Using a Bayesian Framework to Assess Overall Models

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What Bayesians do is...

$$p(y \mid \theta) \Rightarrow p(\theta \mid y)$$

$$p(\theta \mid y) = p(y \mid \theta) \bullet \frac{p(\theta)}{p(y)}$$

Standard statistics used to assess model as a whole

R^2 as a measure of fit

$$F = \frac{R^2/df}{(1 - R^2)/df}$$

$$F = \frac{(SS_1 - SS_2/(df_1 - df_2)}{SS_2/df_2}$$ ... for nested models

Bayesian alternative: Testing which "explanation" is more consistent with the data with *Bayes Factors*

M_1 and M_2 are competing models to describe the real world.

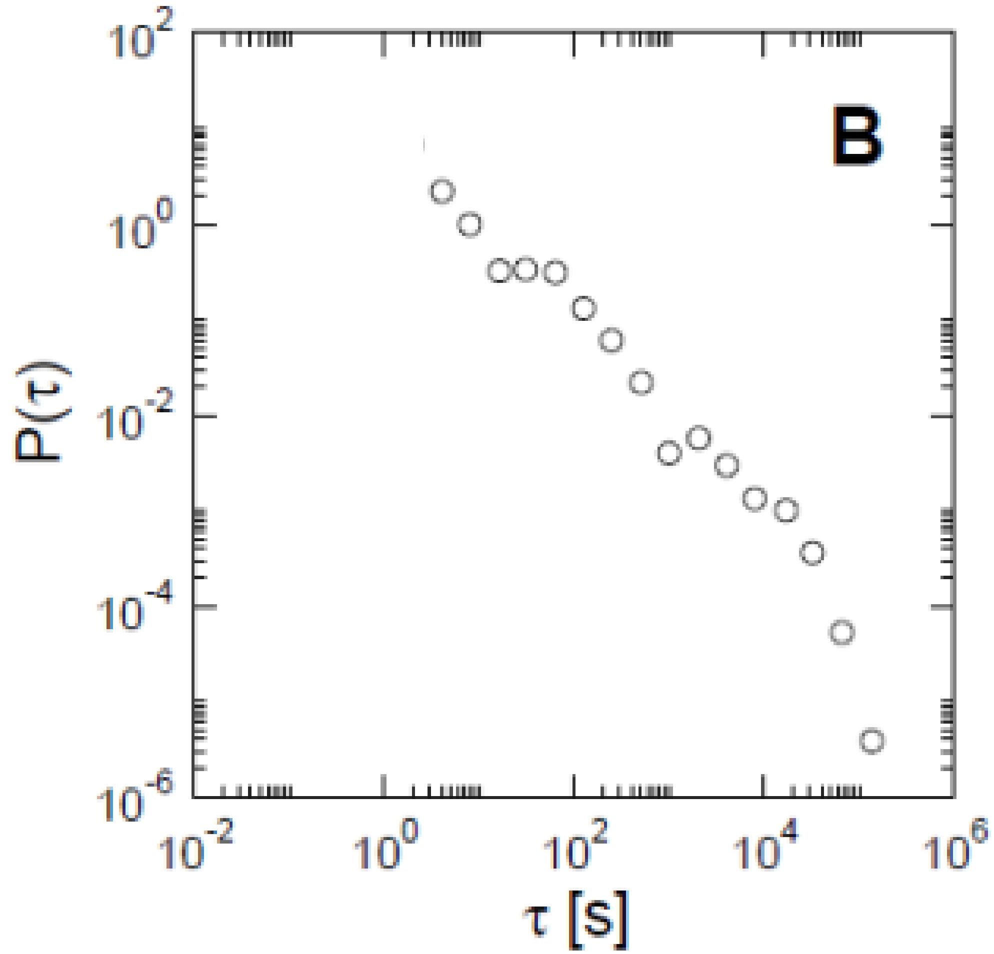
$$\underbrace{\frac{P(M_1 \mid D)}{P(M_2 \mid D)}}_{\text{posterior odds}} = \underbrace{\frac{P(M_1)}{P(M_2)}}_{\text{prior odds}} \underbrace{\frac{P(D \mid M_1)}{P(D \mid M_2)}}_{\text{Bayes factor}}.$$

$$P(M_1) = \frac{odds}{odds+1}$$

Example:

- Barabasi claimed to demonstrate that a set of email response times among 1016 faculty are distributed as a Power Law, described as Power Law queuing.

Barabasi Plot of Distribution of Time Intervals between Emails sent by Faculty



Each circle represents a bin, collecting events that are similar in their time duration.

