A SYSTEM DYNAMICS MODEL OF STOCK PRICE MOVEMENTS

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Abstract: Stock prices are known to exhibit strongly non-linear behaviours, e.g. apparent random volatility, bullish or bearish trends, crashes etc. Using finite difference equations it is easy to generate pseudo-random behaviour patterns. The hope is that richer patterns can be generated in the continuous case using System Dynamics (SD). In the paper we test this possibility. Three basic behavioural attitudes are described by introducing three corresponding families of investors. *α-investors* are rational fundamentalists striving to stabilise the stock price toward a goal value. *Short-term βS-investors* are traders destabilising the market as they follow immediate movements and fads. *Long-term βL-investors* use arbitrage by comparing returns on stock and on risk-free assets. Although they are partly rational they provoke important departures from the fundamentals in an expanding market. It is shown that the behaviour of βL-investors is an important explicative factor of instability on the stock market.

Keywords: System Dynamics modelling, stock price, behavioural finance, fundamentalists, traders

INTRODUCTION AND PURPOSE

The pseudo-chaotic character of financial time series has been an object of fascination to economists for several decades at least. The observed fluctuations evidence a clear

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departure from the traditional equilibrium models of neo-classical theory. An abundant literature is today available on this topic (see for example a compilation of papers and references in Creedy and Martin (Eds.) 1994; Heij et al. (Eds.) 1997). Many authors have explored the statistical properties of the observed non-linear and often chaotic dynamics on the financial markets. It is clear from this literature that behavioural attitudes and bounded rationality are here at work, beyond the efficient market hypothesis founded on rational expectations of the neo-classical view. The Keynesian theme of "animal spirits" has surfaced in the efforts to reconstruct a "market psychology". The present paper tries to go in the same direction for what regards the price evolution of equities on the stock market (SM).

The two next sections describe the elaboration of an original model using the System Dynamics (SD) technique to provide a causal explanation of the investor's behaviour. This point of view is rather different from most financial models based on chaos theory.

The proposed approach belongs to the class of "agent-based model" in the sense defined by Axelrod (1997). Most financial models either use inductive approach to apprehend observed price patterns, or set up deductive models to derive properties of such patterns. Agent-based modelling starts from the assumption of sets of rules to simulate agent behaviours on the market. The interaction of different rules generates data "ab initio", i.e. from first principles. In this sense the present approach is related to "popular models" of finance. However, investors as described in our model are not entirely irrational, or driven by animal spirits. Some rules also relate to the efficient market hypothesis. To our knowledge only few genuinely "ab initio" models have been developed before. The model by R.H. Day (1994) and the recent approach of "artificial stock exchange" (Arthur et al., 1997) have the suitable characteristics in our opinion. They are briefly described in the second section. These models do however not have the same transparency given by simple causal explanations of behaviour like proposed in the present paper. An additional difference of the approach is the use of continuous time by contrast with the most common discrete approach of chaos theory. The basics of our SD-modelling are presented in the third section.

In the fourth section, some typical time series generated by the model are discussed. The simulation show that the model, although it is entirely deterministic, has the capacity of generating volatile evolution patterns. For each situation, reference is made to historical patterns, using the Standard and Poor (S&P) 500 composite index. In the fifth and last section, conclusions are enounced on the usefulness of "ab initio" modelling using SD. While the predictive capabilities are limited, the essentials of a typology of characteristic price patterns are laid bare. This can help investors in staying closer to the rationality ideal or, at least, in providing a better perspective to anticipate the "market psychology". Also prospects for future development are discussed.

"AB INITIO" MODELLING OF THE STOCK MARKET (SM)

To our knowledge the first model drawing from the representation of simple investors' attitude dates back to a classical paper of the former "catastrophe theory" (Zeeman, 1977). By defining basic behaviour rules for investors, Zeeman identified an equilibrium surface of the stock return folded into a cusp catastrophe. The two control parameters of the surface were chosen to be the strength of respectively chartists and fundamentalists investing in the SM. The explicative power of this approach is in the tradition of catastrophe theory strictly limited to discontinuous changes in the evolution of prices: crashes or price upsurges. The qualitative model is still interesting for the psychology of the market. It gives a striking interpretation of common-sense rules expressed in "popular models" applicable to the SM.

R.H. Day was one of the first authors who intensively worked in non-equilibrium models inspired from chaos theory in discrete non-linear systems. Two families of investors are postulated. The first ones, called α-investors, are supposed to behave rationally. Using quantitative valuations, they are basically goal-seekers. Their quest for fundamental values ends up stabilising the market. It is why their investment profile as a function of the price has a reverse shape as it should be. On the contrary the second type, called β-investors, is in phase with the price trend. Combining the two profiles to define an iterative 1-D map of the type $p(t+1) = f[p(t)]$ one can achieve different price evolution patterns depending on the proportion of the two types of investors. Bullish or bearish patterns are generated whenever the 1-D mapping has a stable fixed point. It is a trivial result that the stability in some fixed point requires a slope in the investment profile in the plane $[p(t), p(t+1)]$ smaller than one in absolute value. Under the conditions of unstable fixed points, stable cycles or even pseudo-random behaviour, known as chaos, can be observed. This is well known in the discrete logistic equation. The conditions of chaos are fulfilled in the Day's combined profile shown in Figure 1. The proportion of β-investors is sufficient to position the profile in the right way to achieve the slope properties in the fixed points.

