

2.75

CONSTRAINT

LECTURE

Design of Constraints in Precision Systems

Background

- History
- Reasons
- Requirements
- Problems

Classes of Constraint

- Kinematic
- Quasi-Kinematic
- Variable Geometry
- Partial/Compliance

Hardware/discussion time

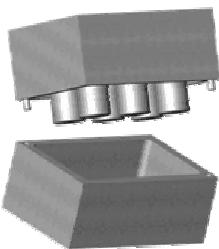
Elastic Averaging will be done next lecture

Common Coupling Methods



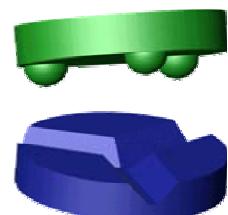
Elastic Averaging

Non-Deterministic



Pinned Joints

No Unique Position



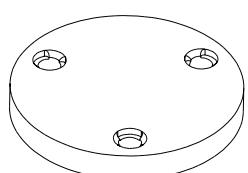
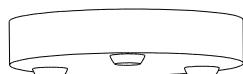
Kinematic Couplings

Kinematic Constraint



Flexural Kin. Couplings

Kinematic Constraint



Quasi-Kinematic Couplings

Near Kinematic Constraint

	$0.01 \mu m$	$0.10 \mu m$	$1.0 \mu m$	$10 \mu m$	$100 \mu m$
Pinned Joints					
Flexural Kinematic Couplings					
Elastic Averaging					
Quasi-Kinematic Couplings					
Kinematic Couplings					

Perspective: What the coupling designer faces...

<u>APPLICATION</u>	<u>SYSTEM SIZE</u>	<u>REQ'D PRECISION</u>
Fiber Optics	Meso	Nano
Optical Resonators	Meso	Nano
Large array telescopes	60 ft diam.	Angstrom
Automotive	3 ft	1 micron

Problems due to strain affects

- Thermal Affects Air, hands, sunlight
- Gravity Sagging
- Stress Relief Time variable assemblies
- Loads Stiffness

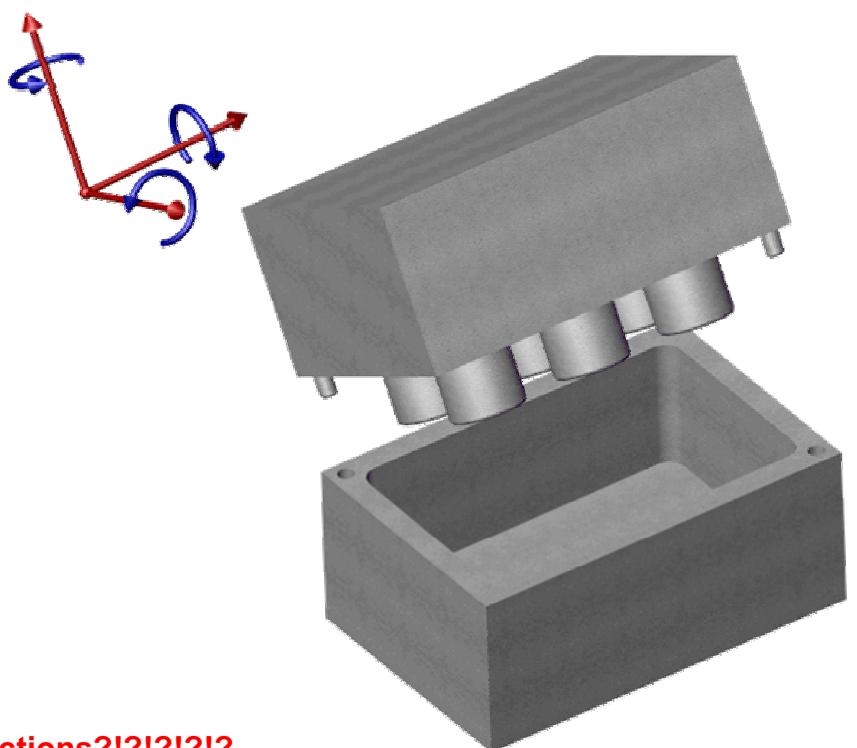
Problems due to sub-optimal designers

- Competing cost vs performance
- Automotive (no temperature control, large parts, resistance to change)

General Service Requirements & Applications

Ideal couplings:

- Inexpensive
- Accurate & Repeatable
- High Stiffness
- Handle Load Capacity
- Sealing Interfaces
- Well Damping



Example Applications:

- Grinding
- Optic Mounts
- Robotics
- Automotive

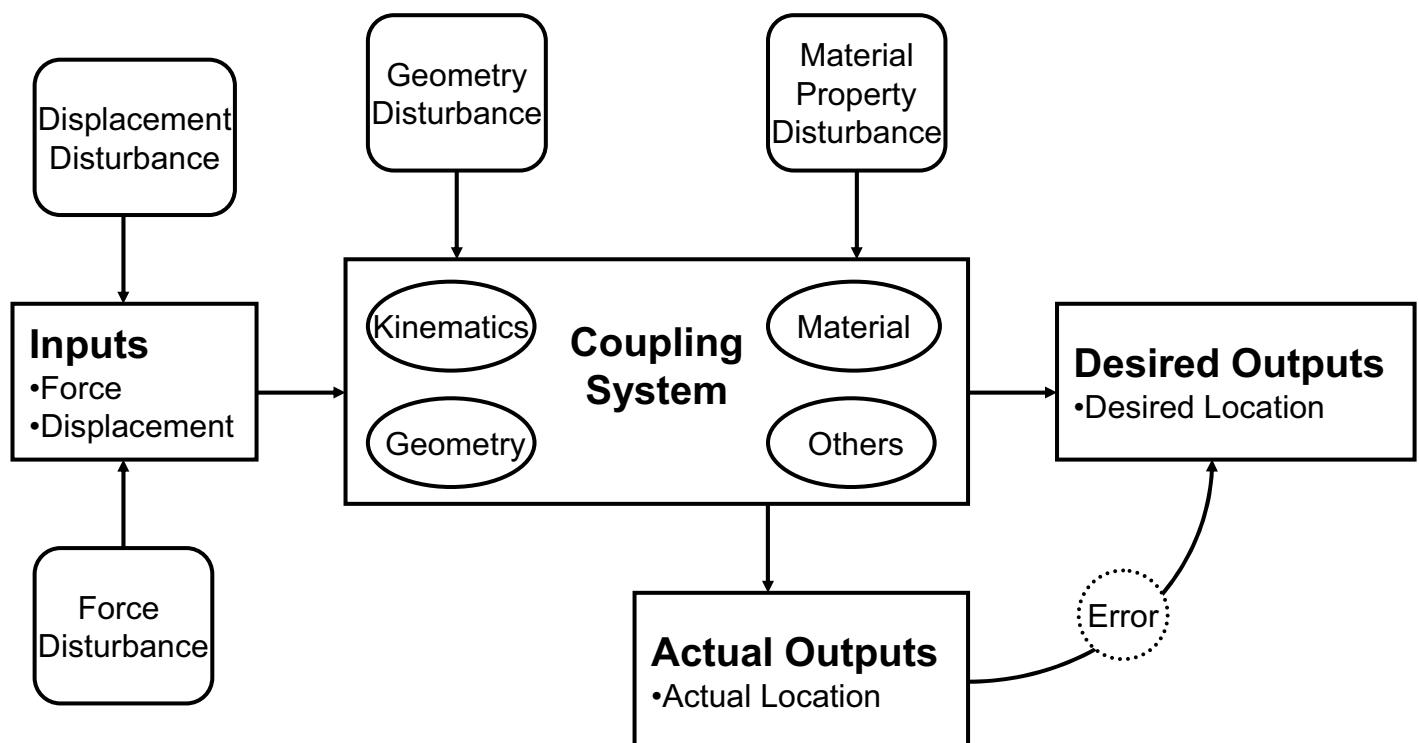
Sensitivity

- What are the sensitive directions?!?!!?!

Couplings Are Designed as Systems

You must know what is going on (loads, environment, thermal)!

Shoot for determinism or it will “suck to be you”



KINEMATIC COUPLINGS

The good, the bad, the ugly....

Defining Constraint

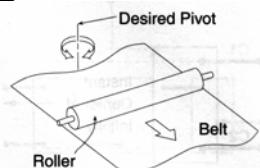


FIGURE 1.9.1

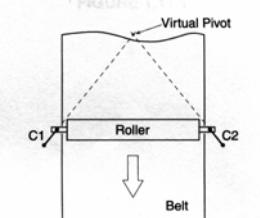


FIGURE 1.9.2

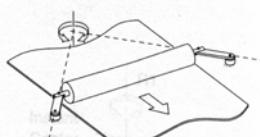


FIGURE 1.9.3

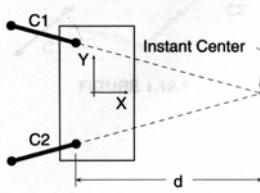


FIGURE 1.10.1

Clever use of constraint

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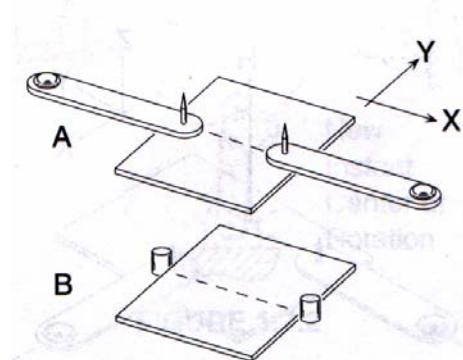


FIGURE 1.5.1

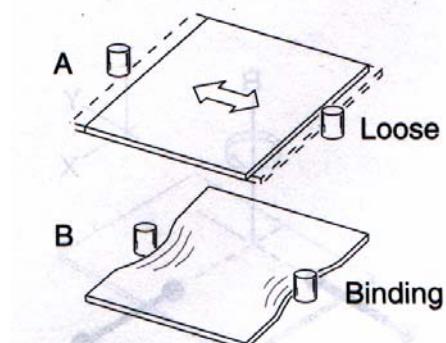


FIGURE 1.5.2

Penalties for over constraint

Exact Constraint (Kinematic) Design

Exact Constraint: Number of constraint points = DOF to be constrained

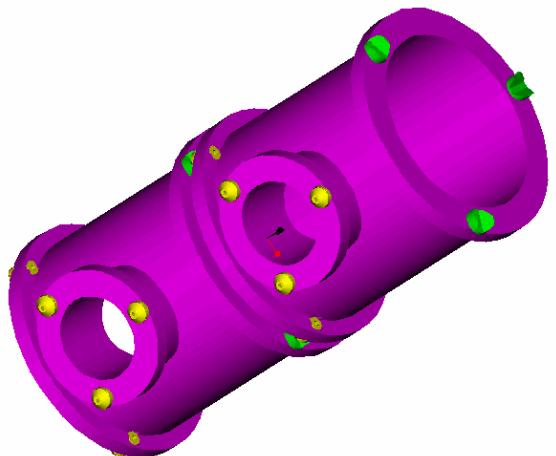
These constraints must be independent!!!

Assuming couplings have rigid bodies, equations can be written to describe movements

Design is deterministic, saves design and development \$

KCs provide repeatability on the order of parts' surface finish

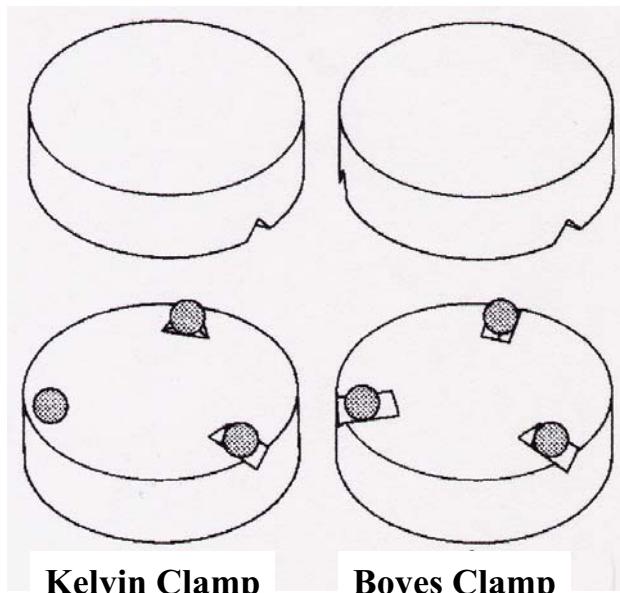
- ¼ micron repeatability is common
- Managing contact stresses are the key to success



Making Life Easier

“Kinematic Design”, “Exact Constraint Design”.....the issues are:

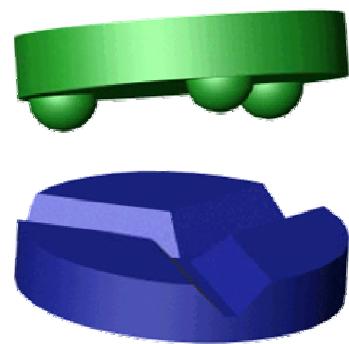
- KNOW what is happening in the system
- Manage forces and deflections
- Minimize stored energy in the coupling
- Know when “Kinematic Design” should be used
- Know when “Elastic Averaging” should be used (next week)



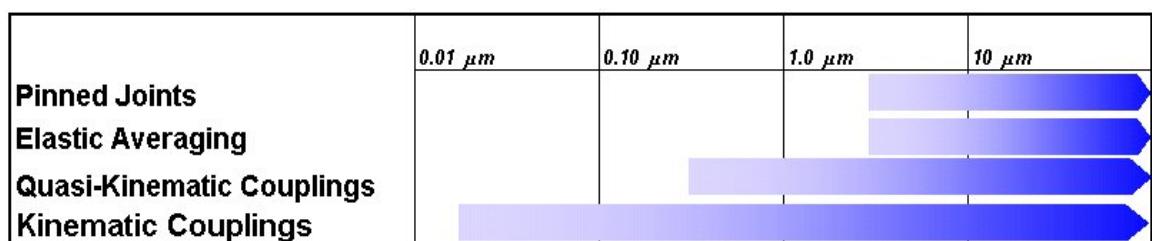
Kinematic couplings

Kinematic Couplings:

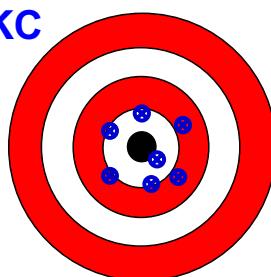
- Deterministic Coupling
- # POC = # DOF
- Do Not Allow Sealing Contact
- Excellent Repeatability



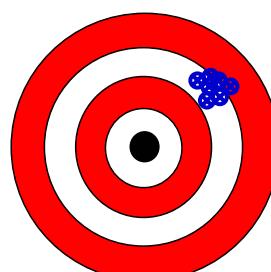
Performance



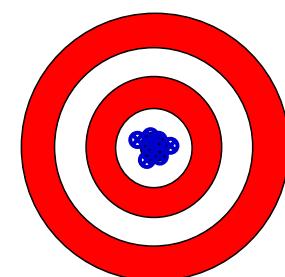
Power of the KC



Accuracy

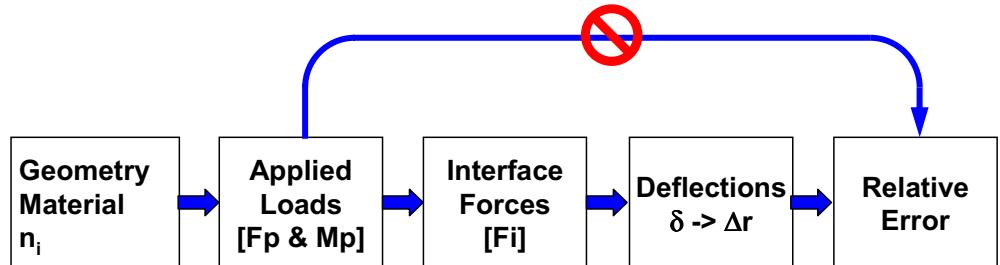
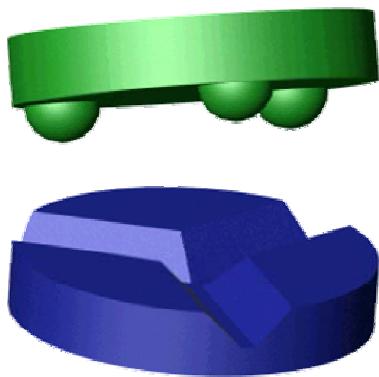


Repeatability



Accuracy & Repeatability

Modeling Kinematic Coupling Error Motions

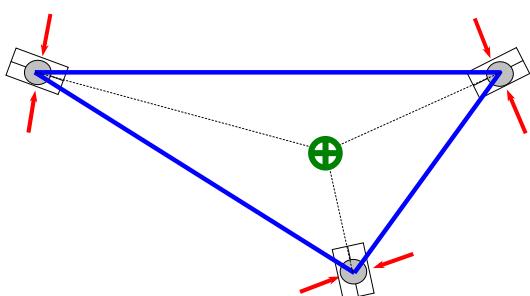


6 Unknown Forces & 6 Equilibrium Equations

$$\sum F_i = F_p \quad \sum M_i = M_p$$

Hertzian Point Contact for Local Displacements

$$\delta_i = f(E_B, E_G, v_B, v_G, R_B, R_G)$$



- Kinematic Coupling Groove
- Mating Spherical Element
- Contact Force
- ⊕ Coupling Centroid
- Angle Bisectors
- Coupling Triangle

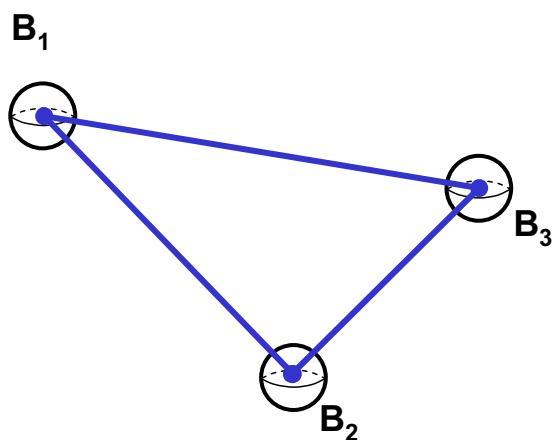
KC Error Motion Analysis

Need $\delta_x, \delta_y, \delta_z, \epsilon_x, \epsilon_y, \epsilon_z$ to predict effect of non-repeatability

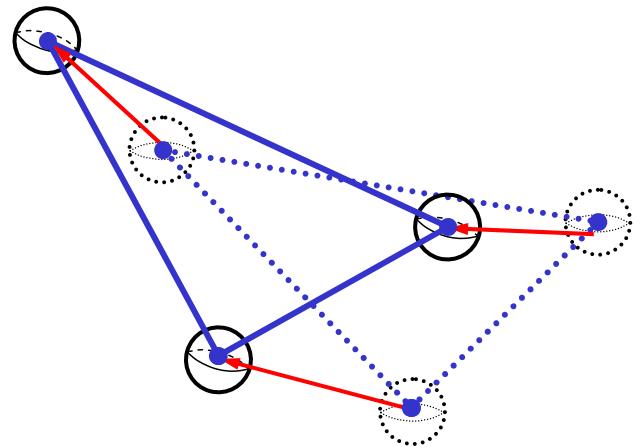
Hertz deflections \rightarrow displacements of ball centers

