MANY INVESTMENTS AND DECISION TREE
T2-2025
DR. SHIVA ABDOLI

Many Investments

Potential investments

Independent projects

Success or failure in one project does not impact other projects

Mining industry, innovative IT products, …

Potential investments

3 Independent mining projects

Mine A

$P_{success-A}$:0.3, NPV: 48$M

$P_{Failure-A}$:0.7, Loss: -10$M

Mine B

$P_{success-B}$:0.3, NPV: 48$M

$P_{Failure-B}$:0.7, Loss: -10$M

Mine C

$P_{success-C}$:0.3, NPV: 48$M

$P_{Failure-C}$:0.7, Loss: -10$M

Expected value of mining decisions

Outcome 1:
0.3 × (48=58-10)=14.4

Expected value of success

+

Outcome 2:
0.7 × (-10)=-7

Expected value of failure

Expected value of mining decision A

14.4-7=7.4 M

Expected value of mining decision B

14.4-7=7.4 M

Expected value of mining decision C

14.4-7=7.4 M

Expected value of mining decisions

Expected value of mining decision A

14.4-7=7.4 M

Expected value of mining decision B

14.4-7=7.4 M

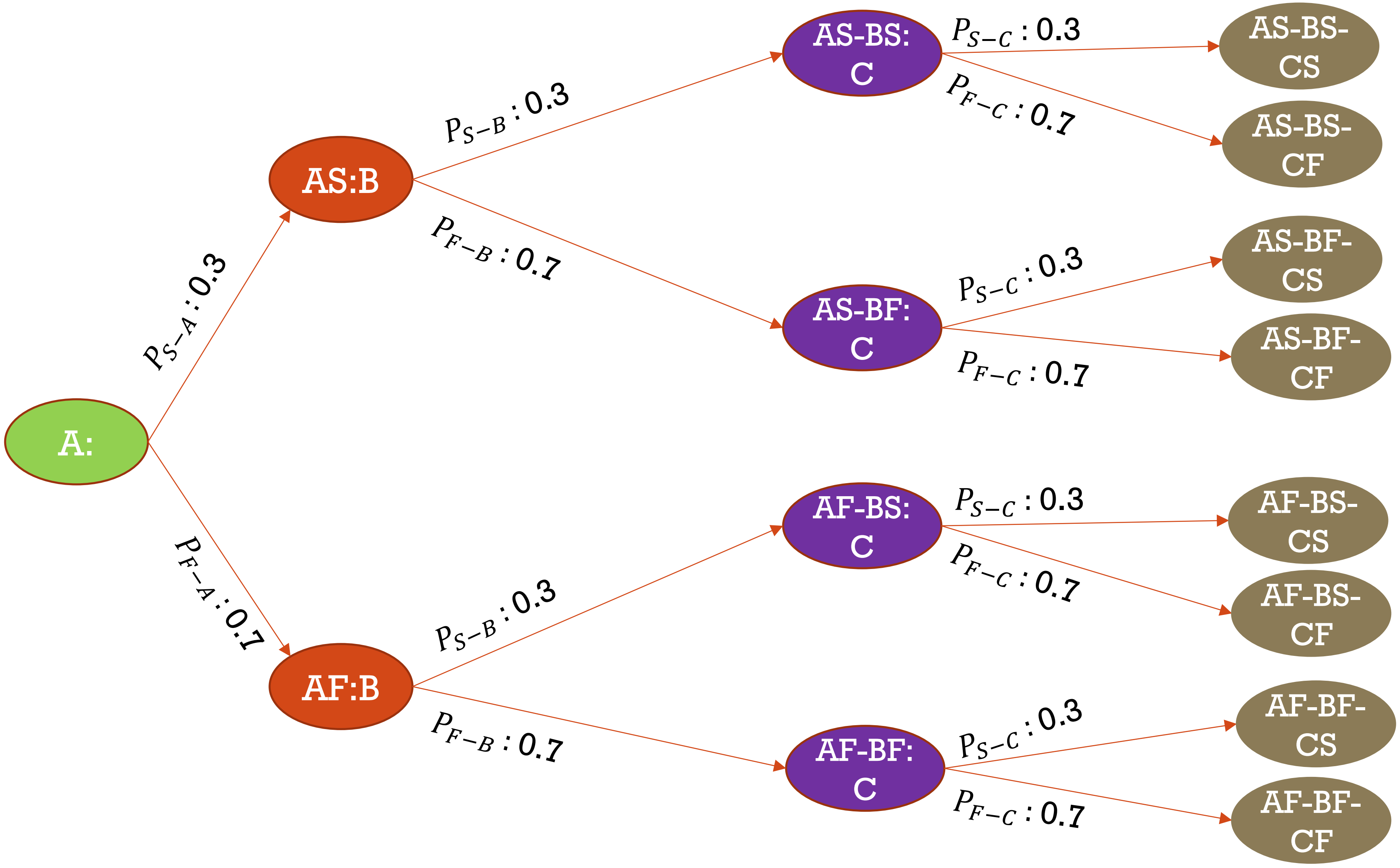
Expected value of mining decision C

14.4-7=7.4 M

3 independent projects

EV=3*7.4=22.2

Expected value of mining decisions



P= 0.3*0.3*0.3=0.027
NPV: 48*3=144,
EV:144*0.027=3.888

P= 0.3*0.7*0.7=0.147
NPV: 48-10-10=28,
EV:28*0.147=4.116

P= 0.7*0.7*0.7=0.343
NPV: -10-10-10=-30,
EV:-30*0.343=-10.29

Expected value of mining decisions

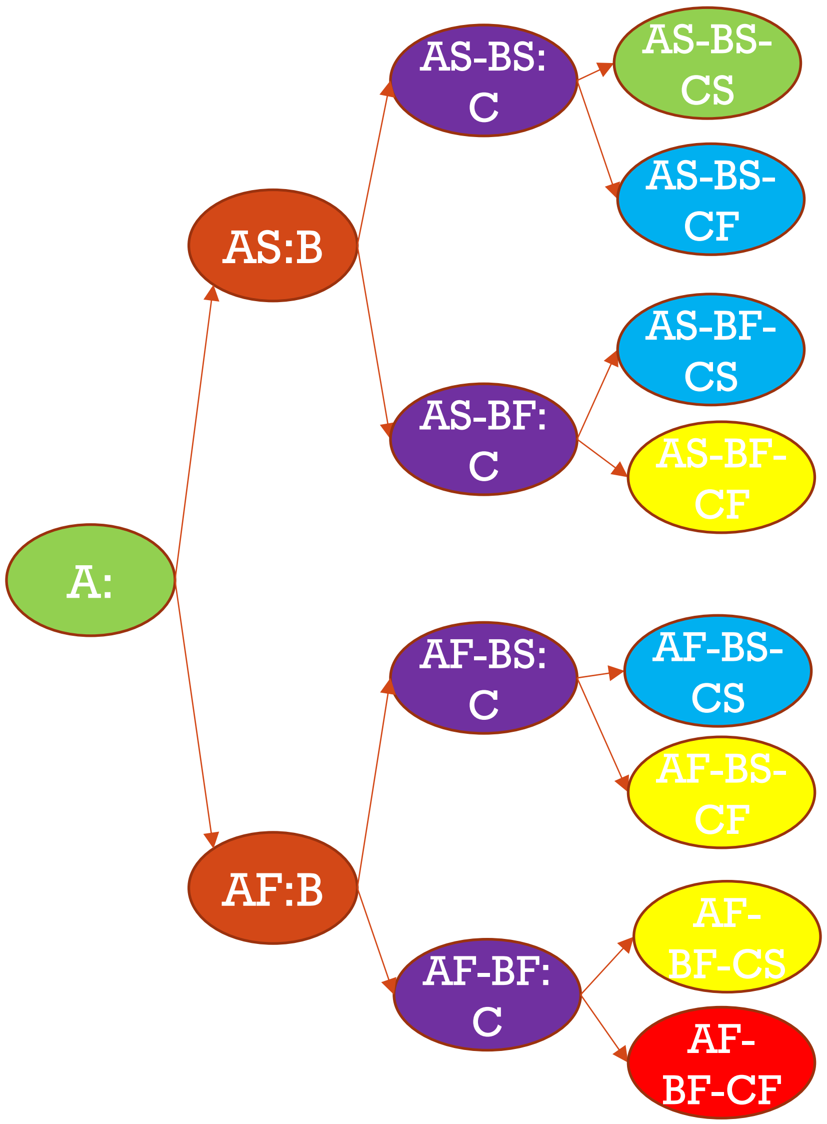
	Outcome	Probability	Outcome	Probability	Outcome	Probability	Probability of set	NPV	EV
	A-Success	0.3	B-Success	0.3	C-Success	0.3	0.027	144	3.888
					C- Failure	0.7	0.063	86	5.418
			B- Failure	0.7	C-Success	0.3	0.063	86	5.418
					C- Failure	0.7	0.147	28	4.116
A									
			B-Success	0.3	C-Success	0.3	0.063	86	5.418
					C- Failure	0.7	0.147	28	4.116
	A-Failure	0.7	B- Failure	0.7	C-Success	0.3	0.147	28	4.116
					C- Failure	0.7	0.343	-30	-10.29

Probability analysis

- Number of success in three mining projects: 3, 2, 1, 0

- Probability of 3 success: 1*0.027=0.027 (1 circumstance)
- Probability of 2 success: 3*0.063=0.189 (3 circumstance)
- Probability of 1 success: 3*0.147=0.441 (3 circumstance)
- Probability of 0 success: 1*0.343=0.343 (1 circumstance)

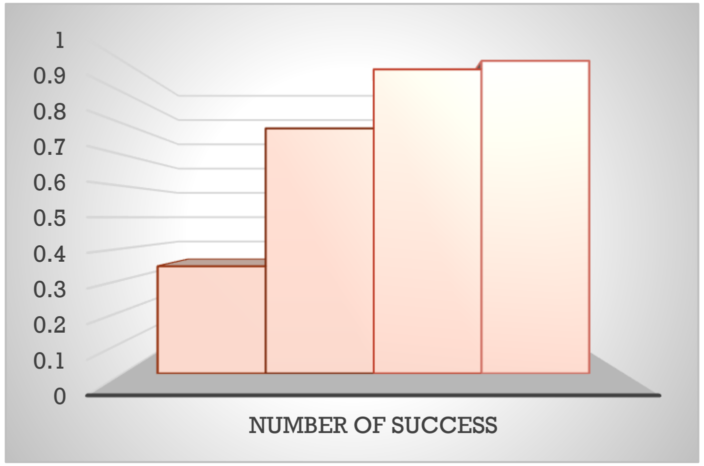
1

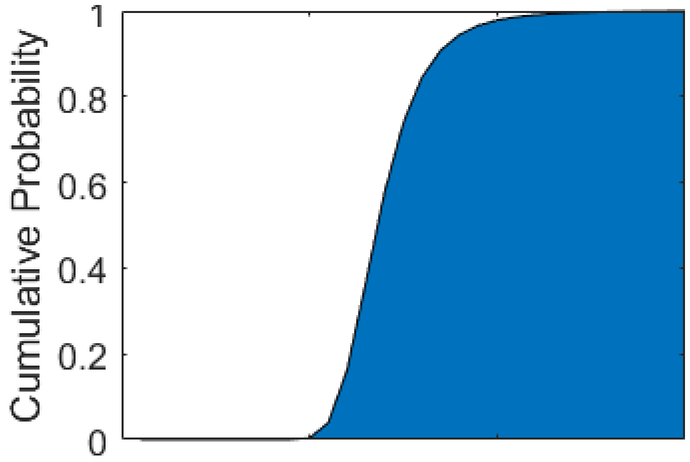


Probability analysis

Number of success	Probability
3 success	0.027
2 success	0.189
1 success	0.441
0 success	0.343

