T2-2025

Dr. Shiva Abdoli

Uncertain decisions and risk sharing

Uncertain decisions and risk sharing/Review

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Review

What is the mode and median of the following dataset?

3,7,2,5,7,8,6,1,10

A) mode: 3, median:7

B) mode: 5, median:5

C) mode: 7, median:6

D) mode: 2, median:7

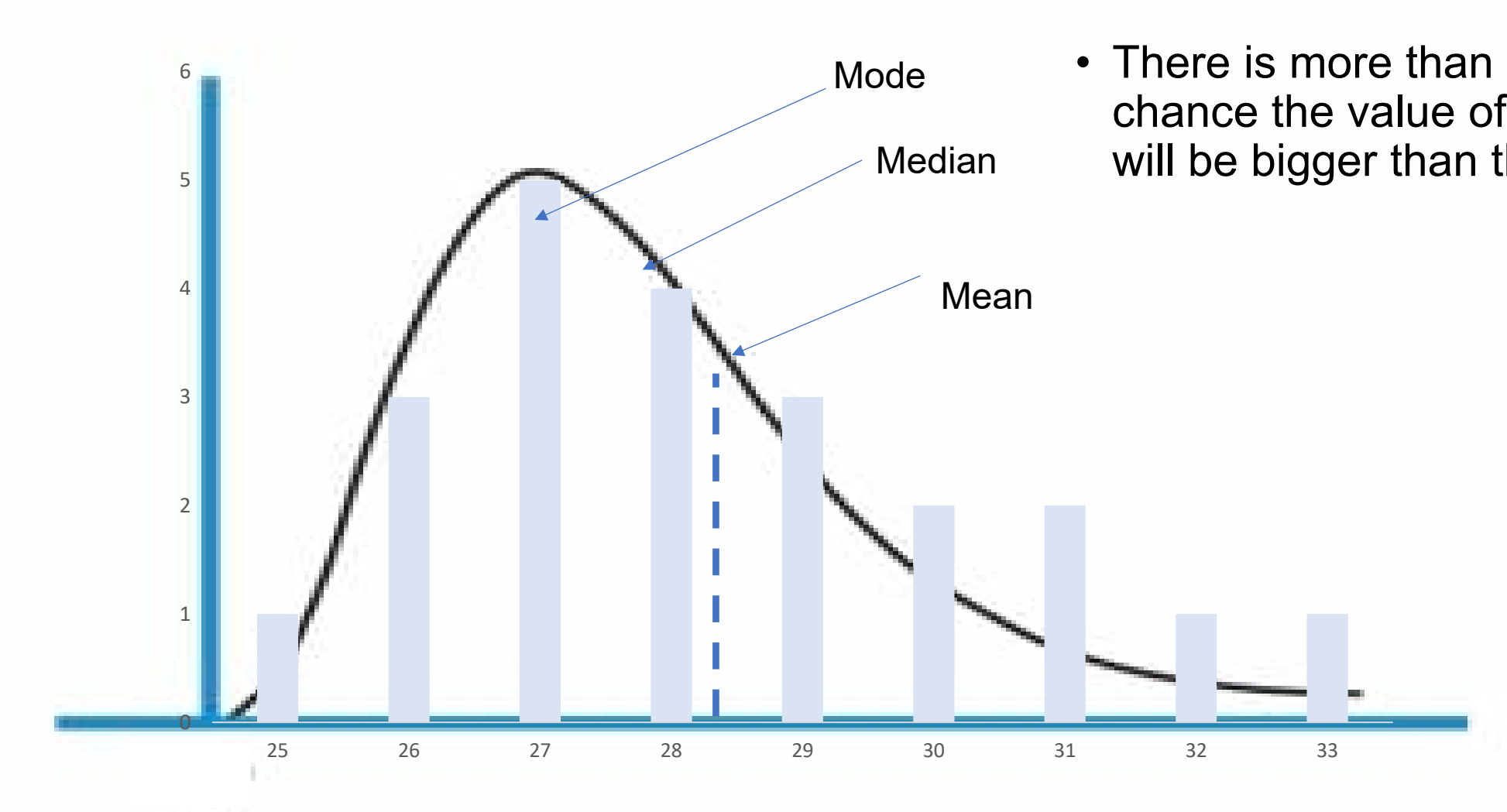
Review

- If the 90th percentile (P90) of a dataset is 85, what does this indicate?
  A) 90% of the data points are below 85.
  B) 10% of the data points are above 85.
  C) Both A and B are correct.
  D) None of the above.
- If a cumulative probability distribution shows that P(X≤50)=0.7P(X \leq 50) = 0.7P(X≤50)=0.7, what does this mean?
  A) 70% of the outcomes are exactly 50.
  B) 70% of the outcomes are below or equal to 50.
  C) 30% of the outcomes are above 50.
  D) Both B and C are correct.

Review

- What is the problem of using mode to describe the uncertainty of random variable when it has a skewed probability distribution?

Probability distribution of a random variable

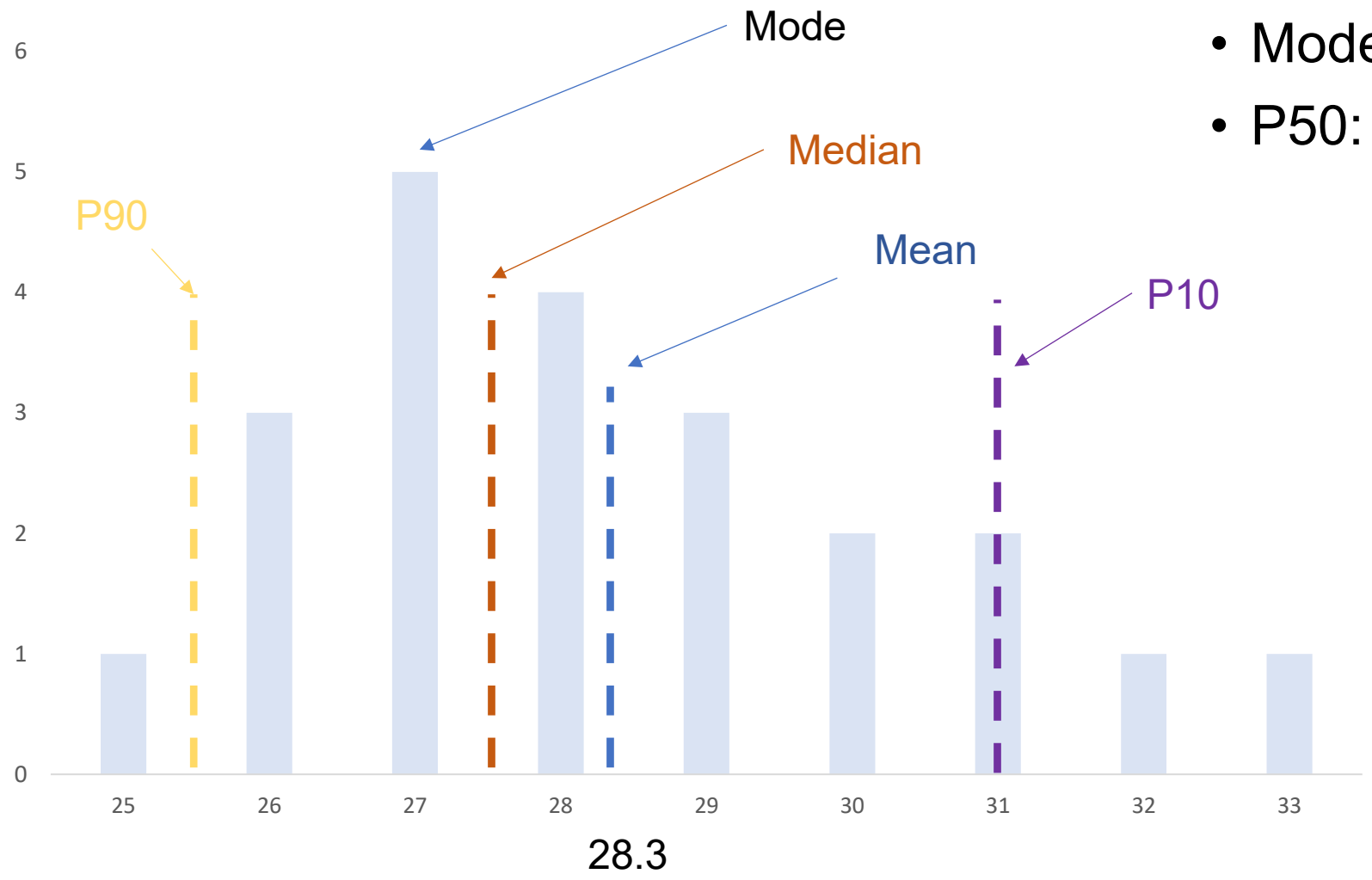


- There is more than 50% chance the value of the viable will be bigger than the mode