Three ball centers form a plane

Analyze relative position of “before” and “after” planes for error motions



Original Positions

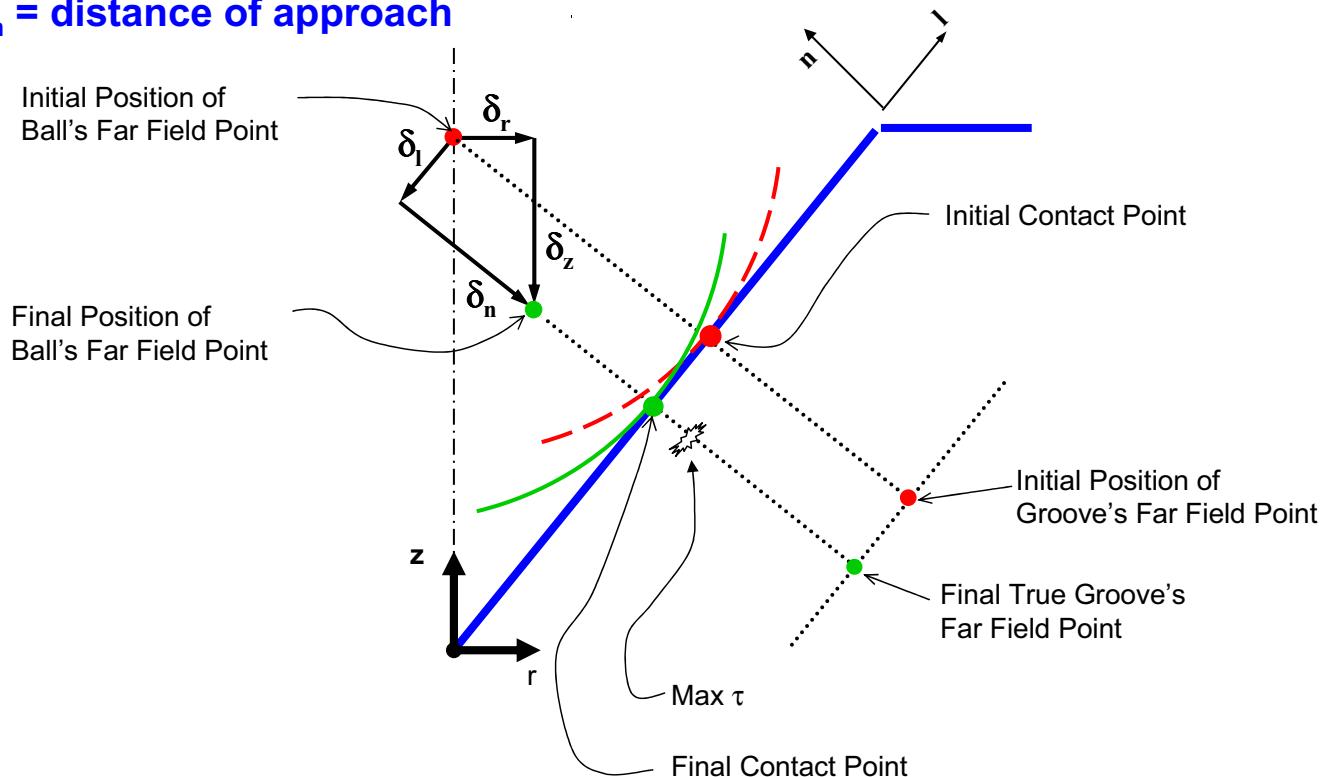


Final Positions

Kinematic Couplings and Distance of Approach

How do we characterize motions of the ball centers?

δ_n = distance of approach



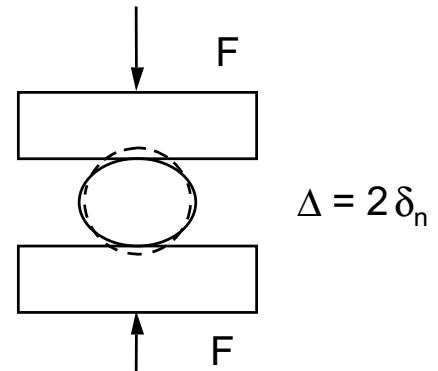
Contact Mechanics – Hertz Contact

Heinrich Hertz – 1st analytic solution for “near” point contact

KC contacts are modeled as Hertz Contacts

Enables us to determine stress and distance of approach, δ_n

Radii	Ronemaj	1.00E+06
	Ronemin	0.06250
	Rtwomaj	0.25000
	Rtwomin	-0.06500
Load	Applied load F	13
	Phi (degrees)	0
	Max contact stress	10,000
Modulus	Elastic modulus Eone	3.00E+07
	Elastic modulus Etwo	4.40E+05
v Ratio	Poisson's ratio vone	0.3
	Poisson's ratio vtwo	0.3
	Equivalent modulus Ee	4.77E+05
Stress	Equivalent radius Re	0.2167
	Contact pressure	12,162
	Stress ratio (must be less than 1)	1.22
	Deflection at the one contact interface	
Deflection	Deflection (μ units)	829



Key Hertz Physical Relations

Equivalent radius and modulus:

$$R_e = \frac{1}{\frac{1}{R_{1\text{major}}} + \frac{1}{R_{1\text{minor}}} + \frac{1}{R_{2\text{major}}} + \frac{1}{R_{2\text{minor}}}} \quad E_e = \frac{1}{\frac{1 - \eta_1^2}{E_1} + \frac{1 - \eta_2^2}{E_2}}$$

cos(θ) function (ϕ is the angle between the planes of principal curvature of the two bodies)

$$\begin{aligned} \cos\theta = R_e & \left[\left(\frac{1}{R_{1\text{major}}} - \frac{1}{R_{1\text{minor}}} \right)^2 + \left(\frac{1}{R_{2\text{major}}} - \frac{1}{R_{2\text{minor}}} \right)^2 \right. \\ & \left. + 2 \left(\frac{1}{R_{1\text{major}}} - \frac{1}{R_{1\text{minor}}} \right) \left(\frac{1}{R_{2\text{major}}} - \frac{1}{R_{2\text{minor}}} \right) \cos 2\phi \right]^{1/2} \end{aligned}$$

Solution to elliptic integrals estimated with curve fits

$$\alpha = 1.939e^{-5.2\theta} + 1.78e^{1.0\theta} + 0.723/\theta + 0.221$$

$$\beta = 35.228e^{-0.98\theta} - 32.424e^{-1.047\theta} + 1.486\theta - 2.634$$

$$\lambda = -0.214e^{-4.9\theta} - 0.179\theta^2 + 0.555\theta + 0.319$$

Contact Pressure	Distance of Approach	Major Contact Axis	Minor Contact Axis
$q = \frac{3F}{2\pi cd} \approx 1.5\sigma_{\text{tensile}}$ for metals	$\delta = \lambda \left(\frac{2F^2}{3R_e E_e} \right)^{1/3}$	$c = \alpha \frac{3FR_e}{2E_e}^{1/3}$	$\beta \frac{3FR_e}{2E_e}^{1/3}$

KEY Hertz Relations

Contact Pressure is proportional to:

- Force to the 1/3rd power
- Radius to the -2/3rd power
- Modulus to the 2/3rd power

Distance of approach is proportional to:

- Force to the 2/3rd power
- Radius to the -1/3rd power
- Modulus to the -2/3rd power

Contact ellipse diameter is proportional to:

- Force to the 1/3rd power
- Radius to the 1/3rd power
- Modulus to the -1/3rd power

**DO NOT ALLOW THE CONTACT ELLIPSE TO BE WITHIN ONE DIAMETER OF
THE EDGE OF A SURFACE!**

Calculating Errors Motions in Kinematic Couplings

Motion of ball centers -> Centroid motion in 6 DOF -> $\Delta x, \Delta y, \Delta z$ at X, Y, Z

- **Coupling Centroid Translation Errors**

$$\delta_{\zeta c} = \left(\frac{\delta_{1c}}{L_{1c}} + \frac{\delta_{2c}}{L_{2c}} + \frac{\delta_{3c}}{L_{3c}} \right) \cdot \frac{L_{1c} + L_{2c} + L_{3c}}{3}$$

- **Rotations**

$$\epsilon_x = \frac{\delta_{z1}}{L_{1,23}} \cdot \cos(\theta_{23}) + \frac{\delta_{z2}}{L_{2,31}} \cdot \cos(\theta_{31}) + \frac{\delta_{z3}}{L_{3,12}} \cdot \cos(\theta_{12})$$

$$\epsilon_y = \frac{\delta_{z1}}{L_{1,23}} \cdot \sin(\theta_{23}) + \frac{\delta_{z2}}{L_{2,31}} \cdot \sin(\theta_{31}) + \frac{\delta_{z3}}{L_{3,12}} \cdot \sin(\theta_{12})$$

$$\epsilon_{z1} = \frac{\sqrt{(\alpha_{B1} \cdot \delta_1 + \alpha_{B2} \cdot \delta_2)^2 + (\beta_{B1} \cdot \delta_1 + \beta_{B2} \cdot \delta_2)^2}}{\sqrt{(x_1 - x_c)^2 + (y_1 - y_c)^2}} \cdot \text{SIGN}(\alpha_{B1} \cdot \delta_1 - \alpha_{B2} \cdot \delta_2) \longrightarrow \epsilon_z = \frac{\epsilon_{z1} + \epsilon_{z2} + \epsilon_{z3}}{3}$$

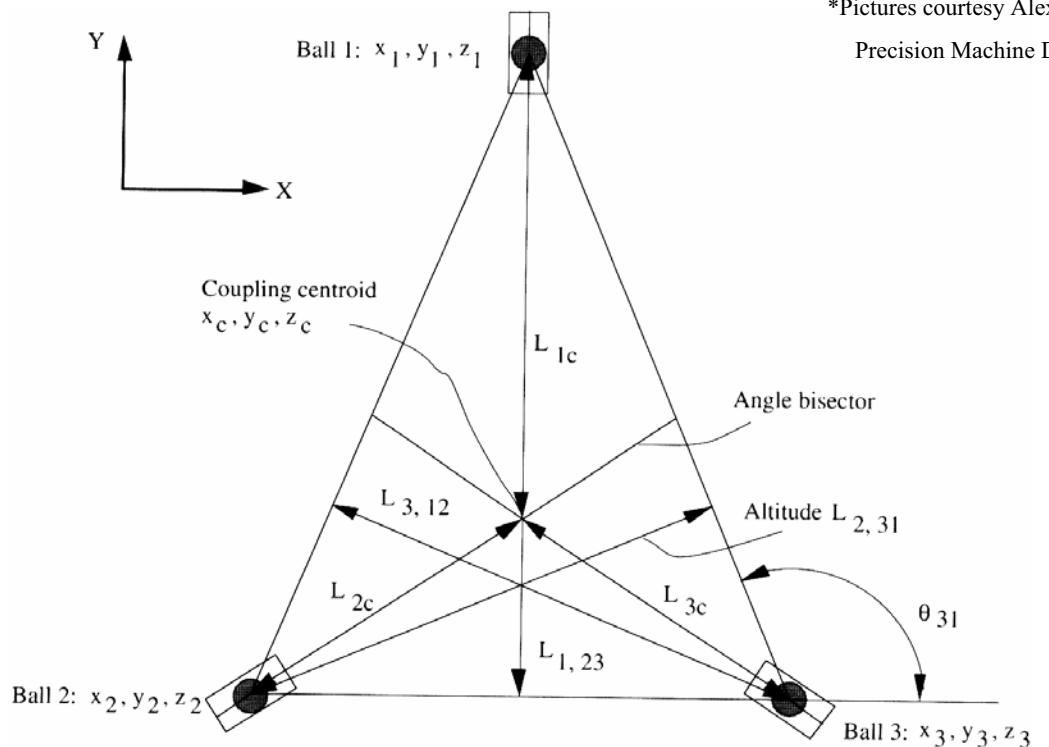
- **Error At X, Y, Z (includes translation and sine errors)**

$$\begin{pmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & -\epsilon_z & \epsilon_y & \delta_x \\ \epsilon_z & 1 & -\epsilon_x & \delta_y \\ -\epsilon_y & \epsilon_x & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X - x_c \\ Y - y_c \\ Z - z_c \\ 1 \end{pmatrix} - \begin{pmatrix} X - x_c \\ Y - y_c \\ Z - z_c \\ 1 \end{pmatrix}$$

Kinematic Coupling Centroid Displacement

*Pictures courtesy Alex Slocum

Precision Machine Design



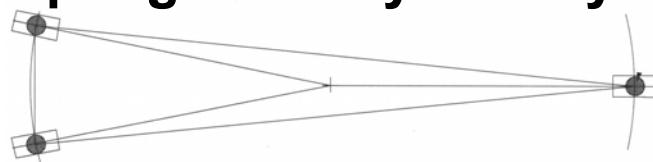
$$\text{Centroid Displacement: } \delta_{\xi c} = \delta_{1\xi} \frac{L_{1c}}{L_{1,23}} + \delta_{2\xi} \frac{L_{2c}}{L_{2,31}} + \delta_{3\xi} \frac{L_{3c}}{L_{3,12}}$$

General Design Guidelines

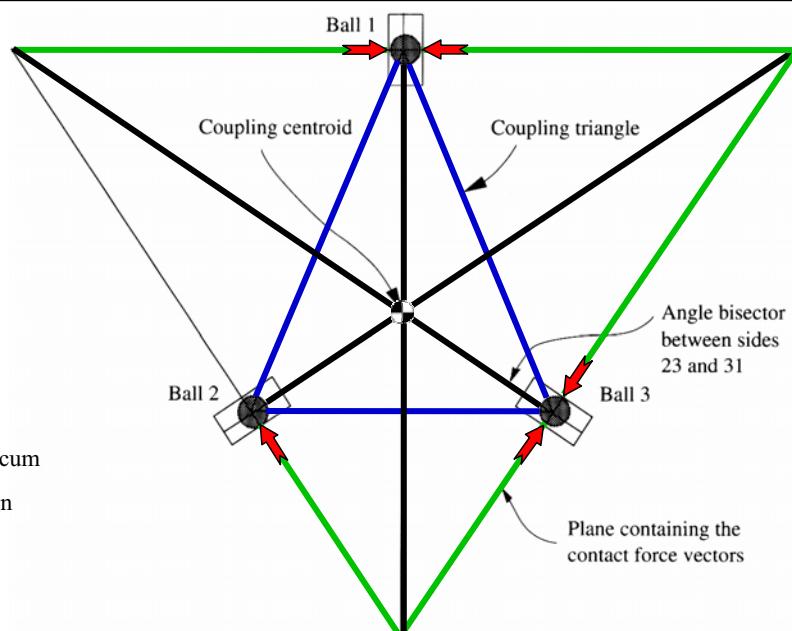
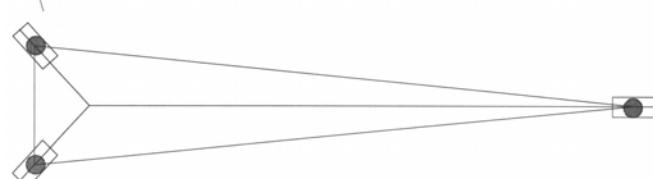
- 1. Location of the coupling plane is important to avoid sine errors**
- 2. For good stability, normals to planes containing contact vectors should bisect angles of coupling triangle**
- 3. Coupling triangle centroid lies at center circle that coincides with the three ball centers**
- 4. Coupling centroid is at intersection of angle bisectors**
- 5. These are only coincident for equilateral triangles**
- 6. Mounting the balls at different radii makes crash-proof**
- 7. Non-symmetric grooves make coupling idiot-proof**

Kinematic Coupling Stability Theory

Poor Design



Good Design

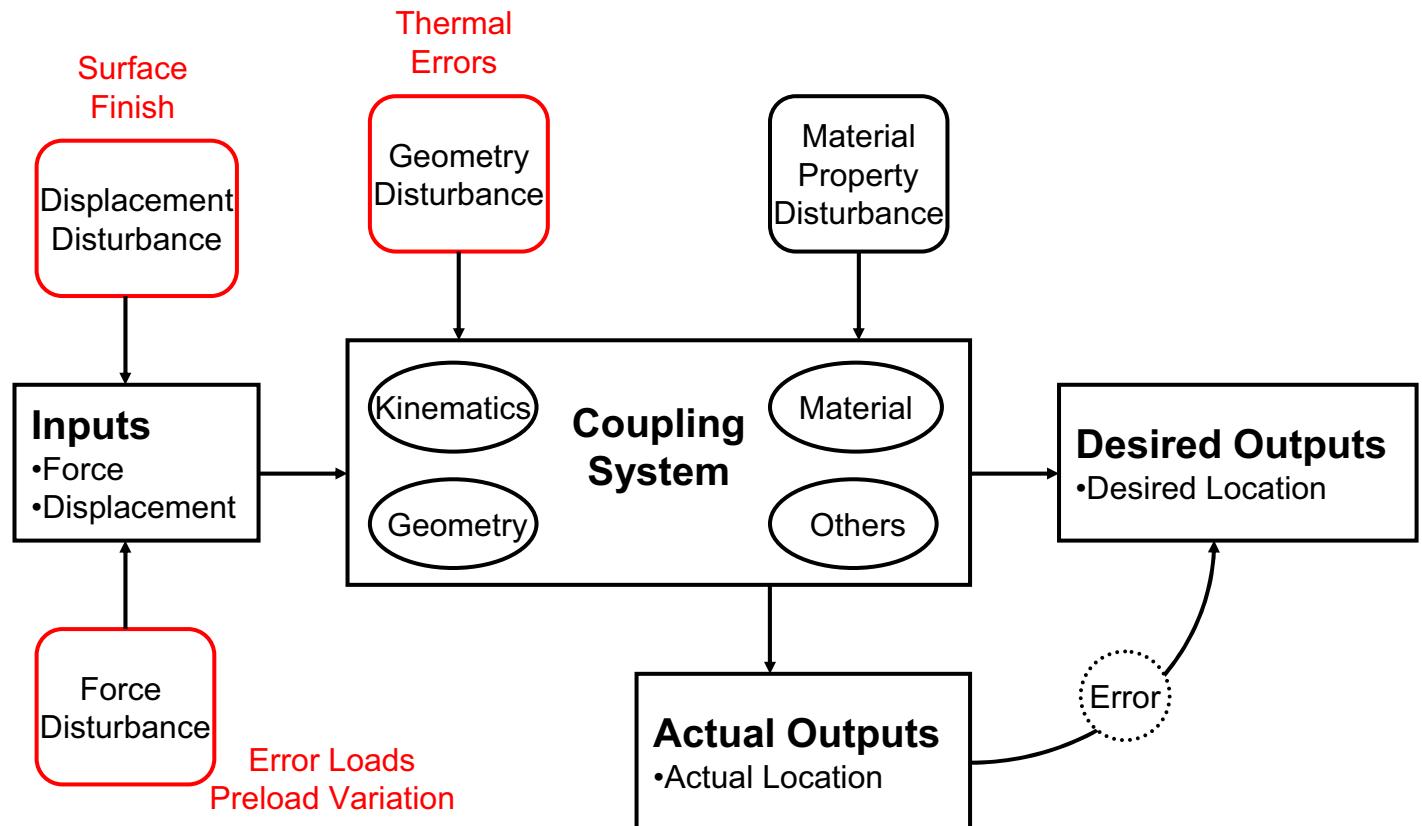


*Pictures courtesy Alex Slocum

Precision Machine Design

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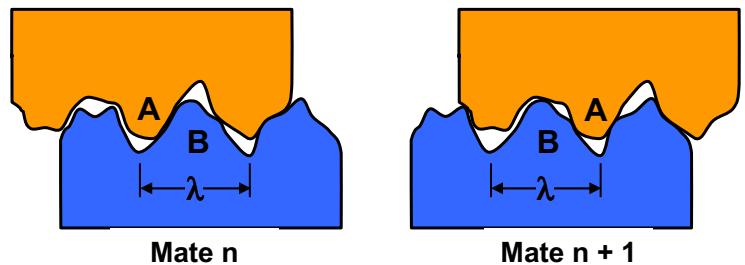
Sources of Errors in KCs



