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A System Dynamics Model of Submarine Operations and Maintenance Schedules

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The extent to which a proposed military force will achieve operational objectives is a prime concern of defence planners. This paper discusses the problem in the context of the exercise of sea power in distant waters and shows that a model of the whole problem would require a feedback analysis, for which an appropriate approach would be system dynamics. Such models have, in general, been continuous, but ships are discrete objects. The paper therefore addresses the construction of discrete system dynamics models as the basis for a model of the whole problem. Two models of a submarine force are presented. The first deals with the construction and major refit programmes, to evaluate the periods of fleet service availability achievable from a submarine force of a given size. The second examines unit usage during periods of fleet service.

Key words: system dynamics, defence analysis, life cycle costing

INTRODUCTION

In a previous analysis of naval operations, Coyle[1] developed an influence diagram of the factors involved in the projection of naval power. This paper concentrates on the role of submarines in such operations.

Figure 1 gives a schematic influence diagram of the whole problem. Political circumstances dictate an operational need to have forces on station in a distant locality, perhaps to exert sovereign control over an area of sea. The operational need is translated by naval planners into a need for submarines on station, complemented by other forces, such as surface ships or long-range

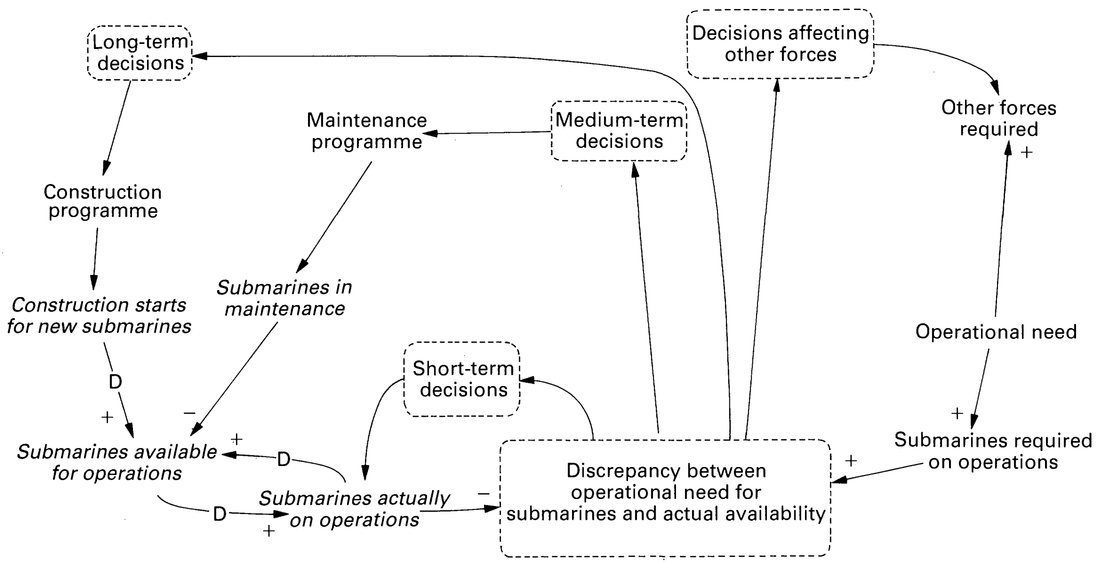

FIG. 1. *Schematic influence diagram for the problem.*

* The views expressed in this paper are entirely the authors' own and do not reflect the official position of the Royal Australian Navy. The numerical data are fictitious values to illustrate an analytical methodology.

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aircraft. If, for some reason, the required number of submarines are not on station then a discrepancy arises between requirement and fulfilment.

Such a discrepancy can, in general, give rise to control actions which may affect other forces, or the submarine force itself. In the latter case, short-term decisions may be made to keep a submarine on operations after its due date for return. This may cause severe problems of supply and may take a submarine past the validity of its certificate that it is safe to dive. While the last factor might be over-ridden in war, it is an important consideration in peace. Medium-term decisions may defer the planned maintenance and overhaul programme, though again with problems being stored up for the future. Finally, long-term decisions may be made to acquire additional boats (the term used for submarines), though at great cost and with much delay before they can have any effect. The frequency with which such circumstances might arise is an interesting indicator of system performance, and the models described below measure such indicators.