Review

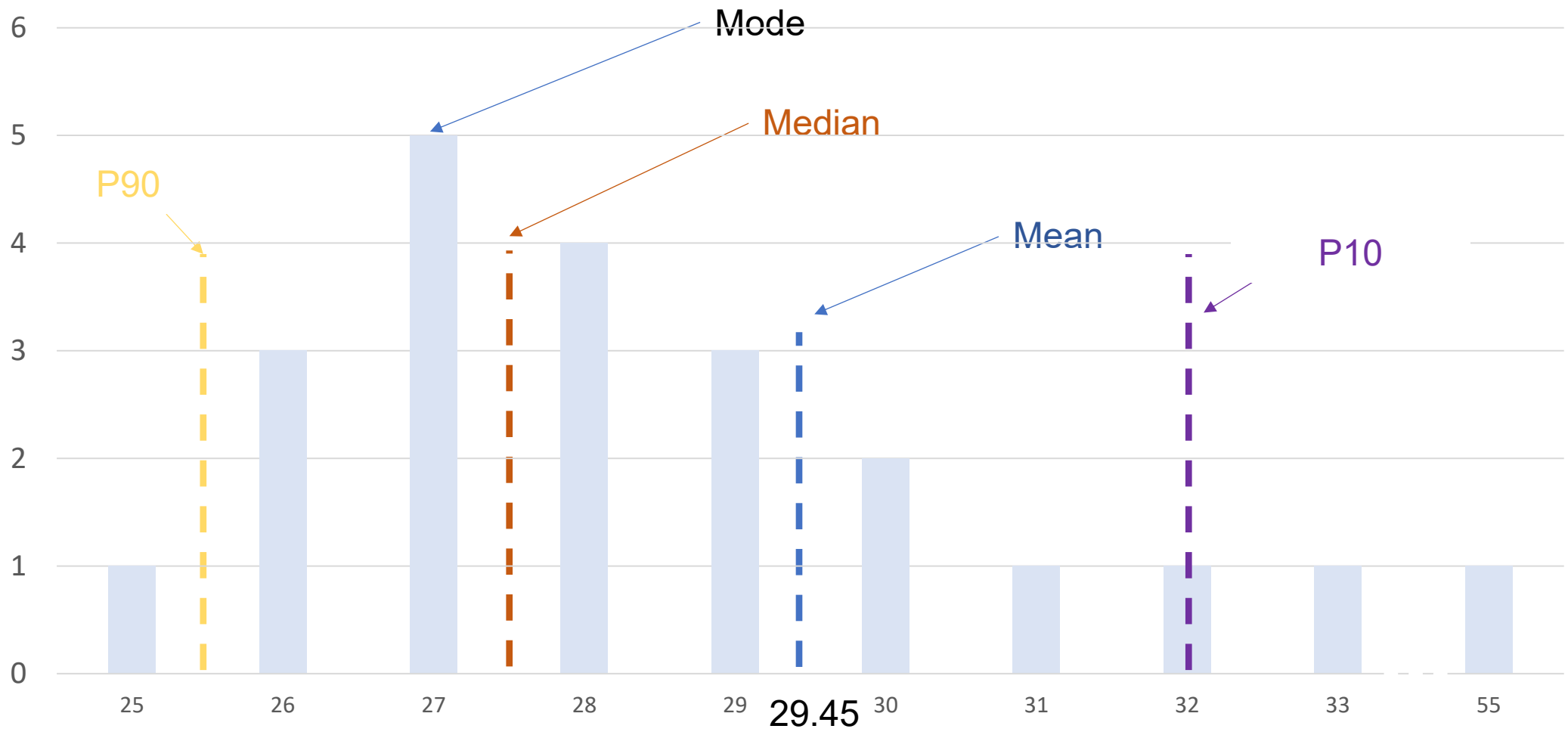
- What is the best single value to describe the uncertainty of a random variable?

Probability distribution of a random variable



- Mode: most likely
- P50: median

Probability distribution of a random variable



Review

- What are the possible approaches to estimate a parameter as a function of several uncertain input variables?

Optimistic-most likely-pessimistic

Monte Carlo simulation

Review

What is optimistic-most likely-pessimistic analysis?

Optimistic-Most Likely-Pessimistic

- Establish optimistic (the most favorable), most likely, and pessimistic (the least favorable) estimates for each factor.
- Perform analysis under each condition for insight into the sensitivity of the solution.
- The results can be seen on a spider plot for further insight.

Revenue (annual)	Optimistic	Most likely	Pessimistic
	45000	38000	32000

Operational Cost (annual)	Optimistic	Most likely	Pessimistic
	3800	4800	6500

Capital initial cost	Optimistic	Most likely	Pessimistic
	90000	150000	160000

Profit after five years	Optimistic	Most likely	Pessimistic

Optimistic-Most Likely-Pessimistic

- Establish optimistic (the most favorable), most likely, and pessimistic (the least favorable) estimates for each factor.
- Perform analysis under each condition for insight into the sensitivity of the solution.
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Revenue (annual) for five years	Optimistic	Most likely	Pessimistic
	45000	38000	32000

Operational Cost (annual) for five years	Optimistic	Most likely	Pessimistic
	3800	4800	6500

Profit after five years	Optimistic	Most likely	Pessimistic
	116000	16000	-32500

Capital initial cost	Optimistic	Most likely	Pessimistic
	90000	150000	160000

Review

- 2-What is Monte Carlo Simulation?

Monte Carlo Simulation

Based on the probability of different outcomes for each random variable, a particular value is randomly generated.

The numbers generated for all random variables constitute an instance or realization reflecting a particular outcome.

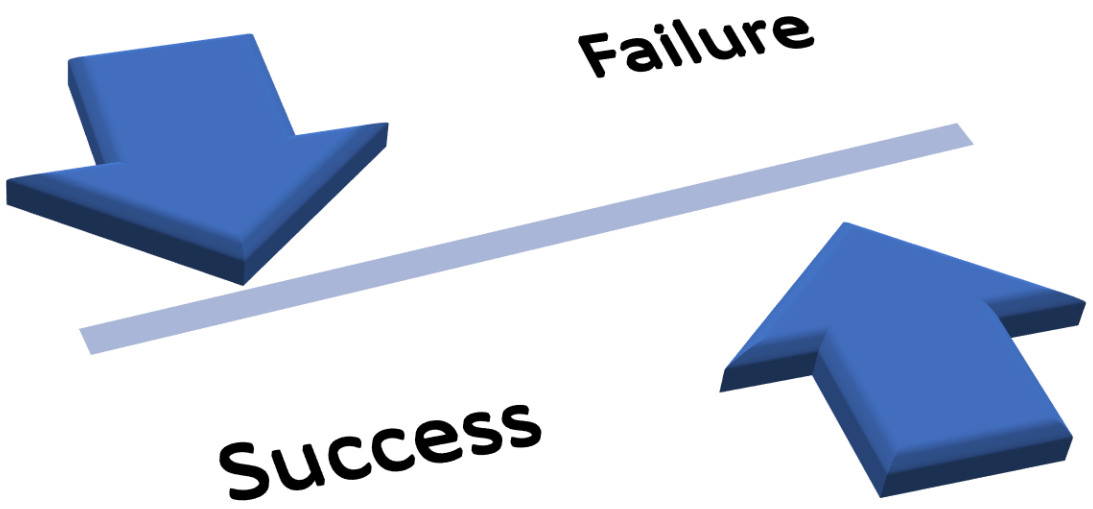
Hundreds or thousands of these *instances* are generated, and these are examined to assist in decision making.

A caution: these will yield long term, average results, and you will be able to see the variation over time. However, your decision may be a one-time decision, so don't expect the "average" outcome to be your outcome.