Example:

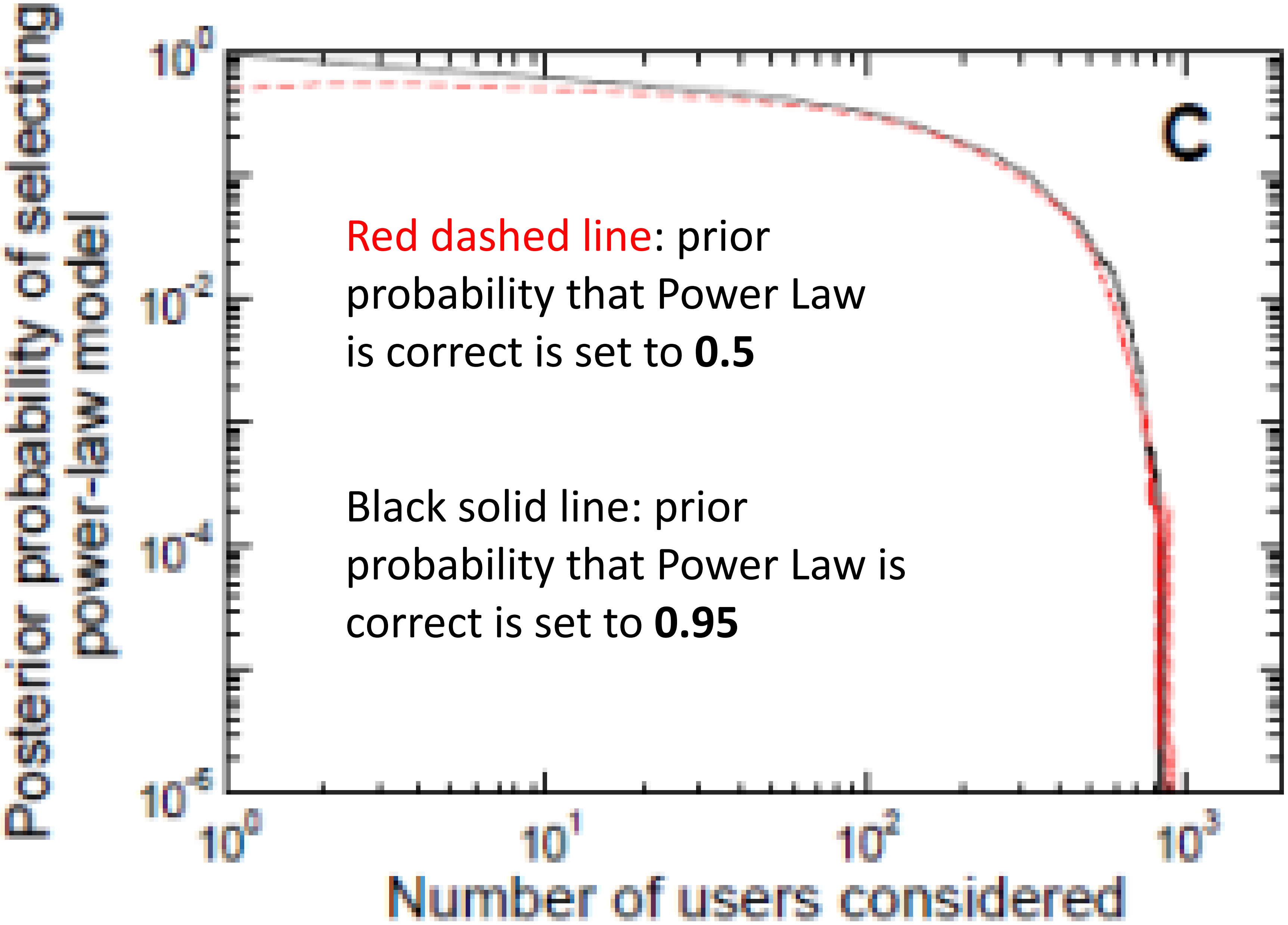
- Barabasi claimed to demonstrate that a set of email response times among 1016 faculty are distributed as a **Power Law**, described as Power Law queuing.

- In a reanalysis of the same data, Stouffer, Malmgren and Amaral (2008)[1] explore whether this is "true"

    - They note that a **log normal** distribution can "look" similar

[1]Stouffer, Malmgren, & Amaral (2008). "Log-normal statistics in e-mail communication pattern." arXiv:Physics/0605027v1.