This model is interesting from the methodological point of view. It is today widely accepted that financial time series have a strong non-linear signature, the GARCH behaviour being an example. Unstable phases of high volatility are observed, betraying the existence of chaotic attractors. Models in Day's fashion will exhibit a complex behaviour whenever the suitable mixture of α- or β-investors has been used. Unfortunately, a discrete 1-D model will only generate chaos and nothing else. Random volatility, bullish or bearish trends, crashes etc. are beyond the potentialities. The reduction of two different behaviours to a simple iterative 1-D map does not permit to reproduce more realistic SM signatures. It is why higher dimensional models are needed. Only recently higher dimensional discrete models have been developed (see Brock and Hommes, 1997). For the sake of convenience, continuous models can be used, though it is more difficult to generate chaos (a minimum of 3-D is needed for continuous autonomous systems of ordinary differential equations). The wide recognition of a non-efficient SM comes in support for the idea of accepting Day's approach. Economists in their majority today accept that stock returns cannot be predicted. The departure from efficient market is caused by the existence of drivers outside the rational response to available information coming from fundamental analysis. As Shiller (1989) states it, smart-money investors, i.e. α-investors, who are responding to rationally expected returns are not alone to drive the market. Ordinary investors, who have many if not all features of β investors, overreact to sudden price moves or to fads. This explains why price patterns look quite random.

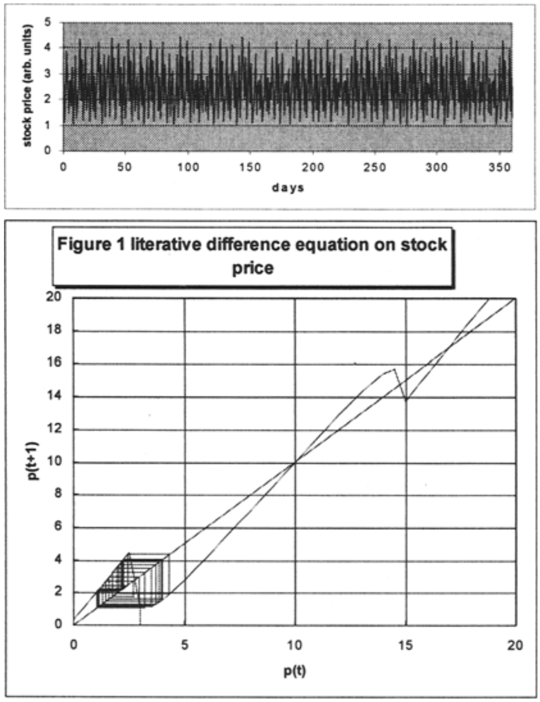

Figure 1. Iterative difference equation on stock price

Before proceeding to elaborate on the continuous simulation model proposed by the authors, it is worthwhile mentioning the recent development towards "artificial stock markets". The latter are located in the realm of artificial life with adaptive and learning capabilities (see for example Marengo and Torjman, 1996; Beltrametti et al., 1997; Arthur et al., 1997). In Arthur et al. (1997) investors are forming expectations by inductive reasoning rather that by deductive reasoning. The latter is shown by the authors to suffer from the indeterminacy characterising the celebrated Keynesian "beauty contest". The learning process initiated by the inhomogeneous set of investors consists in using genetic algorithms to continuously select and update investment rules best suited to maximise the portfolio performances. The market consists of many agents trading in a universe of stochastic dividends. It is clearly an "ab initio" approach of the type we have described above. But the aim of the present paper is not the use of classifier techniques to make stock price predictions in a turbulent environment. It is rather to distil some very basic "psychological behaviour rules" which help understand the complexity of the SM. In fact, we like to show in the following that complexity can also arise in completely frozen universes. It is why we work with a deterministic model in which economic parameter values are kept constant in each scenario. Even then unpredictability and randomness are shown to emerge. At the same time, it is hoped to gain a clear causal insight into the basic cogwheels of the SM. This is particularly useful at the time of writing: the stock prices in the Western world are apparently well above fundamentals.

SYSTEM DYNAMICS MODELLING OF THE STOCK MARKET (SM)

System Dynamics (SD) is a simulation technique using ordinary differential equations for approaching complex systems. As Richardon and Pugh III (1981) put it: "The system dynamics approach to complex problems focuses on feedback processes. It takes the philosophical position that feedback structures are responsible for the changes we experience over time". This whole approach identifies intrinsic and structural explanations of systems rather than exogenous causes. The first stage of the approach is to set up a still qualitative mental model to gain insight into causes and structures within the system. A mental model is obviously akin to the mental frame of "popular models" in behavioural finance. The influence diagram details the mental model by providing the main elements of the feedback dynamics. Typically this conceptualisation, prior to quantitative modelling, evolves through experience gathering during a typical learning-by-doing process conducted in many steps. This is not unlike the way investment strategies develop in the minds of investors. Quantitative modelling comes later. It consists in translating the influence diagram into a system of non-linear ordinary differential equations (ODE) to be solved numerically. The outputs are the time paths of the model variables: stocks, flows and auxiliaries. VENSIM 3.0 developed by Ventana Systems (1997) has been used in the present case for computing the stock price evolution in different scenarios.