Take home exercise

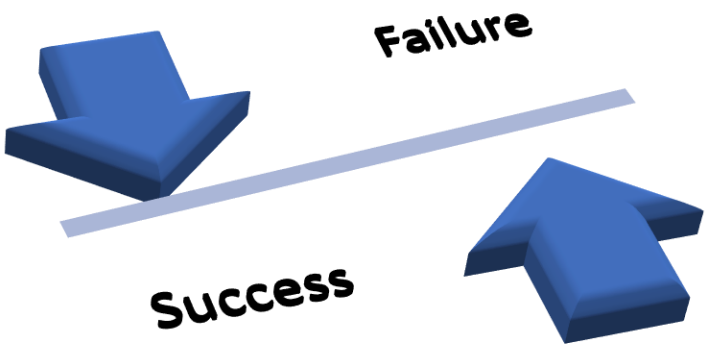
- A company is evaluating a new product and wants to estimate its potential profit. They believe that the profit can be modeled using the following assumptions:
- The product's selling price is normally distributed with a mean of $50 and a standard deviation of $5.
- The variable costs per unit is normally distributed with a mean of $20 and $3.
- Using a Monte Carlo simulation, describe the steps you would take to estimate the potential profit for this product over 1,00 simulations. Additionally, explain how you would interpret the results of the simulation.

Decisions with risks



Decisions with risks

- Two dimensions to consider
  - Value of success
  - Cost of failure
  - Success probability
  - Failure probability



Investment in Silver mining project is a Risky Decision

- Do we find enough resources?
- Do we find high quality resources?
- What happens if the mind collapse?

Should we invest or not?

Playing a game

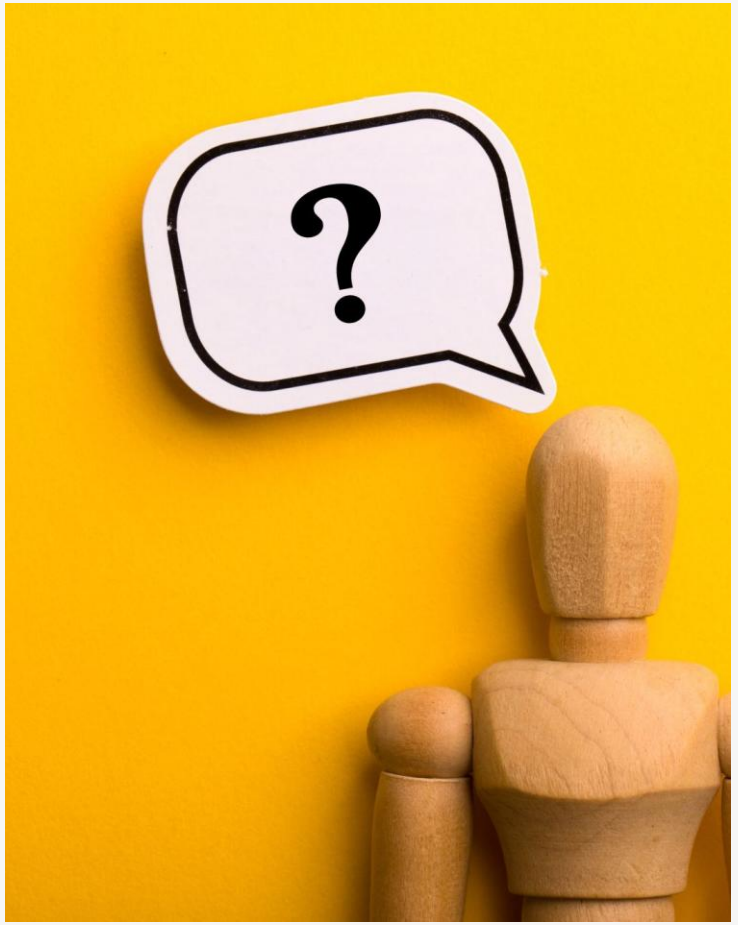
- Heads: $10
- Tails: $0.
- 50% chance of heads and a 50% chance of tails.
- For what price x would you choose to play the game?

Expected value of Playing the game

- The average payoff is: 0.5(10 - x) + 0.5(- x) = 5 - x
- Thus the value of the game is $5 - x.

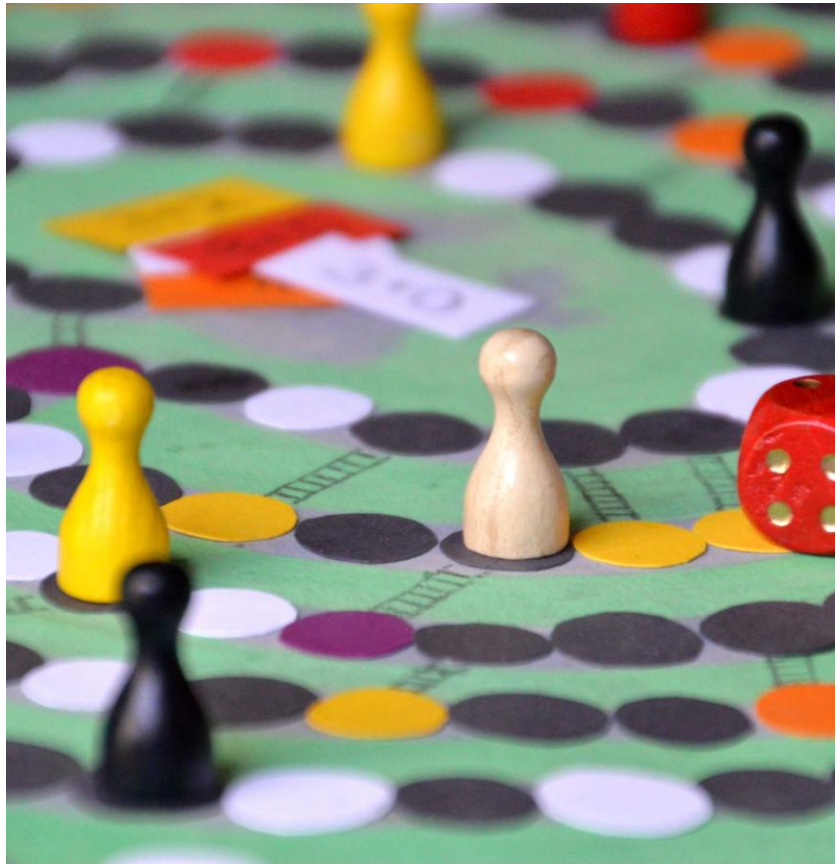
Decision making under risk

- A process of choosing between different lotteries.
- A lottery (or prospect) consists of a number of possible outcomes with their associated probability
  It can be described as:
- $\boldsymbol{q} = (x_1 , p_1 ; x_i , p_i ; \dots x_n , p_n)$
- x_i represents the i th outcome
- p_i is its associated probability
- $\sum p_i = 1$