Number of success	Cumulative probability
0 success	0.343
1 or less success	0.784
2 or less success	0.973
3 or less success	1

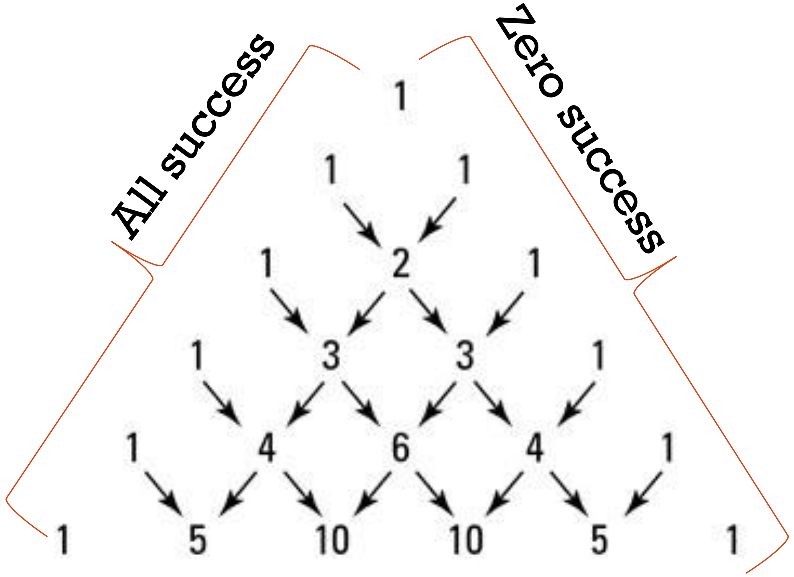




Probability analysis

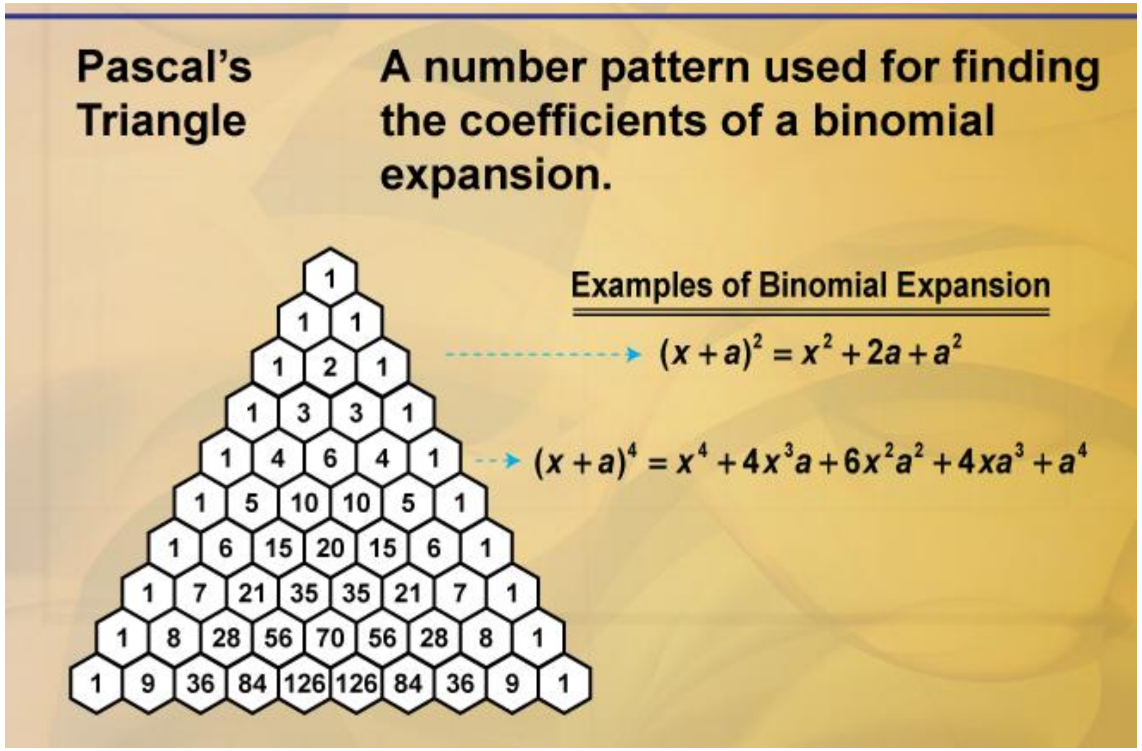
- Number of success in 4 projects: 4, 3, 2, 1, 0
- Number of outcomes: 2 ×2× 2×2= 16
- Probabilities :
- $(1\times P_{success}^{4}) + (4\times P_{success}^{3} \times P_{failure}^{1}) + (6\times P_{success}^{2} \times P_{failure}^{2}) + (4\times P_{success}^{1} \times P_{failure}^{3}) + (1\times P_{failure}^{4}) = 1$

Pascal triangle for coefficients of binomial event



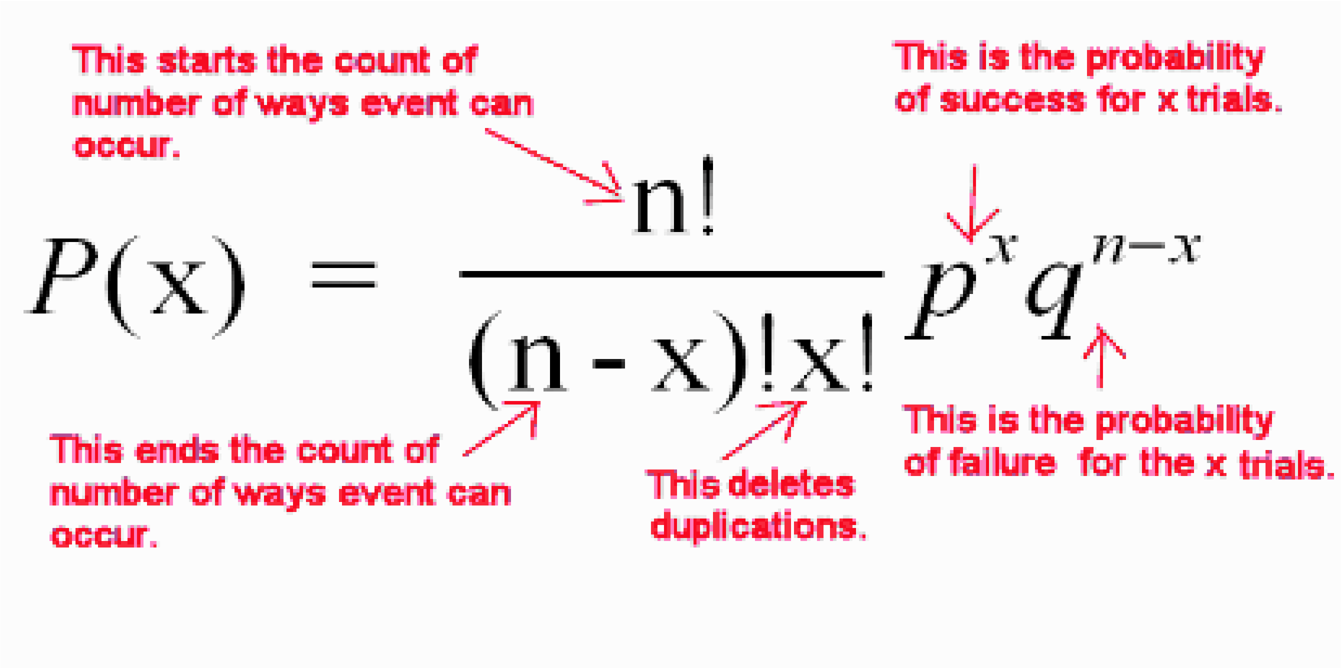
1 event : number of outcomes: $2^1 = 2$

2 events : number of outcomes: $2^2 = 4$



Binomial model

- Total Number of successes in N events: s
- Number of events : N
- All Possible number of circumstances to have s number of successes in N events:
- C(N, S)=$\frac{N!}{S!\times(N-S)!}$
- Probability of S success in N events = C(N, S) $*(P_{success})^{(S)}(1-P_{success})^{(N-S)}$

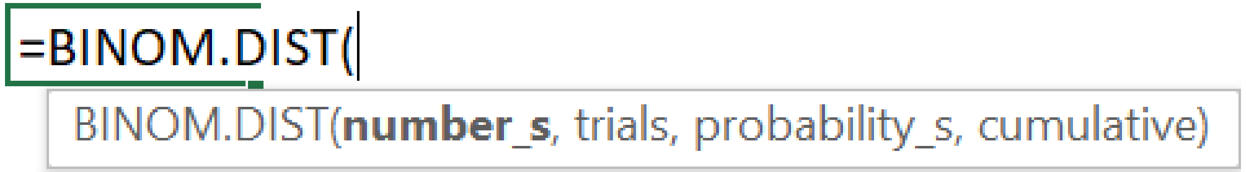


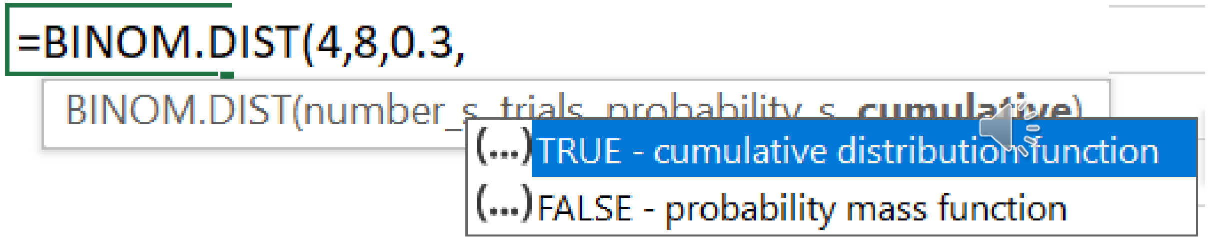
Binomial model

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

$$C(8,4) = \frac{8!}{4! \times (8-4)!} = \frac{8*7*6*5*4!}{4! \times 4*3*2} = 70$$