For reasons of clarity the influence diagram of the stock price model has been split into three figures, 2 to 4. Each figure describes one out of three homogeneous families of investors to be detailed in the next section: α-, βS-, and βL-investors. They are extending the discrete Day's model, which had only two such families. Several important feedback loops are visible. They assist the understanding of basic behavioural rules developed in the investors' minds. Let us note some characteristics. Negative loops assist the goal-seeking approach of fundamentalists. They therefore help stabilising the stock prices. By contrast positive loops, often activated by short-term traders, are responsible for amplifying perturbation or rumours. Sometimes such a loop can act as a virtuous circle, in case it triggers a desired growth effect in prices. Sometimes it will act as a vicious circle, because the loop amplifies the market volatility or it triggers crashes. In the SM model, the two roles will be played in turn.

The universe in the SM model is very simple. There are only two assets: a risky asset at a variable homogeneous price P and a risk-free asset represented by a constant interest rate. This universe is frozen for a given simulation run. This means first that the total number of equity shares is fixed in all scenarios. Second, all economic parameters of the model are constant, including the growth rate of the fundamental value and the risk-free rate. Some fixed constraints are imposed on the available budget of the investors and their borrowing capacity.

Before we start presenting several main quantitative results, obtained with VENSIM, we now define the three families of investors and we discuss their respective behaviours.

Family of α- investors (Figure 2)

α-investors are "smart investors" as the rational goal-seekers in Day's model. Their aim is to achieve convergence towards the current goal price for the stock, minimising herewith the price discrepancy. It is why the unique feedback loop visible in figure 2 is negative. Note that in general overshooting above the equilibrium price will cause damped oscillations. The alpha goal price is the sum of two terms: fundamental value and arbitrage value:

- The fundamental value results from fundamental analysis, for example Dividend Discount Model (DDM). In our model dividends are deterministic. They could easily be made stochastic, but this would not be essential for understanding causal mechanisms in the model. They are growing with the given constant growth rate of the industry, assumed to be 7% per year in the model.
- The arbitrage value builds up as money flows to the SM away from risk-free assets. This appears as βL-investors (see below) observe a stock return above the risk-free rate (see below). In this case α-investors will adjust their long-term expectation above the fundamental value to follow the positive trend. In practice arbitrage value is incorporated into the goal price by α-investors only up to a certain point. In the model it is assumed that in a bullish market mainly driven by the arbitrage term, α-investors will cap their goal price by a maximum arbitrage value. The latter corresponds to their expectation of a reasonably high premium added to the fundamental value.

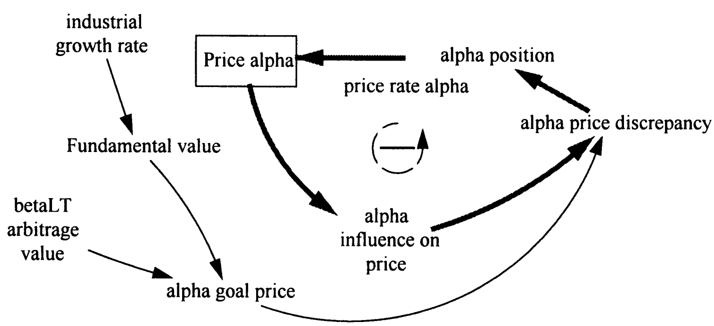

Figure 2. Negative Feedback loop of α-investors (tracking behaviour)

In the model α-investors do not experience any liquidity constraints. This is a reasonable assumption, as they stop anticipating further price growth, as the gap with the fundamental value becomes exceedingly large.

Families of β- investors

The β-investors introduced by Day are ordinary investors in Shiller's sense. They are not entirely rational with respect to the use of information coming from the market. They use different approaches for processing information, from rules of thumb to advanced technical analysis. An important aspect is the time horizon of anticipation covering a continuum between short- to long-term. The authors have extended Day's approach by considering two extremes in this continuum: βS-investors have a short-time horizon (S) while βL-investors have a long-term horizon (L).

Family of βS- investors (Figure 3)

βS-investors are myopic traders. They follow immediate price movements, ups and downs. In the model they change their stock position in proportion of the first derivative of the price. Therefore they destabilise the goal-seeking efforts of α-investors, causing permanent noise. To confirm this, a positive feedback loop is visible in the right part of figure 3. It manifests the destabilising investment approach. The driver in this loop is the first derivative of the price, initiating a vicious circle of growth or decay. A negative loop is visible in the left part of the diagram. It becomes active as the available budget drops to zero, forcing βS- investors to limit their stock position or even to liquidate part of their portfolio.