Problems With Physical Contact (and solutions)

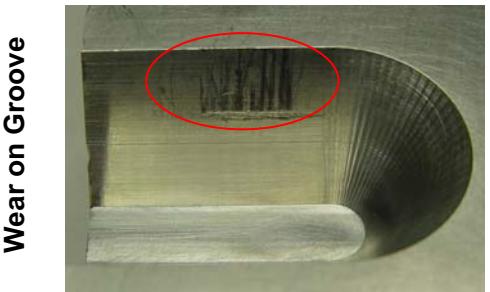
Surface topology (finish):

- 50 cycle repeatability ~ $1/3 \mu\text{m Ra}$
- Friction depends on surface finish!
- Finish should be a design spec
- Surface may be brinelled if possible



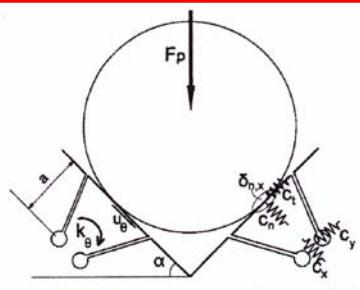
Wear and Fretting:

- High stress + sliding = wear
- Metallic surfaces = fretting
- Use ceramics if possible (low μ and high strength)
- Dissimilar metals avoids “snowballing”



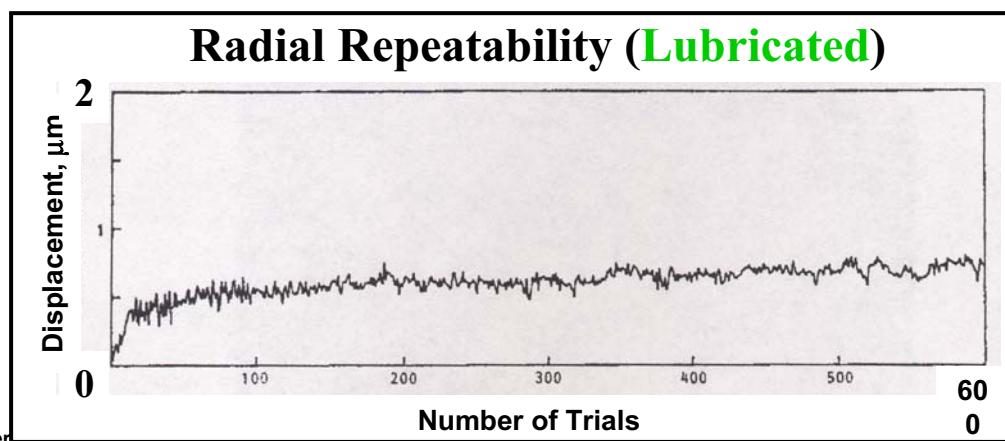
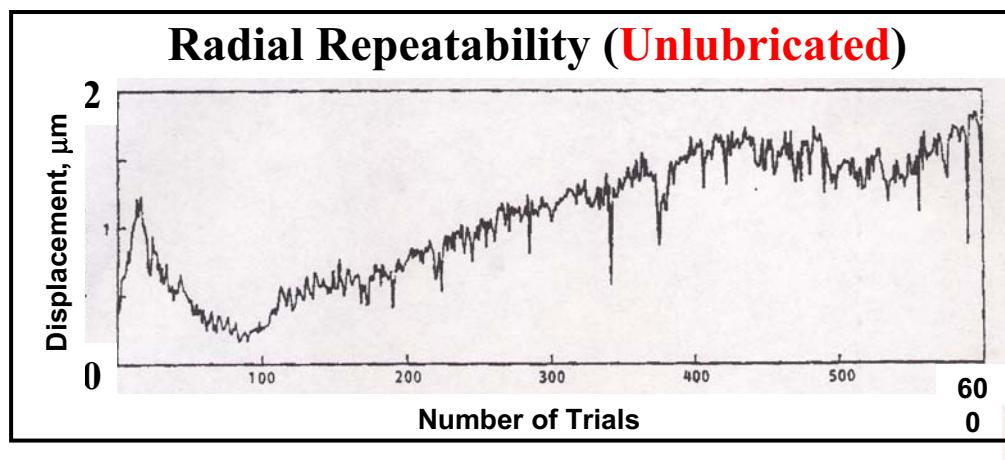
Friction:

- Friction = Hysteresis, stored energy, overconstraint
- Flexures can help (see right)
- Lubrication (high pressure grease) helps
 - Beware settling time and particles
- Tapping can help if you have the “magic touch”



Ball in V-Groove with Elastic Hinges

Experimental Results –Repeatability & Lubrication



Practical Design of Kinematic Couplings

Design

- Specify surface finish or brinell on contacting surfaces
- Normal to contact forces bisect angles of coupling triangle!!!

Manufacturing & Performance

- Repeatability = f (friction, surface, error loads, preload variation, stiffness)
- Accuracy = f (assembly) unless using and ARKC

Precision Balls (ubiquitous, easy to buy)

- Baltec sells hardened, polished kinematic coupling balls or.....

Grooves (more difficult to make than balls)

- May be integral or inserts. Inserts should be potted with thin layer of epoxy

Materials

- Ceramics = low friction, high stiffness, and small contact points
- If using metals, harden
- Use dissimilar materials for ball and groove

Preparation and Assembly

- Clean with oil mist
- Lubricate grooves if needed

Example: Servo-Controlled Kinematic Couplings

Location & automatic leveling of precision electronic test equipment

Teradyne has shipped over 500 systems

Example: Canoe-Ball Kinematic Interface Element

The “Canoe Ball” shape is the secret to a highly repeatable design

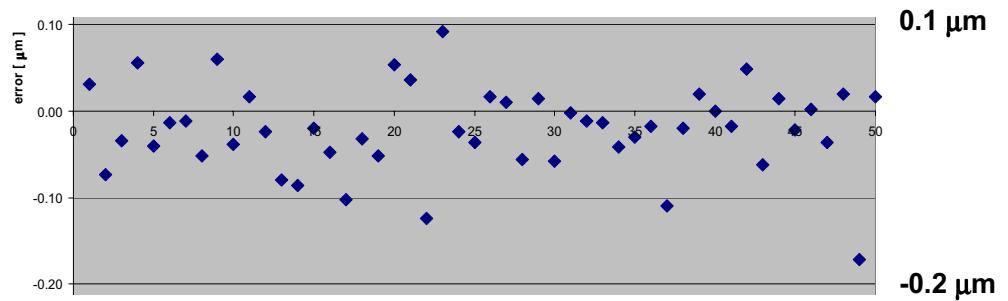
- It acts like a ball 1 meter in diameter
- It has 100 times the stiffness and load capacity of a *normal 1"* ball

Large, shallow Hertzian zone is very (i.e. < 0.1 microns) repeatable

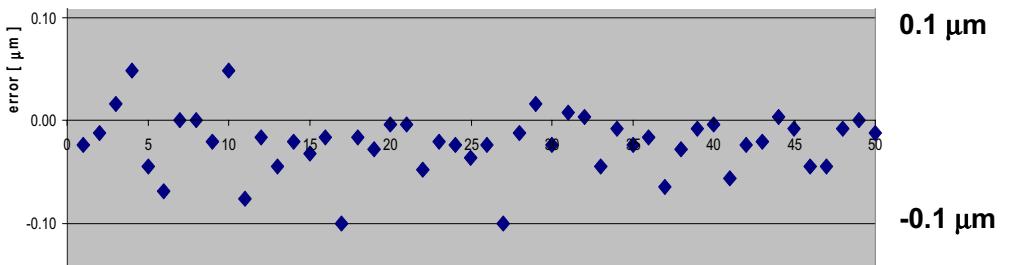
Canoe Ball Repeatability Measurements

Test Setup

Coupling



Meas. system



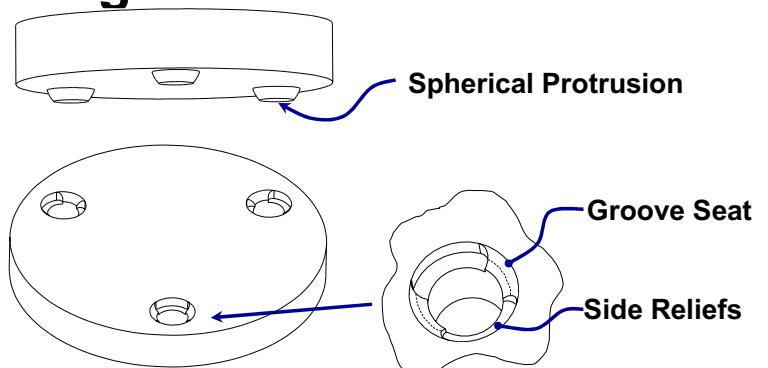
QKCs

Why do it the easy way when you can do it the lazy way?

Quasi-Kinematic (QKC) Alignment

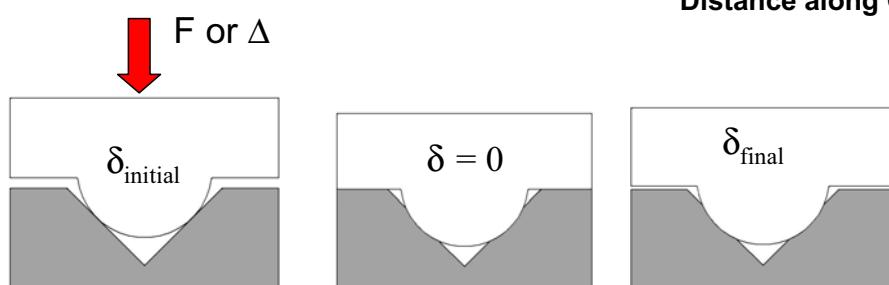
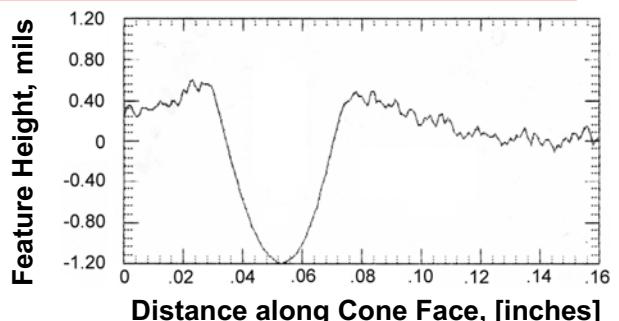
QKC characteristics:

- Arc contact
- Submicron repeatability
- Stiff, sealing contact
- Less expensive than KCs
- Easier to make than KCs



QKC Function:

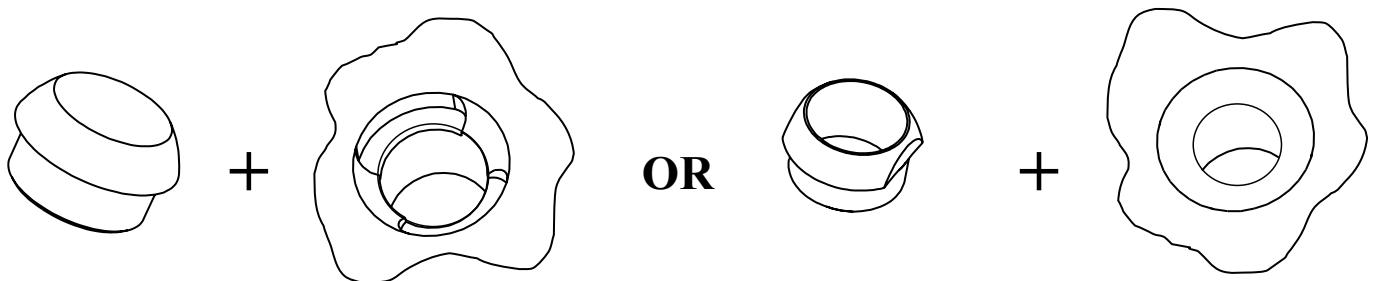
- Ball & groove comply
- Burnish surface irregularities
- Elastic recovery restores gap



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Details of QKC Element Geometry

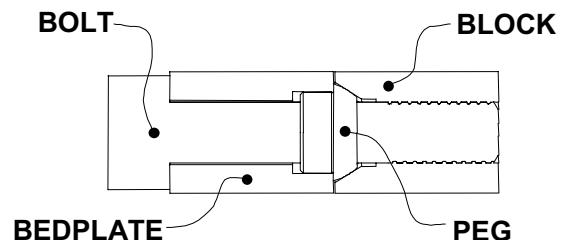
PAIRS OF QKC ELEMENTS



TYPE 2 GROOVE MFG.

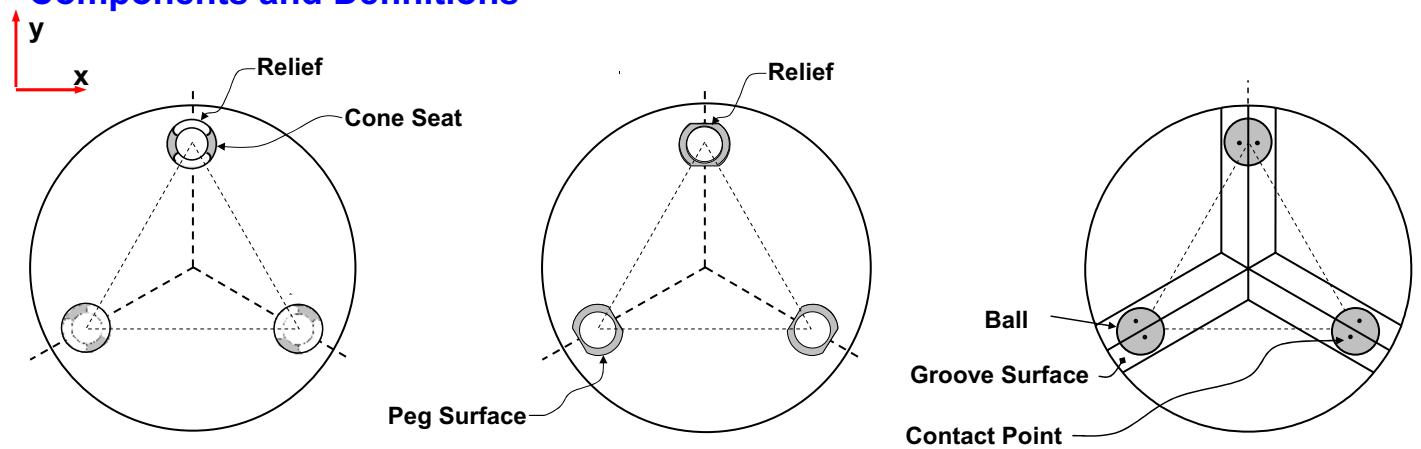
CAST FORM TOOL FINISHED

ASSEMBLED JOINT

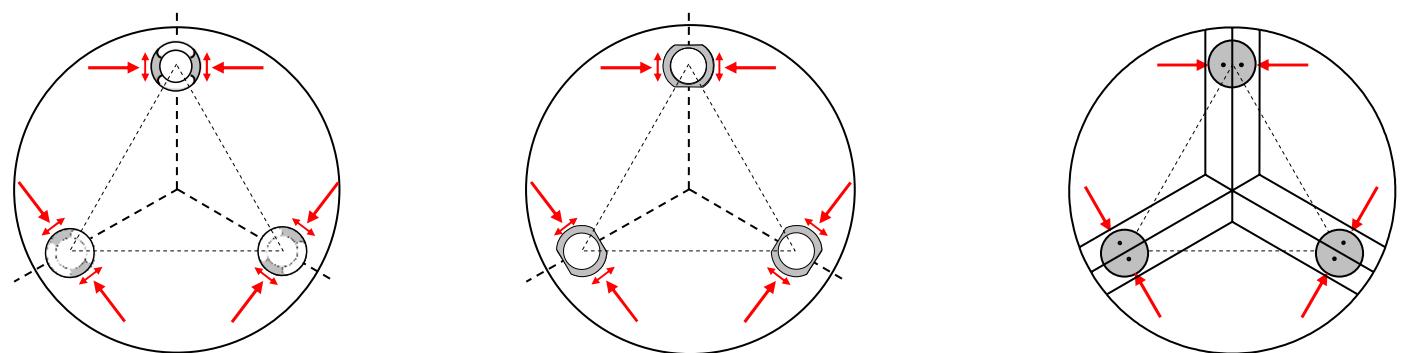


QKC Methods vs Kinematic Method

Components and Definitions

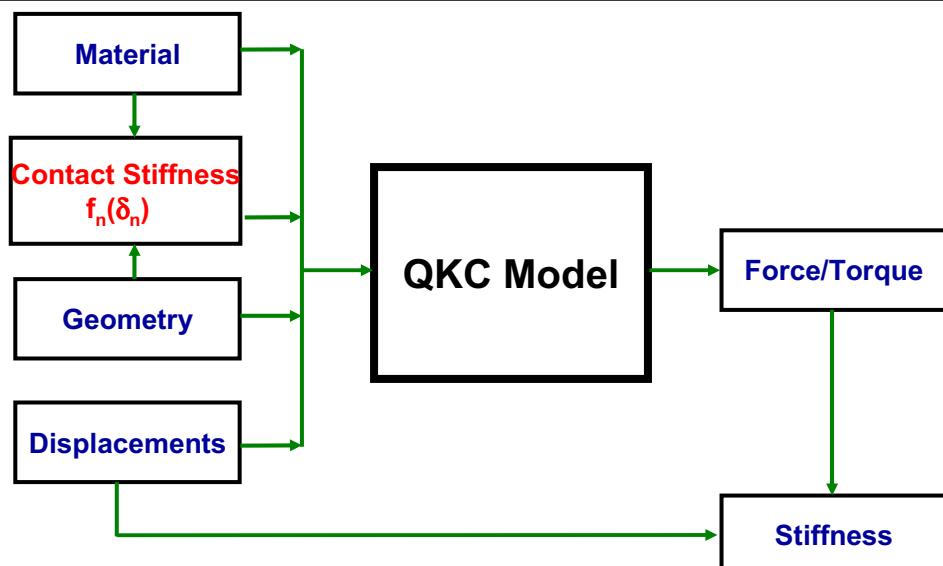
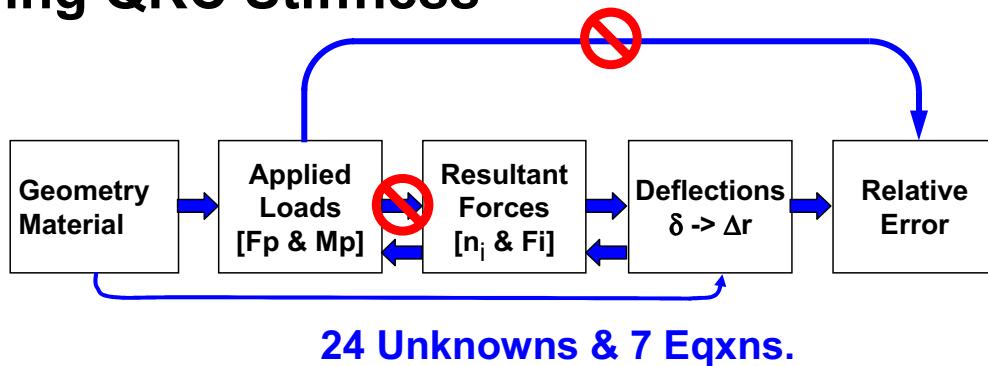