Results…



Red dashed line: prior probability that Power Law is correct is set to **0.5**

Black solid line: prior probability that Power Law is correct is set to **0.95**

Second Example: Specifying whether our prior beliefs determine what we find…

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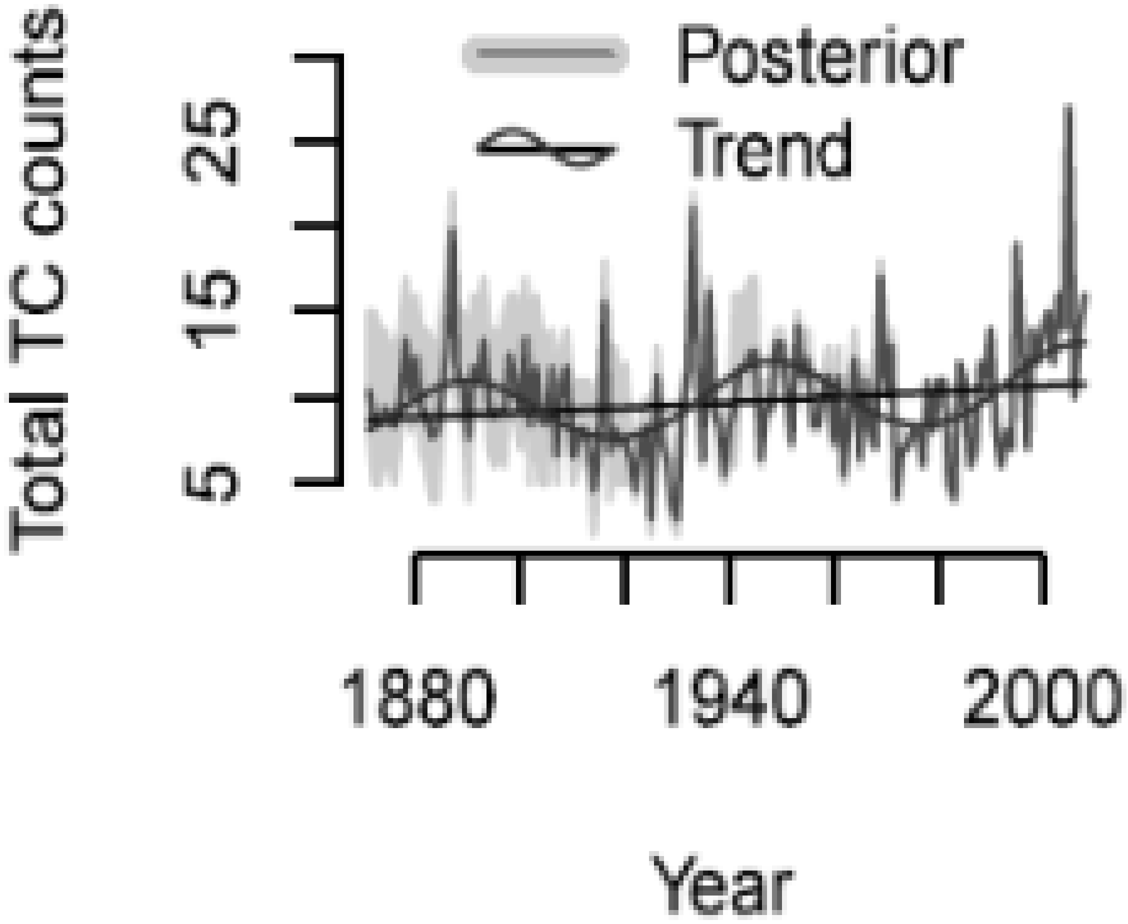
- Question: Is climate change observed in an increase in the frequency and strength of hurricanes over the past 150 years?[2]

[2] Surya T. Tokdar, Iris Grossmann, Joseph B. Kadane, Anne-Sophie Charest, and Mitchell J. Small (2011). "Impact of beliefs about Atlantic tropical cyclone detection on conclusions about trends in tropical cyclone numbers." *Bayesian Analysis*, **6**(4):547-572.