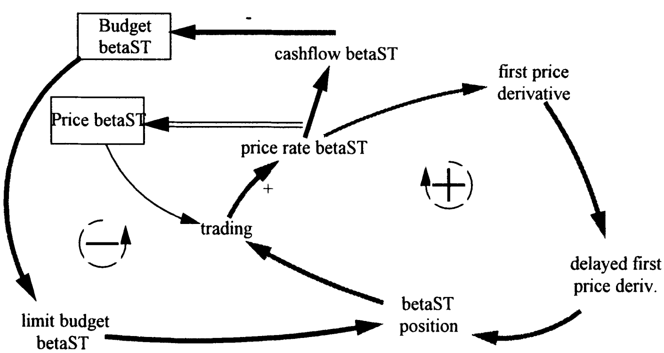

Figure 3. Feedback structure of short-term βS-investors

Family of βL- investors (Figure 4)

βL-investors have a more sophisticated approach to the market than βS-investors. Contrary to the latter, they have a long-term perspective. They permanently make arbitrage between the long-term stock return and the risk-free rate (called irate in the diagram). Their strategy is adaptive, i.e. updated at each change in the spread between the two rates. Note that the risk-free rate is fixed in each simulation run of the model. In case of a positive spread, in favour of risky asset positions, additional money is invested, curbing on the growth of the stock price. Therefore a positive feedback loop is visible in the upper part of the diagram in figure 4. It is driven by the return spread between risky and risk-free assets. In case of a positive spread, this loop drives the positive loop discussed in the α-diagram. As said above, α-investors will adjust in part their goal price to follow the growing price trend caused by βL-investors' virtuous circle. In contrast to βS- investors, βL- investors have some financial robustness. They are ready to borrow money up to a certain extend anticipating further price increases. Doing so, they reinforce the growing trend by transforming it into a vicious circle. In the model as in the real world, it is therefore important, in case of a positive spread, to impose limits on an otherwise boundless growing stock price. Negative feedback loops have to be awakening in order to keep the price explosion within bounds. The main control loop is visible in the lower part of the diagram in figure 4. βL-investors have an initial budget and some important borrowing capacity up to a given limit of permissible debt level. In any case, their willingness to reimburse their loans will grow with the relative level of their debt expressed as a percentage of their stock position. This willingness is represented in the model as a barrier hindering further investment. Below a given debt threshold, βL-investors continue strengthening their stock position. Above the threshold, they experience an incentive to liquidate at least part of their stock position for limiting their debt. How much and how fast they sell depend in the model on the parameters of the debt barrier i.e. the threshold, the importance and rate of the reimbursement. In addition to this negative loop, another control is not visible in figure 4. It finds its origin in the cap imposed by α-investors on the permitted price growth above fundamentals. Beyond this point α-investors will no longer follow the trend set by βL-investors.

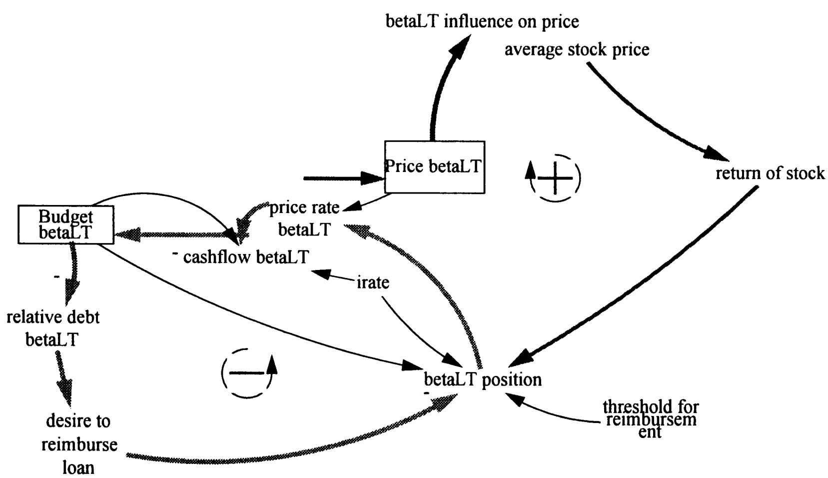

Figure 4. Feedback structure of long-term βL- investors

MAIN SIMULATION RESULTS

All following figures 5 to 9 show different conditions of the stock market over a time horizon of 1,000 days. The numerical integration brings some challenge, as the stock price evolution is all too often chaotic. Robustness in details seems, therefore, to be out of reach. A Runge-Kutta 4th order-integration scheme and a fixed time step of 0.0625 days have been adopted as giving sensible results. Deriving general patterns in several situations has deemed to the authors to be more important than achieving numerical perfection, which is anyway outside of reach in the actual chaotic conditions of the stock market.

The total stock price in Belgian francs (BEF) appears at the top of each time-diagram. The choice of absolute price values is arbitrary. This price appears in the diagrams as the sum of the three basic components attributed to the positions of the three families, i.e. of α-, βS-, and βL-investors. The different conditions in each figure correspond to varying parameters in the model, i.e. for the figure 5 to 9 the risk-free rate, the initial price, and the characteristic parameters of the debt barrier of βL-investors which appears to be the main drivers for many price patterns.