Figure 1 is clearly a feedback system and one might analyse its dynamic behaviour to study, in particular, how well a force of a given size is likely to meet the operational need and what policies might guide decision-making to cope with difficulties as and when they arise. An appropriate modelling tool would be system dynamics[2,3], but such models have traditionally been seen as involving continuous simulation (though see, for example, Coyle[4] for the incorporation of stochastic and discrete effects in a model of coal production). In the present case, submarines must be treated as integers if the final model is to carry conviction. This paper therefore deals with the formulation of integer models of the parts of the schematic influence diagram in Figure 1 that are in italics. (The technical aspects of the departures from continuous system dynamics modelling are dealt with in the appendix.)

Two models are described, both using hypothetical data. One appraises long-term construction and major dockyard refit periods, while the other accounts for the dynamics of unit availability for operational tasking. The models are applicable both to peacetime operations and to periods of tension before the outbreak of hostilities. They do not include the effect of losses of the patrolling force, though they could be extended to represent combat.

Before discussing the models and their results, it is necessary to explain something of the technical background to submarine operations. The explanation is broadly applicable to surface vessels, so the models are of some generality.

CYCLES IN SUBMARINE OPERATIONS

The procurement cycle, from identifying a need, through naval staff planning and budget controls, government decision and programme development, results in a dockyard construction order being let for a batch of ships. The ships may be either from a new class or an addition to existing fleet units. The rate of delivery of new ships becomes a function of the size and complexity of the individual units, the number of units in the batch, dockyard, crewing and fiscal limitations. The cost to the nation is the sum of planning, developing, constructing, maintaining, supporting, crewing, administering, updating, operating and finally disposing of the batch. That is the full *life cycle cost* rather than merely the initial outlay on purchase.

A unit, once built, enters a period of fleet service. During the fleet service period the unit is supported by its parent base or a forward national base while carrying out operational duties. For the purpose of the model, this period is 16 months. On completion of the first fleet service period the submarine returns to a dockyard for an *intermediate docking* of 2 months' duration. It repeats a second fleet service period of 16 months and follows this by the *mid-cycle survey docking*, a 4-month maintenance period. It then enters the third fleet service period followed by an intermediate docking and fourth service period. At the end of the fourth service period it returns to the dockyard for a major refit of 2 years' duration.

The first to the fourth fleet periods comprise Cycle A. On completion of the refit the submarine repeats the fleet service process as Cycle B. It completes a third cycle on completion of a second major refit and then decommissions and is scrapped. The operational life of the unit is some 22 years from the completion of construction to its 'paying-off'. This cycle is shown in Figure 2(a). The maintenance periods within a fleet service cycle are shown in Figure 2(b).

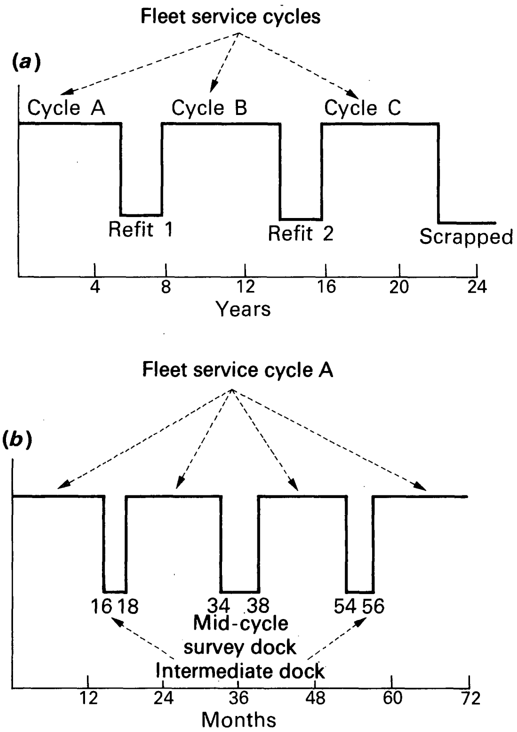

FIG. 2. *Cycles in submarine operations. (a) The submarine refit schedule. (b) The submarine docking cycle.*

THE LONG-TERM MODEL

Modelling of construction and refits

At the beginning of the simulation, construction commences on the first submarine in the batch. It is assumed that the construction yard is tooled, manned and stocked ready to commence construction and that the Navy will have men trained and ready to take it to sea when it is built. The construction yard starts building new submarines at 18-month intervals, with a 4-year period between starting construction and the submarine being available to enter its first fleet service period.