Counts of observed Tropical Cyclones per year



Tropical Cyclones – Technologies for Detecting

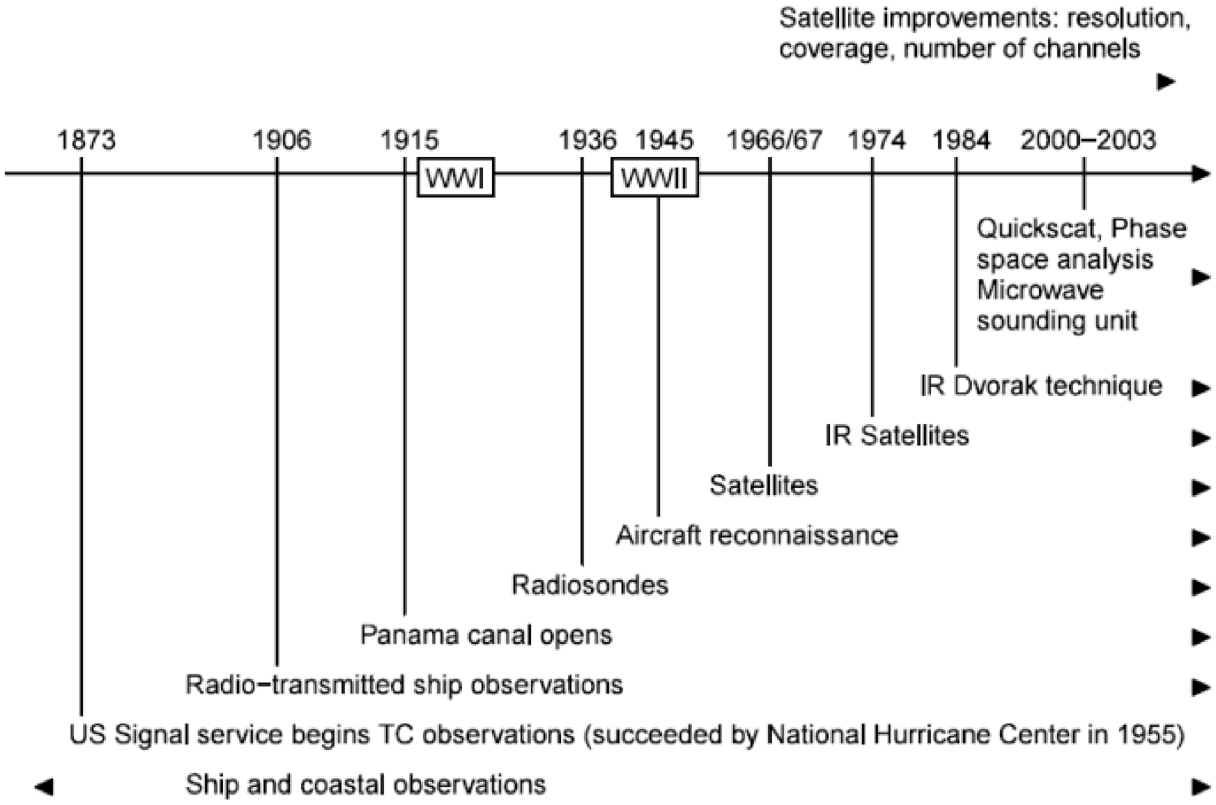
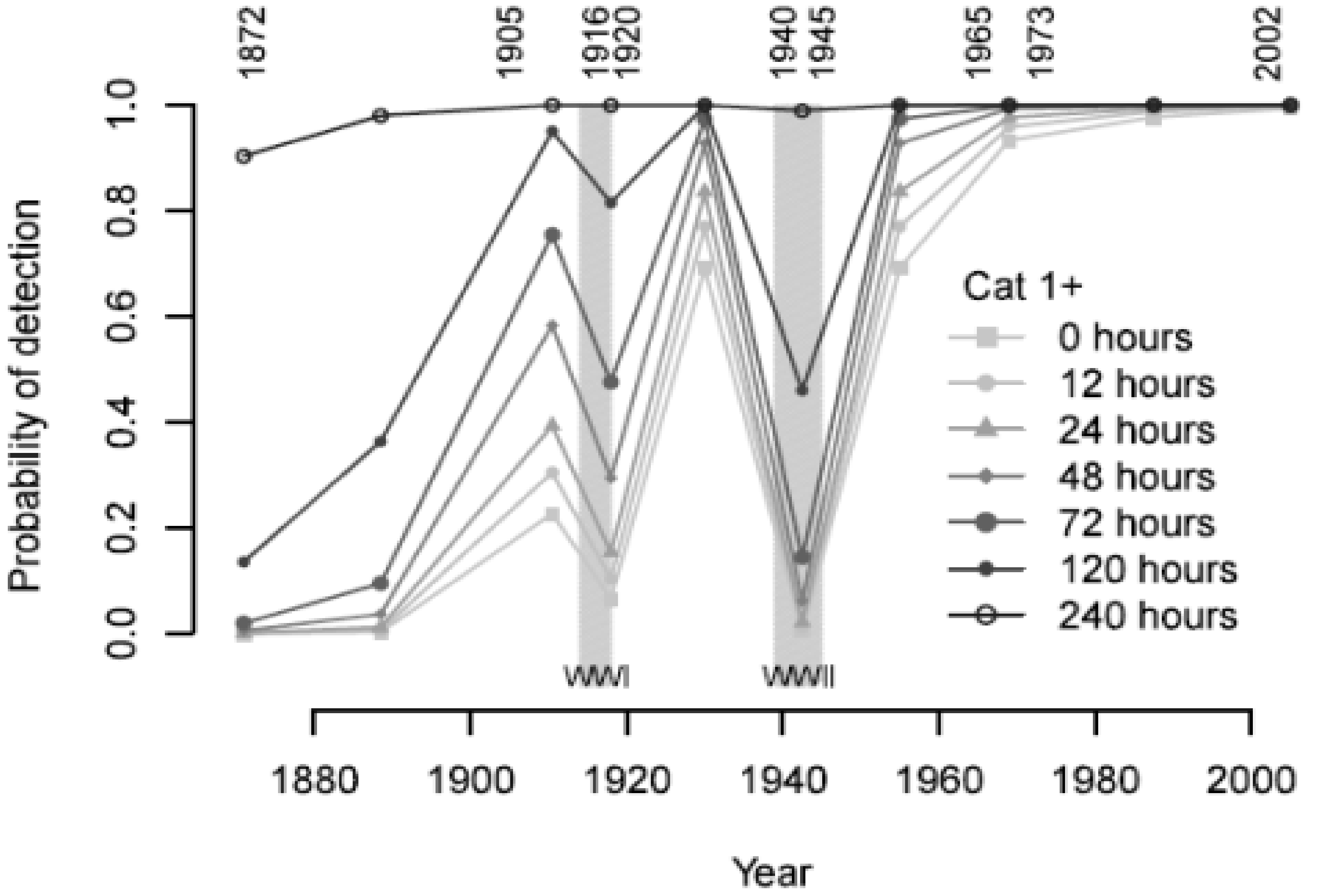
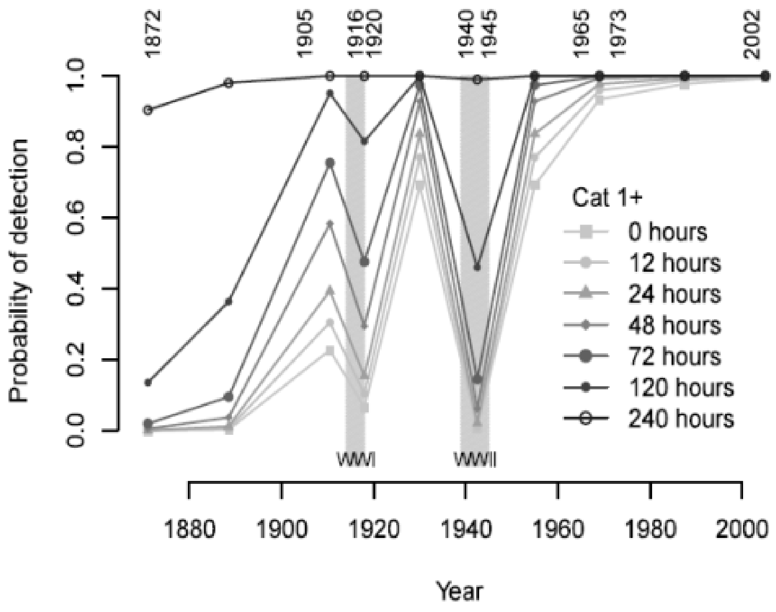