Force Diagrams



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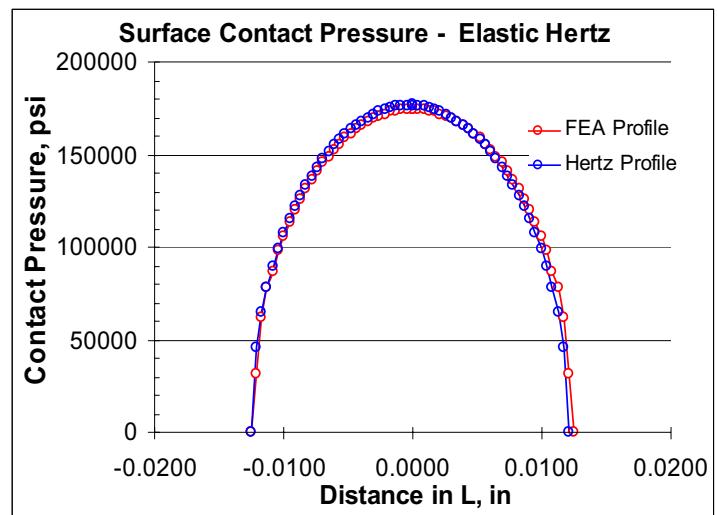
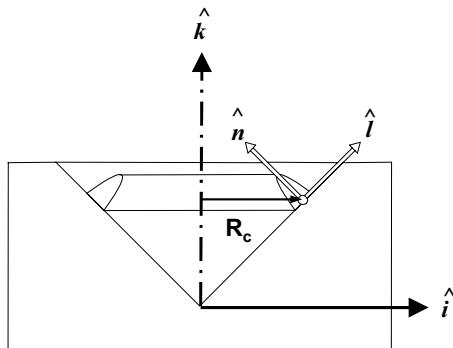
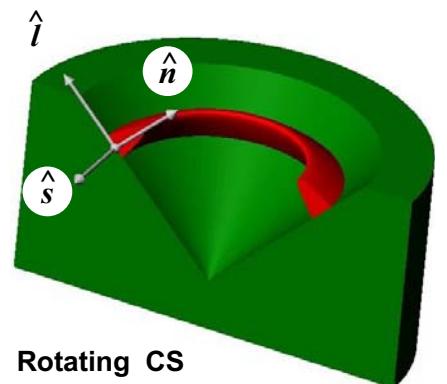
Modeling QKC Stiffness



Contact Mechanics

MECHANICS:

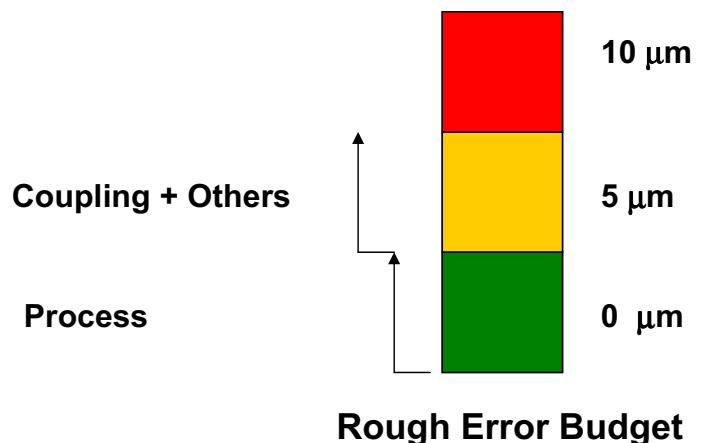
- Use Rotating Coordinate System
- Assume Sinusoidal Normal Distance of Approach
- Obtain Contact Stress Profile as Function of Above
- Integrate Stress Profile in Rotating CS thru contact



Example: Duratec Assembly

Characteristics:

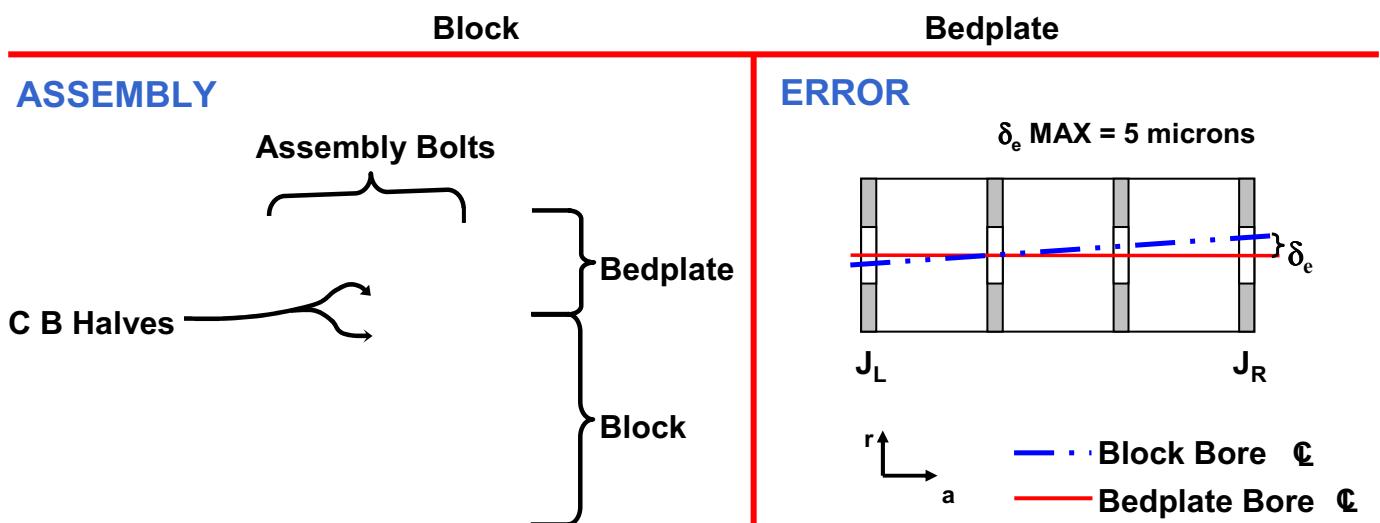
- Ford 2.5 & 3.0 L V6
- > 300,000 Units / Year
- Cycle Time: < 30 s



	0.01 μm	0.10 μm	1.0 μm	10 μm
Pinned Joints				
Elastic Averaging				
Quasi-Kinematic Couplings				
Kinematic Couplings				

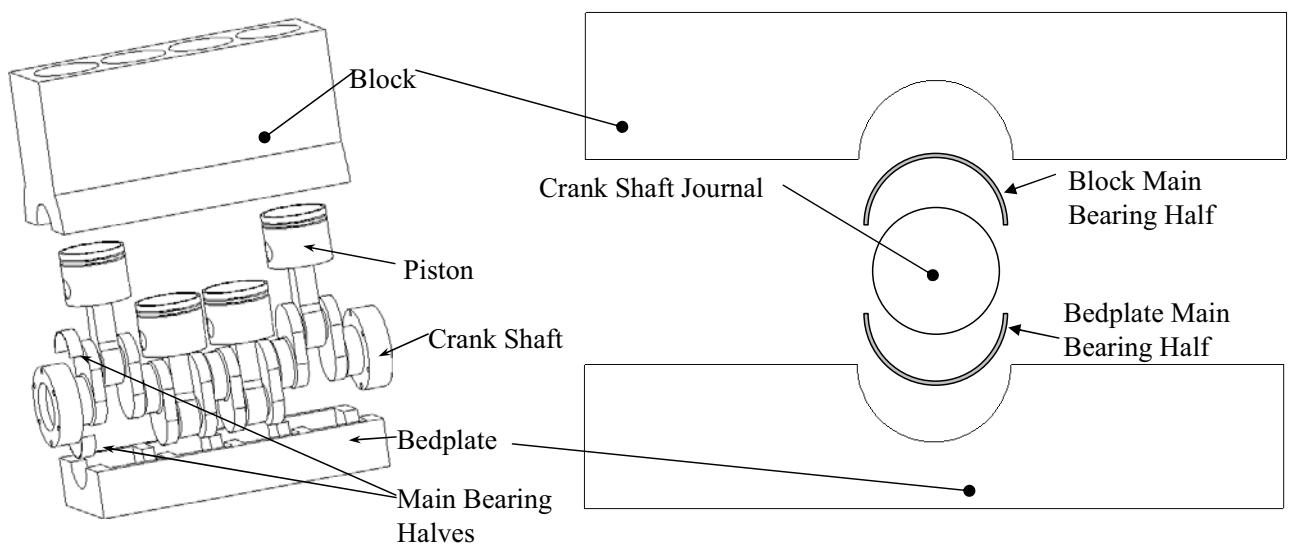
Example: Assembly of Duratec Block and Bedplate

COMPONENTS



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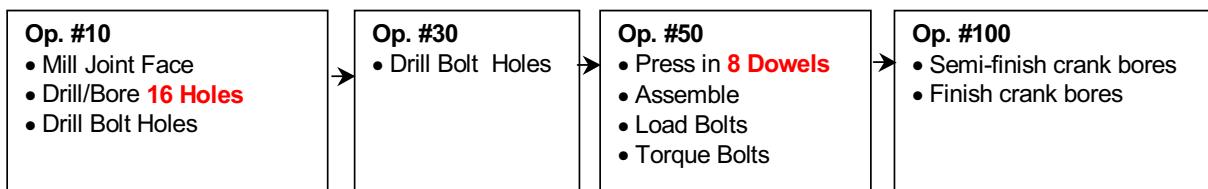
Bearing Assemblies in Engines



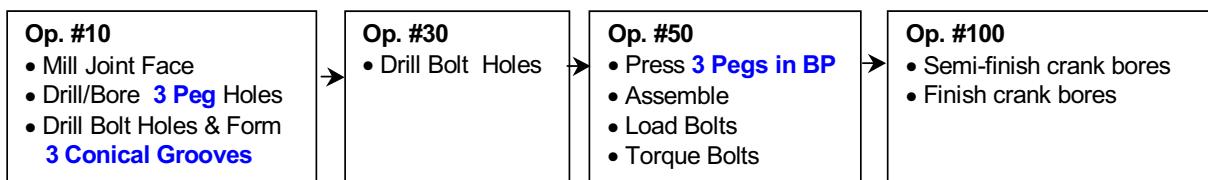
Results of Duratec QKC Research

MANUFACTURING:

Engine Manufacturing Process With Pinned Joint



Modified Engine Manufacturing Process Using Kinni-Mate Coupling

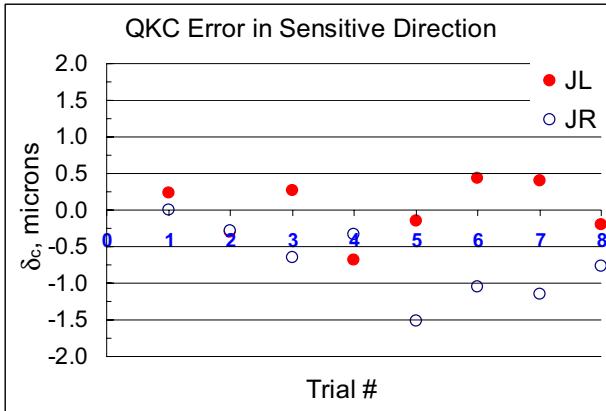
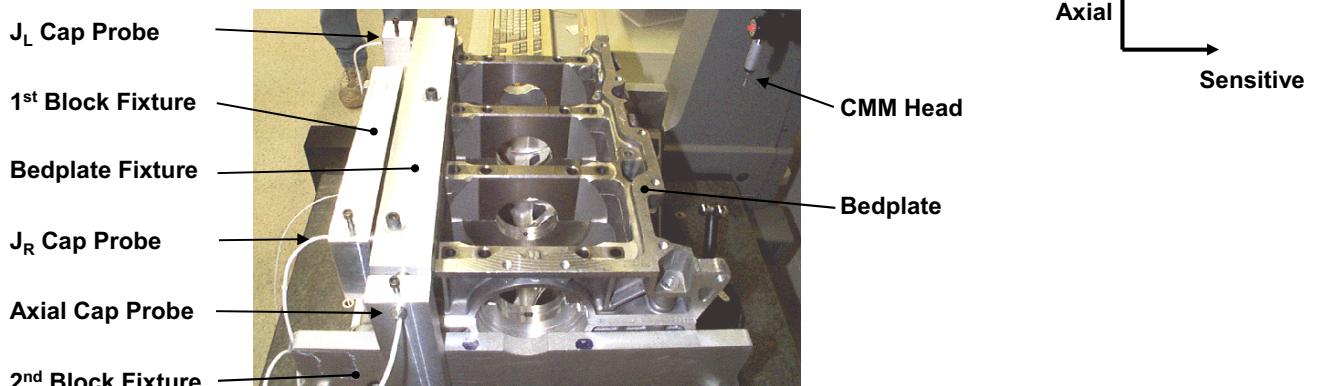


DESIGN:

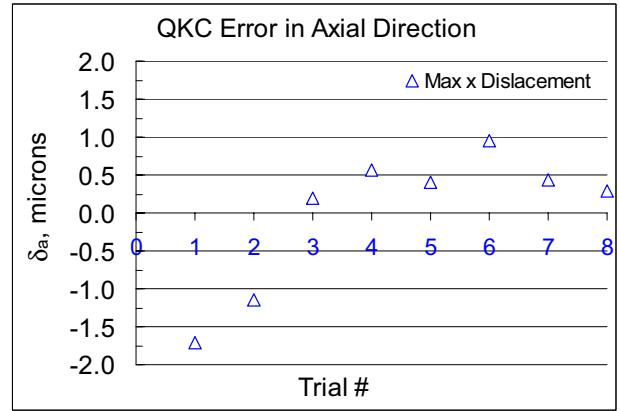
ITEM	QKC	Pinned Joints
# Precision Pieces	3	8
# Precision Features	3	16
Feature Placement Tolerance	+/- 0.08mm	+/- 0.04mm
Average Centerline Repeatability	0.65 µm	4.85 µm
Normalized \$/Engine	0.64	1

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Engine Assembly Performance



$$(Range/2)|_{AVG} = 0.65 \mu\text{m}$$



$$(Range/2) = 1.35 \mu\text{m}$$

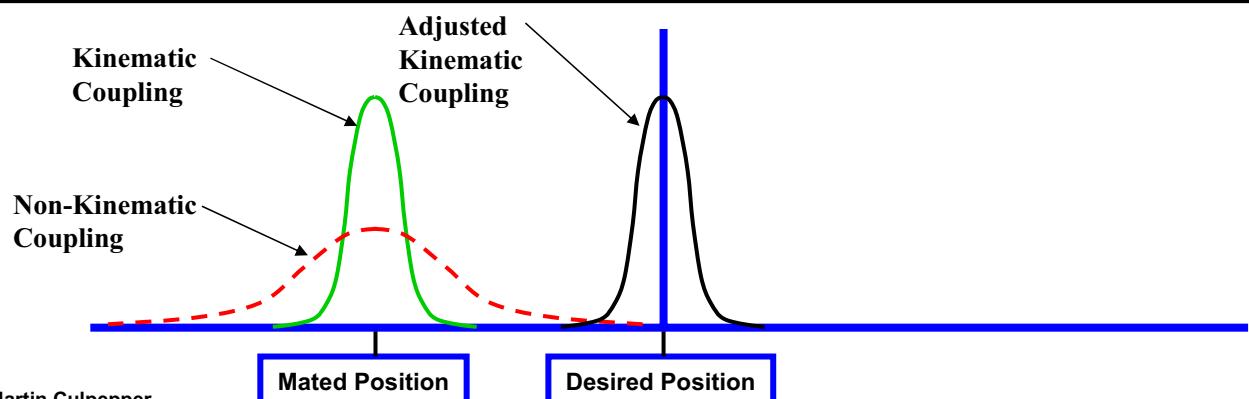
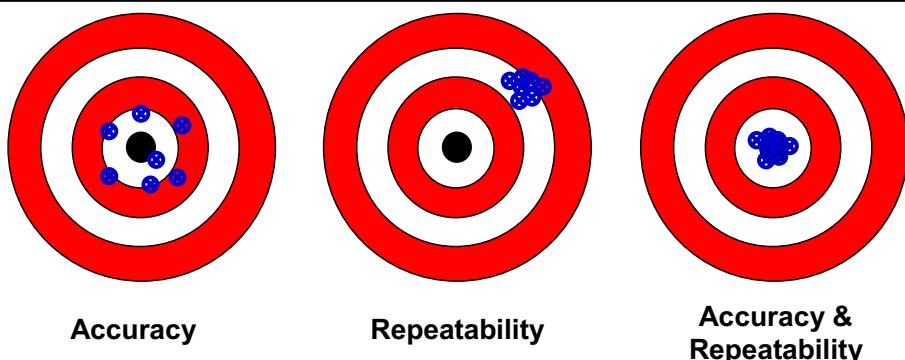
ADJUSTABLE GEOMETRIC CONSTRAINTS

**Hundreds of years of use/development and the
‐@!*&% thing is not yet accurate!?!?!**

Why adjust kinematic couplings?