Probability of $C(8,4) = 70*(0.3)^{(4)}(0.7)^{(8-4)} = 0.136$





Binomial model

=BINOM.DIST(

BINOM.DIST(**number_s**, trials, probability_s, cumulative)

=BINOM.DIST(4,8,0.3,

BINOM.DIST(number_s, trials, probability_s, **cumulative**)

(...) TRUE - cumulative distribution function

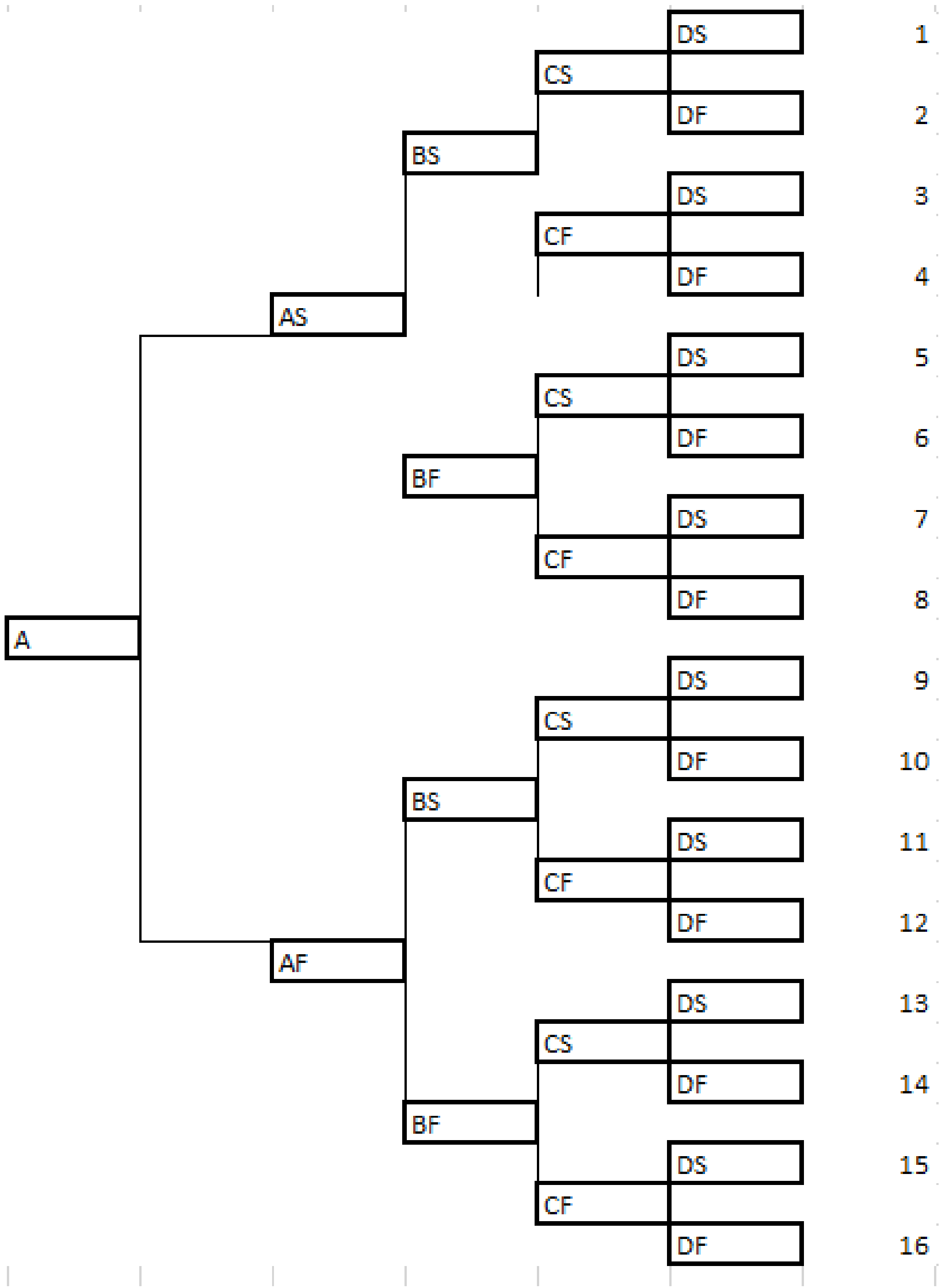
(...) FALSE - probability mass function

Number of success	Mass distribution function
4	0.1361367
3	0.25412184
2	0.29647548
1	0.19765032
0	0.05764801
	0.94203235