System dynamics modelling normally uses continuous rates of change of levels or states, but it would be unrealistic to model the building of 0.5 of a submarine. Thus, integer values, in this case 1 or 0, are required. Similarly, the dockyard would have 1, 2 or 3 units in construction or maintenance, rather than 0.75, 1.35 or 2.65. To avoid this one must aggregate the effects of continuous dynamic variables to integer values (see, for example, Coyle,[2] chapter 4). The equations to provide these effects are explained in the appendix.

The model differentiates each type of maintenance; thus with minimal change it can show the workload of the repair dockyards versus the constructing yards if the submarines are not maintained in the same yard in which they were built.

Performance indicators

The code records the total number of units available for fleet service, UAFS, as well as the total in dockyard hands, UID. It has the ability to calculate the periods when only 1 or 2 or 3 etc. submarines are available and the percentage of coverage given the time period from the time the first unit commissions to the time the last unit is scrapped.

The coding represents the full life cycle availability for a particular class or batch of submarines ordered. If these units are replacing existing units, or if other units are phased in as these units are themselves scrapped, then fleet availability figures need to be reconciled with the other types of submarines' life cycle availability.

Results from the long-term model

Figure 3 shows a typical pattern of output from the long-term model, in this case for a fleet of six vessels. The solid line shows the progressive increase in boats available for sea service. The first

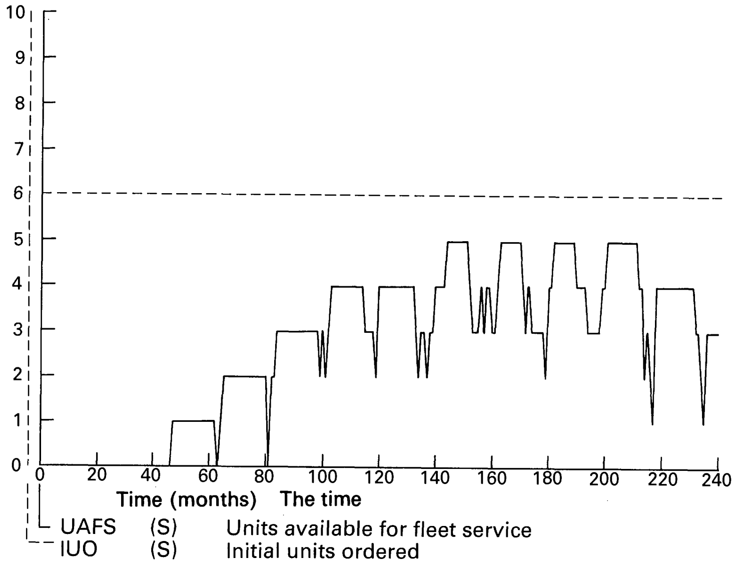

FIG. 3. *Typical output from the long-term model.*

construction is completed at about month 47, with that vessel returning for major maintenance at month 64, just as the second is completed, these times corresponding to within a month to the theoretical values. Thereafter, submarines move out of construction and back in for maintenance or refit, over a period of 20 years.

The accuracy of the model may be judged from the fact that, for example, hand calculation shows that 3 submarines should be available for duty at time 230. In fact, four are available then, but 3 become available at time 232. This is a good correspondence between this simple model and a detailed calculation, and lends credence to the view that these models could, indeed, be used as part of a more sophisticated analysis, representing the policy factors mentioned in Figure 1.

The simulation calculates units available for sea duty, given the number of hulls within the major maintenance cycle, and Table 1 gives these results for fleets of 4, 6, 8 or 10 hulls. It shows the percentages of the total period when at least 1 or 2 or 3 etc. submarines are available for fleet service. The model allows for the fact that the different batch quantities result in varying durations of operational life. For example, a batch of 4 would be in service for 8 × 18 months (the delay in the construction programme) less than a batch of 12 hulls. The figures for months available includes the percentage time this represents for a batch of that size. The time starts when the first submarine of the batch commissions and stops when the last unit of the batch decommissions.