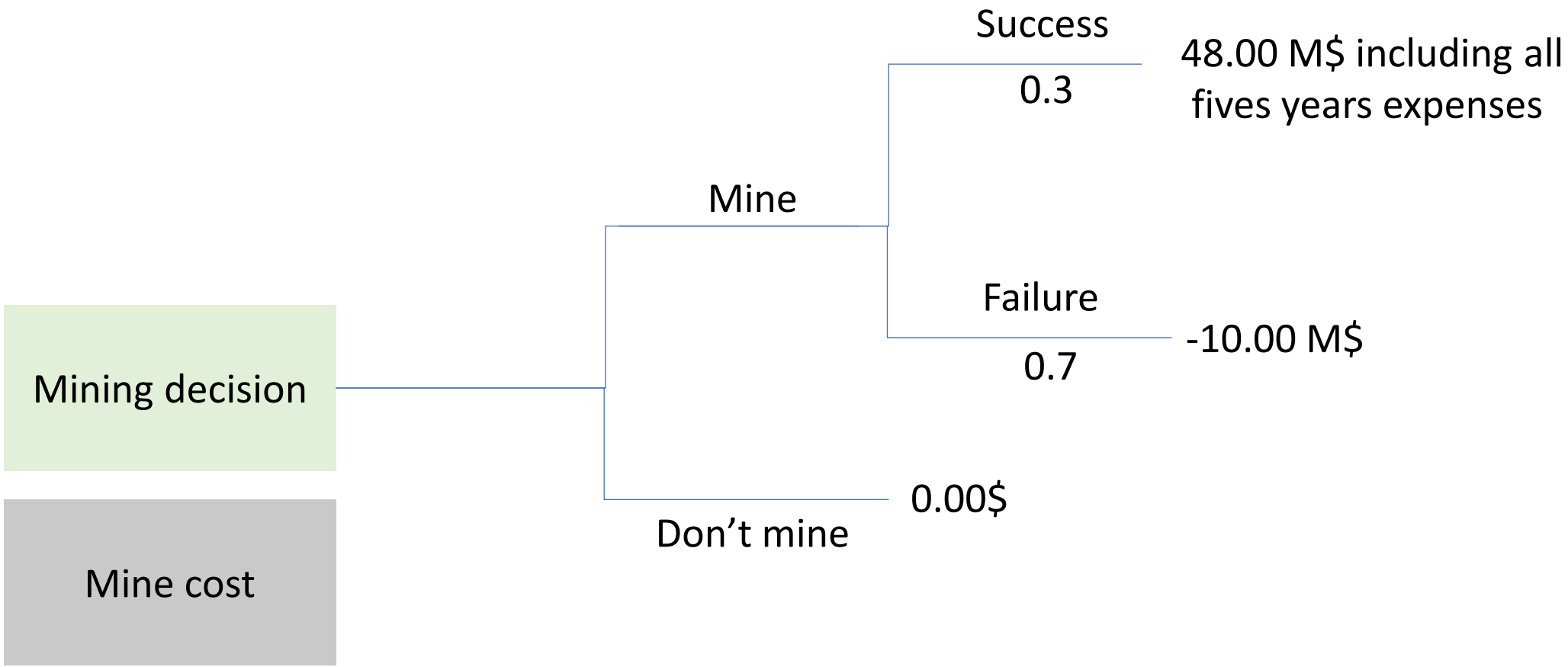


Expected Value Calculation

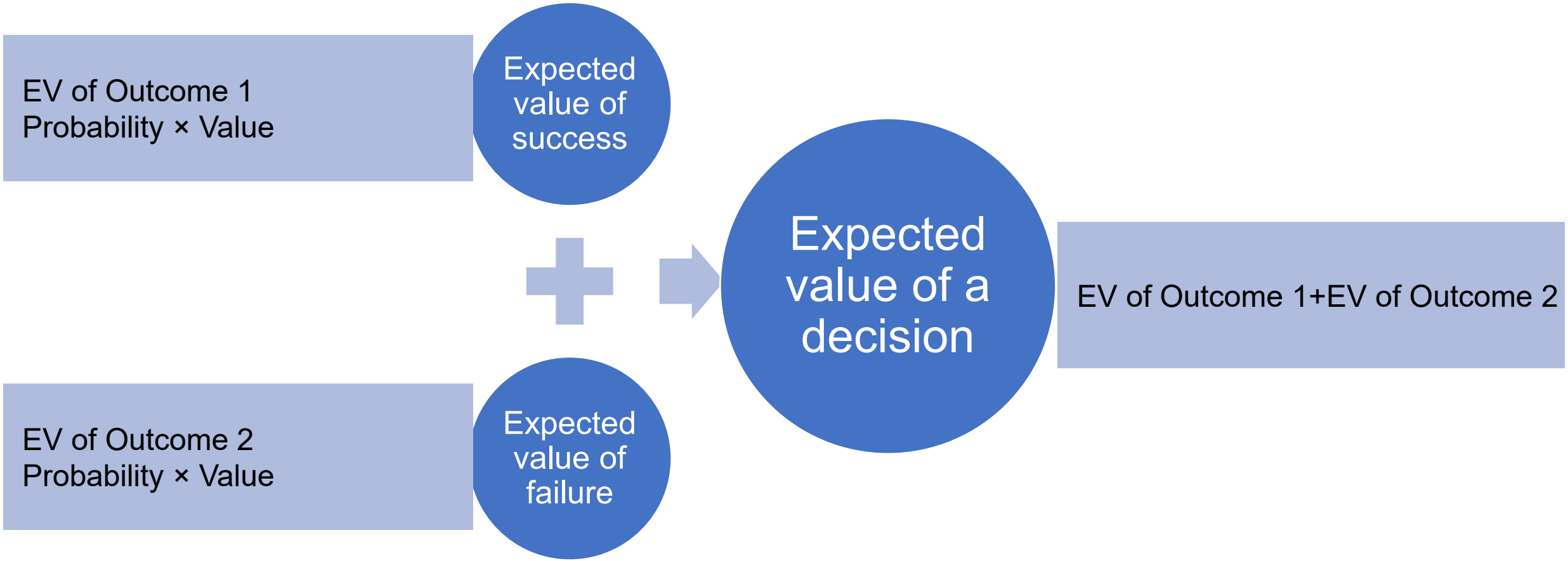
- **1. Define Outcomes and Probabilities**
- List all possible outcomes of the game.
- Assign a probability to each outcome. Make sure the total probabilities sum up to 1.
- **2. Determine Payoffs**
- Identify the payoff associated with each outcome (this could be a win amount or a loss).
- **3. Calculate Expected Value**
- Use the formula:
- $EV = \sum(\text{Probability} \times \text{Payoff})$
- This means you multiply each outcome's probability by its payoff and then sum these values.



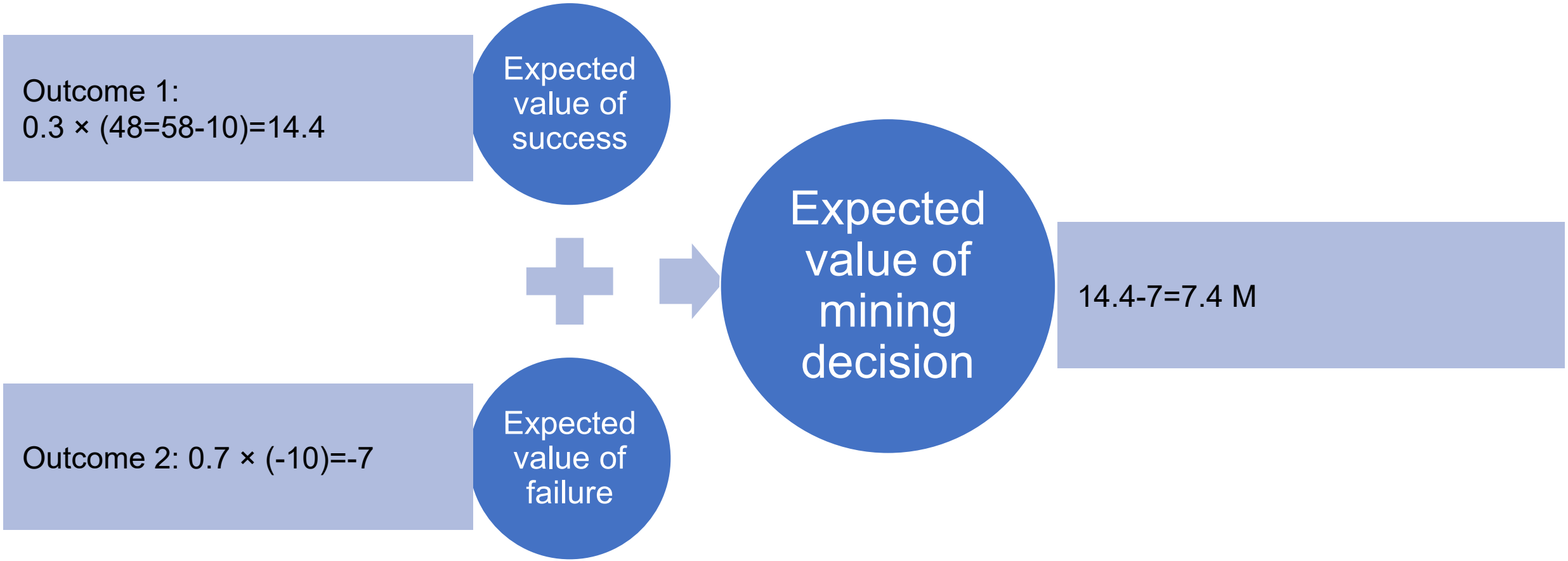
Investment in Silver mining project is a Risky Decision