KC Repeatability is orders of magnitude better than accuracy

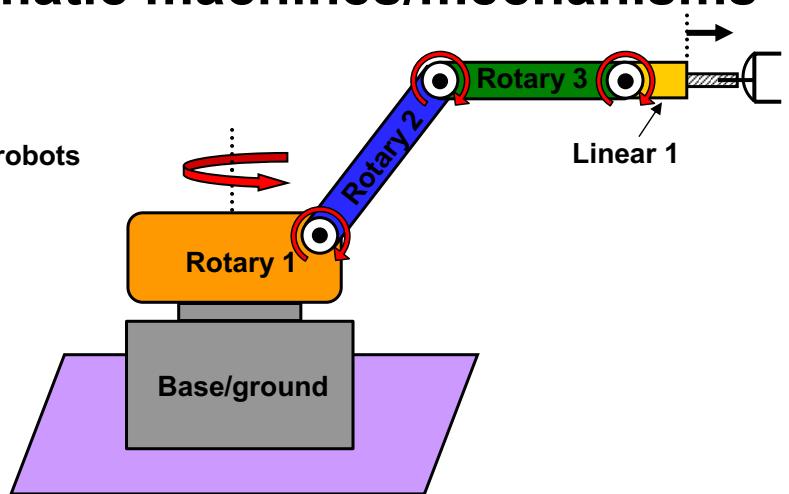
Accuracy = f (manufacture and assemble)



Serial and parallel kinematic machines/mechanisms

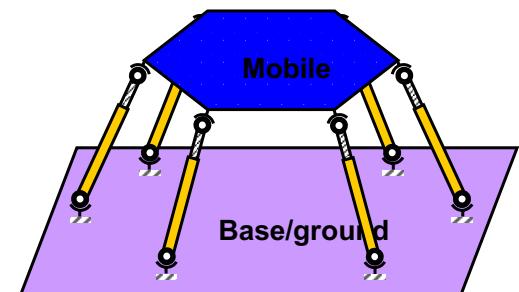
SERIAL MECHANISMS

- Structure takes form of open loop
- I.e. Most mills, lathes, “stacked” axis robots
- Kinematics analysis typically easy



PARALLEL MECHANISMS

- Structure of closed loop chain(s)
- I.e. Stewart platforms & hexapods
- Kinematics analysis usually difficult
- 6 DOF mechanism/machine
- Multiple variations on this theme



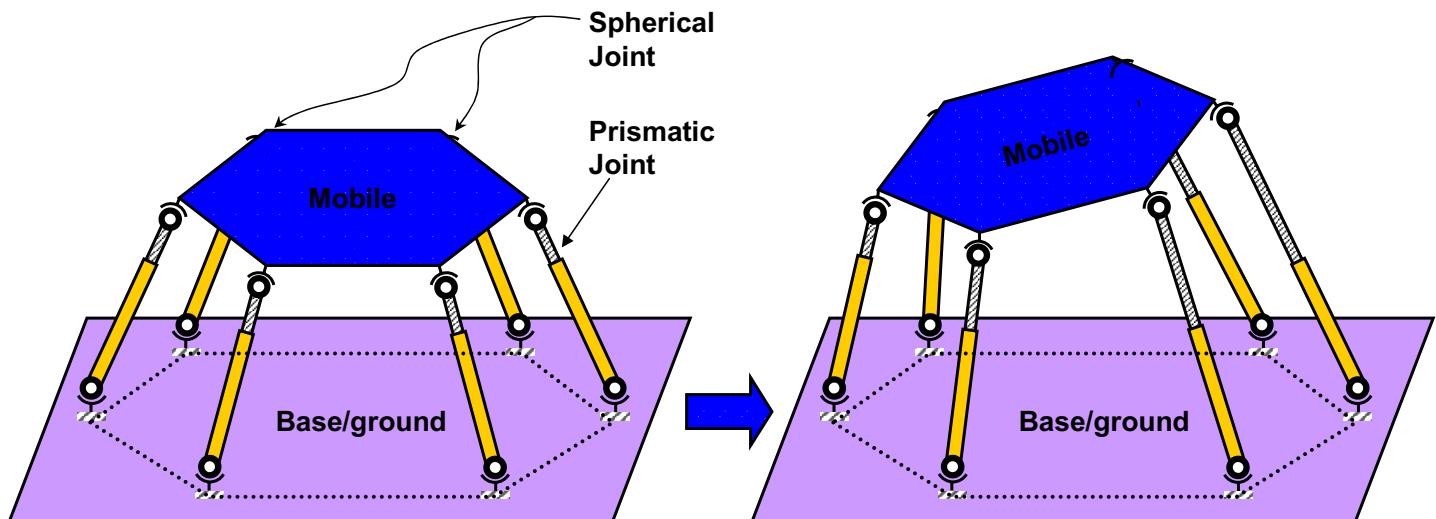
Parallel mechanism: Stewart-Gough platform

6 DOF mechanism/machine

Multiple variations on this theme with different joints:

- | | | | |
|---------------------|------|------------------------------------|-----------------|
| ◎ Spherical joints: | 3 Cs | Permits 3 rotary DOF | Ball Joint |
| ◎ Prismatic joints: | 5 Cs | Permits one linear DOF | Sliding piston |
| ◎ Planar: | 3 Cs | Permits two linear, one rotary DOF | Roller on plane |

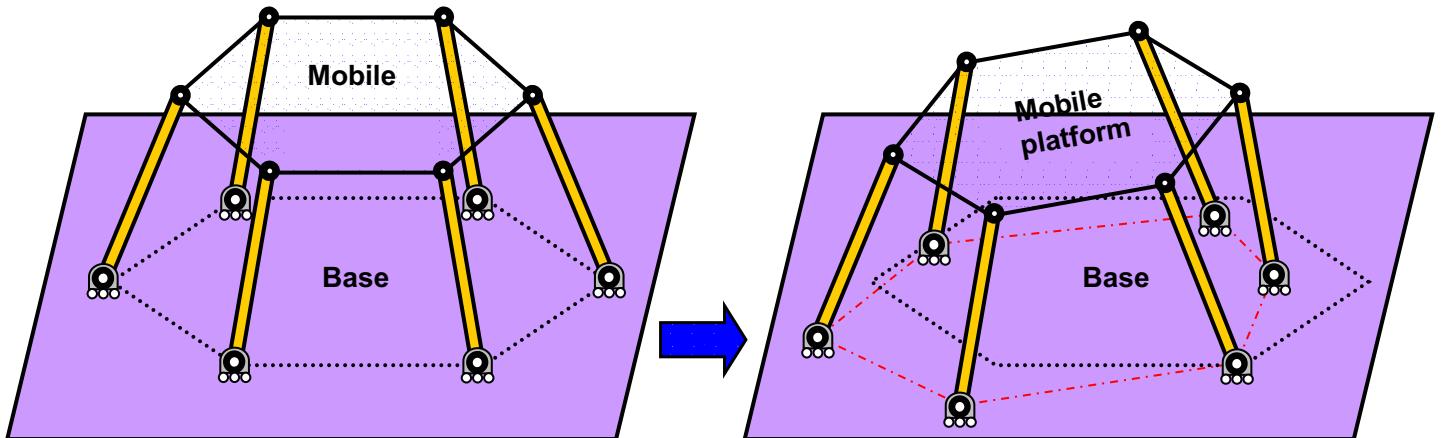
E.g. Changing length of “legs”



Parallel mechanism: Variation

6 DOF mechanism/machine by changing position of joints

Can have a combination of position and length changes



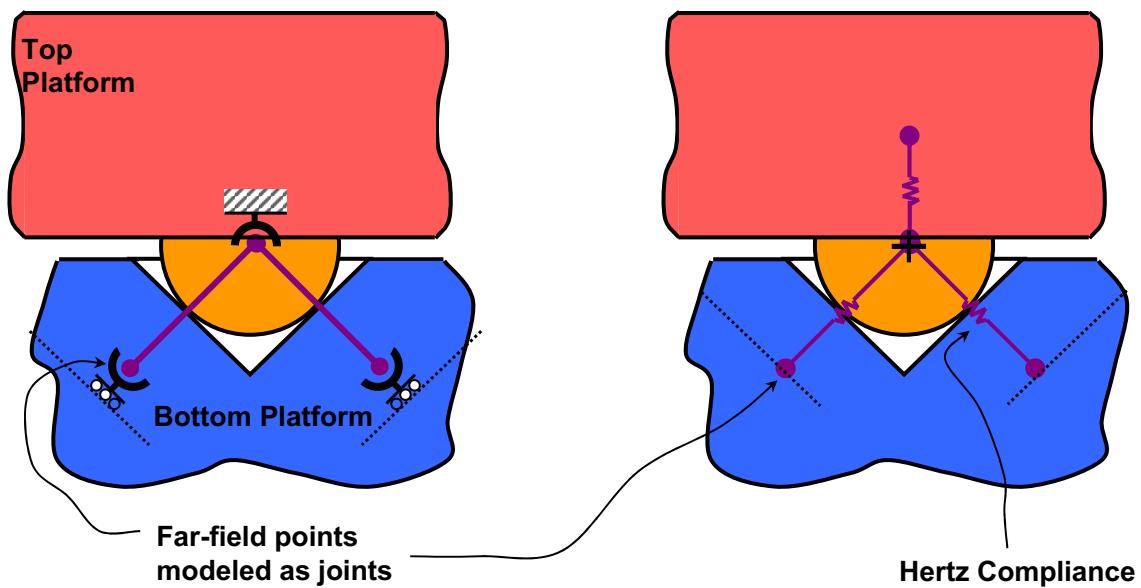
Kinematic couplings as mechanisms

Ideally, kinematic couplings are static parallel mechanisms

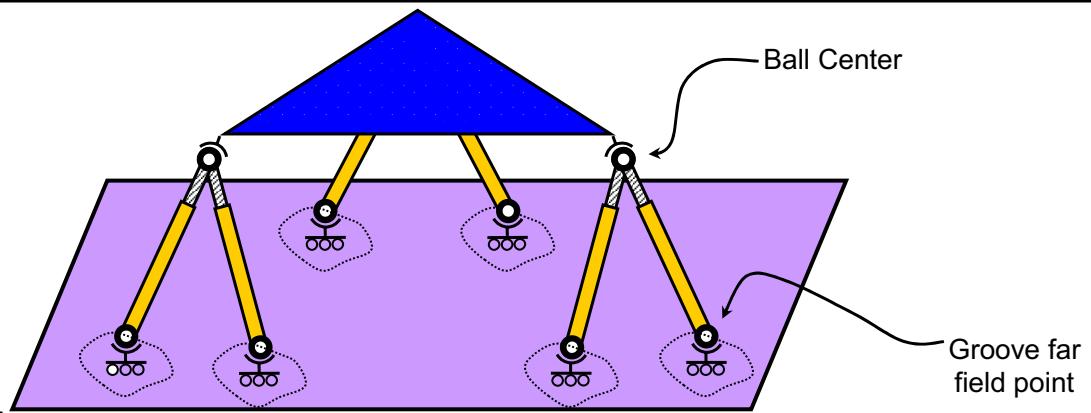
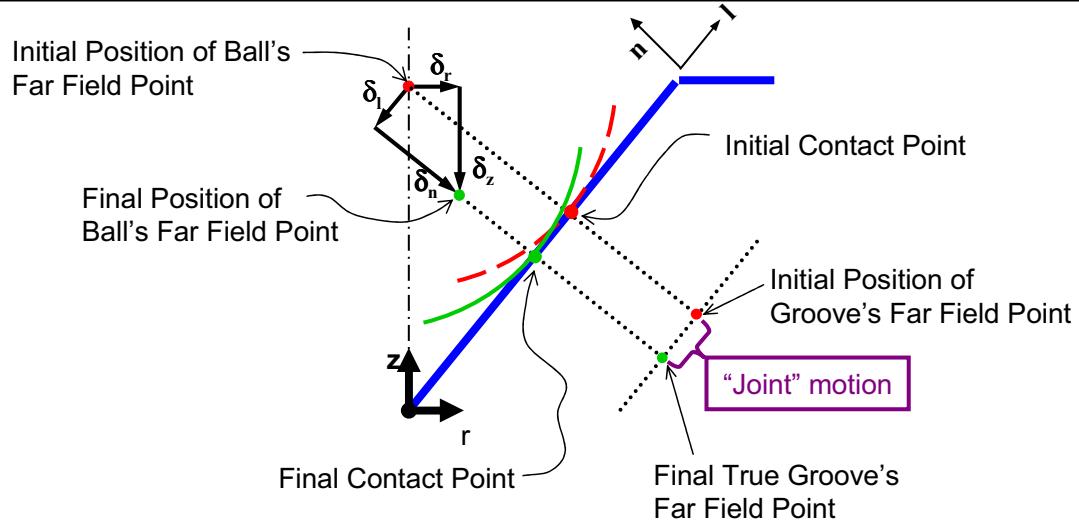
IRL, deflection(s) = mobile parallel kinematic mechanisms

How are they “mobile”?

- Hertz normal distance of approach ~ length change of leg
- Far field points in bottom platform moves as ball center moves ~ joint motions



Model of kinematic coupling mechanism



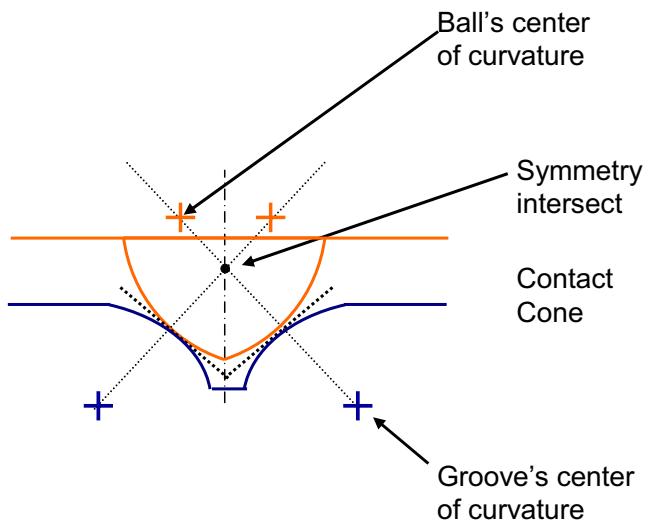
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Accuracy of kinematic mechanisms

Since location of platform depends on length of legs and position of base and platform joints, accuracy is a function of mfg and assembly

Parameters affecting coupling centroid (platform) location:

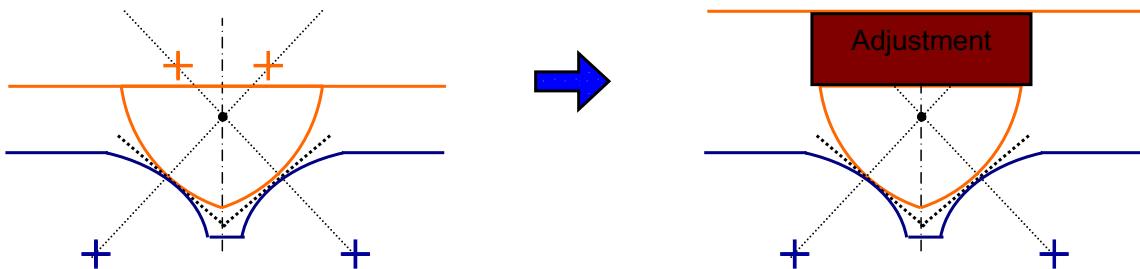
- Ball center of curvature location
 - Ball orientation (i.e. canoe ball)
 - Ball centerline intersect position (joint)
 - Ball radii
-
- Groove center of curvature location
 - Groove orientation
 - Groove depth
 - Groove radii



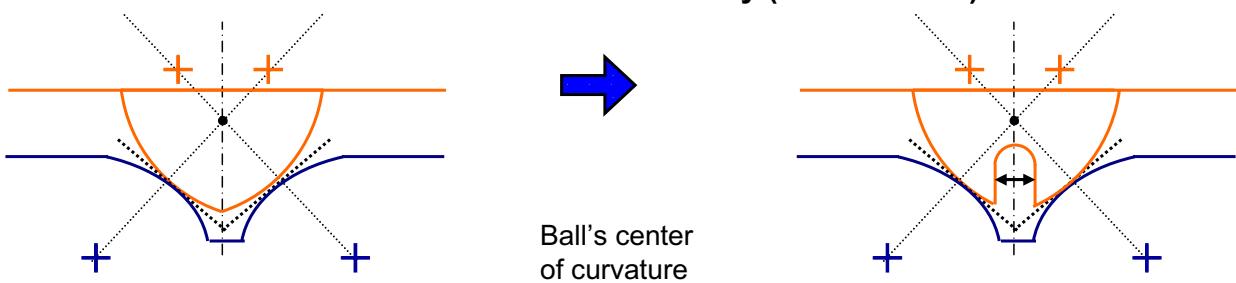
Utilizing the parallel nature of kinematic couplings

Add components that adjust or change link position/size, i.e.:

- Place adjustment between kinematic elements and platforms (joint position)



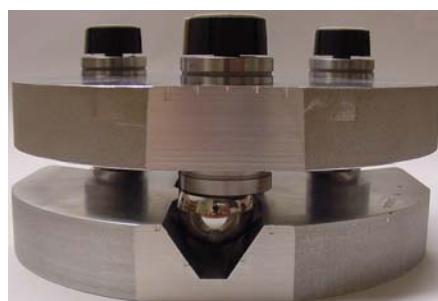
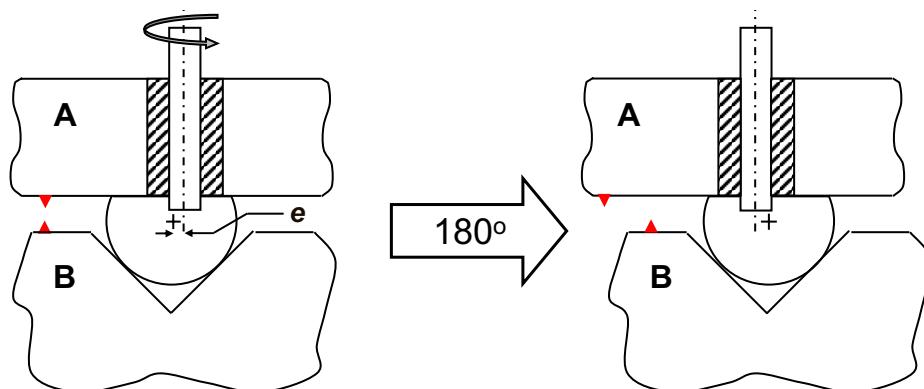
-
- Strain kinematic elements to correct inaccuracy (element size)



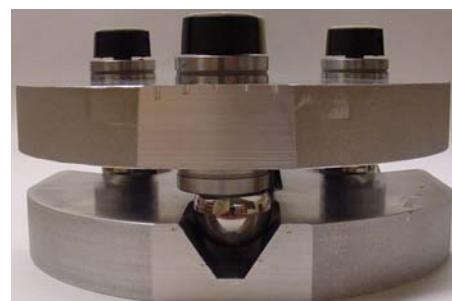
Example: Adjusting planar motion

Position control in x, y, θ_z :

- Rotation axis offset from the center of the ball



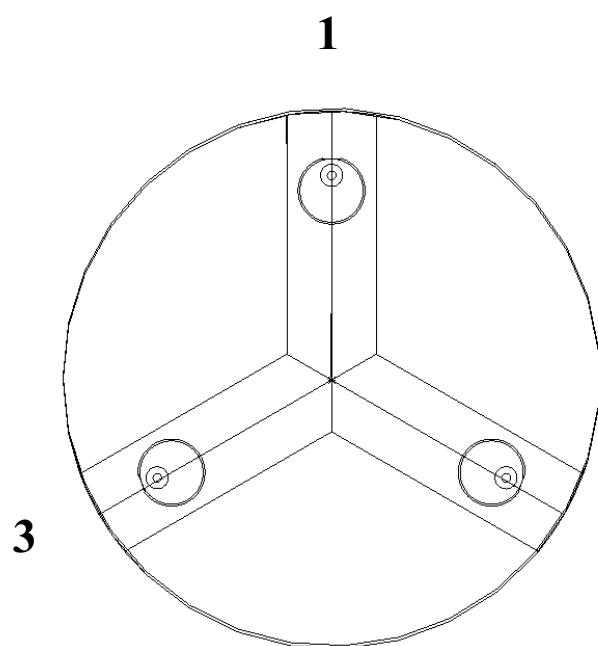
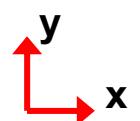
Eccentric left