TABLE 1. *The percentage of the total period when at least 1, 2, 3 or 4 submarines are available in a fleet of 4, 6, 8 or 10 hulls*

Total in batch		4	6	8	10
		%	%	%	%
At least	1	97	98	99	99
	2	82	89	90	90
	3	56	74	79	79
	4	0	50	69	69

The same results are shown graphically in Figure 4, in which the lines sloping down from left to right are the percentage availabilities achieved from batch sizes of 4, 6, 8 and 10. Those sloping upwards from left to right are the effects on percentage availability for a given number of boats of the successive increase in batch size.

A navy must determine its operational requirements for unit availability, then determine how many units need to be built in order to achieve that target. Table 1 predicts a rapid tapering-off of unit availability regardless of the batch size. This may be seen more clearly in Figure 4. The results suggest the possibility that it may be more cost-effective to consider a series of short buying

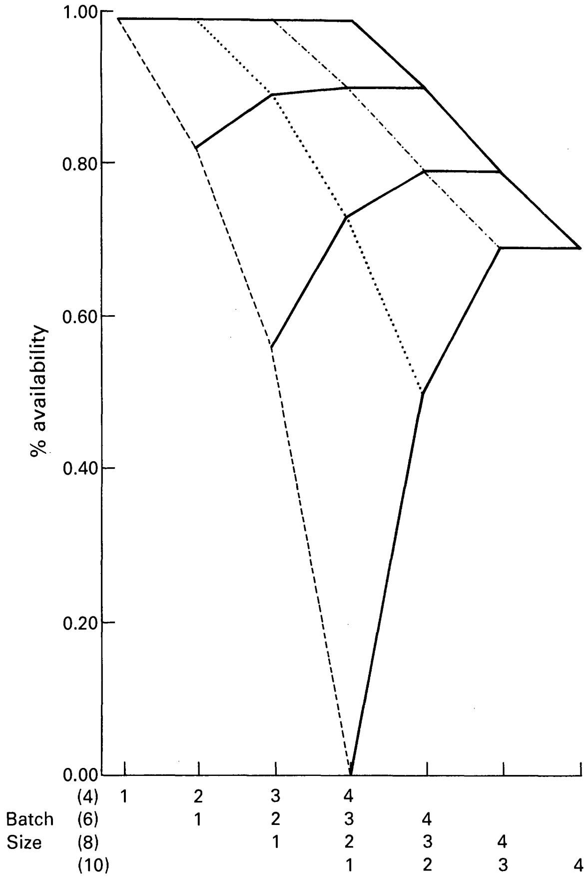

FIG. 4. *Submarine availability achieved.*

programmes rather than one large one. That would allow technological updates to be incorporated into new designs rather than retro-fitting such modifications into existing hulls, which may be difficult and expensive. The spreading out of work by a sequential series of small orders could prove to be of benefit to the nation's shipbuilding industry.

This point is illustrated in Figure 5, which shows the total shipyard load as time passes. As expected, there are long periods of steady load as the building programme progresses, followed by much greater instability during the refit periods. Such data may give guidance on which shipyards should receive which contracts for refits or construction.

INTERMEDIATE-TERM MODEL

Within the intermediate term model, units may be in any one of the following states:

(a) preparing to sail on patrol;
(b) on the outward transit to the patrol area;
(c) in the patrol area;
(d) on the return transit having completed a patrol;
(e) in maintenance having returned to port; or,
(f) in port, available for sea duty having completed maintenance.

During fleet service periods a unit may be tasked with a variety of training roles and take part in both national and allied exercise commitments.