Expected value of a decision



Expected value of mining decision



Expected value of an investment decision

Probability × NPV

Reward

Exchange rate

Grade

Silver Market price

Cost of development

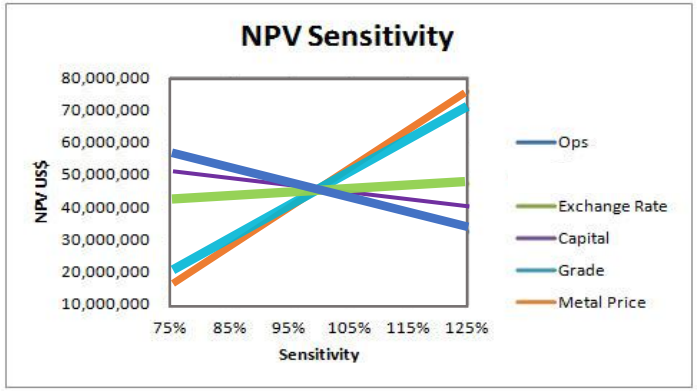
Geology

Risk
Probability × NPV

Expected value of failure

Expected value of a decision

EV of Outcome 1+EV of Outcome 2



Exercise

- 1-Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)

Exercise

- 1-Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)
- 120 x 0.15+(-5) x(0.85)=13.75

Expected value of Playing the game

- The average payoff is: p_1(10-x) + (1- p_1)(- x) =
- $10p_1$- xp_1-x+ xp_1 = $10p_1$-x
- Expected value=$10p_1$-x

Expected value of an investment decision

- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times p_{Failure})$
-
- $V_{Success}$: Conditional value of success
- $V_{Failure}$: Conditional value of Failure
- $P_{Success}$: Probability of success
- $P_{Failure}$: Probability of success

- $P_{Success} + P_{Failure} = 1$
- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1 - p_{Success}))$

Expected value of mining decision

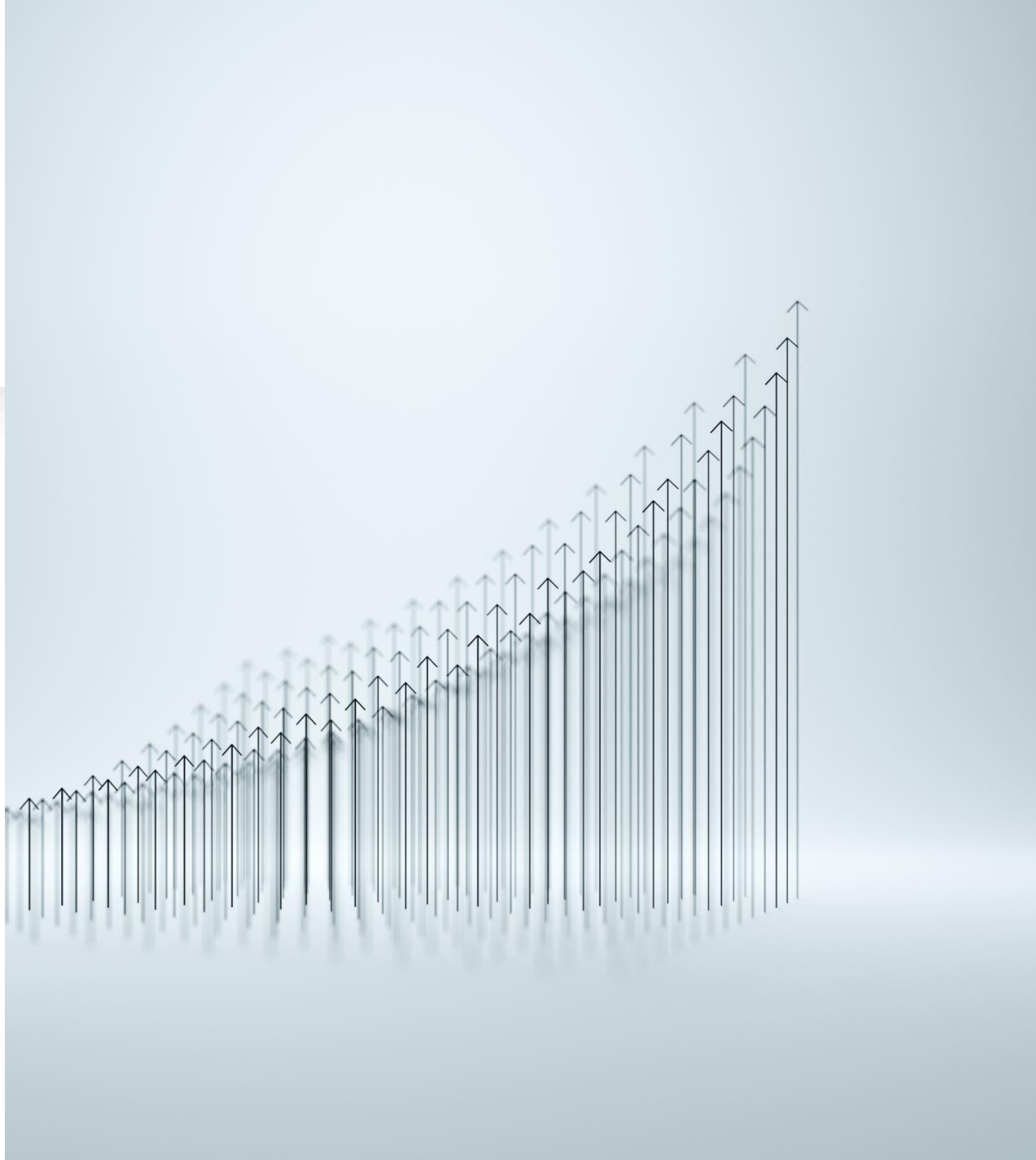
- $EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1\text{-}p_{Success}))$
- $EV = (48 \times p_{Success}) + (-10 \times (1\text{-}p_{Success}))$
- $EV = 58 \times p_{Success} - 10$

Minimum probability of success for indifference

- The minimum probability of success for indifference is a key concept in decision-making under uncertainty, particularly in scenarios involving risk and investment choices. It represents the probability at which a decision-maker is indifferent between two options: one with a risky payoff and another with a guaranteed (certain) payoff.

Minimum probability of success for indifference

- The minimum probability of success is the threshold probability at which the expected value of the risky option equals the value of a sure (certain) outcome.
- The minimum probability of success, where the expected value of the risky option equals the guaranteed value of no investment :
- $p_{Success} \times V_{Success} + (1 - p_{Success}) \times V_{Failure} = 0$