	Cumulative distribution function
F(4)	0.942032

Decision tree in economic analysis

Event tree


A
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14
15
16


Event Tree

Decision tree with information

High value outcome: High NPV

Low value outcome: Low NPV

Cost of getting information about a possible outcome: cost of testing the rocks

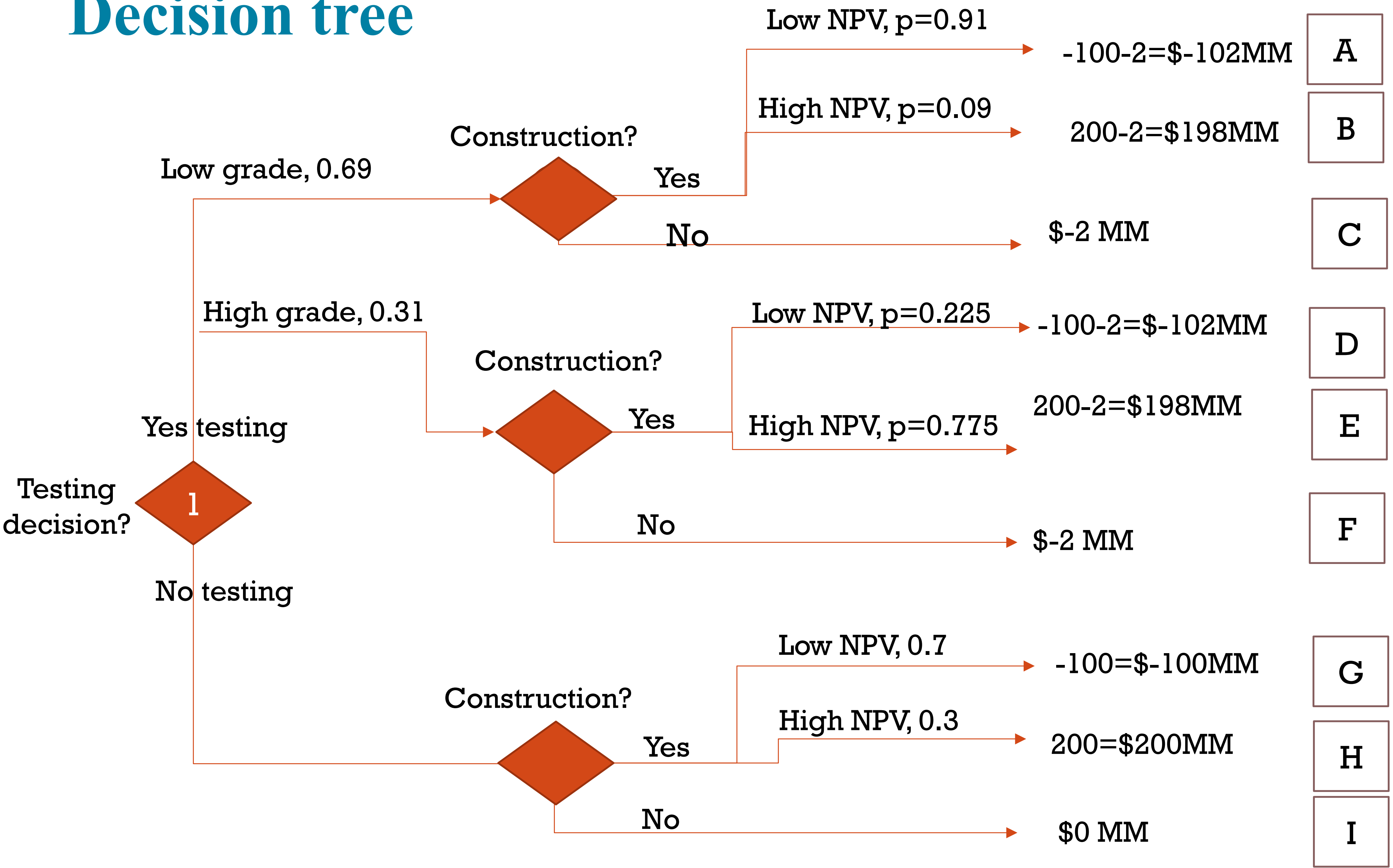
Decision tree

High NPV: 200 MM

Low NPV: -100 MM

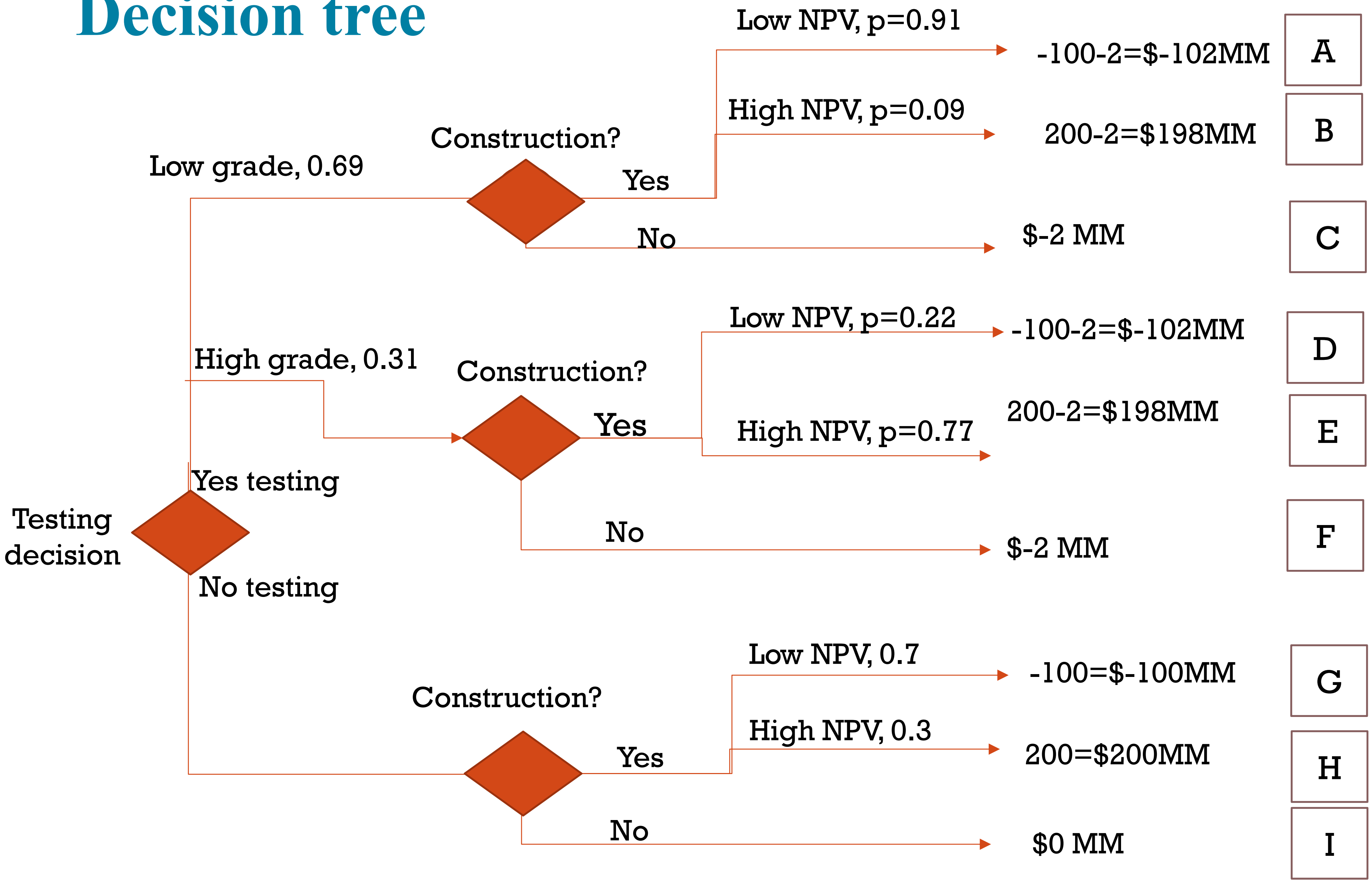
Cost of testing: -2$MM

Decision tree



Expected value of A and B:
-102*0.9+198*0.1-=-75

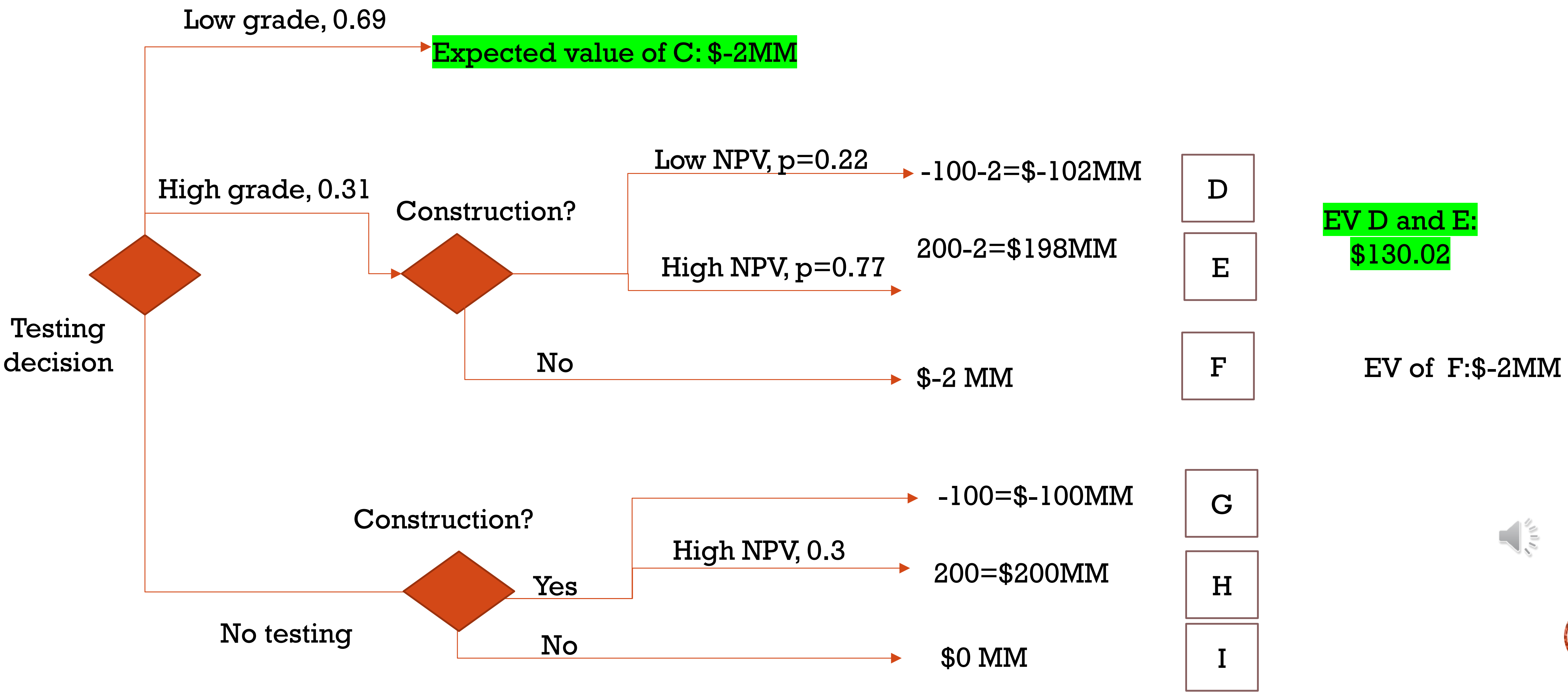
Decision tree



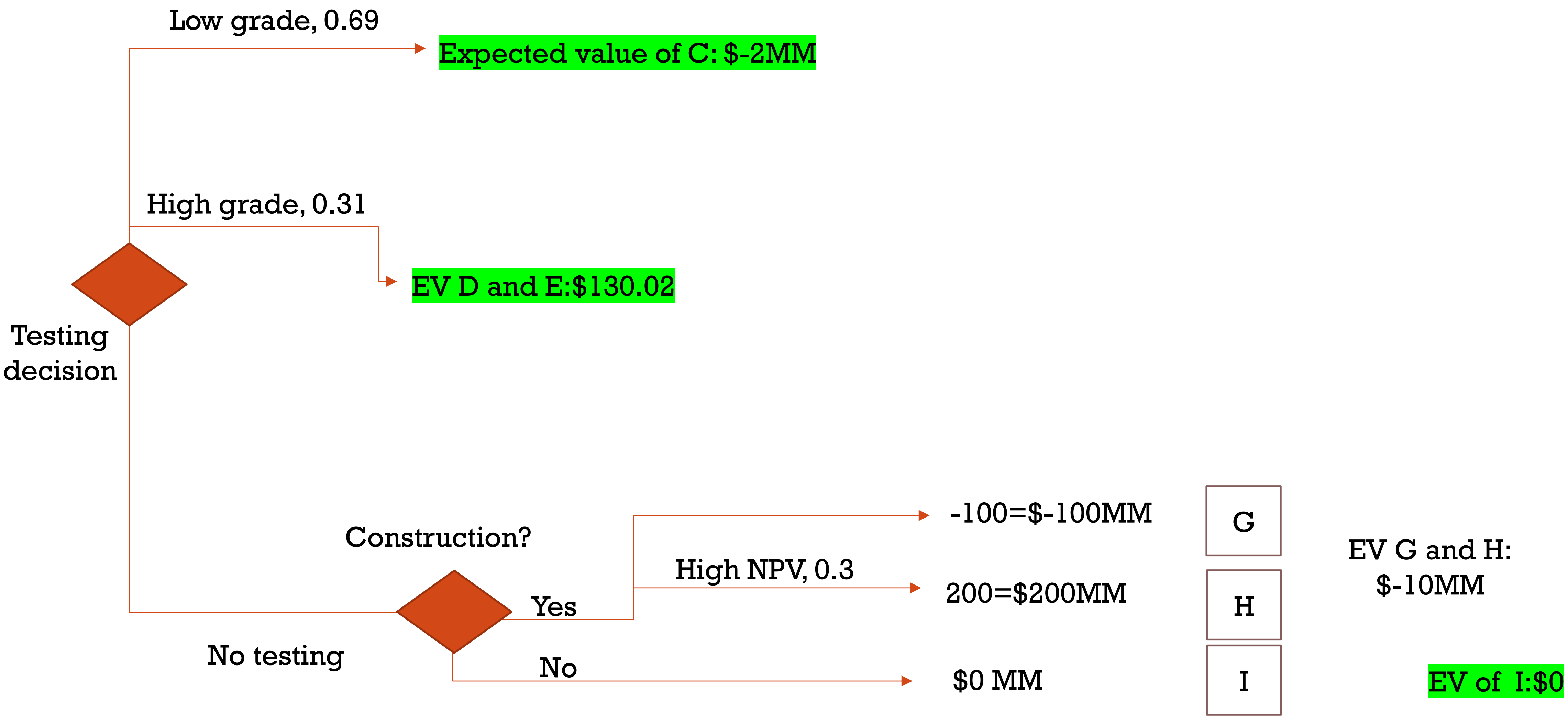
Expected value of A and B:
-102*0.9+198*0.1-=-75