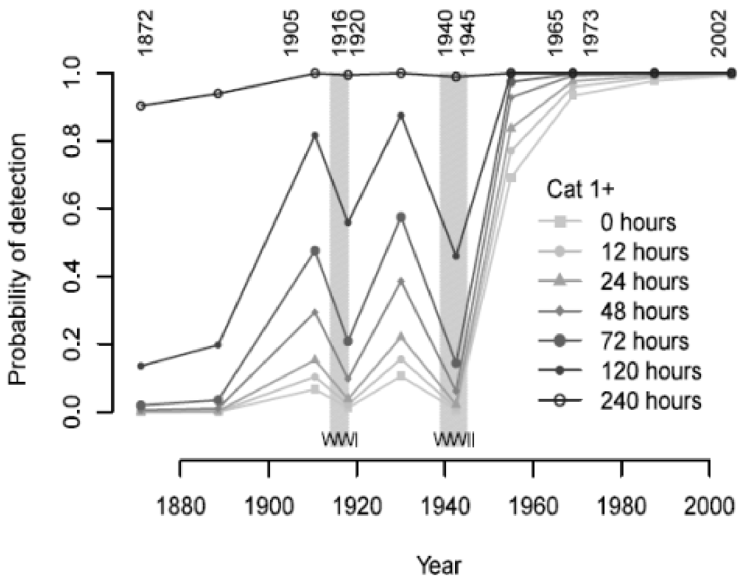
Figure 1: A timeline of Atlantic TC observation cataloging main changes in observation technology along with other major events that may have impacted TC recording.