Minimum probability of success for indifference

- EV=0
- $0 = 58 \times p_{Success}$-10
- $p_{Success}=\frac{10}{58}=0.17$

Expected value line

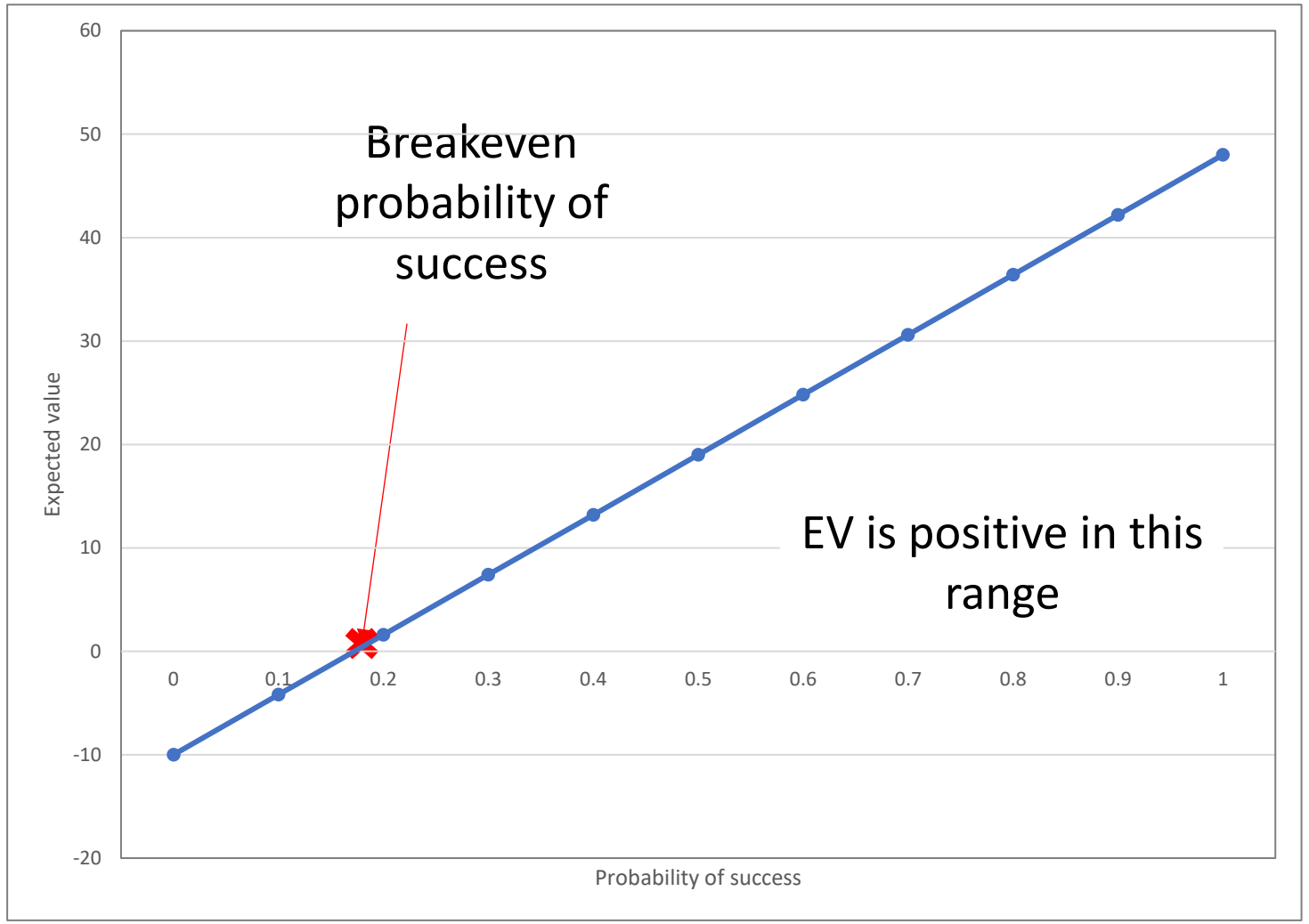
- **Creating an Expected Value Line**
- **1. Define**
- **Outcomes**: Different possible results of an investment.
- **Probabilities**: The likelihood of each outcome occurring.
- **2. Calculate the Expected Value**

The expected value line effectively communicates how various outcomes and their probabilities impact the decision-making process. By visualizing this information, decision-makers can better understand the risk-reward trade-off

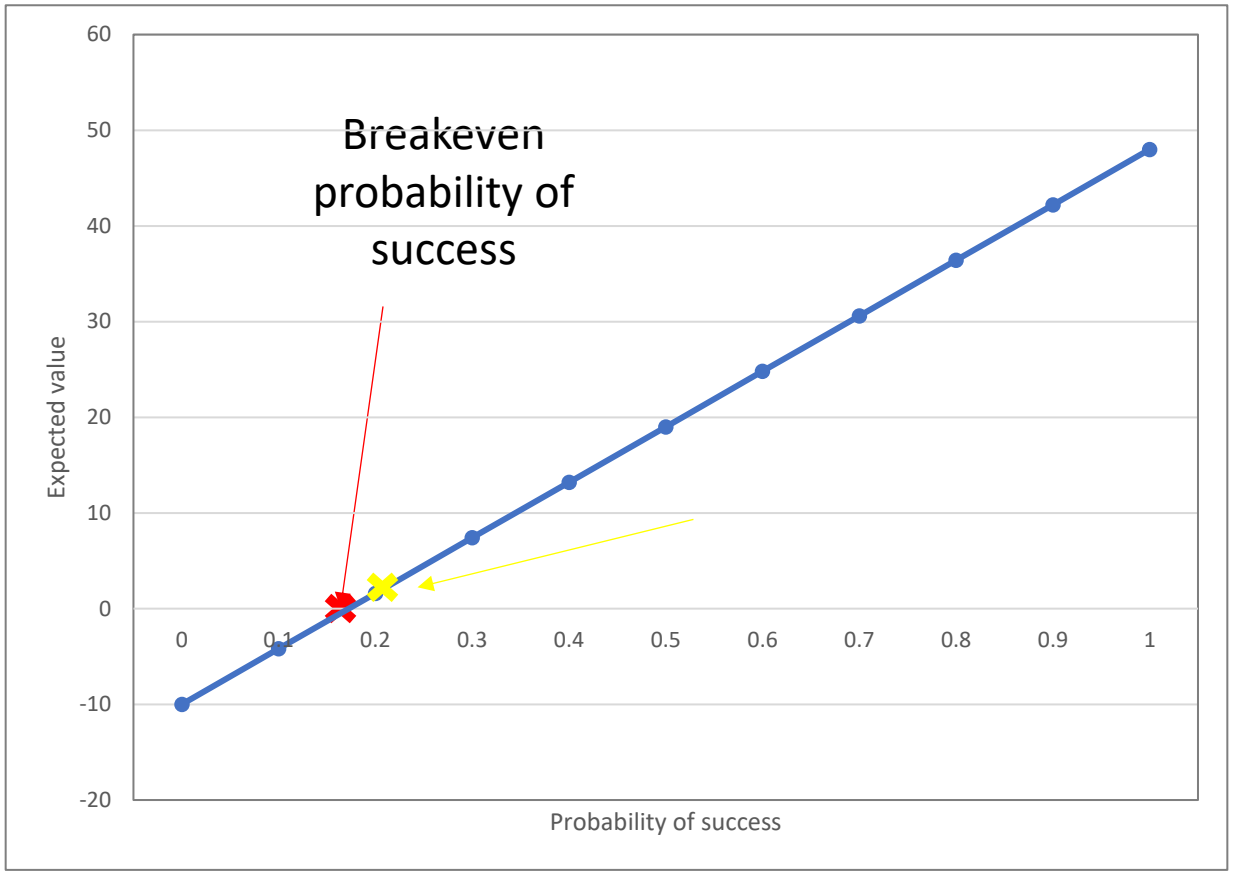
Expected value line

- EV = 58 × $p_{Success}$-10

The **break-even probability of success** is the probability at which the expected value of a risky investment or decision equals zero. In other words, it's the point where the potential gains from the investment are just enough to offset the potential losses, making the decision neutral in terms of expected value.