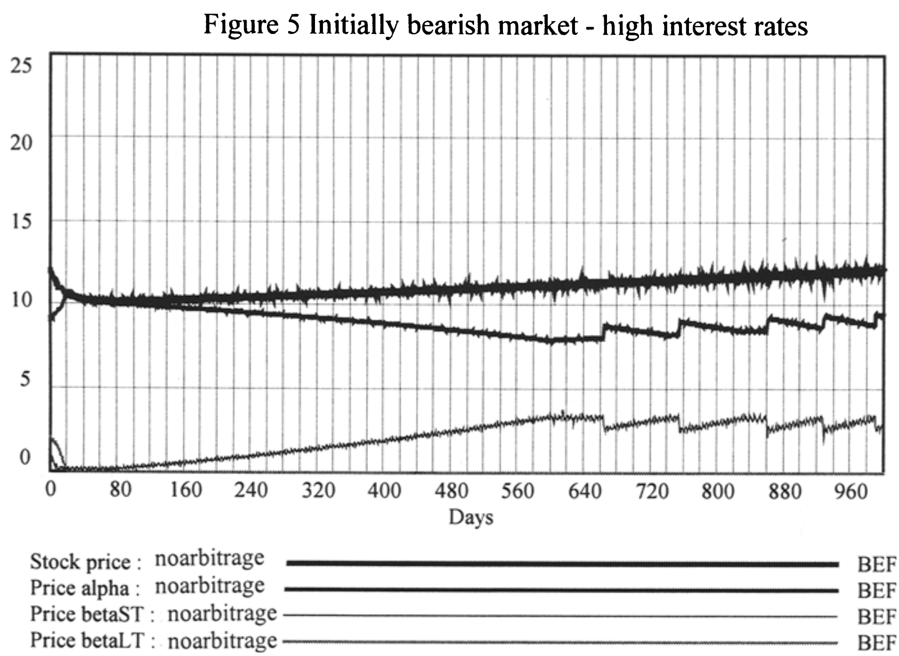

Figure 5. Test run assuming a very high risk-free rate and therefore no βL-investors

A first run shown in figure 5 is used as a test of validity. The βL-investors are absent as arbitrage investment is completely switched off. This is achieved by keeping the risk-free rate very high. In this way, the stock market is very unattractive for long-term investors. The game is then entirely fought between the goal-seeking α-investors and the destabilising β noise-traders. The result is a regular wavy price evolution with a trend equal to the long-term industrial growth assumed to be 7% per annum (p.a.). This evolution is roughly comparable to the situation at European SM for some years after the 1987 crash. Long-term interest rates were high; growth rates were rather low, which made stock investment less attractive than conservative bond portfolios or even cash-accounts.

For the following simulation runs, the authors have looked in historical time series for rough points of comparison. The reference is the S&P 500 composite price index. Further finer calibration work could be done on the parameters of the model. This has not been attempted so far.

Figure 6. Initially bearish market - high + stiff barrier

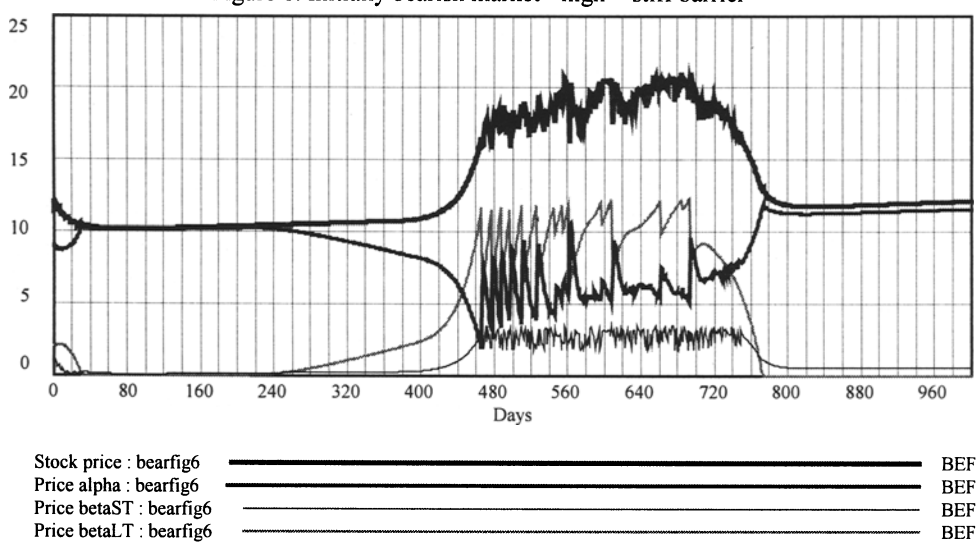

Figure 6. Low risk-free interest rates attract βL-investors to the SM. The latter accumulate a high debt level they are forced to reimburse rapidly and intensively. The market starts being bearish.

The second run in figure 6 assumes a low interest rate, by contrast with the previous run. The initial condition is a bearish market. The return spread is negative, i.e. in favour of risk-free investment. For quite a long time, the market is in near-equilibrium at the goal price set by α-investors, the growth rate being equal to the industrial growth rate (7% p.a.). However, the steady industrial growth brings about a fresh-born wave of βL-investors. As a result the goal price also shifts up. βL-investors soon find their limits. The debt barrier is reached. In this run, the threshold for maximum acceptable debt is set at 40% of the total current value of the stock position, a quite important value. In addition, it is assumed that the "debt barrier" is rather stiff and also high. This means that βL-investors have to rapidly liquidate the largest part of their stock portfolio in order to bring down their loan debt to a much lower level. This reimbursement constraint has the same effect as a reflecting barrier on the price. The price bounces back creating chaotic ups and downs of the price in search for a new equilibrium value. The larger the debt threshold, the larger will be the bouncing back of the price. The stiffness of the barrier is another element amplifying the plunge. βS-investors amplify the appeared volatility. The price volatility becomes so large that at some point the long-term return drops below the risk-free rate. βL-investors disappear from the scene after a crash of limited amplitude. The market moves to a new equilibrium following the natural trend of fundamental values.