Eccentric right

Patent Pending

ARKC demo animation



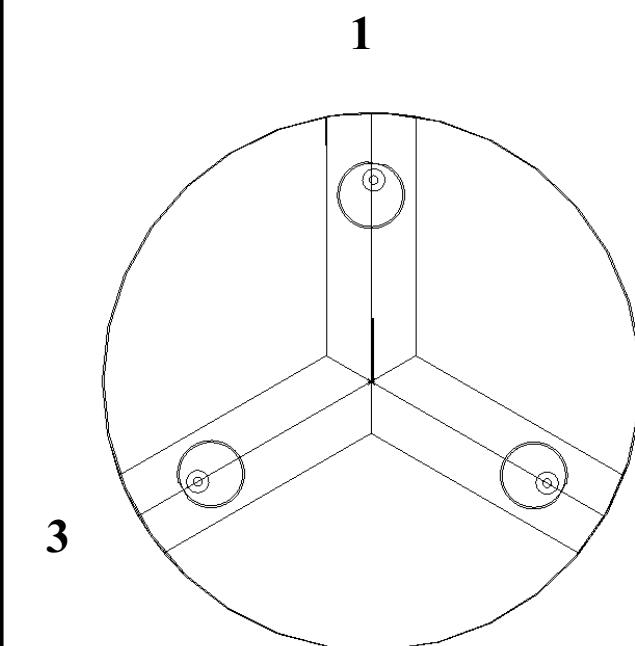
3

1

2

Input: Actuate Balls 2 & 3
Output: Δy

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3

1

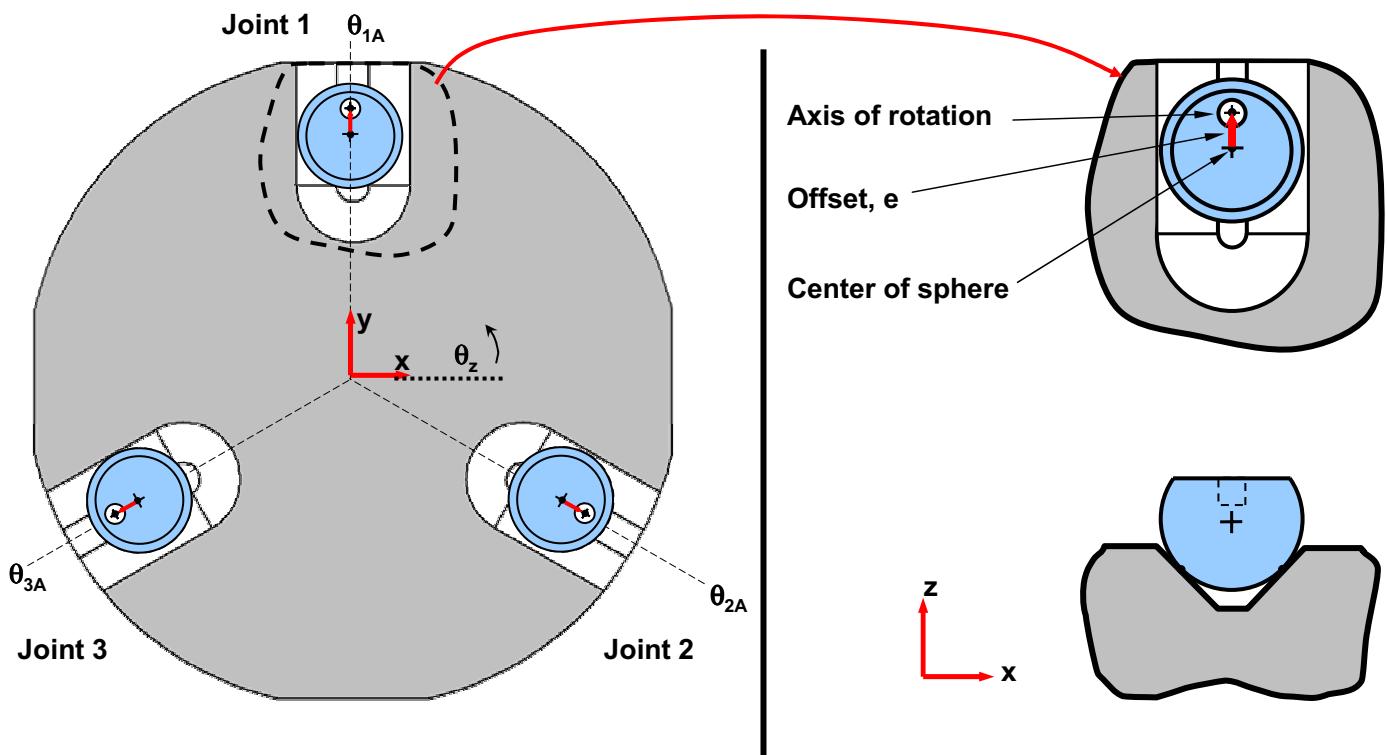
2

Input: Actuate Ball 1
Output: Δx and $\Delta \theta_z$

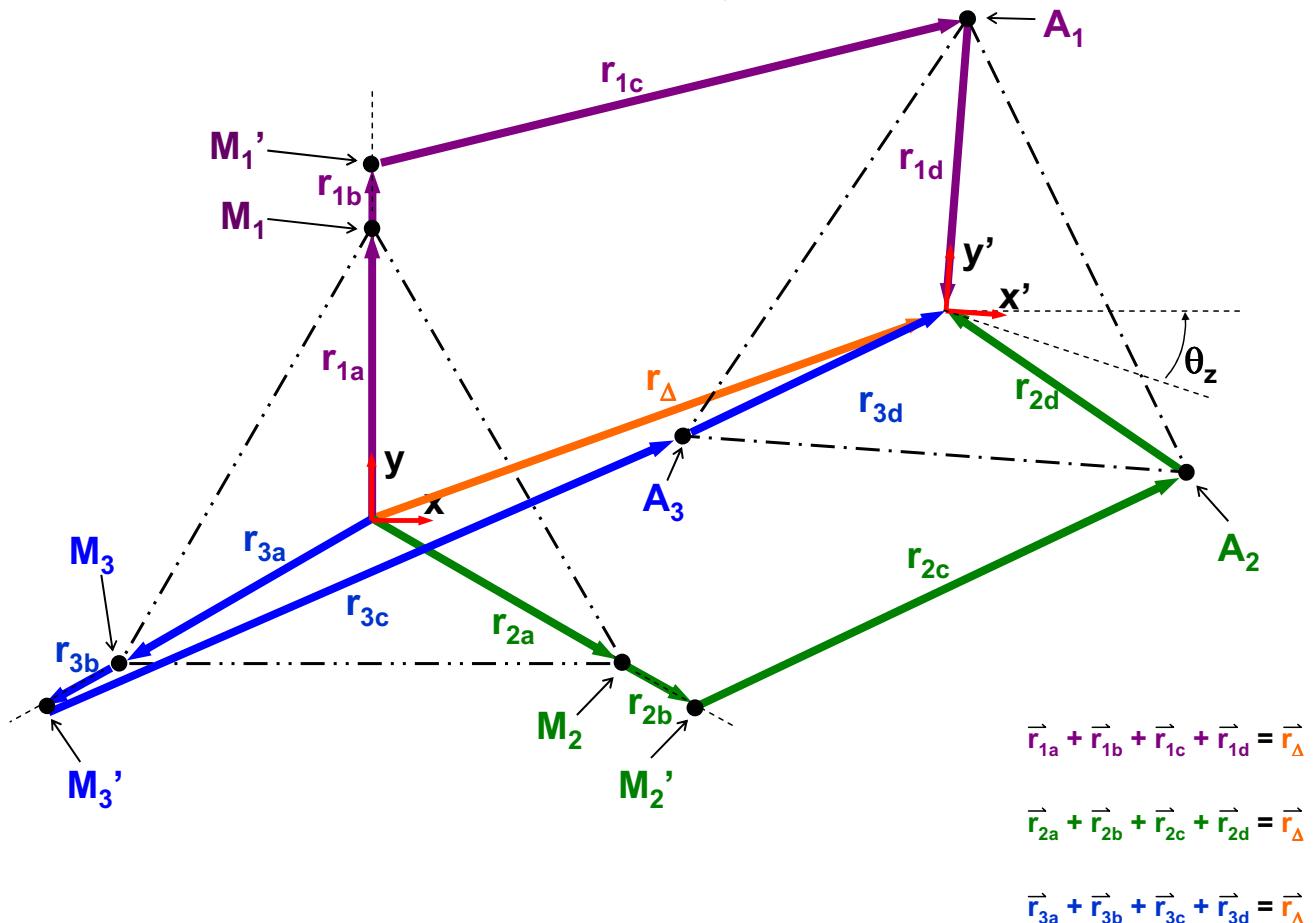
Planar kinematic model

Equipping each joint provides control of 3 degrees of freedom

View of kinematic coupling with balls in grooves (top platform removed)



Vector model for planar adjustment of KC

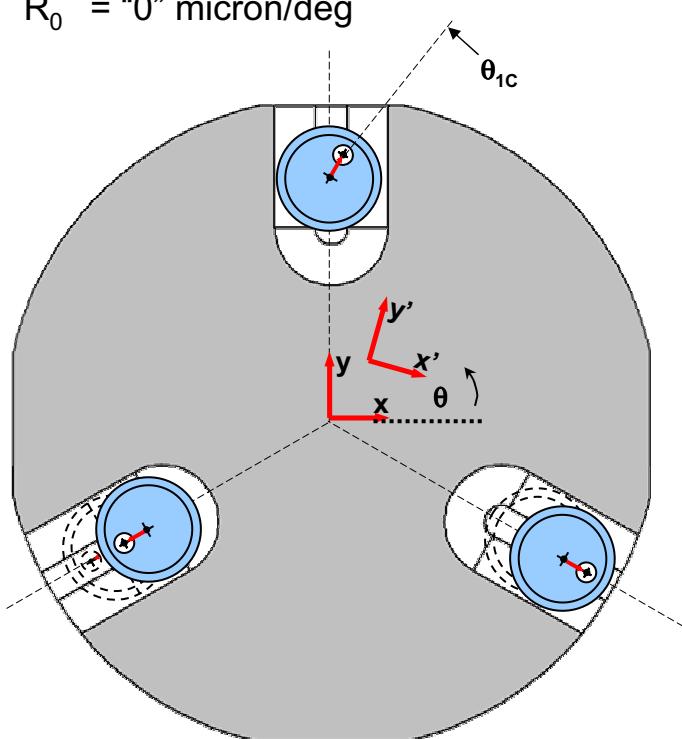


ARKC resolution analysis

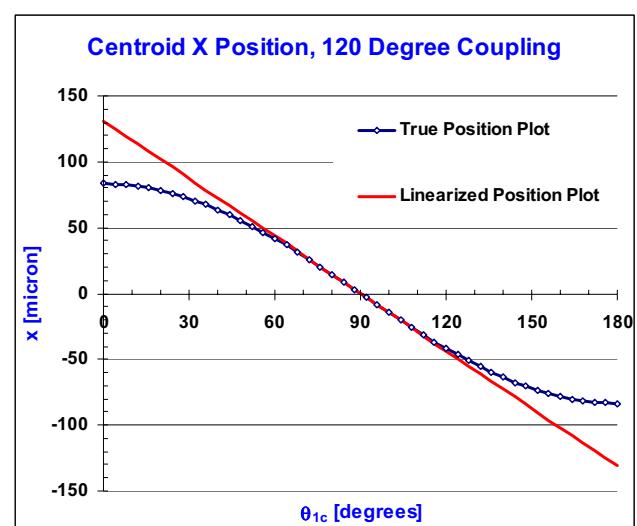
For $E = 125$ microns

$R_{90} = -1.5$ micron/deg

$R_0 = "0"$ micron/deg



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Limits on Linear Resolution Assumptions

% Error	θ_{1c}		
	Lower Limit [Degree]	Upper Limit [Degree]	Half Range [Degree]
1	75	105	+/- 15
2	70	110	+/- 20
5	60	120	+/- 30
10	47	133	+/- 43

Forward and reverse kinematic solutions

ARKC Kinematic Analysis Spread Sheet

Do not change cells in red, only change cells in blue

PART I: COUPLING CHARACTERISTICS

Use this to specify groove angles and input ball rotations and heights used to calculate coupling position in part II

θ_{1A}	90	degrees	z_1	5	microns
	1.571	radians		0.0001969	inches
θ_{2A}	330	degrees			
	5.760	radians	z_2	500	microns
	0.0196850	inches			
θ_{3A}	210	degrees			
	3.665	radians	z_3	200	microns
				0.0078740	inches
θ_{1C}	87.7073	degrees			
	1.531	radians			
θ_{2C}	331.1455	degrees			
	5.780	radians			
θ_{3C}	211.1467	degrees			
	3.685	radians			
E	125	microns			
	0.0049	inches			
R_T	57150	microns	Note: $R_T = L_D$		
	2.2500	inches			

PART II: CALCULATED MOVEMENTS

This takes input from part I to calculate the position of the top part of the coupling.

θ_z	0.0000	radians	θ_x	-0.004024	radians
	0.0006	radians		-4024.4889	radians
	0.000000	degrees		-0.230586	degrees
x	5.0005	microns	θ_y	-0.003031	radians
	0.000197	inches		-3030.7040	radians
				-0.173647	degrees
y	-0.0014	microns	z	235.0000	microns
	0.000000	inches		0.009252	inches

PART III: REVERSE SOLUTION

Use this to input a desired position and goal seek to solve for the position of the balls

DESIRED POSITION POSITION ERROR

θ_z	0.000	0.00	0.0	μrad
x	5.000	-0.001		microns
y	0.000	0.001		microns

BALL SETTINGS

See part I for modified ball angles, the following angles are the difference between groove and ball angles

θ_1	-2.2927	degrees
θ_2	1.1455	degrees
θ_3	1.1467	degrees

Low-cost adjustment ($10 \mu\text{m}$)

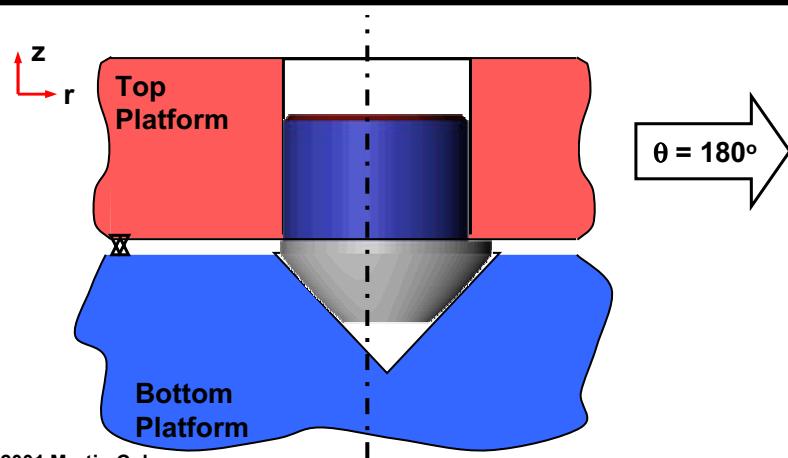
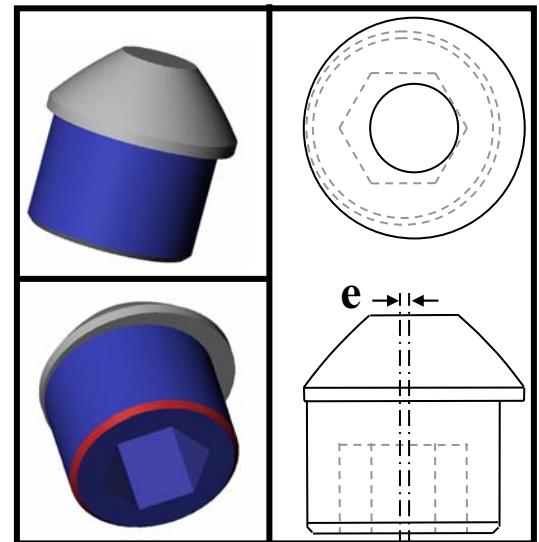
Peg shank and convex crown are offset

Light press between peg and bore in plate

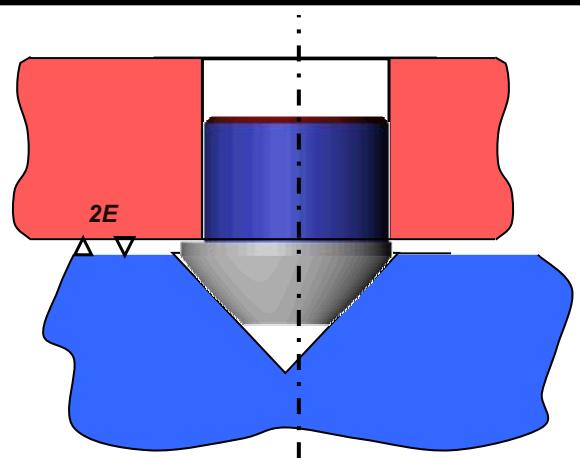
Adjustment with allen wrench

Epoxy or spreading to set in place

Friction (of press fit) must be minimized...



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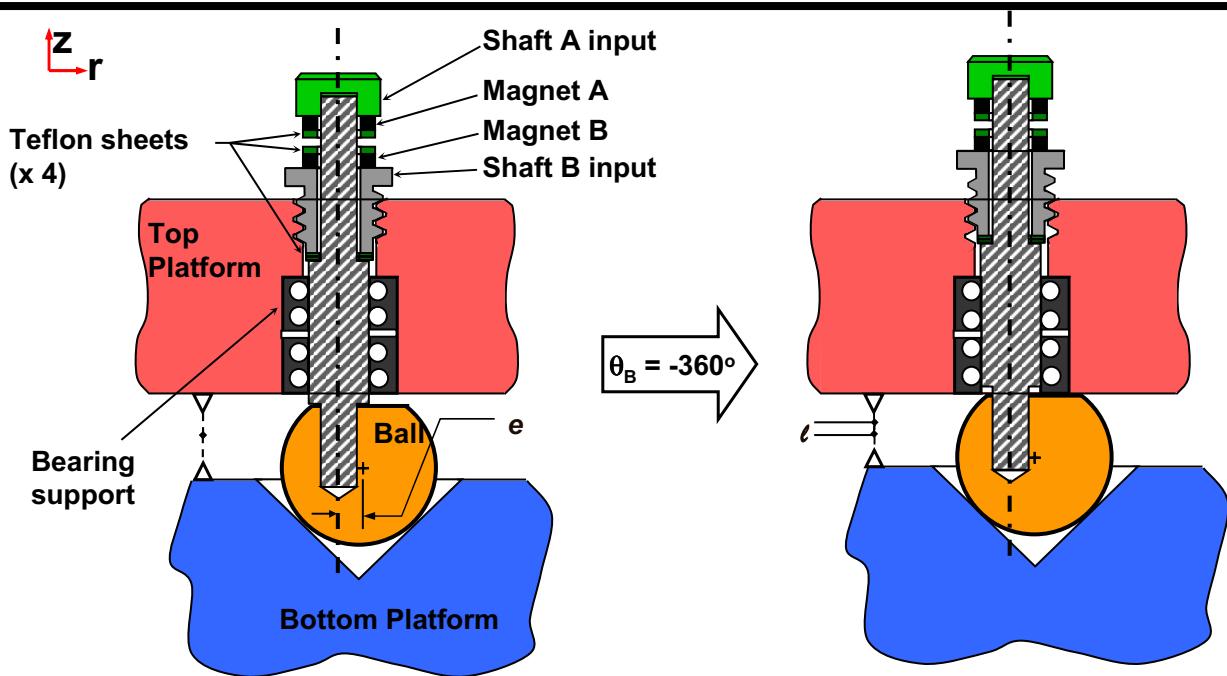
Moderate-cost adjustment (3 micron)

Shaft B positions z height of shaft A [z, θ_x, θ_y]

Shaft A positions as before [x, y, θ_z]

Force source preload

i.e. magnets, cams, etc..