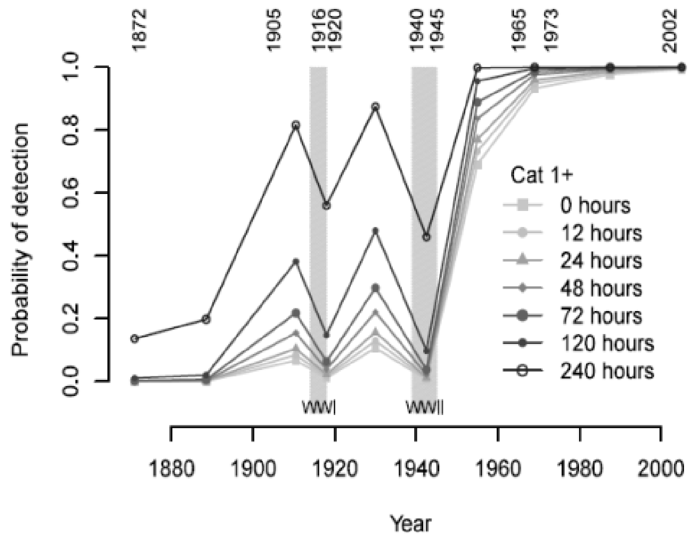
Probability of Detecting Cyclones



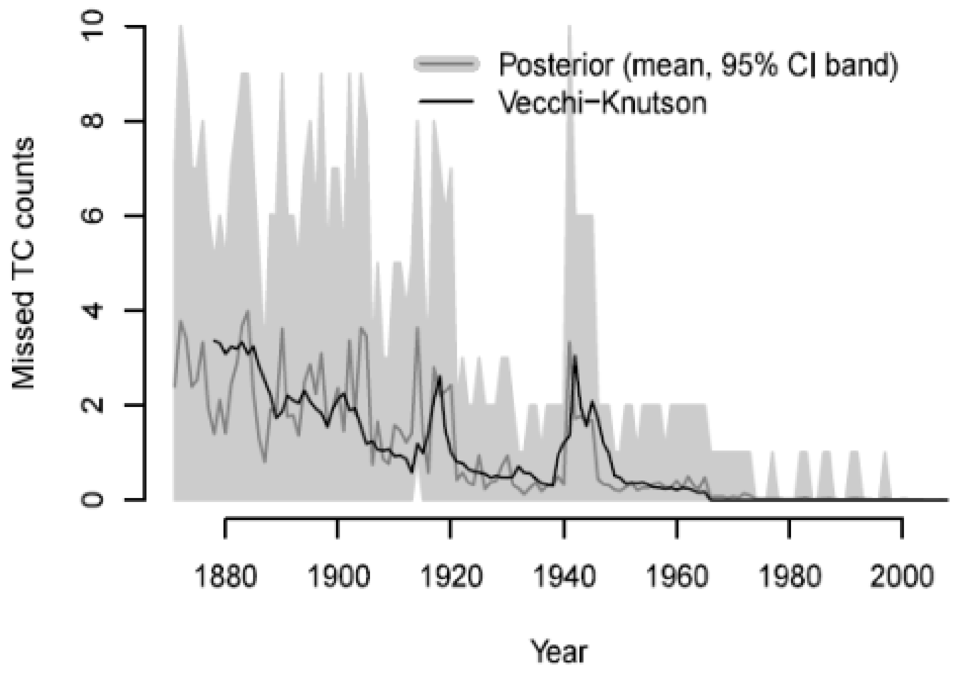
Probability of Detecting Cyclones

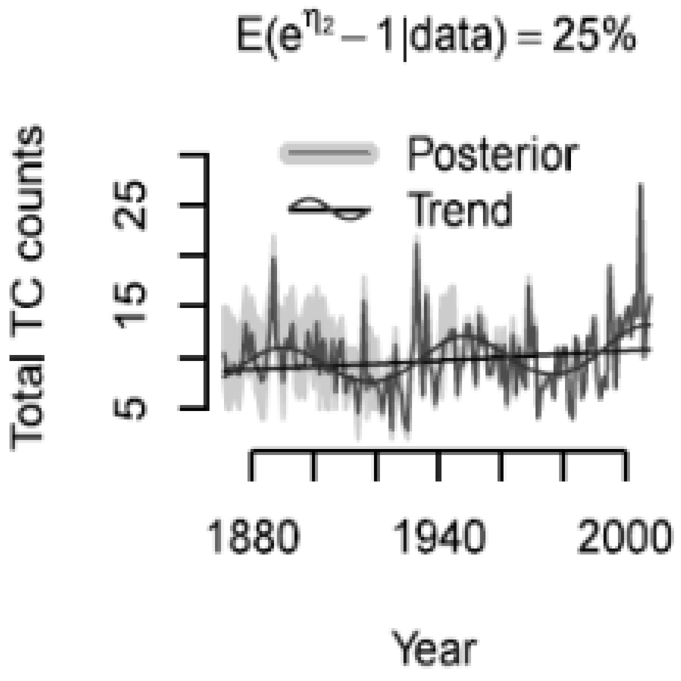