Expected value line



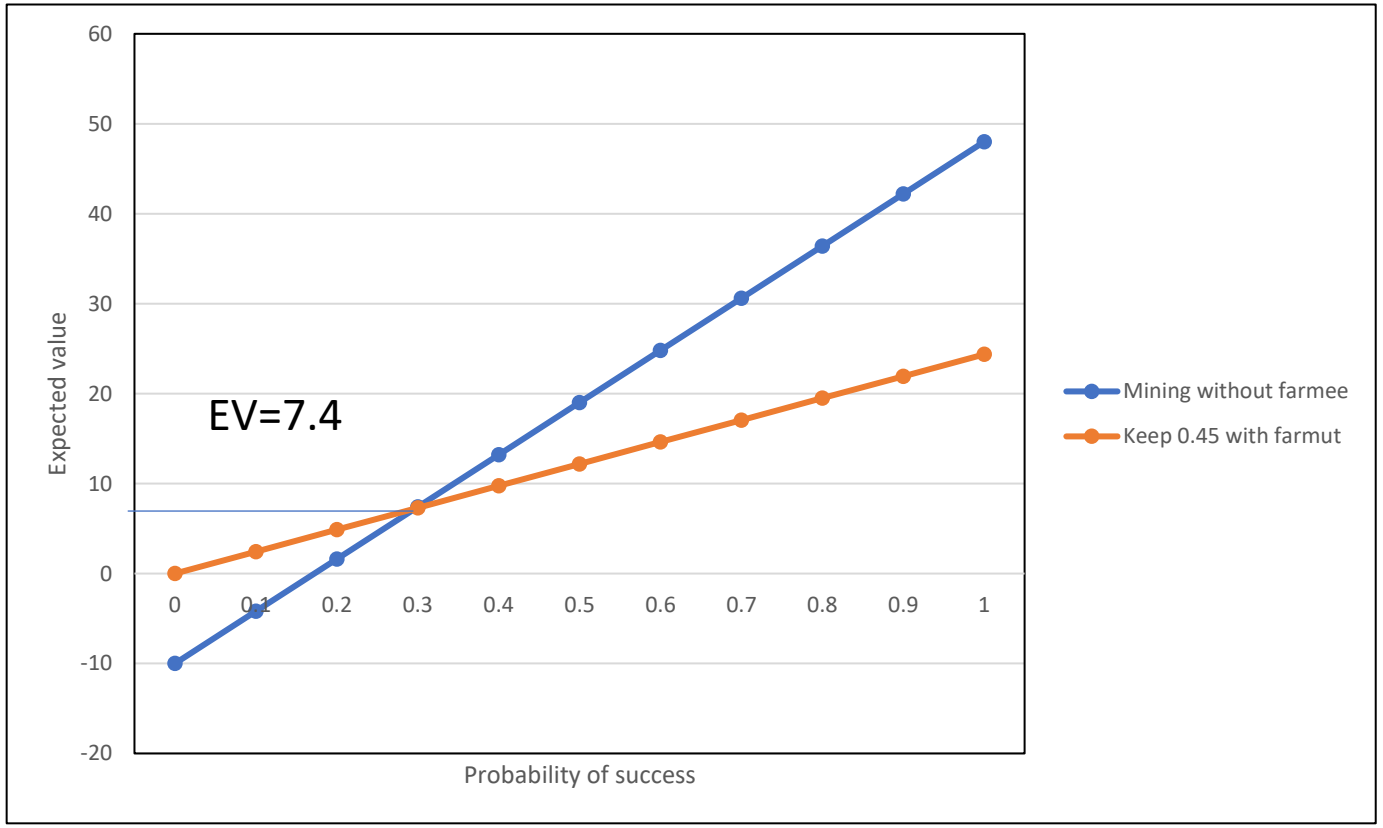
Risk sharing

- **farm-out agreement:**
  An entity (the farmor) agrees to provide a working interest in a mining property to a third party (the farmee), provided that the farmee makes a cash payment to the farmor and/or incurs certain expenditures on the property to earn that interest.

Farmout the mining

- *EV of decision to mine* $= (48 \times 0.3) + (-10 \times 0.7) = 7.4$
- *EV of decision to farmout* $= (58 \times 0.3) \times (1-x) + (0 \times 0.7)$
- Maximize working interest to farmout
- $7.4 = (58 \times 0.3) \times (1-x) + (0 \times 0.7)$
- X=0.57

Risk sharing in mining decision



Farm-out agreement:

- Key Components of a Farm-Out Agreement:
  - Farmor: The party that owns the rights to the property and is looking to reduce its financial exposure or share the risk.
  - Farmee: The party that takes on the interest and associated responsibilities, usually in exchange for covering some costs.
  - Interest Transfer:Specifies the percentage of interest being transferred from the farmor to the farmee.
  - Cost Sharing: Details on how costs will be allocated between the parties, e.g., all upfront payments or some of it being paid by Farmee.

Exercise

- 1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)
- 2-Find the break even probability point
- 3-Find the farmout value that provides same EV for the original company?

Exercise

1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities

- Success: (0.15, 120)
- Failure: (0.85, -5)
- 2-Find the breakeven probability point

[120 * ps]+[(-5) * (1-ps)]=0 →ps=0.04

- 3-Find the farmout value that provides same EV for the original company?

[(120-(-5))*(0.15)] *((1-X)] + [(0.85) * (0)]=13.75

→X=0.2665

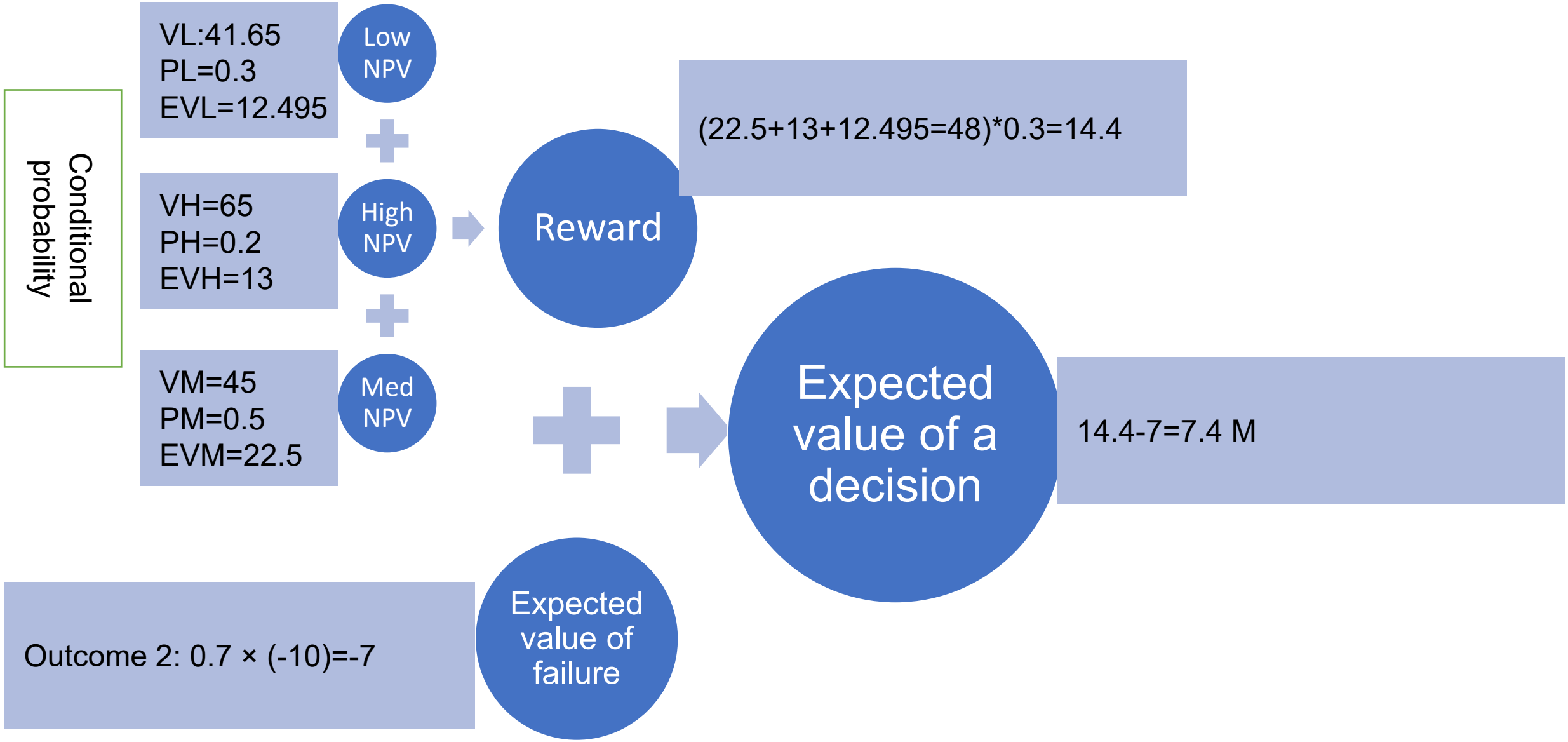
Decision making under risk: more than two outcomes

- A process of choosing between different lotteries.
- A lottery (or prospect) consists of a number of possible outcomes with their associated probability It can be described as:
- $\boldsymbol{q} = (x_1 , p_1 ; x_i , p_i ; \dots x_n , p_n)$
- x_i represents the ith outcome
- p_i is its associated probability
- $\sum p_i = 1$