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“Premium” adjustment (sub-micron)

z_r

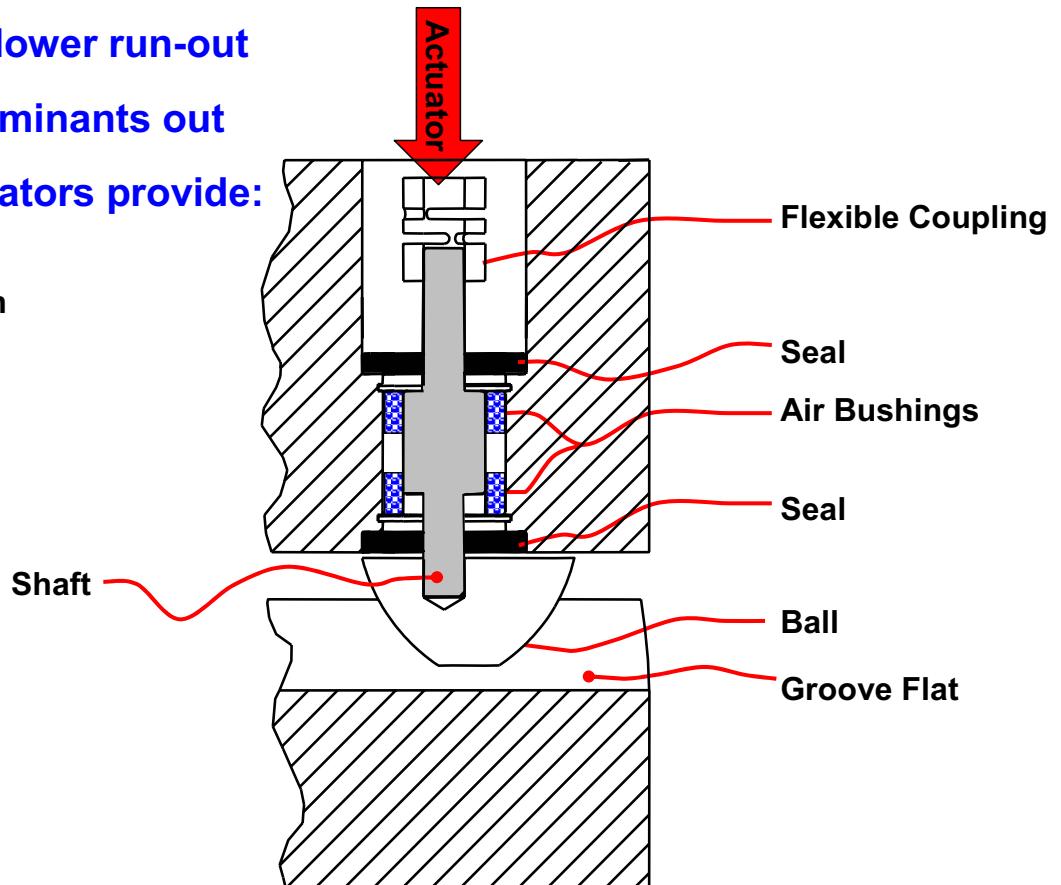
Run-out is a major cause of error

Air bushings for lower run-out

Seals keep contaminants out

Dual motion actuators provide:

- Linear motion
- Rotary motion

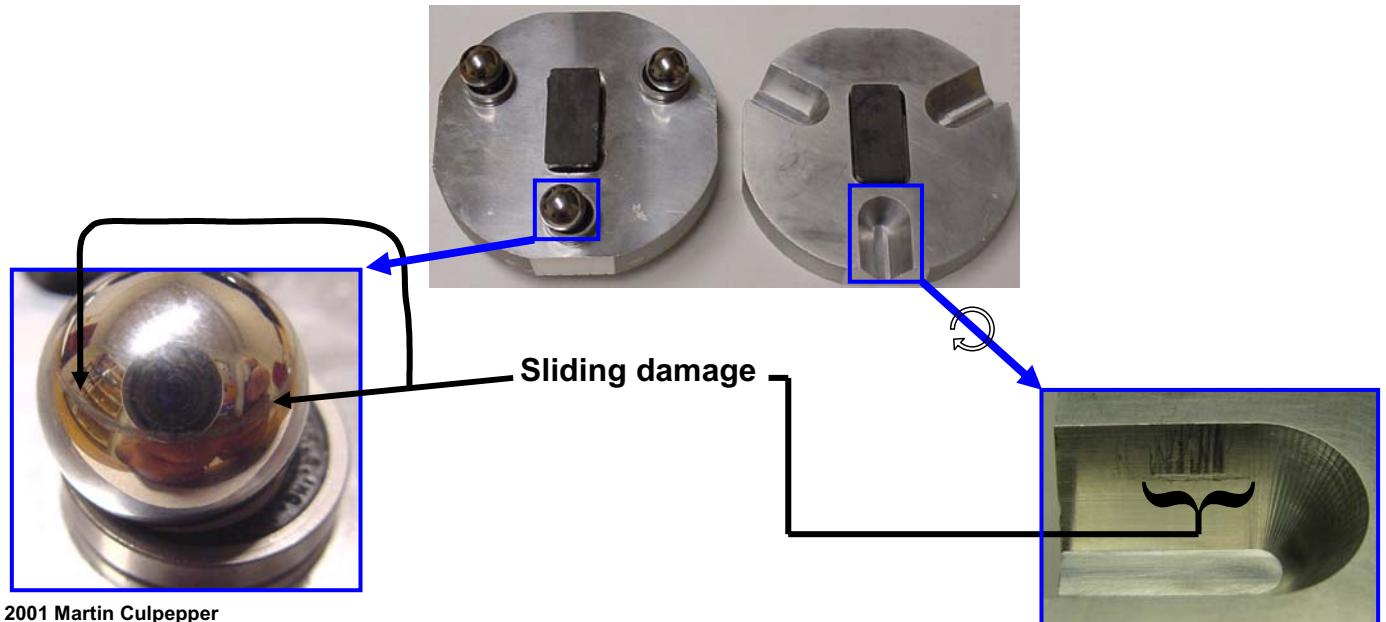


Mechanical interface wear management

Wear and particle generation are unknowns. Must investigate:

- Coatings [minimize friction, maximize surface energy]
- Surface geometry, minimize contact forces
- Alternate means of force/constraint generation

At present, must uncouple before actuation



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PARTIAL CONSTRAINT

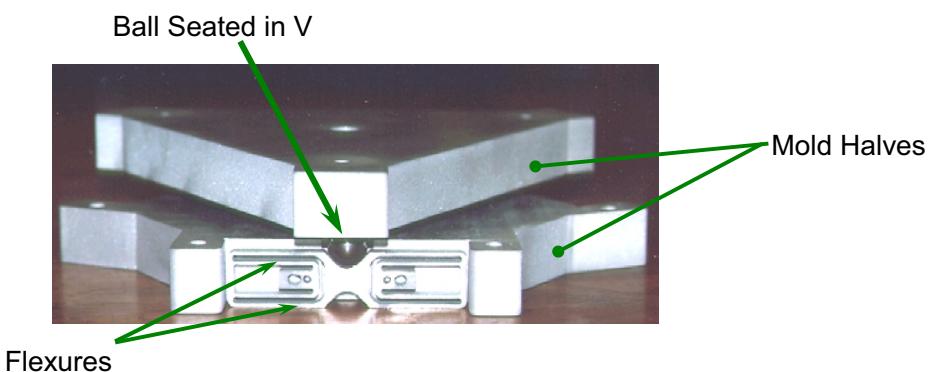
Motivated by coupling envy.....

Adding and taking away constraints

It may be helpful to add/remove DOF in coupling applications

For instance, KCs can not form seals

- We can add compliance to KCs to allow this to happen
- This is equivalent to adding a Degree of Freedom



Care must be taken to make sure

- compliant direction is not in a sensitive direction
- Parasitic errors in sensitive directions are acceptable

Stiffness ratio

Actuation loads should be:

- Applied through center of stiffness
- In compliant direction

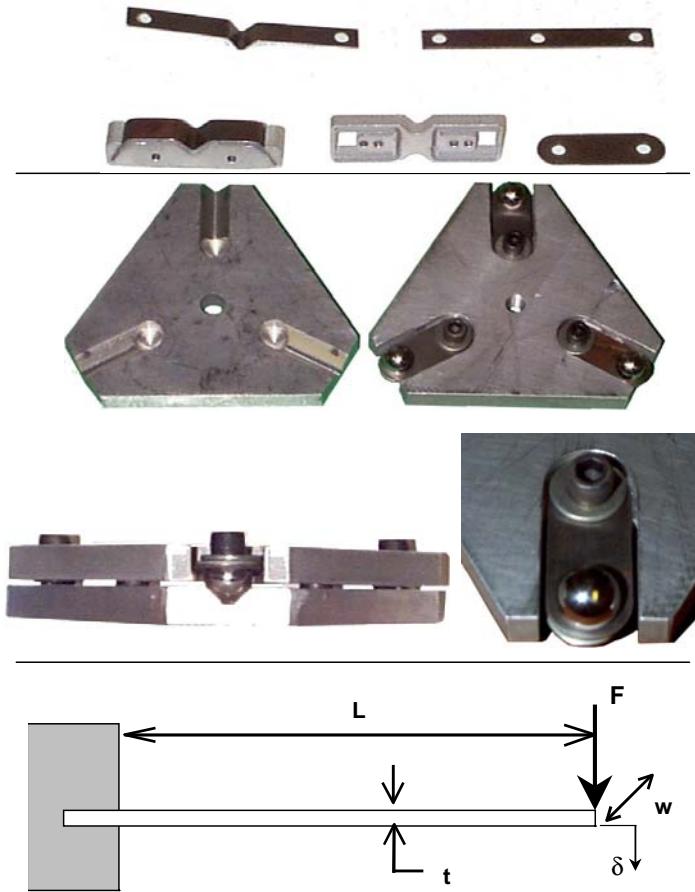
Error loads are often proportional to applied loads

- Example: Bolt head friction
- $T_B \sim F_B R_B \mu$
- Design for $k_{\text{sensitive}} \gg k_{\text{non-sensitive}}$

Practical metric is stiffness ratio:

$$\frac{k_{\text{sensitive}}}{k_{\text{non-sensitive}}} \gg 1$$

Stamped compliant kinematic couplings



Characteristics

Stroke ≤ 0.25 inches

Repeatability 5 -10 microns

Ball movement in non-sens. direction

Applications/Processes

1. Assembly

2. Casting

Design Issues (flexure)

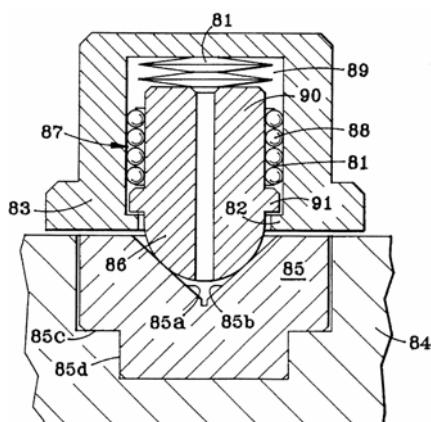
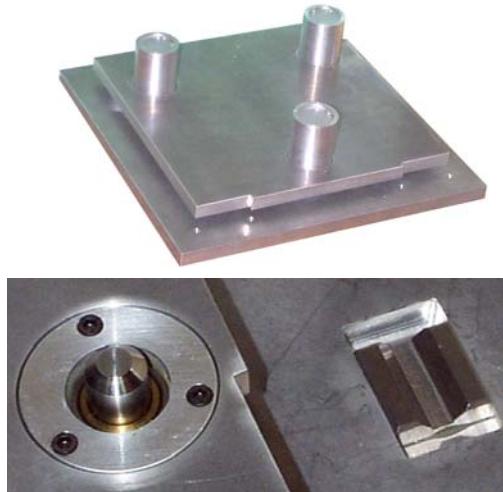
$$1. K_r \sim \frac{w^2}{t^2}$$

2. Tolerances affect K_r

Cost

\$ 10 - 200

Integral spring compliant kinematic couplings



Characteristics

1. Repeatability (2.5 micron)
2. Stroke ~ 0.5 inches

Applications/Processes

1. Assembly
2. Casting
3. Fixtures

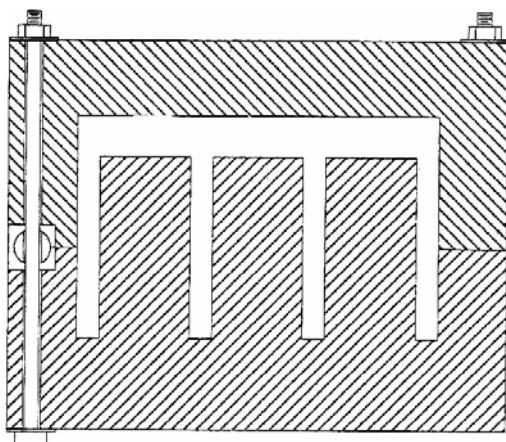
Design Issues (flexures)

1. $K_r = \frac{K_{\text{guide}}}{K_{\text{spring}}}$
2. Press fit tolerances

Cost

\$ 2000

Plastic compliant kinematic couplings



Characteristics

1. 180 microns
2. ~ 0.125 inches
3. 1 Time Use

Applications/Processes

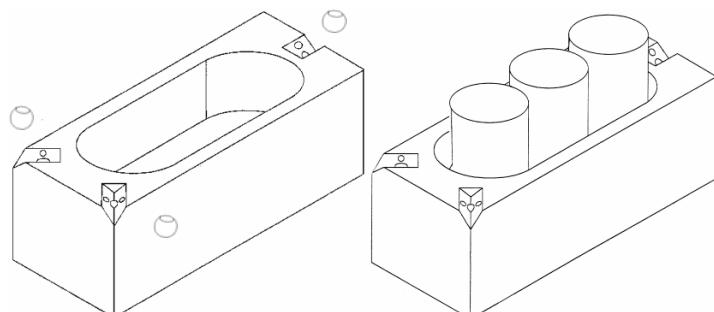
1. Sand Casting

Design Issues

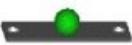
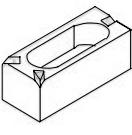
1. Loose Sand
2. K_r application specific

Cost

1. Modify Pattern
2. Purchase Balls
3. Tie Rods



Experimental results

<i>Coupling Type</i>		<i>Prototype</i>	<i>Radial</i> μinch (microns)	<i>Average Repeatability</i>		<i>Standard Deviation</i> μinch (microns)	<i>Angular</i> $\mu\text{radians}$	<i>Standard Deviation</i> $\mu\text{radians}$	<i>Manufacturing Cost</i> \$
Stamped	1.		Water Jet (mate-ball)	300 (7.6)	100 (2.5)	35	21	23	
	2.		Stamped (mate-ball)	400 (10.2)	200 (5.1)	120	70	10	
	3.		Stamped (mates-V)	200 (5.1)	100 (2.5)	52	30	10	
	4.		Stamped (mates-V)	300 (7.6)	100 (2.5)	57	23	10	
Springs	5.		Ultra Die Set Coupling	100 (2.5)	N/A	N/A	N/A	2000	
	6.		Grooves made in mold sand	19,000 (480)	7000 (180)	3900	1300	(cost to modify pattern)	

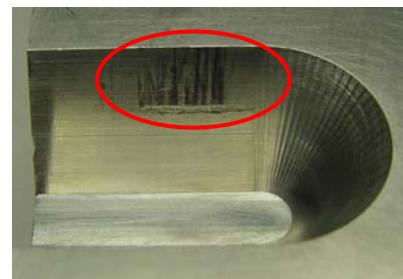
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USING CONSTRAINTS IN MECHANISM DESIGN

Alternatives to motion with physical contact

Problems you can not avoid with contact:

- Surface topology (finish)
- Wear and Fretting
- Friction
- Limited resolution, at best on order of microns....



Wear on Groove

Next generation applications require nanometer level fixtures, i.e.:

- Fiber optics
- Photolithography

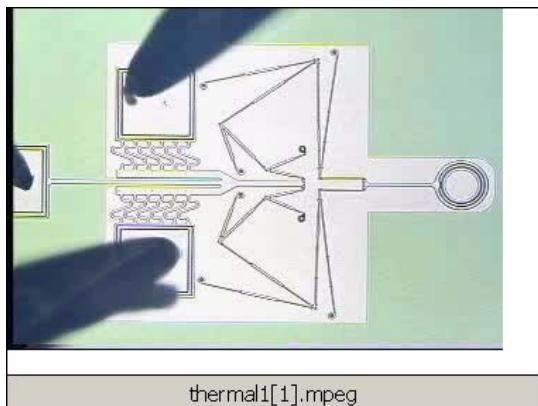
Compliant mechanisms:

- Mechanical reduction to interface with larger scale actuators
- Motion through strain
- Small and moderately sized motions in comparison to mechanism size
- Can be made to emulate machines

Compliant mechanism examples

University of Michigan: Prof. Sridhar Kota

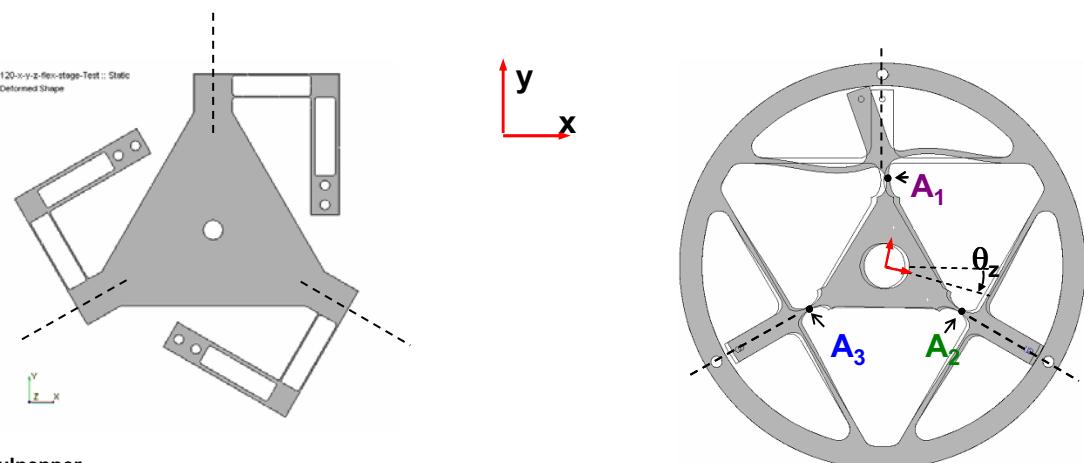
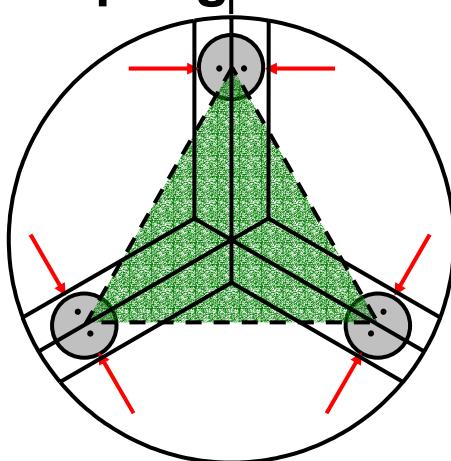
- <http://www.engin.umich.edu/labs/cSDL/index.htm>