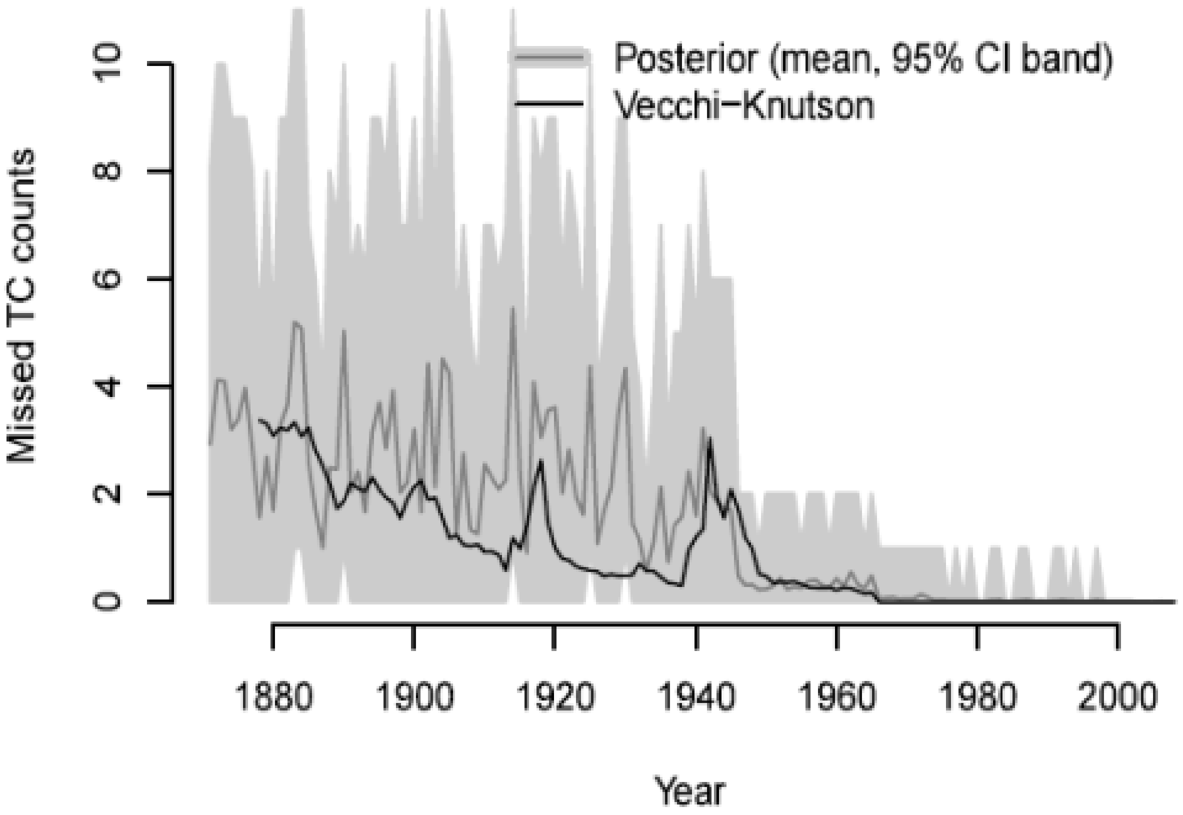


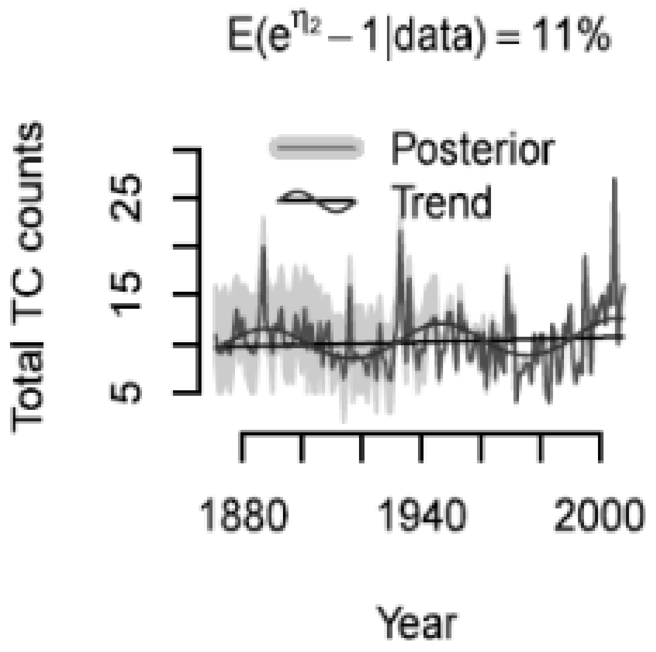
Number of Tropical Cyclones Missed, and Implications for Trend under Assumptions E1