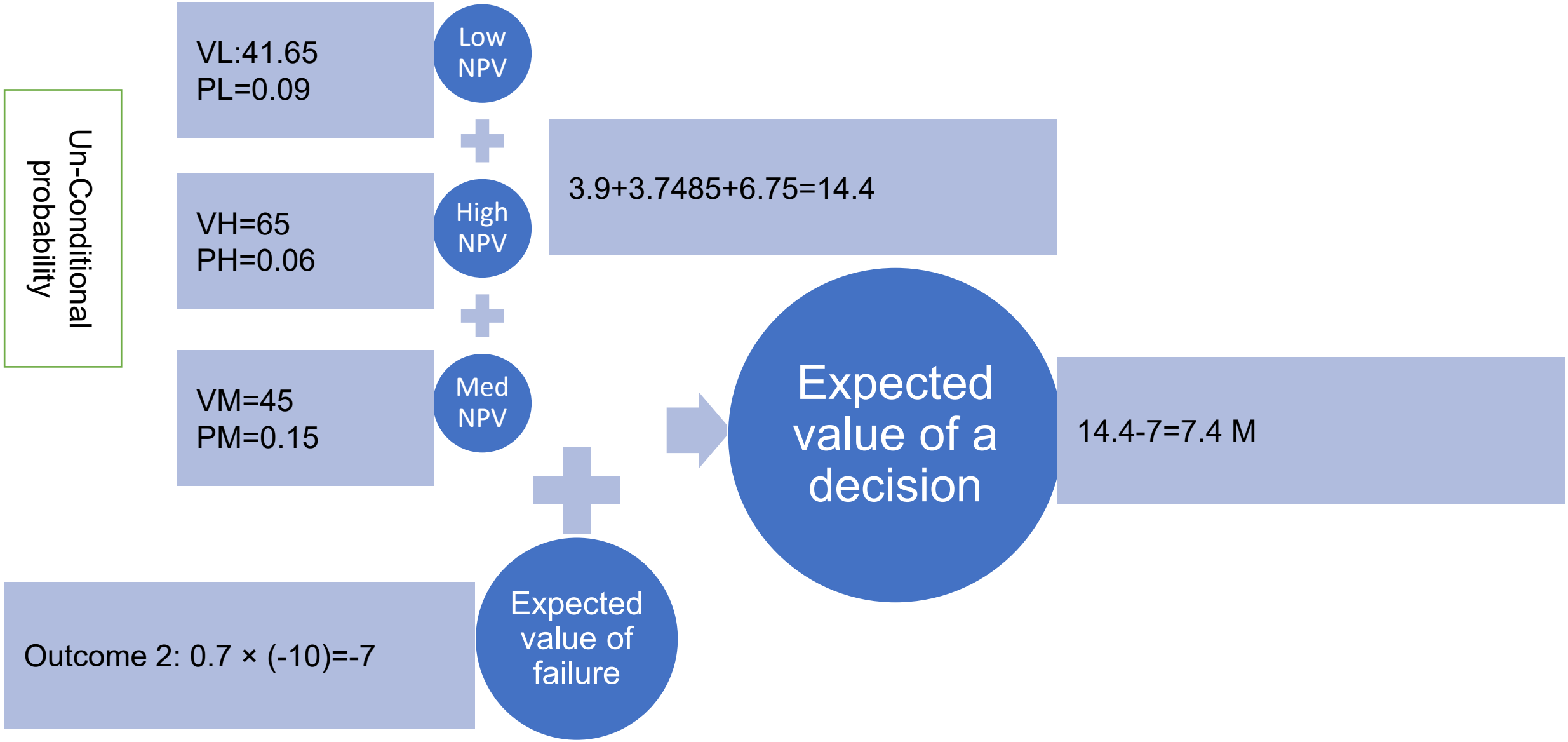
Risky decisions:more than two outcomes

- Outcomes
- x_1 : Success:
  - x_{11} Low success
  - x_{12}: Medium success
  - x_{13} High success
- x_2 : Failure
- $1=p_1 + p_2$
- $p_{11} + p_{12} + p_{12}=1$

Expected value of mining



Expected value of mining



Applying fiscal relief

Outcome 1:
0.3 × (48=58-10)=14.4

Expected value of success

Outcome 2: 0.7 × (-10)=-7

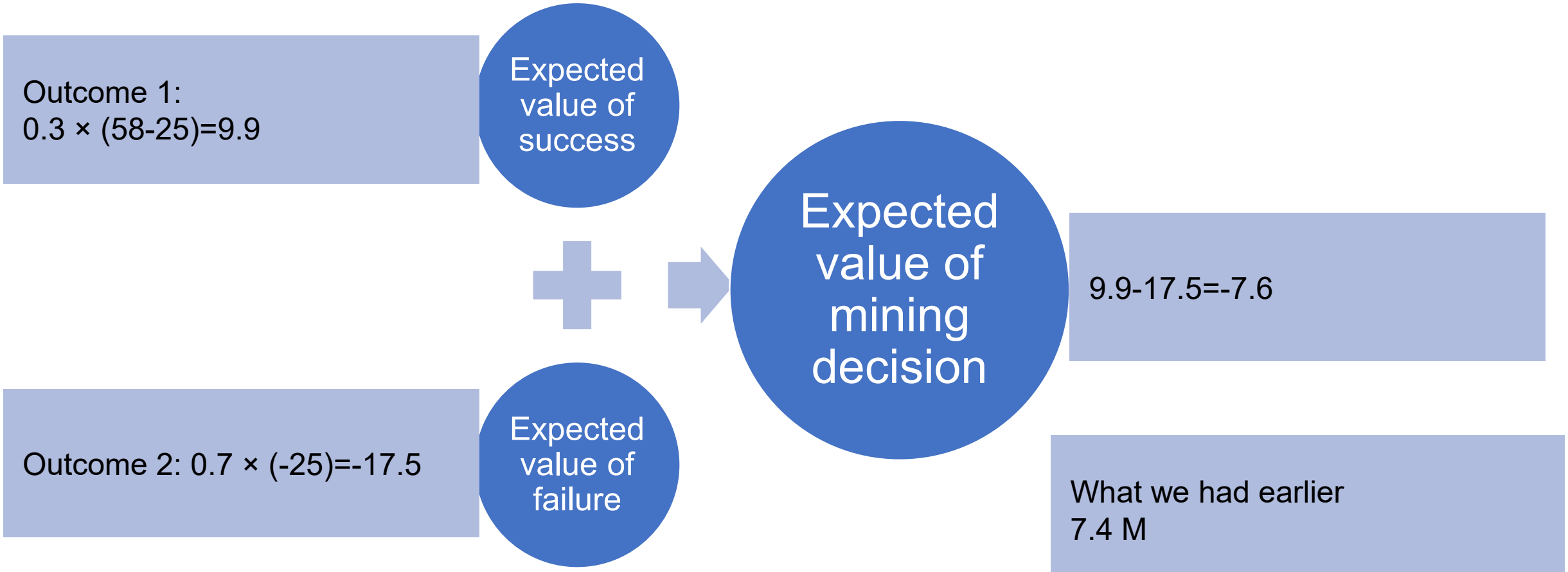
Expected value of failure

Expected value of mining decision

Fiscal relief

- Overall rate of tax: 60%
- The cost after tax with full fiscal relief: 10 M
- Cost with no fiscal relief:
- $\frac{10}{1-60\%}$ =25M

Fiscal relief



Fiscal relief

$EV = (V_{Success} \times p_{Success}) + (V_{Failure} \times (1\text{-}p_{Success}))$

$EV=((58\text{-}\ 25) \times p_{Success})+(\text{-}25\times (1\text{-}p_{Success}))$

$EV = 58 \times p_{Success}\text{-}25$

EV=0

P=25/58

P=43.1%

Compared to 0.17 without applying the effect fiscal relief

Exercise

- 1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, 120)
- Failure: (0.85, -5)
- 2-Find the break even probability point
- 3-Find the farmout value that provides same EV for the original company?
- 4- Recalculate the values of question 1-3 without fiscal relief when the overall rate of tax is 50%. Compare the results with calculated values in questions 1-3.

Exercise

- 1- Calculate the expected value of a mining project with following NPV outcomes and their conditional probabilities
- Success: (0.15, (125-5=120))
- Failure: (0.85, -5)
- 2-Find the break even probability point
- 3-Find the farmout value that provides same EV for the original company?
- Recalculate the values of question 1-3 without fiscal relief when the overall rate of tax is 50%. Compare the results with calculated values in questions 1-3.
- -5/(1-50%)=-10
- New NPV for success: (125-10=115
- Expected value= 115*0.15+(-10)*0.85=8.75
- New breakeven point:115*p+(-10)*(1-p)=0
- P=0.076925
- Farmout agreement: (115-(-10))*0.15*(1-x)+(0)*0.85=8.75
- X=0.533