Expected value of C: $-2MM

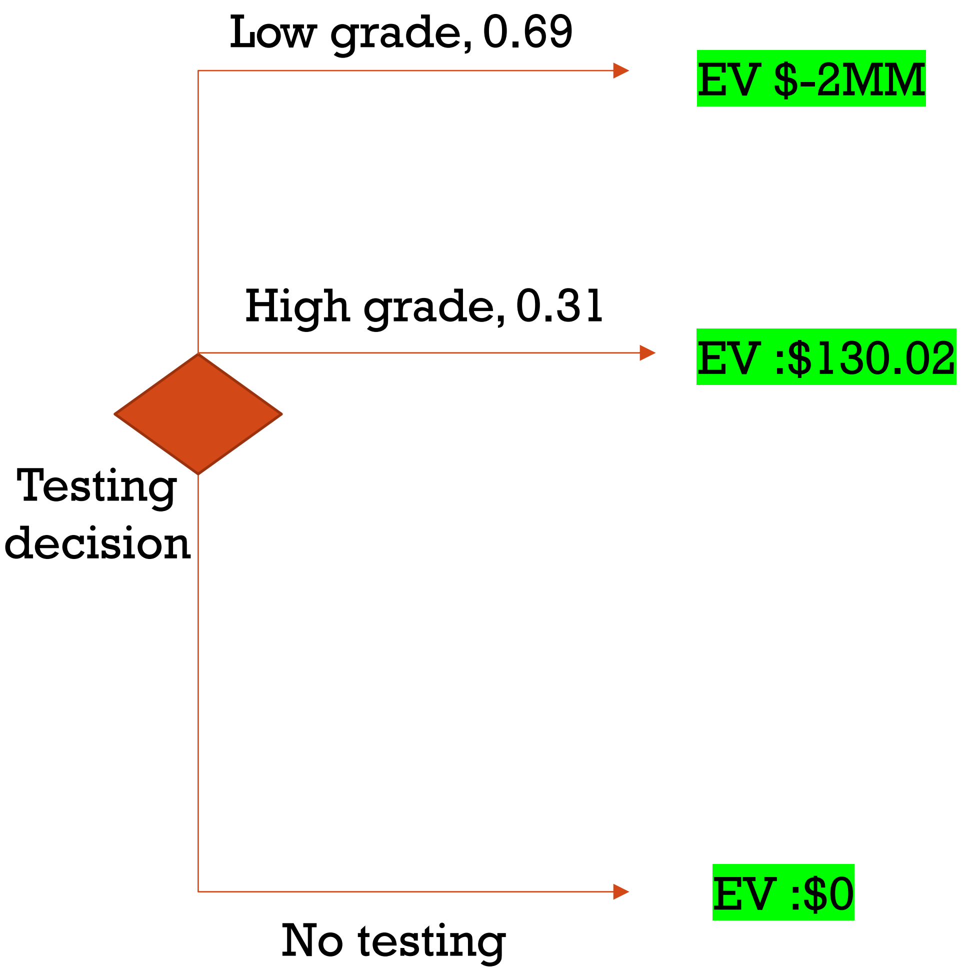
Decision tree



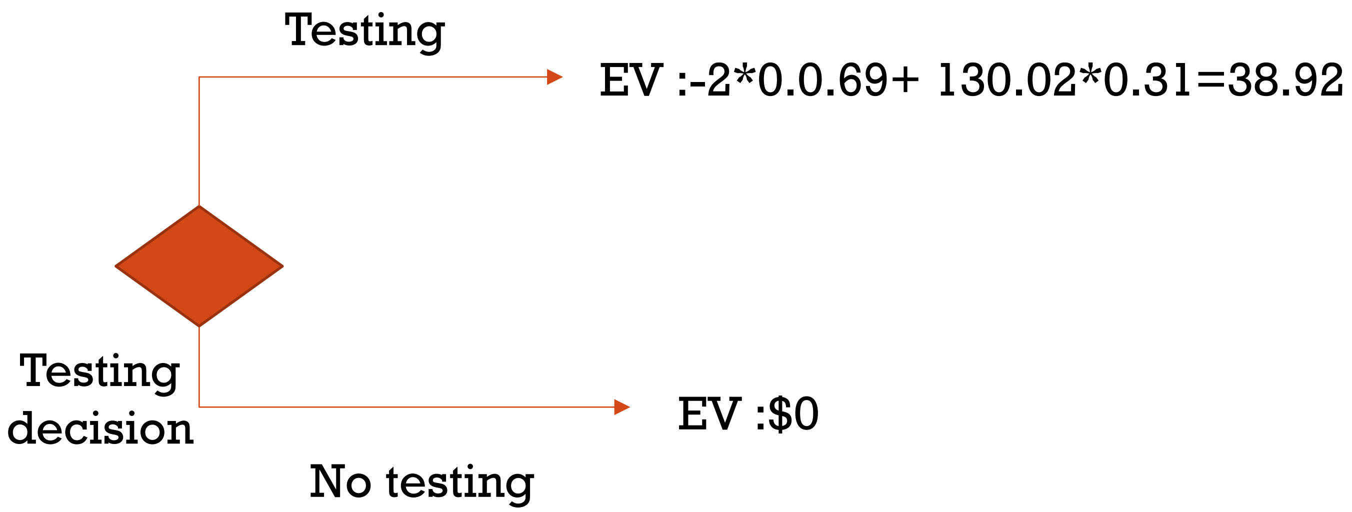
Decision tree



Decision tree



Decision tree

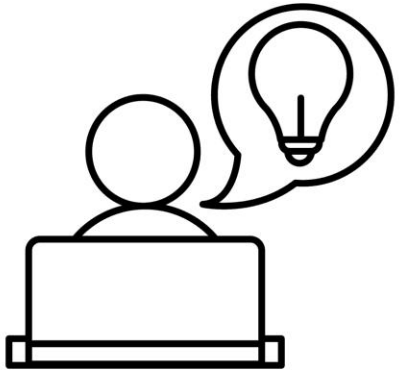


Conclusion: run the test

Value of information: 38.9-0=38.9

Exercise-10-15 mins

- We want to introduce a new product to the market. We are not sure about the market reaction to this product.
- We can run a survey on a sample customer group to analyse their response. The cost of running the survey is $1000.
- If the product fails in the market, we lose S10000 but if it is successful we earn 160000.
- The probabilities of different events are given in the excel sheet.



Revising probabilities

- Information to enhance our confidence

Test result: 0.8

- High grade metal recovery: 0.20

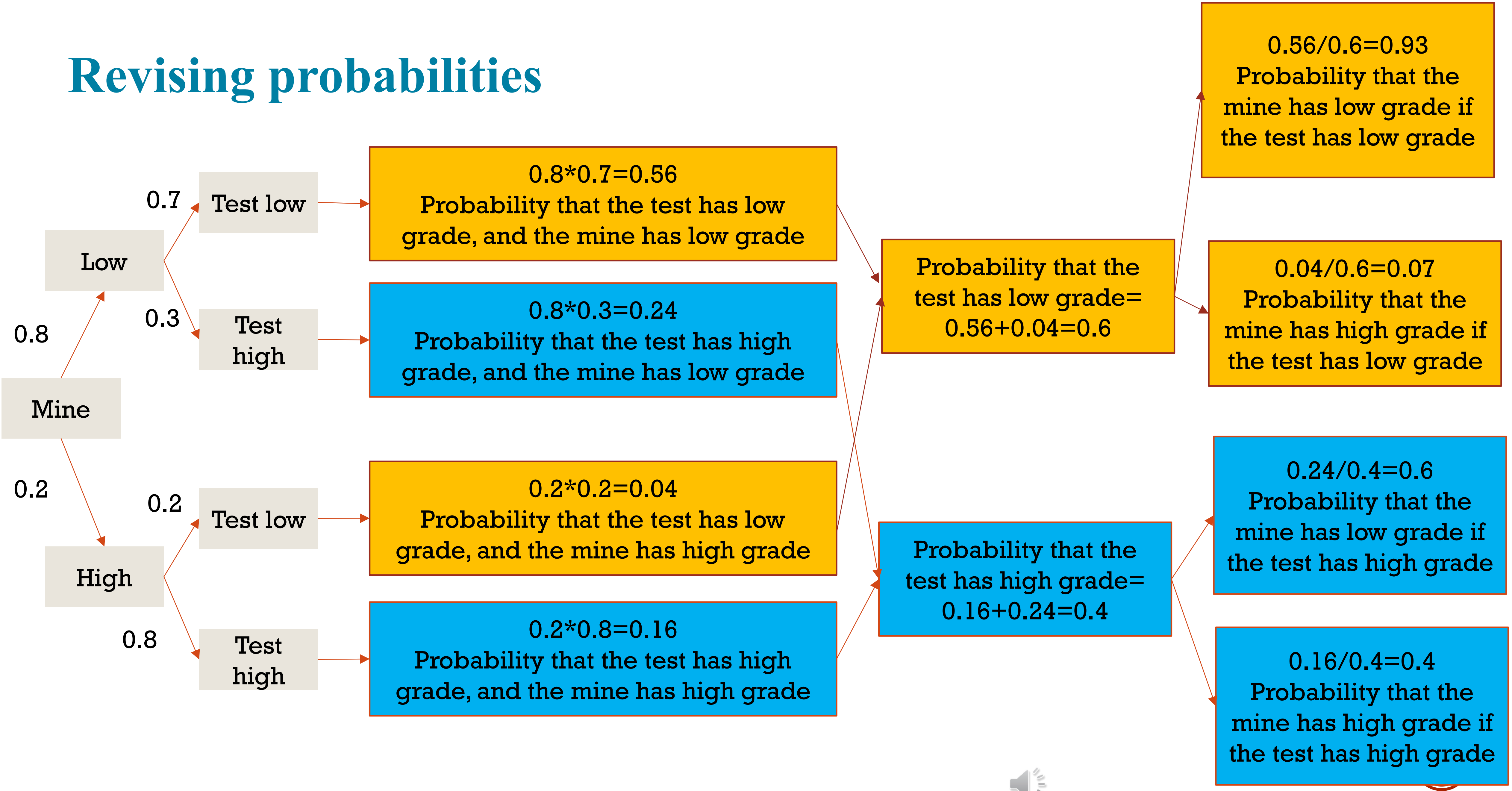
Test result: 0.2

Test result: 0.7

- Low grade metal recovery : 0.80

Test result: 0.3

Revising probabilities



Revising probabilities

Mine Condition		Reliability		Combined probabilities		
Mine Grade		Test grade		Mine grade		
Value	Probability	Low	High	Low	High	
Low	0.8	0.7	0.3	0.56	0.24	
High	0.2	0.2	0.8	0.04	0.16	
Total	1			0.6	0.4	

Mine Condition	Low	High
Low	0.56/0.6=0.93	0.24/0.4=0.6
High	0.04/0.6=0.07	0.16/0.4=0.4
Total	1	1

USING DECISION TREES IN A SUPPLY CHAIN DECISIONS

DISCOUNTED CASH FLOWS

Decisions must be evaluated as a sequence of cash flows over time

Discounted cash flow (DCF): evaluates the present value of any stream of future cash flows and allows comparing different cash flow streams against financial value

DISCOUNTED CASH FLOW ANALYSIS

$$\text{discount factor} = \frac{1}{1+k}$$

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \left(\frac{1}{1+k} \right)^t C_t$$

Where:

$C_0, C_1, \ldots, C_T$ is stream of cash flows over T periods

NPV = net present value of this stream

K = rate of return

- Compare NPV of different supply chain design options
- The option with the highest NPV will provide the greatest financial return

TRIPS LOGISTICS EXAMPLE

- Trips Logistics, a third-party logistics firm that provides warehousing and other logistics services, is facing a decision regarding the amount of space to lease for the upcoming three-year period. The general manager has forecast that Trips Logistics will need to handle a demand of 100,000 units for each of the next three years. Historically, Trips Logistics has required 1,000 square feet of warehouse space for every 1,000 units of demand. For the purposes of this discussion, the only cost Trips Logistics faces is the cost for the warehouse.
- Trips Logistics receives revenue of $1.22 for each unit of demand. The general manager must decide whether to sign a three-year lease or obtain warehousing space on the spot market each year. The three-year lease will cost $1 per square foot per year, and the spot market rate is expected to be $1.20 per square foot per year for each of the three years. Trips Logistics has a discount rate of k = 0.1. Should the general manager sign a lease?

