1	10	0.1	40	0.01
30	0.1	20	0.8	50	0.08
30	0.1	30	0.1	60	0.01

Assuming independence



Monte Carlo Simulation in economic analysis

Monte Carlo Simulation in economic analysis

- Monte Carlo simulation is a statistical technique that uses random sampling and repeated computation to model and analyze complex systems or processes.
- It helps in understanding the impact of risk and uncertainty in economic forecasts and decision-making.

Another Way to Handle Uncertainty Is to Use Monte Carlo Simulation

Based on the probability of different outcomes for each random variable, a particular value is randomly generated.

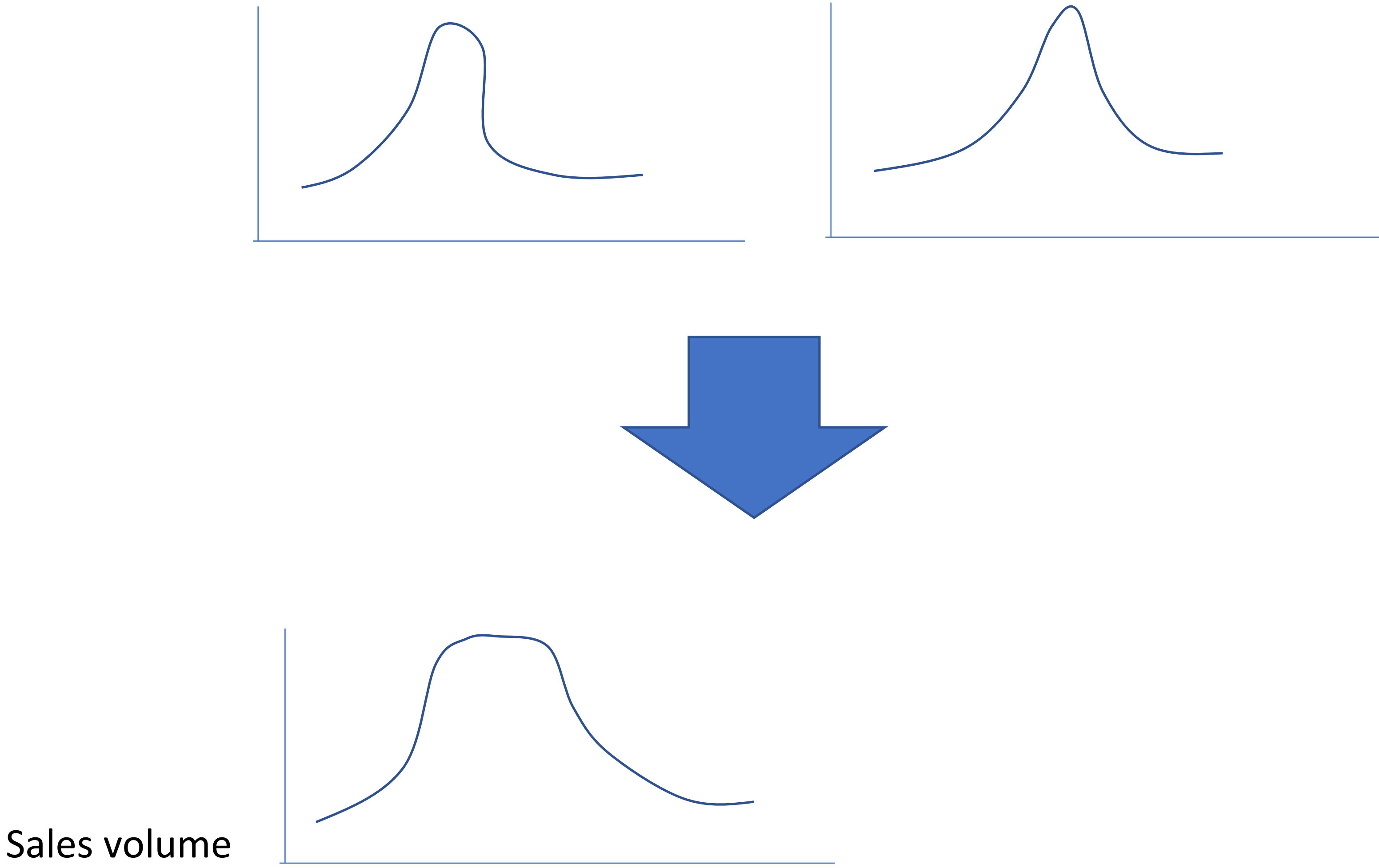
The numbers generated for all random variables constitute an instance or realization of reflecting a particular outcome.

Hundreds or thousands of these *instances* are generated, and these are examined to assist in decision making.

A caution: these will yield long term, average results, and you will be able to see the variation over time. However, your decision may be a one-time decision, so don't expect the "average" outcome to be your outcome.

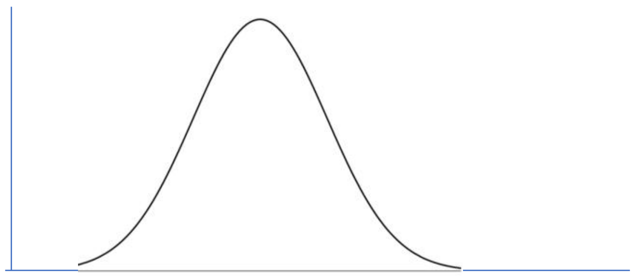
Monte Carlo Simulation

- Construct sales volume
- Random variables:
  - Market size
  - Market share

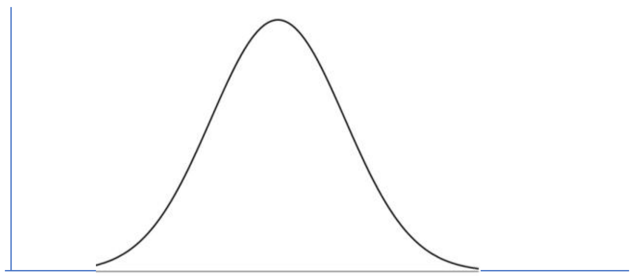


Monte Carlo Simulation

- Construct Revenue as a function of Market-product sales
- Random variables:
  - Product-Market A sales volume
  - Product-Market B sales volume



Market A-Sales volume probability distribution function



Market B-Sales volume probability distribution function

Steps in Monte Carlo Simulation:

Define the Model: Identify the economic model and variables of interest.

Assign Probability Distributions: Determine the distributions for uncertain parameters.

Run Simulations: Execute the simulation multiple times to generate possible outcomes.

Analyze Results: Summarize findings through statistical measures (mean, variance, percentiles).

Monte Carlo Simulation

Exercise

Perform the Monte Carlo simulation on three product-markets to analyse the revenue stream