Why compliant mechanisms in precision fixtures

- Repeatable/low hysteresis
- No assembly
- No contact

From kinematic couplings to compliant stages



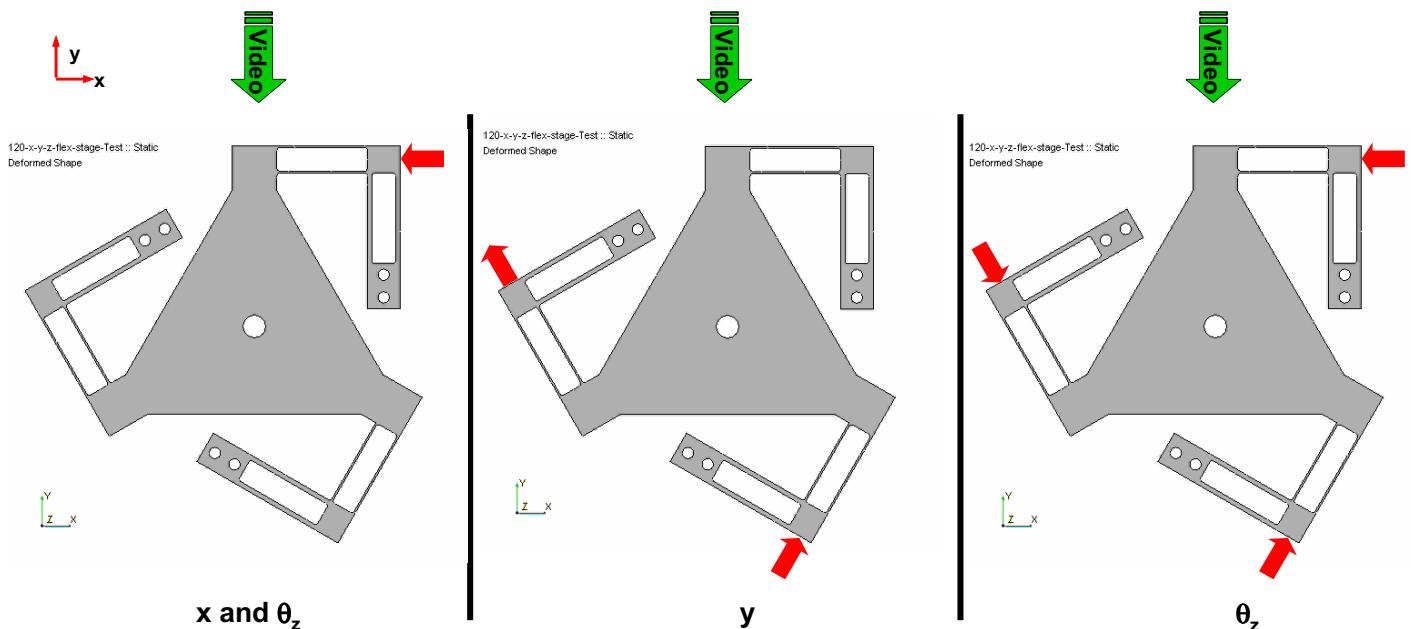
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Constraint based compliant mechanisms

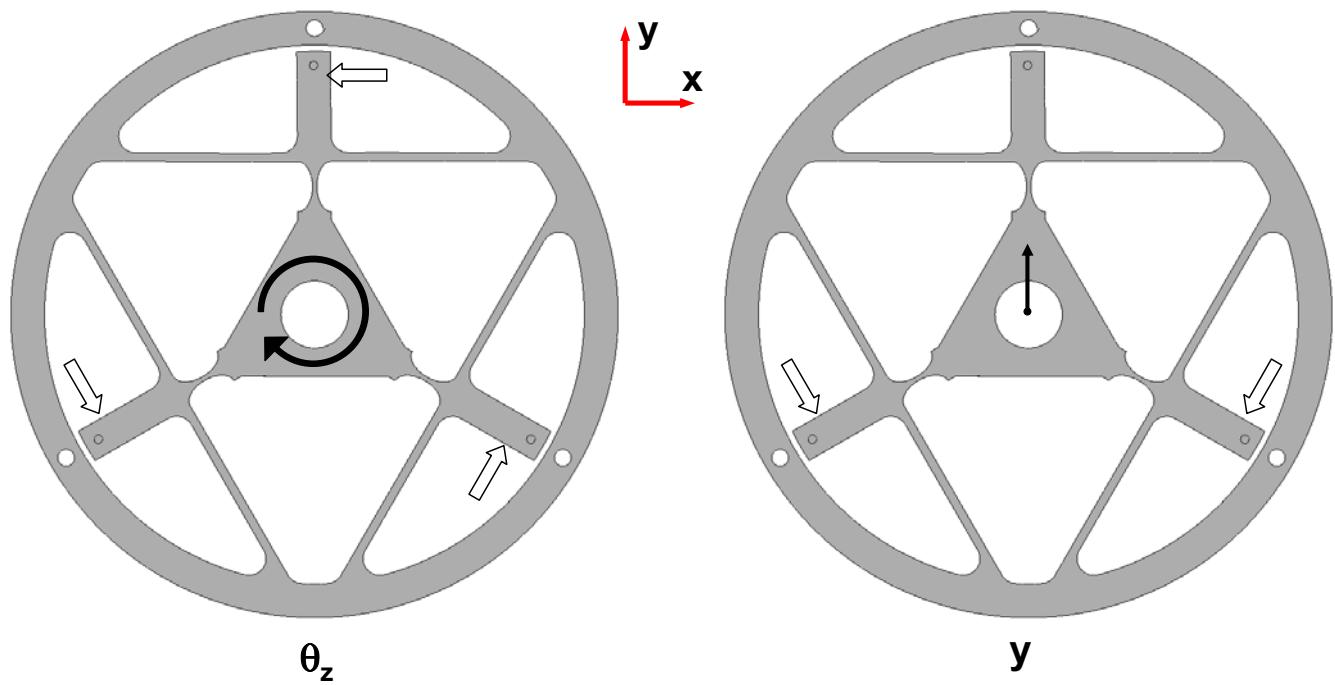
High volume, low cost, multi-degree of freedom alignment

Example 3 DOF flexure system:

Target applications: Opto-electronic packaging/alignment



Constraint based compliant mechanisms cont.

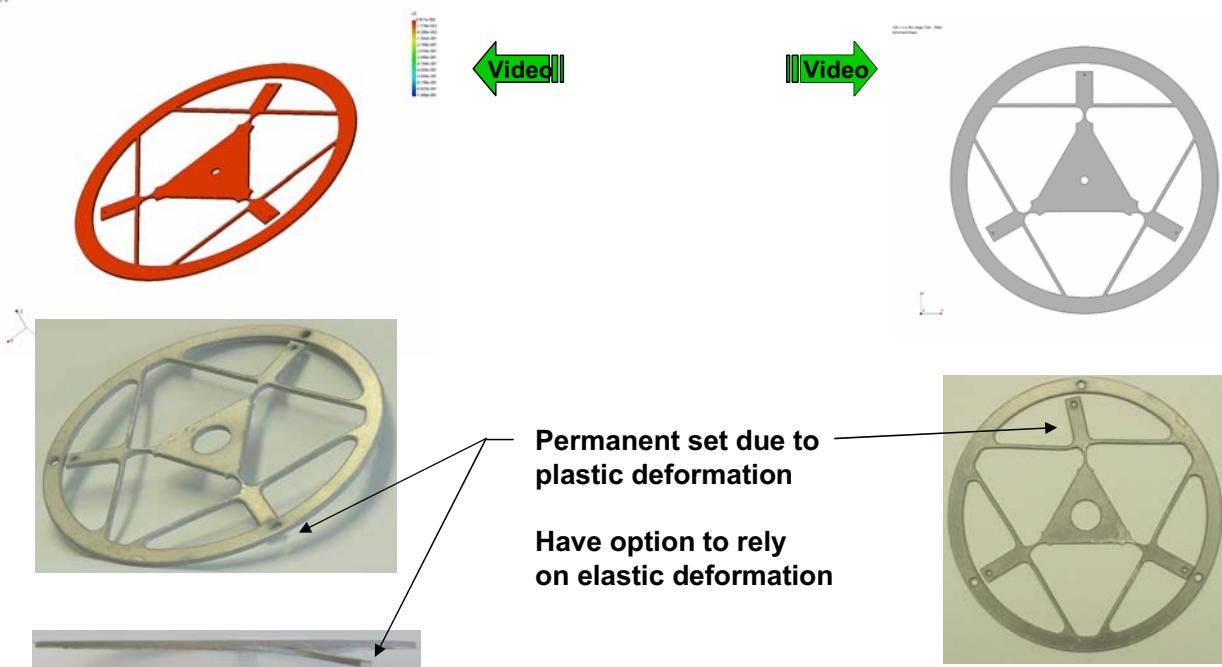


Constraint based compliant mechanisms cont.

Example 6 DOF alignment capability

Target app.: Micro and meso scale positioning (i.e. opto-electronics)

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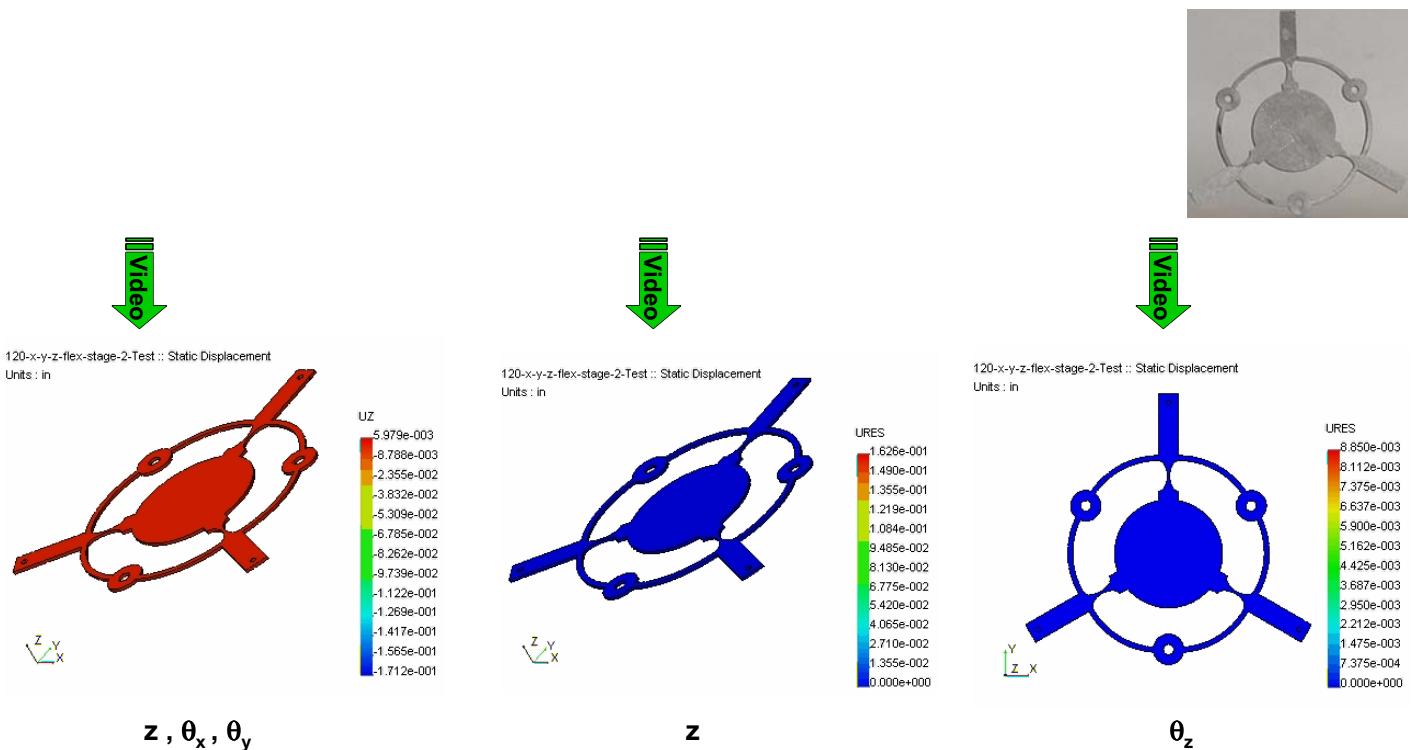
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z , θ_x , θ_y

x and θ_z
Patent Pending

Constraint based compliant mechanisms cont.

Example 6 DOF alignment capability

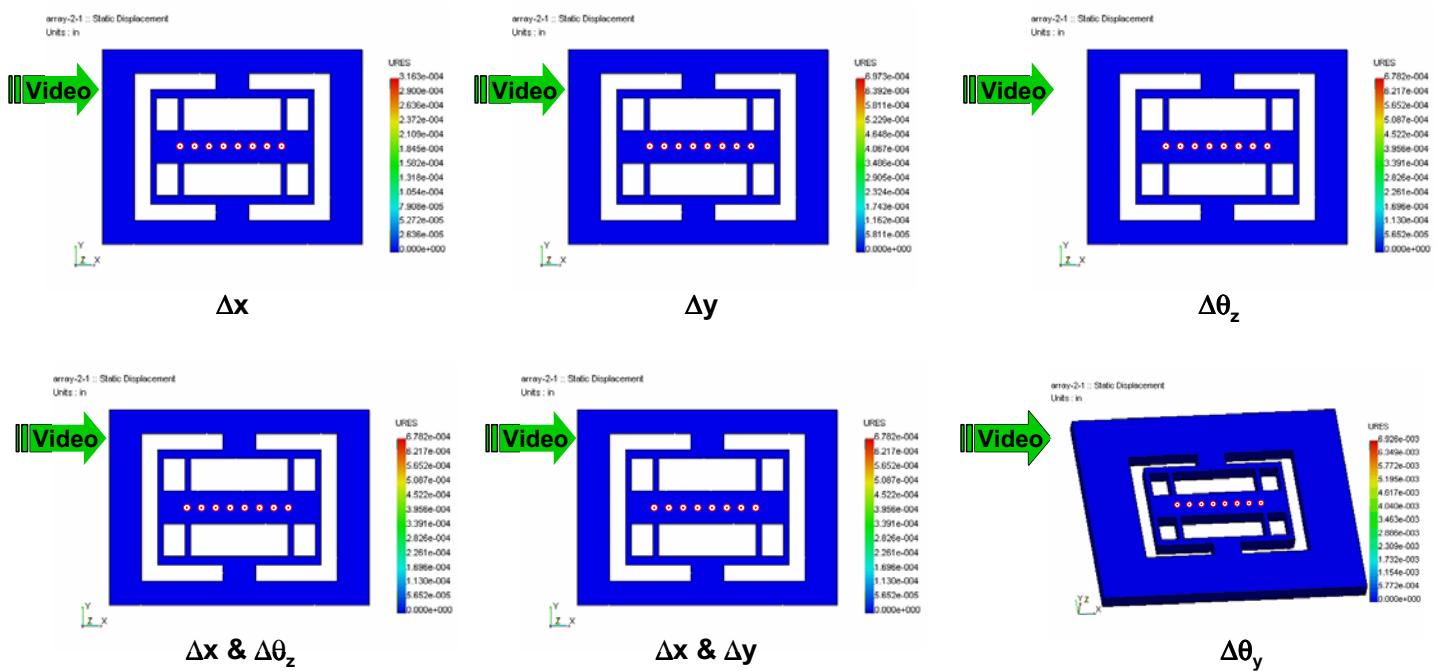
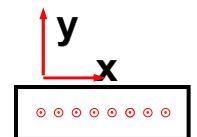
Target app.: Micro/meso scale positioning (i.e. opto-electronics)



Constraint based compliant mechanisms cont.

3 DOF active alignment [x, y, z] & 2 DOF passive alignment [z, θ_y]

Good fit for wire-EDM (stacked sheets) ~ order of \$ 1 - 10

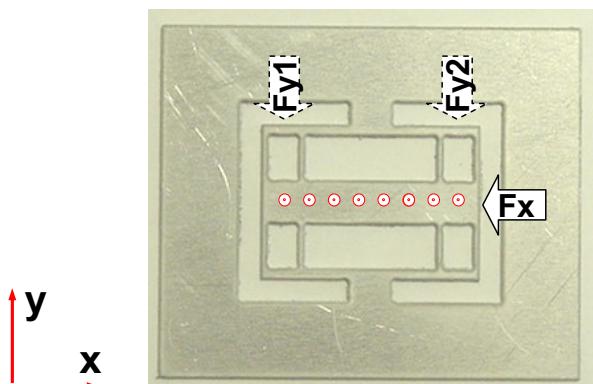


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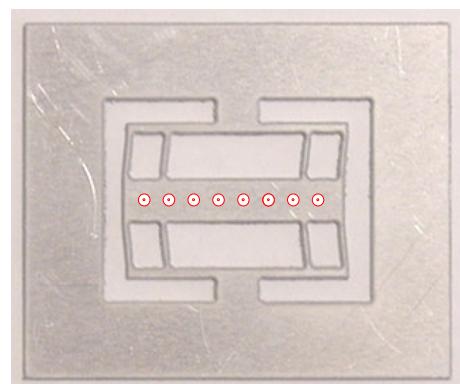
Constraint based compliant mechanisms cont.

Plastic deformation can be utilized for position keeping

Device should be potted in place to avoid stress relief



Initial position



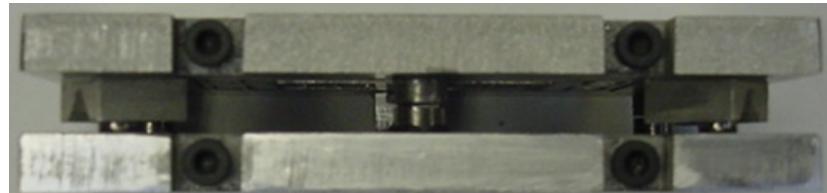
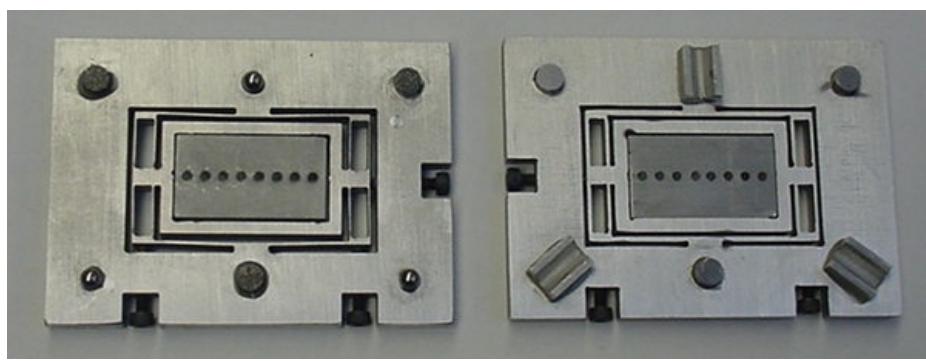
Plastically flexed

Constraint based compliant mechanisms cont.

Static or flexible kinematic coupling

Components biased toward each other

Flexure takes up bias, provides mating force in z direction



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