A roughly comparable type of evolution in the S&P 500 composite price index has been observed in the period 1974 to 1977. As a consequence of the first oil crisis and the resulting inflation, the nominal interest rates were particularly high, the real rates being negative. The spread between governmental and corporate bonds was widening. As a consequence, the supply rate of money was largely declining, imposing new strains on the borrowing barrier.

In figure 7 the same assumptions are used for simulating an initially bullish market. The same assumptions on the debt barrier are used as in the previous case. A burst in price, accompanied by high volatility, sets up immediately. It manifests the same type of behaviour as GARCH in the stochastic theory. The bouncing back of the price against the debt barrier induces more volatility than in the previous case. This turbulent behaviour cannot maintain itself very long. The market moves to its fundamental equilibrium as in figure 6. After a while, a new price upsurge is observed with still more volatility than previously in the growing phase. It lasts for quite some time, exhibiting swings of considerable amplitude. As before, a crash brings back the price to its natural equilibrium. When pursuing the computation regular replica with similar shapes are periodically observed.

Fig. 7 Initially bullish market - high+stiff debt barrier

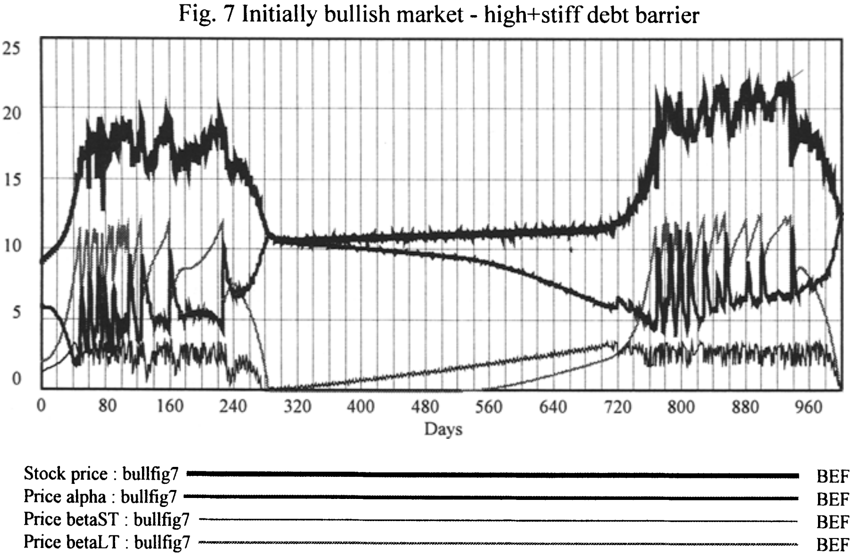

Figure 7. Low risk-free interest rates attract βL-investors to the SM. The latter are accumulating a high debt level. They are forced to reimburse rapidly and intensively their loans. The market starts being bullish.

Each replica in its growing phase has a similar shape to the evolution of the S&P 500 composite price index in the period starting at mid-1978 till end 1981. Following the previous period, the inflation rate had dropped from 12% to 4% p.a. The real interest rate was again positive and the prospects were therefore better. At start, the market was bullish. However the second oil crisis caused a new burst in inflation bringing down the real rates to negative values. The spread between governmental and corporate bonds widened again, slowing down the supply rate of money. A borrowing constraint appeared as in the previous period.

These two last simulations are not entirely convincing as they show an excessive volatility and too rapid crash conditions after the growth period. Adjustments in the basic parameters of the debt barrier of βL-investors, i.e. threshold, height and stiffness, are giving additional degrees of freedom as now illustrated. As an example, reducing the threshold reduces the volatility, because there is less reflection on the debt barrier.

The next run, illustrated in figure 8, is obtained from the previous one by lowering the debt threshold to 10% of the total current value of the stock position. As a result, the debt barrier is rather low, compared to the previous case. In addition the barrier is made softer by decreasing the slope in the willingness function triggering the reimbursement of loans. As a result less volatility is observed in the high price phase. The latter becomes also more sustainable and extends for a longer period than in the two previous phases.