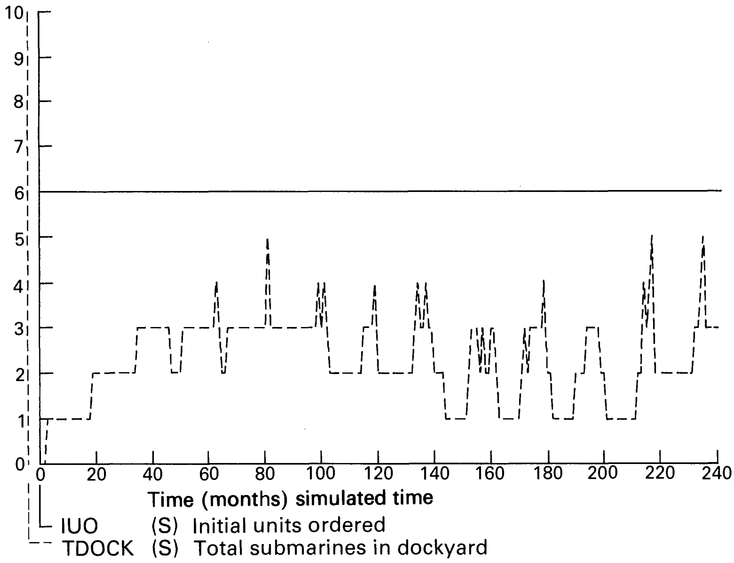

FIG. 5. *Work loadings in the dockyards.*

The simulation assumes the area to be patrolled is 10 days' sailing time from the operating base and uses a total patrol duration of 60 days, giving a maximum of 40 days on task in the patrol area. Once a unit returns to port, the simulation uses a 15-day maintenance norm before the unit is available for sea duty again. These minor maintenance periods are not shown in Figure 2(b).

In practice, the nominal patrol duration may not be achieved, owing to minor breakdowns which cannot be repaired with the limited facilities available in a submarine at sea. To represent this, two random numbers are generated on each occasion a submarine sails. The first will determine whether or not an early return will occur, the second governs the ensuing patrol duration.

The model forces 50% of the units to return early, a hypothetical figure which is easily changed to actual values. For simplicity, the breakdowns occur only in the patrol phase but could be extended to the transit delays. The simulation credits a 'broken' submarine with a 10-day return transit time; this is easily extended if desired.

In the normal course of events a submarine should sail every 40 days to ensure that one arrives as its predecessor leaves the patrol area, thus providing continuous coverage. The breakdown facility complicates this simple cyclic process. The sailing rate becomes a response to sending a unit as soon as one is needed, provided one is available to be sent. A unit is needed if the 40-day cycle since the last sailing time occurs or if a submarine signals it must return early. A unit is available once it has completed post patrol maintenance.

Some typical behaviour of the intermediate model is shown in Figure 6, for a fleet of two operational submarines. The solid line represents the times when a submarine is at its assigned station, the gaps between representing failure to meet the operational requirement. The severity of the 50% breakdown assumption is easily seen, with very few submarines, during this short period, achieving the nominal 40 days on station. The gaps of 10 days between submarines on station arise because one vessel has only just completed its return voyage and 15 days' minor maintenance before the other returns, so 10 days must elapse before the relief boat can reach its station. On one occasion, at time 130, there is a gap of 10 days during which no submarine is available for departure.

Whether a submarine could be quickly despatched to replace the broken one depends on availability and on such aspects as whether radio silence is to be maintained. In practice it is a useful indicator of performance to measure how often such exceptional demands might arise.

Intermediate results

The simulation examines operational availability within the minor maintenance cycle of the submarine's usage plan. For a given number of submarines within this phase of their usage, one is able to determine overall operational availability. The measure of success is to patrol continuously