DECISION TREE – TRIPS LOGISTICS

- 1000 sq. ft. of warehouse space needed for 1000 units of demand
- Current demand = 100,000 units per year
- Binomial uncertainty: Demand can go up by 20% with $p = 0.5$ or down by 20% with $1 - p = 0.5$
- Lease price (three-year approach) = \$1.00 per sq. ft. per year
- Spot market price = \$1.20 per sq. ft. per year
- Spot prices can go up by 10% with $p = 0.5$ or down by 10% with $1 - p = 0.5$
- Revenue = \$1.22 per unit of demand
- $k = 0.1$

Decision Tree – Trips Logistics

Warehouse lease options

1. Get all warehousing space from the spot market as needed
2. Sign a three-year lease for a fixed amount of warehouse space and get additional requirements from the spot market

METHODOLOGY

Identify the duration of each period (month, quarter, etc.) and the number of periods T over which the decision is to be evaluated

Identify factors whose fluctuation will be considered

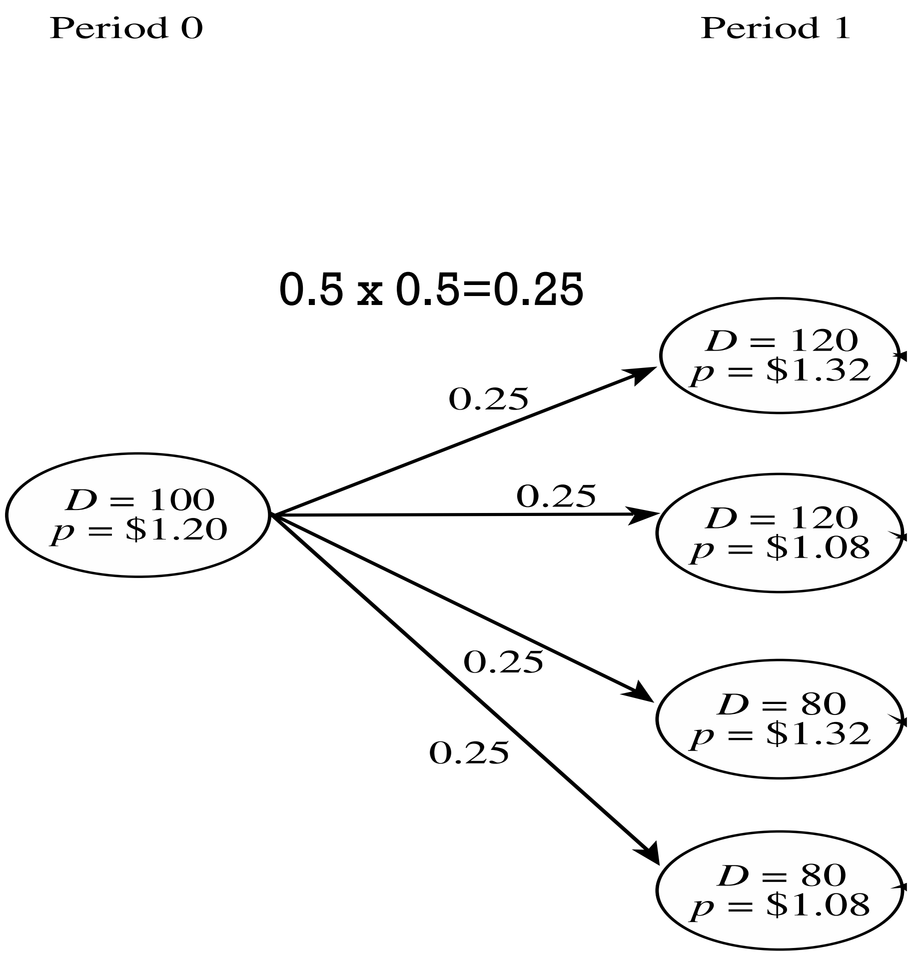
Identify representations of uncertainty for each factor

Identify the periodic discount rate k for each period

Represent the decision tree with defined states in each period as well as the transition probabilities between states in successive periods

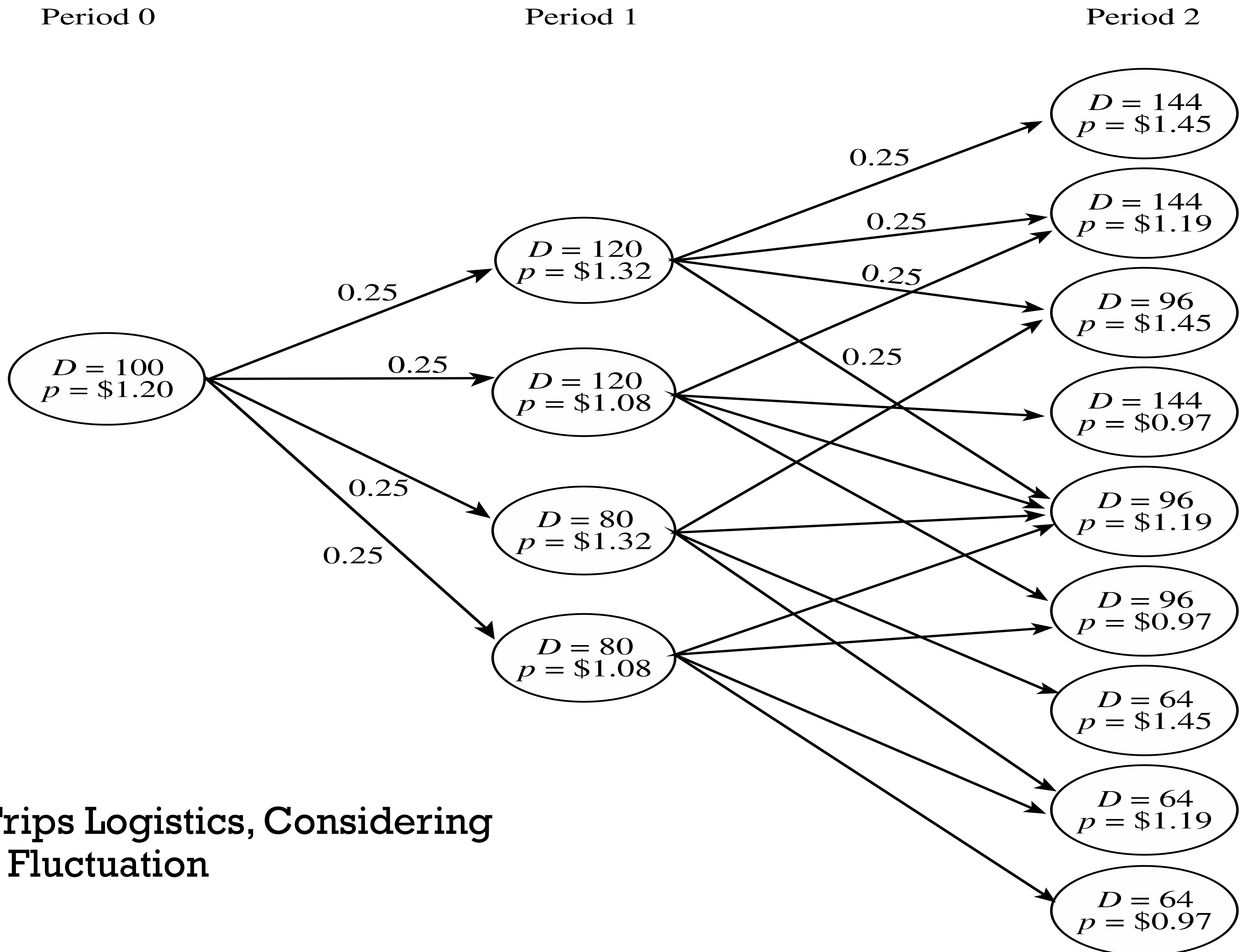
Starting at period T, work back to Period 0, identifying the optimal decision and the expected cash flows at each step

DECISION TREE



Decision Tree for Trips Logistics,
Considering Demand and Price Fluctuation

DECISION TREE



Decision Tree for Trips Logistics, Considering Demand and Price Fluctuation

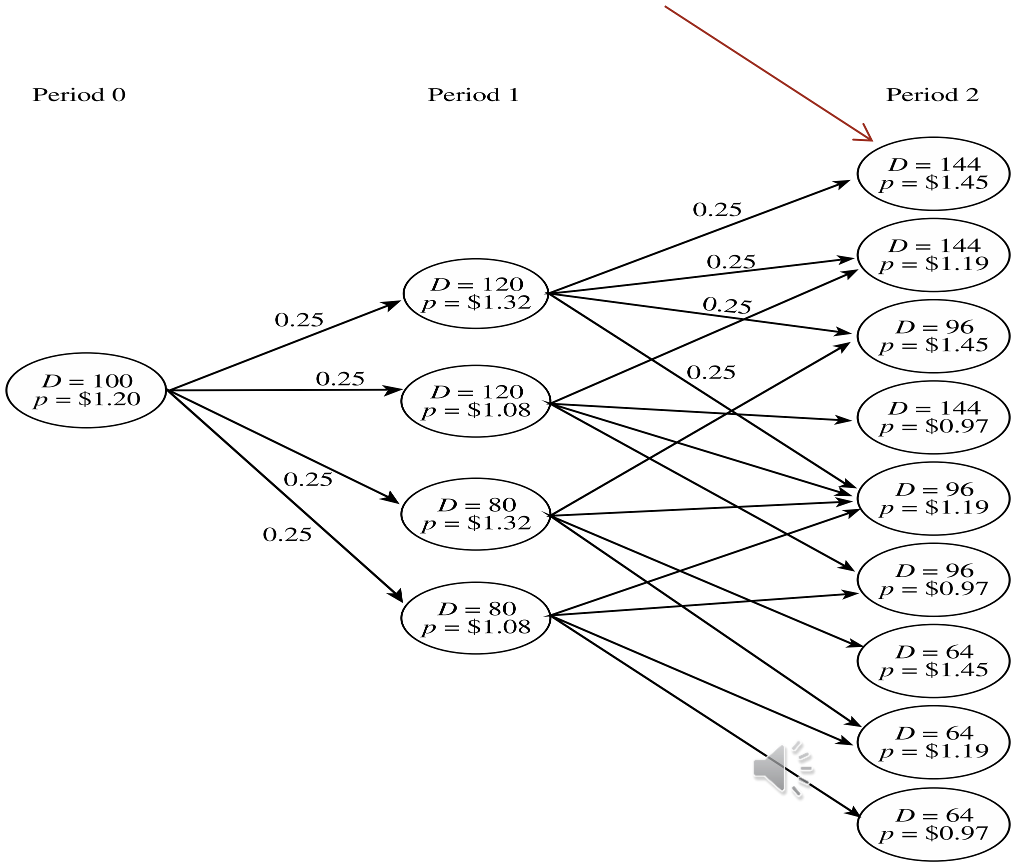
Evaluating the Spot Market Option

- **Analyze the option of not signing a lease and using the spot market**
- Start with Period 2 and calculate the profit at each node

For D = 144, p = \$1.45, in Period 2:

$$Cost(D = 144, p = 1.45, 2) = 144{,}000 \times 1.45$$
$$= \$208{,}800$$

$$Profit(D = 144, p = 1.45, 2) = 144{,}000 \times 1.22$$
$$- Cost(D = 144, p = 1.45, 2)$$
$$= 175{,}680 - 208{,}800$$
$$= -\$33{,}120$$