Figure 8 Initially bullish market - low debt barrier

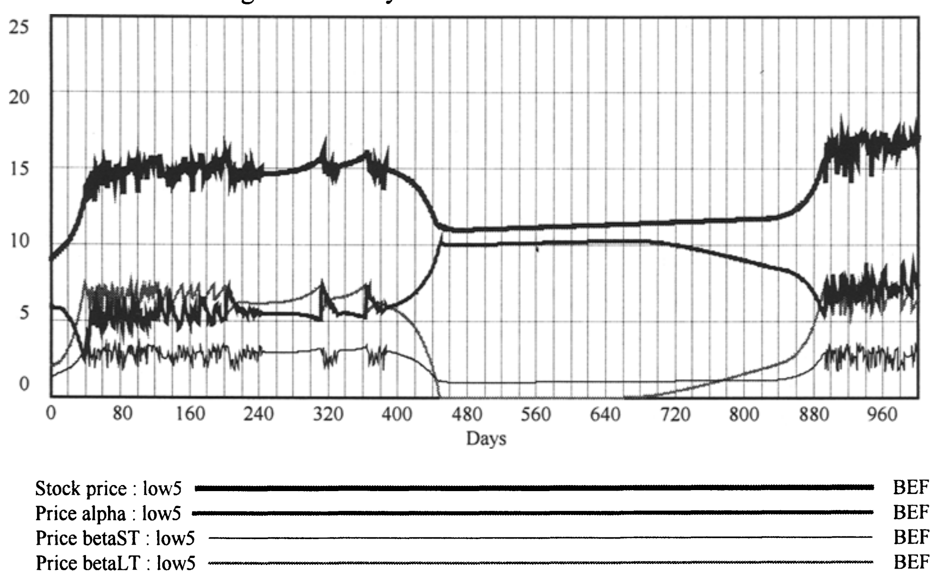

Figure 8. Low risk-free interest rates attract βL-investors. In contrast to the previous case it is easier to borrow money, and the reimbursement rate is moderate. The market starts being bullish

A comparable evolution of the S&P 500 composite price index was observed in the period mid-1991 to end 1993. The US economy was just leaving a period of recession. The growth rate of the supply of money was close to vanishing. By contrast, the spread between governmental and corporate bonds was small. This did indicate that the obstacles to money borrowing were not structural and did not really create a barrier for money supply.

The next and last run, illustrated in figure 9, still goes a step further in achieving a sustainable price increase. The debt threshold is lowered to zero value, giving a zero height to the barrier. At the same time the barrier is kept soft, i.e. the reimbursement rate is slow. As result, a much lower volatility is achieved as the evolution sticks at the edge of the zero-barrier in a sustainable way. The stock price now incorporates the arbitrage value created by the positive spread in returns. The trend is given by the growth rate of fundamentals. The only permanent instability is caused by small βS-movements. The price is maintained at its upper level and it continues to grow at the industrial growth rate of 7% p.a.

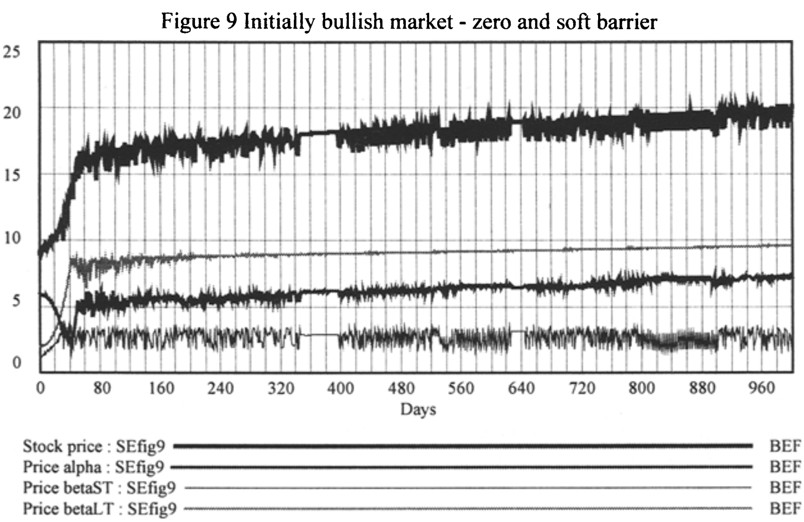

Figure 9. Low risk-free interest rates attract βL-investors to the SM. It is easy to borrow money and the reimbursement rate is slow. The market starts being bullish

This situation of long-term sustainable growth with limited volatility in the price is also roughly comparable to the situation in the Western SM described by the S&P 500 composite price index for the period mid-1995 to mid-1998, before the limited crash in the second half of 1998. During that period, a continuous growth of the stock price has been recorded. As assumed in the present simulation, a sustainable flow of liquidity has been invested in the SM for arbitrage reasons by "ordinary investors". As an example, U.S. pensioners facing low risk-free interest rates have been moving to the SM. In Europe the "European Monetary Unit" convergence criteria leading to a common currency unit

(EURO) has imposed low interest rates, limited public deficits, and reduced public debts. High-rate governmental bonds have been called back, while saving accounts became unattractive. A lot of money has been flowing to the stock market heating up the prices.

CONCLUSIONS AND FURTHER WORK

The main objective of the proposed "ab initio" approach is to provide a causal model in behavioural finance. In the opinion of the authors the applied SD-methodology departs from the usual interpretations using discrete chaos theory. The latter are far from transparent (see for recent surveys of the literature Heij and al. (Eds.) 1997; Creedy et al. (Eds.) 1994). By contrast the present analysis stresses causal interpretations. It therefore provides straightforward explanations by identifying simple feedback structures created by few investors' attitudes. Though the proposed model is completely deterministic, it is able to portray several complex patterns of real stock markets. Refined SD models of this type are thinkable to capture and to anticipate SM dynamics. Adding stochastic aspects to the models would be a trivial think to do. In the author's opinion it is not likely to provide deeper insights into fast market dynamics.