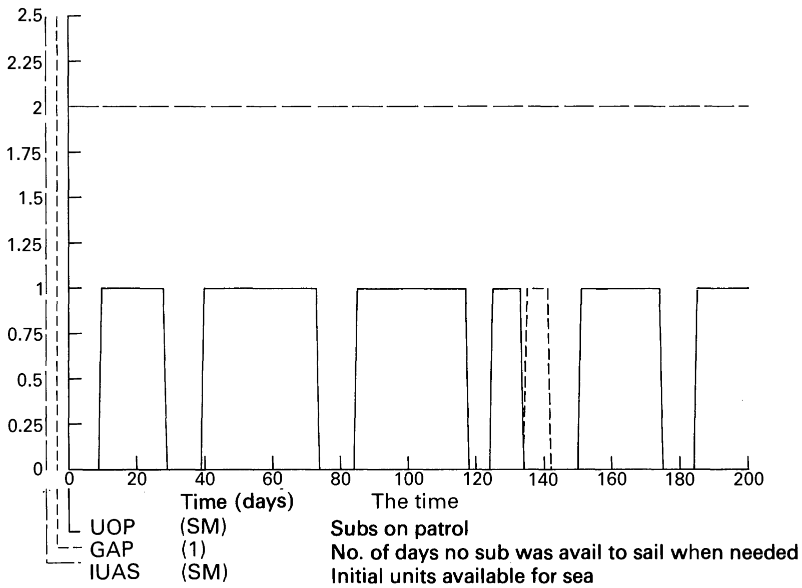

FIG. 6. *Typical output from the intermediate term model.*

an area of sea, some 10 days' sailing distance from the unit's base. As a corollary to this operational 'up-time', one is able to examine support base loadings. Apart from time periods when no unit was on patrol it is important to establish when a submarine should have sailed but none was available to be dispatched. It should be noted that an artificially high mechanical failure rate has been used, and the model monitors the system's ability to recover from these severe shocks.

The total number of days on station in the patrol area (DOS) and the total number of days on which a submarine should have been sent, but none were available (TGAP), for the simulation is given in Table 2. The length of the runs used was 10,000 days (some 27 years).

TABLE 2. *Results of the simulation giving number of days on station (DOS) and total number of days when required but not available (TGAP) for 1, 2, 3 and 4 units available*

	Units available for fleet service			
	1	2	3	4
DOS	4790	7882	8117	8117
TGAP	4289	262	0	0

INTERMEDIATE MODEL RESULTS

Clearly, with only one submarine the area is patrolled for less than 50% of the time; this figure rises significantly when two units are available for duty but then does not rise as dramatically for any further increase in the number of hulls. The rise to three merely reduces the number of days on which no unit was available when one should have been sent. These figures are valid for one area only and would change if the simulation allowed a unit to break down in transit either to or from a patrol, or increased the transit time for a broken unit, or if the percentage of breakdowns on patrol was amended.

CONCLUSIONS

The models described in this paper have successfully represented the behaviour of integer submarines over the long-term construction and refit and over the medium-term operational cycles. The results of the long-term model indicate unit availability within the major dockyard maintenance schedule for a batch of submarines ordered. The intermediate model examines patrol coverage in a given area of interest and highlights the occasions when units were required, but none were available to satisfy the demand to sail. In this way, these simple models could provide a framework for a broader policy model to embrace the wider issues indicated schematically in Figure 1.

The results support the concept expressed by Wolstenholme,[5] who states that 'the role of system dynamics in defence operational analysis can be currently perceived as being one of creating insights into problem situations prior to conventional modelling'. We regard this as a sound and realistic judgement, which is of wide applicability. In particular, it appears that there is scope for much further work on the detailed aspects of submarine operations in combat.

FURTHER WORK

To take but one example, the commander of a conventional diesel/electric submarine (still by far the most common type of submarine) has to balance several factors in engaging a target. One of the most dramatic is the state of the battery. A short period of high speed to close with a distant target will reduce the battery charge very rapidly, bringing nearer the time when the submarine must come close to the surface in order to run its diesel engines to recharge the batteries. During recharging, the vulnerability of the submarine is considerably increased, especially if enemy vessels and aircraft have been alerted by the submarine attacking a target. The commander must, therefore, balance battery charge required for the attack with that required for withdrawal to regenerate battery charge.

Such command decisions involve dynamics of an hour or so, but others, such as the management of food and water supplies, the control of the relationship between speed and noise and the decisions to expend weapons are of longer duration. The commander must, in addition, manage these balances during the voyage to an operational station, during the patrol itself, and on the return journey. He may, for instance, need to conserve supplies and weapons in order to fight his way into port through the possible cordon of hostile submarines in the vicinity of his own home base if hostilities have broken out during his voyage.

These are immensely complicated questions and it seems likely that a system dynamics model, with its ability to represent the interplay of factors over time, might have much to contribute as an architecture within which more detailed models might be developed. Such a model would probably need to use the system dynamics optimization methodology (Coyle;[6,7] Keloharju and Wolstenholme[8]) to maximize appropriate measures of submarine performance against equipment under the severe constraints of space and electrical power supply which exist in such a vessel.