EVALUATING THE SPOT MARKET OPTION

Period 2 Calculations for Spot Market Option

	Revenue	Cost $C(D =, p =, 2)$	Profit $P(D =, p =, 2)$
$D = 144, p = 1.45$	144,000 × 1.22	144,000 × 1.45	–$33,120
$D = 144, p = 1.19$	144,000 × 1.22	144,000 × 1.19	$4,320
$D = 144, p = 0.97$	144,000 × 1.22	144,000 × 0.97	$36,000
$D = 96, p = 1.45$	96,000 × 1.22	96,000 × 1.45	–$22,080
$D = 96, p = 1.19$	96,000 × 1.22	96,000 × 1.19	$2,880
$D = 96, p = 0.97$	96,000 × 1.22	96,000 × 0.97	$24,000
$D = 64, p = 1.45$	64,000 × 1.22	64,000 × 1.45	–$14,720
$D = 64, p = 1.19$	64,000 × 1.22	64,000 × 1.19	$1,920
$D = 64, p = 0.97$	64,000 × 1.22	64,000 × 0.97	$16,000

EVALUATING THE SPOT MARKET OPTION

- Expected profit at each node in Period 1 is the profit during Period 1 plus the present value of the expected profit in Period 2
- Expected profit EP(D =, p =, 1) at a node is the expected profit over all four nodes in Period 2 that may result from this node
- PVEP(D =, p =, 1) is the present value of this expected profit and P(D =, p =, 1), and the total expected profit, is the sum of the profit in Period 1 and the present value of the expected profit in Period 2

EVALUATING THE SPOT MARKET OPTION

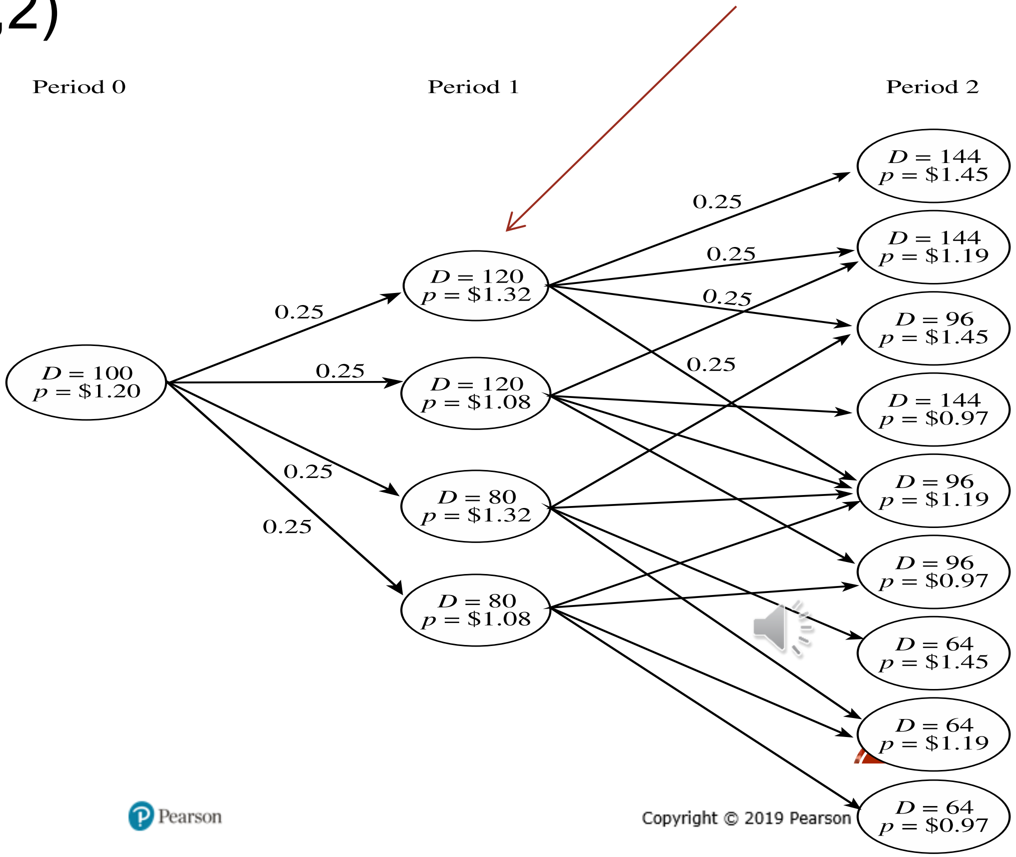
- From node D = 120, p = \$1.32 in Period 1, there are four possible states in Period 2
- Evaluate the expected profit in Period 2 over all four states possible from node D = 120, p = \$1.32 in Period 1 to be

$$
\begin{aligned}
EP(D = 120, p = 1.32, 1) = 0.25 \times [&P(D = 144, p = 1.45, 2) \\
&+ P(D = 144, p = 1.19, 2) \\
&+ P(D = 96, p = 1.45, 2) \\
&+ P(D = 96, p = 1.19, 2)
\end{aligned}
$$

$$= 0.25 \times [-33{,}120 + 4{,}320$$

$$- 22{,}080 + 2{,}880]$$

$$= -\$12{,}000$$



EVALUATING THE SPOT MARKET OPTION

- The present value of this expected value in Period 1 is

$$PVEP(D = 120, p = 1.32, 1) = \frac{EP(D = 120, p = 1.32, 1)}{(1 + k)}$$

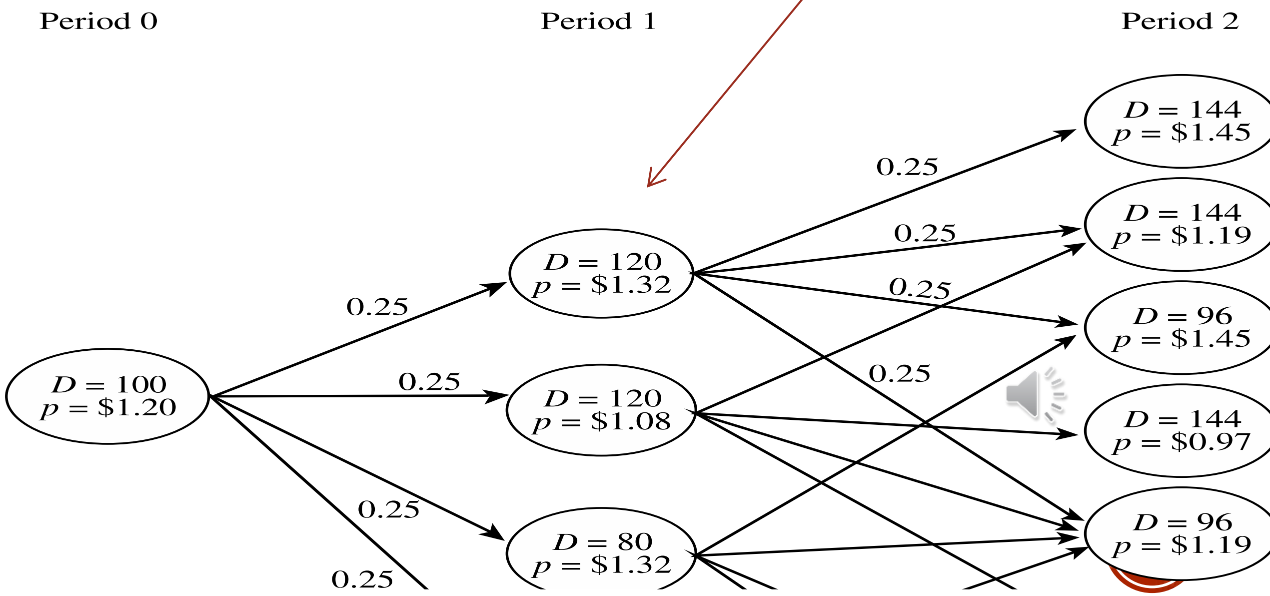
$$= \frac{-\$12{,}000}{(1.1)}$$

$$= -\$10{,}909$$

EVALUATING THE SPOT MARKET OPTION

- The total expected profit *P(D* = 120, *p* = 1.32,1) at node *D* = 120, *p* = 1.32 in Period 1 is the sum of the profit in Period 1 at this node, plus the present value of future expected profits possible from this node

$$P(D = 120, p = 1.32,1) = (120{,}000 \times 1.22) - (120{,}000 \times 1.32)$$
$$+PVEP(D = 120, p = 1.32,1)$$
$$= -\$12{,}000 - \$10{,}909 = -\$22{,}909$$



EVALUATING THE SPOT MARKET OPTION

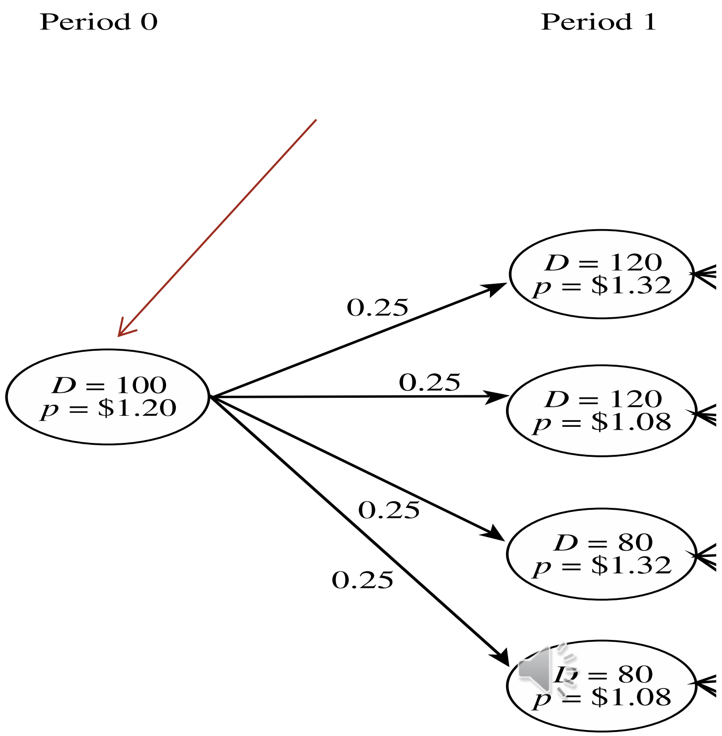
Period 1 Calculations for Spot Market Option

Node	$EP(D =, p =, 1)$	$P(D =, p =, 1) = D \times 1.22 - D \times p + \dfrac{EP(D =, p =, 1)}{(1+k)}$
$D = 120, p = 1.32$	−\$12,000	−\$22,909
$D = 120, p = 1.08$	\$16,000	\$32,073
$D = 80, p = 1.32$	−\$8,000	−\$15,273
$D = 80, p = 1.08$	\$11,000	\$21,382