The main by-product of our modelling is the confirmation of the existence of "animal spirits" next to rational behaviours supposed in the efficient market hypothesis. At a time of volatile markets, the analysis of the behavioural price drivers seems to be of some interest. In particular the increased volatility of the SM due to day trading on the Internet suggests further SD-modelling in the same direction.

As a main result of the present approach it appears that the behaviour of "ordinary" βL-investors could be an important explicative factor of the volatile price pattern on the SM for the following two main reasons:

- The psychological willingness to contract loans to chase the growing price trends appears as a key factor (the SM as a gambling casino as it is sometimes popularly stated). Perhaps the origin of the ominous "financial bubble" must be found in this risk prone behaviour. There is a vicious circle here as confidence grows with the stock price. So do the threshold and the stiffness of the "debt barrier" causing hard landing. Note also that the barrier is dynamic rather than static. As the recent Asian crisis has shown, fresh money supply might continue to sustain the price growth, moving aside the barrier.
- At the same time, the fear of the bubble by anticipation is growing, another psychological and perhaps irrational attitude. In our model, α-investors stop incorporating additional arbitrage growth into their goal prices. While this helps bringing down the price closer to fundamentals, there is an additional risk here. Massive sales of α-stock to β-investors further drive the debt spiral and enhance the ruin-provoking mechanisms.

Of course, one drawback in the model is the clear-cut definition of the group of investors. In the real world, neither institutional investors nor traders are definitely respectively α- or β-investors. Each investor has features of each elementary type, acting alternately as a fundamentalist, technical analyst or noise trader (see Brock and Hommes, 1997). The main difficulty is that the proportion of each type depends on the market conditions. There are additional feedback loops to be drawn in the model inducing these changes in attitudes. The same applies to the parameters of the frozen universe. In the real world, influence links exist between industry growth and stock price. More elaborate model could focus on these weak spots in modelling. Also, in the real world, the availability of liquidity for investment is not fixed. For example, in the first part of 1998, additional monies were flowing to Western markets away from the Asian markets as the latter were caught in turmoil. Herewith the sustainability of the growth is maintained for a longer period, or a later "hard landing" is somewhat softened.

The relative strength of each group of investors must be handled as an additional parameter to be calibrated under the given economic conditions. This is part of the improvements and tuning of the model. Other possibilities not yet explored are the definition of softer debt barriers and the role of α-investors in caring for a damping in the anticipated price crash.

In conclusion we feel that a refined SD modelling of basic investors' attitudes is a rewarding task for future development. Though it is probably illusory to expect some forecasting from such rough models, the causal interpretation has a great value per se. In particular it can deliver a warning signal to ordinary investors driven by unbounded Keynesian "animal spirits". Unlimited growth is impossible on earth and, by consequence, on the stock market.

REFERENCES

Arthur, W.B., Holland, J.H., LeBaron, B., Palmer, R. and P. Tayler (1997) Asset Pricing Under Exogeneous Expectations in an Artificial Stock Market in Arthur, W.B., Durlauf, S.N., and Lane D.A. (Eds.) *The Economy as an Evolving Complex System II.* Proceedings Vol. XXVII, Santa Fe Institute Studies in the Sciences of Complexity. Reading MA: Addison Wesley.

Axelrod, R. (1997) *The Complexity of Cooperation, Agent-Based Models of Competition and Cooperation*, Princeton: University Press.

Beltrametti, L., Fiorentini, R., Marengo, L. and R. Tamborini (1997) A learning-to-forecast experiment on the foreign exchange market with a classifier system. *Journal of Economic Dynamics and Control* (21): 1543-1575.

Brock, W.A., and C.H. Hommes (1997) Models of Complexity in Economics and Finance" in C. Heij et al. (Eds.) op. cit.: 3-44. Chichester, England: John Wiley & Sons, Ltd.

Creedy, J., and V.L. Martin (Eds., 1994) *Chaos and Non-Linear Models in Economics- Theory and Applications.* Brookfield, Vermont: Edward Elgar Publ. Co.

Day, R.H. (1994) *Complex Economic Dynamics, Vol. I An Introduction to Dynamical Systems and Market Mechanisms.* Cambridge, MA: MIT Press,

Heij, Ch., Schumacher, H., Hanzon B., and K. Praagman (Eds., 1997) *System Dynamics in Economic and Financial Models.* Chichester, England:John Wiley & Sons, Ltd.

Marengo, L. and H. Tordjman (1996) Speculation, Heterogeneity and Learning: A Simulation Model of Exchange Rates Dynamics. *Kyklos*, 49(3): 407-438.

Richardson, G.P., and A.L. Pugh III (1981) *Introduction to System Dynamics Modeling.* Productivity Press, Portland, Oregon.

Shiller, R.J. (1989) *Market volatility.* Cambridge, MA & London, England: The MIT Press.

Ventana Systems, Inc. (1997) *VENSIM 3.0 software*, Harvard, MA.

Zeeman, E.C. (1977) *Catastrophe Theory – Selected Papers 1972-1977.* Reading, MA: Addison-Wesley Publ. Co.