APPENDIX

The Equation Formulation Aspects

Introduction

As mentioned above, it was necessary to depart from normal system dynamics modelling practice in order to represent the integer nature of submarine construction and to create random conditions for breakdowns during operational trips. This appendix discusses those aspects of the work. The models were written using the COSMIC system dynamics software package, which is one of several languages using similar syntax and essentially differing only in the number of user functions they offer (COSMIC has 36), and whether or not they provide automatic dimensional analysis (COSMIC and DYSMAP2), optimization facilities (COSMOS and DYSMOD), array handling (DYNAMO) or icons for model construction (STELLA).

Integer construction

Given an equation for a rate variable, scs, the submarine construction commenced, one calculates a third-order delay for dsscr, a dummy submarine construction rate, in the usual way:

R dsscr.kl = DELAY3(scs.jk,del1)

where del1 is the nominal construction period.

The flow of completed work on a boat, which is what dsscr represents, is accumulated into a level, dssc, which represents the work completed on a submarine:

$$1 \text{ dssc.k} = \text{dssc.j} + \text{dt*(dsscr.jk} - \text{sscr.jk)}$$

The outflow from dssc will represent the completion of construction, but if one waited for dssc to reach unity, the tail of DELAY3 would involve a very long delay, far longer than del1, which is the real time for construction. One gets round this difficulty by allowing a boat to be completed when dssc reaches some fraction, frac, of unity. The values for frac are determined empirically, and are different for the first and subsequent submarines, to ensure that submarines are completed a period del1 after they were started. Thus

$$\text{r sscr.kl} = \text{CLIP(1/dt,0,dssc.k,frac.k)}$$

$$\text{a frac.k} = \text{CLIP(0.85,0.95,cscom.k,1)}$$

where cscom is the integral of sscr and is the cumulative submarines completed.

The preceding equation will make sscr have the value 1/dt lasting for 1 dt to generate an integer submarine completion. The level dssc will still contain $-(1 - \text{frac})$ of a submarine, which will gradually be turned positive by the next one in the construction sequence.

An influence diagram for these equations, and the dynamic behaviour of the variables are shown in Figures A1 and A2, respectively.