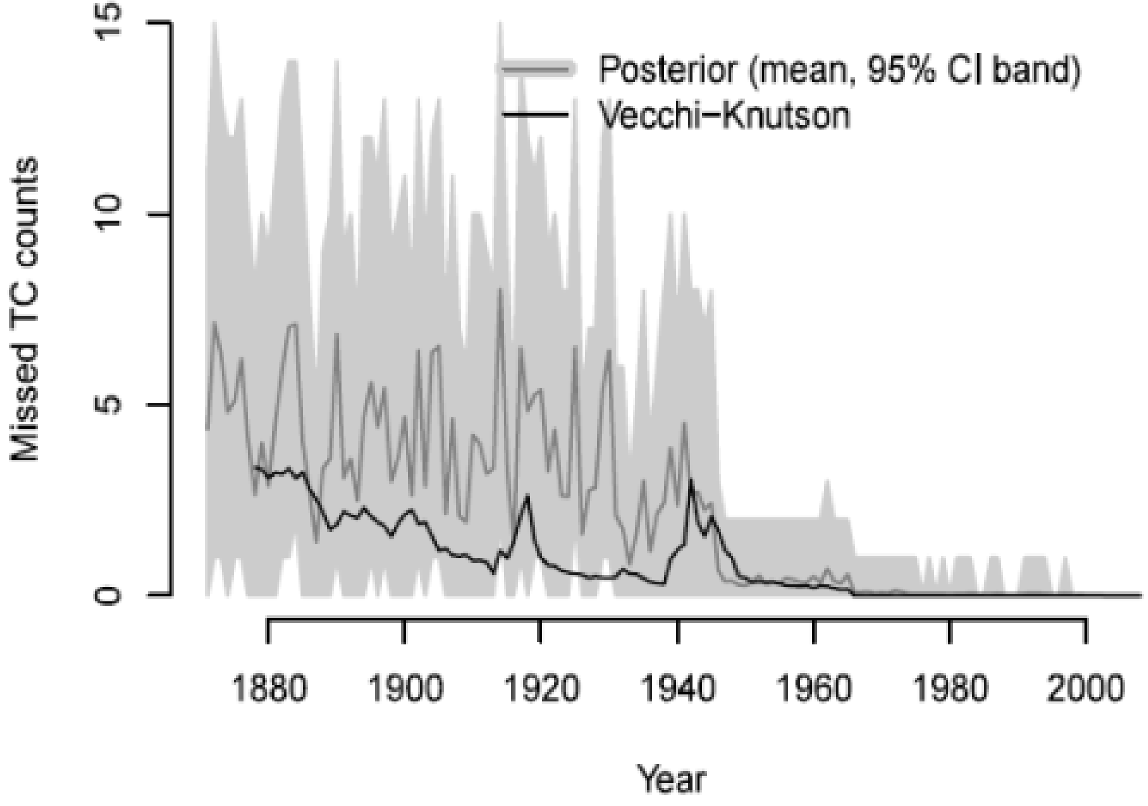


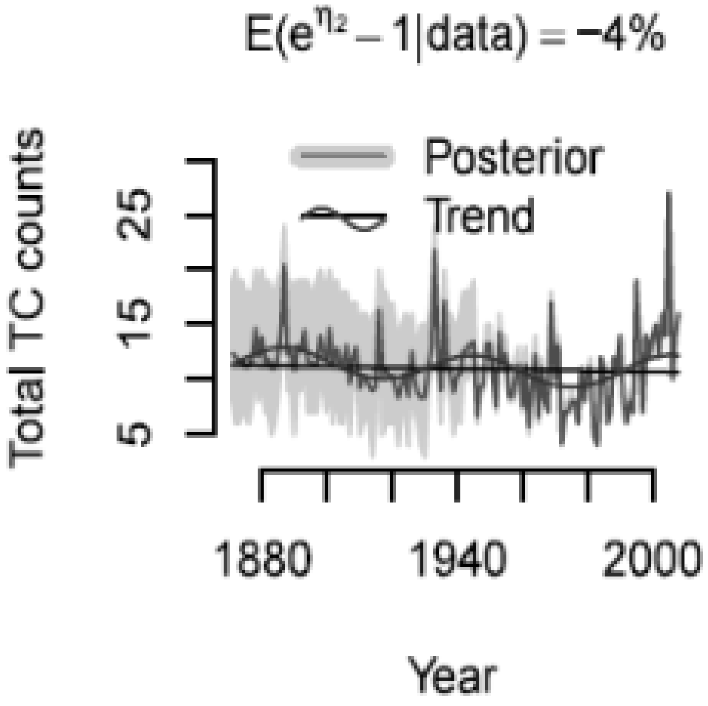
Number of Tropical Cyclones Missed, and Implications for Trend under Assumptions E2



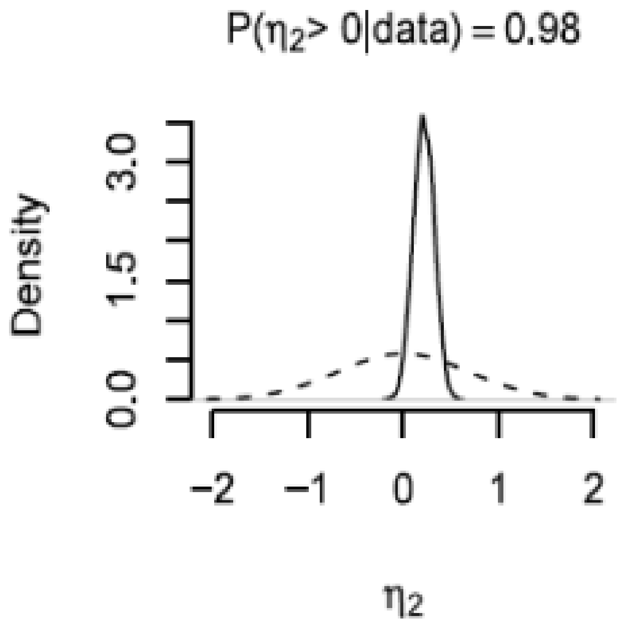


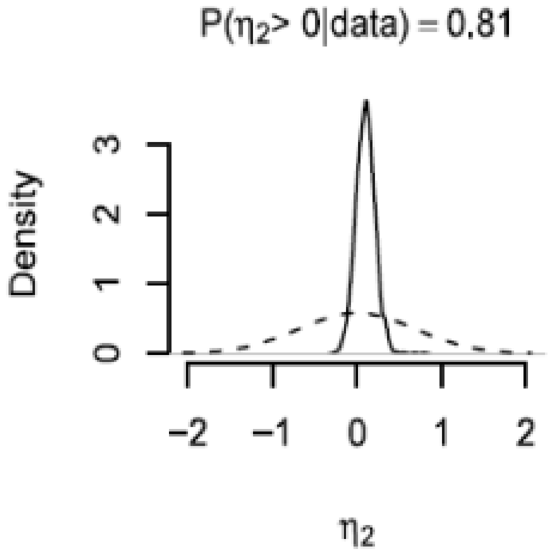
Number of Tropical Cyclones Missed, and Implications for Trend under Assumptions E3

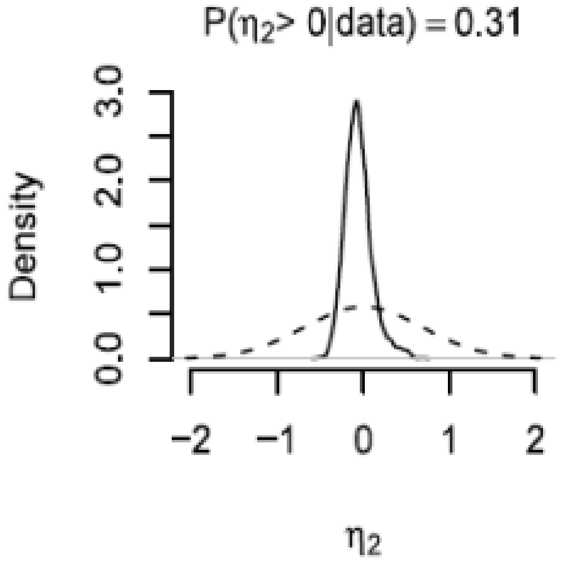




Posterior probability of Increasing Trend for Assumptions E1, E2, and E3







Their Conclusion

- There is no sense in using these data to argue whether or not we are experiencing global warming, or that it is causing more hurricanes. Move onto other evidence…