EVALUATING THE SPOT MARKET OPTION

- For Period 0, the total profit *P(D* = 100, *p* = 120,0) is the sum of the profit in Period 0 and the present value of the expected profit over the four nodes in Period 1

$$
\begin{aligned}
EP(D = 100, p = 1.20,0) = 0.25 \times [&P(D = 120, p = 1.32,1) \\
&+ P(D = 120, p = 1.08,1) \\
&+ P(D = 96, p = 1.32,1) \\
&+ P(D = 96, p = 1.08,1)] \\
= 0.25 \times [&-22{,}909 + 32{,}073 \\
&- 15{,}273) + 21{,}382] \\
= \$3{,}818&
\end{aligned}
$$



EVALUATING THE SPOT MARKET OPTION

$$PVEP(D = 100, p = 1.20,1) = \frac{EP(D = 100, p = 1.20,0)}{(1+k)}$$

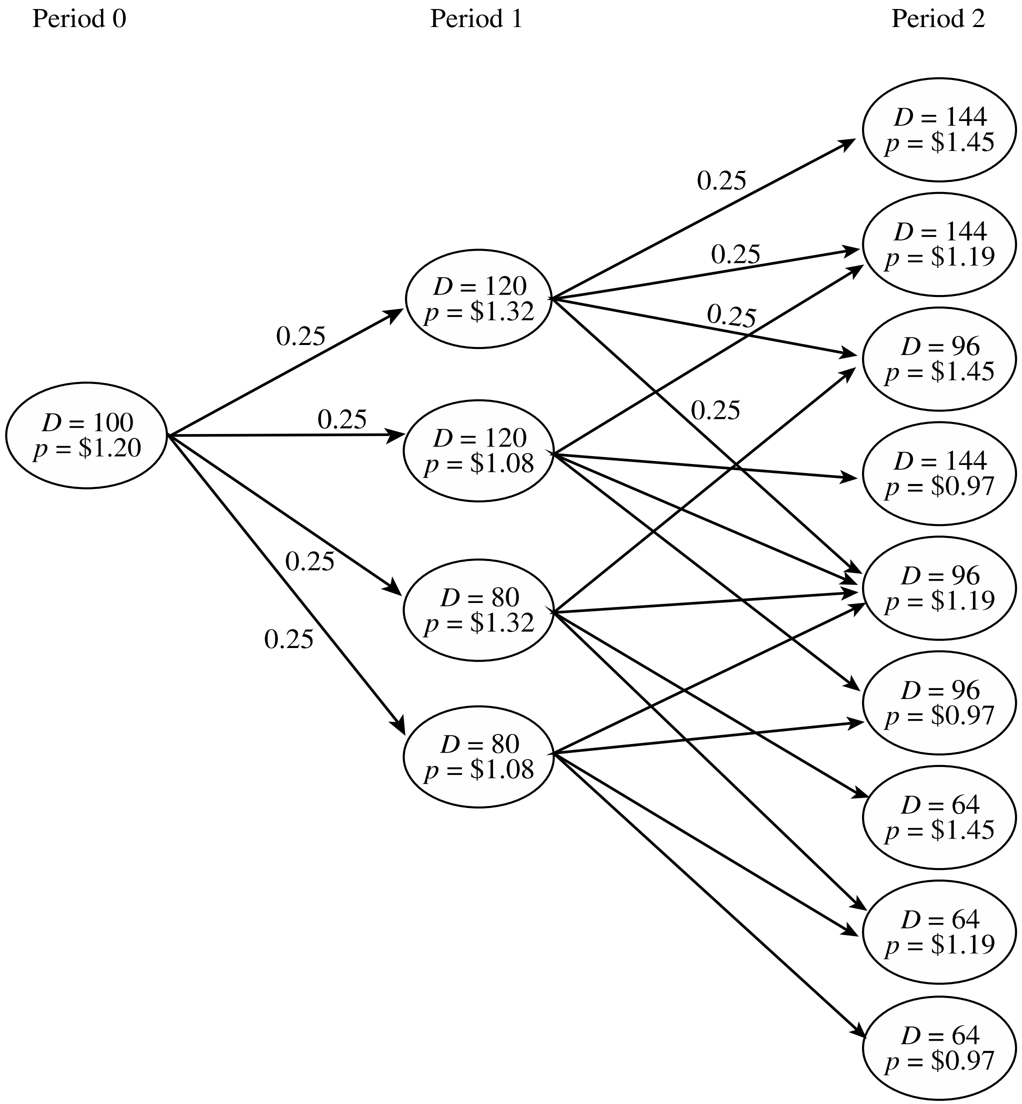
$$= \frac{\$3,818}{(1.1)} = \$3,471$$

$$P(D = 100, p = 1.20,0) = (100,000 \times 1.22) - (100,000 \times 1.20) + PVEP(D = 100, p = 1.20,0)$$

$$= \$2,000 + \$3,471 = \$5,471$$

Therefore, the expected NPV of not signing the lease and obtaining all warehouse space from the spot market is given by NPV (Spot Market) = $5,471

EVALUATING THE FIXED LEASE OPTION



EVALUATING THE FIXED LEASE OPTION

Period 2 Profit Calculations at Trips Logistics for Fixed Lease Option

Node	Leased Space	Warehouse Space at Spot Price (S)	Profit $P(D =, p =, 2) = D \times 1.22 - (100{,}000 \times 1 + S \times p)$
$D = 144, p = 1.45$	100,000 sq. ft.	44,000 sq. ft.	\$11,880
$D = 144, p = 1.19$	100,000 sq. ft.	44,000 sq. ft.	\$23,320
$D = 144, p = 0.97$	100,000 sq. ft.	44,000 sq. ft.	\$33,000
$D = 96, p = 1.45$	100,000 sq. ft.	0 sq. ft.	\$17,120
$D = 96, p = 1.19$	100,000 sq. ft.	0 sq. ft.	\$17,120
$D = 96, p = 0.97$	100,000 sq. ft.	0 sq. ft.	\$17,120
$D = 64, p = 1.45$	100,000 sq. ft.	0 sq. ft.	–\$21,920
$D = 64, p = 1.19$	100,000 sq. ft.	0 sq. ft.	–\$21,920
$D = 64, p = 0.97$	100,000 sq. ft.	0 sq. ft.	–\$21,920

=144000 x 1.22-(100,000 +(44000 x 1.45))

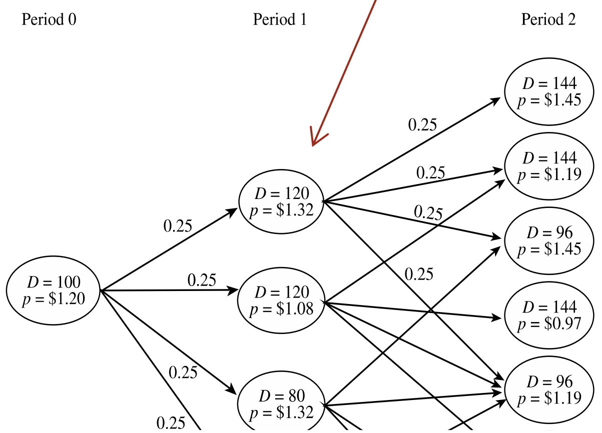
=64000 x 1.22-(100,000 +(0x 0.97))

EVALUATING THE FIXED LEASE OPTION

Period 1 Profit Calculations at Trips Logistics for Fixed Lease Option

Node	***EP*(*D* =, *p* =, 1)**	**Warehouse Space at Spot Price (*S*)**	***P*(*D* =, *p* =, 1) = *D* x 1.22−(100,000 x 1 + *S* x *p*) + *EP*(*D* =, *p* = ,1)(1 + *k*)**
$D = 120, p = 1.32$	0.25 × [$P(D = 144, p = 1.45,2) + P(D = 144, p = 1.19,2) + P(D = 96, p = 1.45,2) + P(D = 96, p = 1.19,2)$] = 0.25 × (11,880 + 23,320 + 17,120 + 17,120) = \$17,360	20,000	\$35,782
$D = 120, p = 1.08$	0.25 × (23,320 + 33,000 + 17,120 + 17,120) = \$22,640	20,000	\$45,382
$D = 80, p = 1.32$	0.25 × (17,120 + 17,120−21,920 − 21,920) = −\$2,400	0	−\$4,582
$D = 80, p = 1.08$	0.25 × (17,120 + 17,120 − 21,920 −21,920) = −\$2,400	0	−\$4,582

=(17,360/1.1)+[(120,000*1.22)-(100,000 + (20000*1.32))]



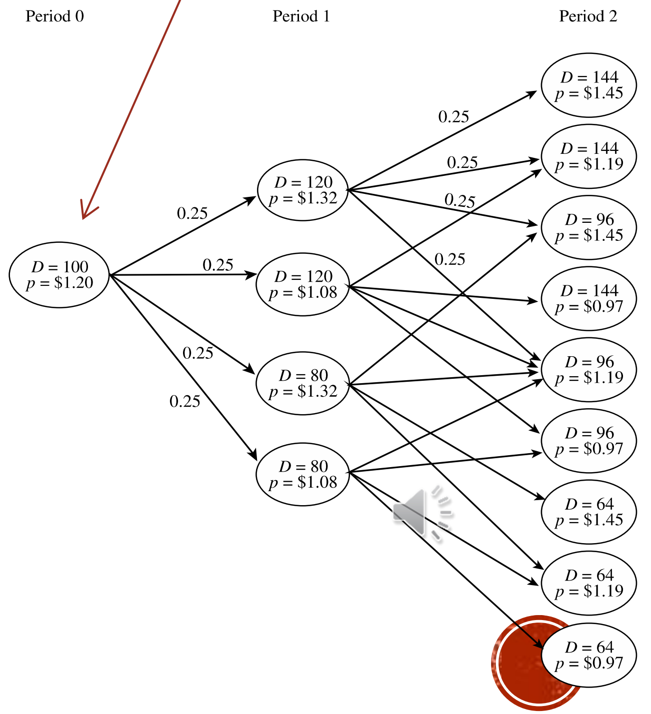
EVALUATING THE FIXED LEASE OPTION

- Using the same approach for the lease option, NPV (Lease) = \$38,364

$$EP(D = 100, p = 1.20,0) = 0.25 \times [P(D = 120, p = 1.32,1) + P(D = 120, p = 1.08,1) + P(D = 80, p = 1.32,1) + P(D = 80, p = 1.08,1)]$$

$$= 0.25 \times [35{,}782 + 45{,}382 - 4{,}582 - 4{,}582]$$

$$= \$18{,}000$$



EVALUATING THE FIXED LEASE OPTION

$$PVEP(D = 100, p = 1.20,1) = \frac{EP\left(D = 100, p = 1.20,1\right)}{\left(1 + k\right)}$$

$$= \frac{\$18,000}{\left(1.1\right)} = \$16,364$$

$$P(D = 100,\ p = 1.20,0) = (100,000 \times 1.22) - (100,000 \times 1) + PVEP(D = 100,\ p = 1.20,0)$$

$$= \$22,000 + \$16,364 = \$38,364$$

EVALUATING THE FIXED LEASE OPTION

- The manager would probably prefer to sign the three-year lease for 100,000 sq. ft. because this option has the higher expected profit