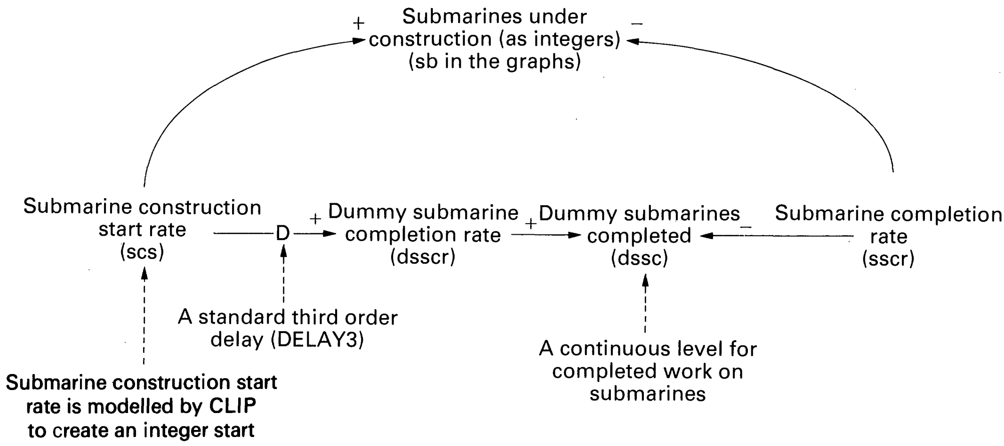

FIG. A1. *Influence diagram for the delays.*

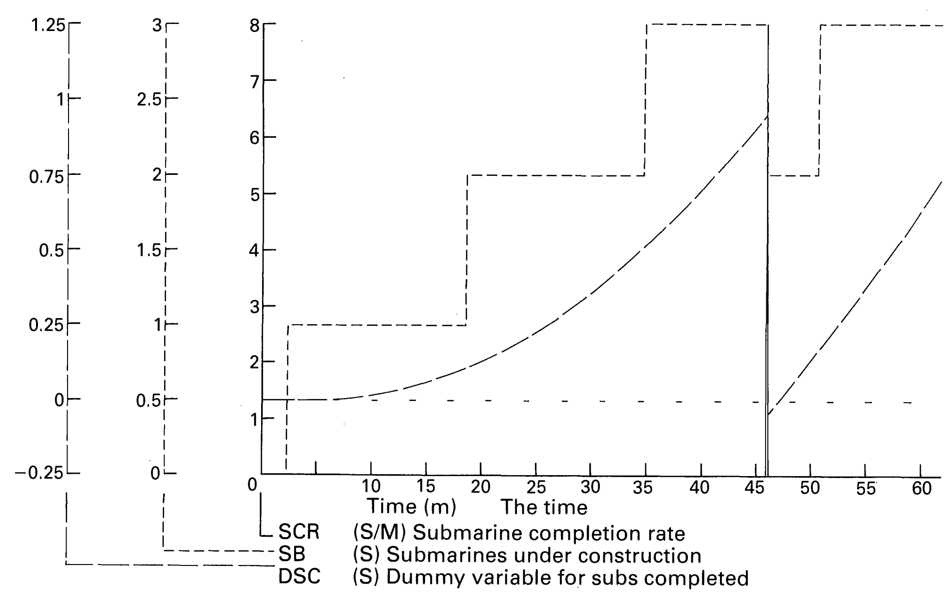

FIG. A2. *Dynamics of submarine construction equations.*

Random patrol durations

In the intermediate model, submarines leave port and transit to the patrol area, integer submarines being modelled by equations similar to those used above for integer construction. The modelling problem arises when they are to leave their patrol areas early because of mechanical breakdown.

Submarines return from patrol after a duration, pd, the patrol duration. They must, however, be represented as integers, which is achieved using a dummy level, as described above:

r dudpar.kl = DELAY3(uepar.jk,pd.k)

l dudpa.k = dudpa.j + dt*(dudpar.jk − udpar.jk)

r udpar.kl = CLIP(1/dt,0,dudpa.k,frac.k)

The first stage is to store a random number, a, which will represent whether or not a given submarine will break down. If a is negative then the given boat will break down, which allows for the unrealistic 50% failure rate. However, note that a new value for a is determined only at the dt for which sr, the sailing rate, is non-zero:

l a.k = a.j + dt*CLIP((NOISE(4) − a.j)/dt,0,sr.jk,0.1)

The patrol duration, pd, will be determined by the normal duration, basedur, or a sampled duration, tp, depending on whether a is positive:

a pd.k = CLIP(basedur,tp.k,a.k,0)

A new duration, newdur, is sampled to be anywhere between 1% and 99% of the base duration:

a newdur.k = basedur*(one-scale*(col − NOISE(seed)))

c basedur = 40

c col = 0.5

c scale = 0.99

c seed = 27

and is stored as a level so that it correctly controls pd, the patrol duration, as long as that submarine has been scheduled, by the sampling, to have a breakdown and return early from its voyage:

l tp.k = tp.j + (dt/dt)*((newdur.j − tp.j)

× *CLIP(1,0,sr.jk,0))

Relatively simple modelling approaches such as these can make all the difference between the client's acceptance of a model as adequately realistic and his rejection of work as over-simple.

REFERENCES

1. R. G. Coyle (1983) Who rules the waves? *J. Opl Res. Soc.* **34,** 885–899.
2. R. G. Coyle (1979) *Management System Dynamics.* Wiley, Chichester.
3. E. F. Wolstenholme (1990) *System Enquiry.* Wiley, Chichester.
4. R. G. Coyle (1985) Representing discrete events in system dynamics models. *J. Opl Res. Soc.* **36,** 307–318.
5. E. F. Wolstenholme (1988) Defence operational analysis using system dynamics. *Eur. J. Opl Res.* **34,** 10–19.
6. R. G. Coyle (1985) The use of optimisation methods for policy design in a system dynamics model. *Syst. Dynam. Rev.* **1,** 81–91.
7. R. G. Coyle The optimization of defence expenditure. *Eur. J. Opl Res.*, to be published.
8. R. Keloharju and E. F. Wolstenholme (1989) A case study in system dynamics optimization. *J. Opl Res. Soc.* **40,** 221–230.