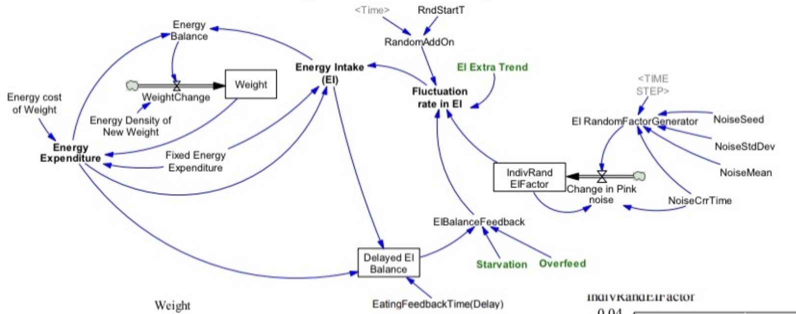
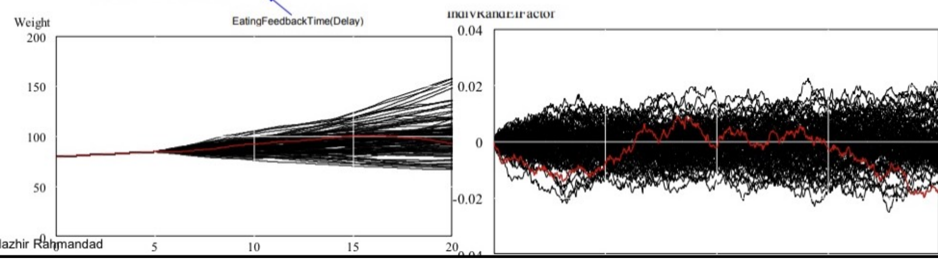
Process Noise

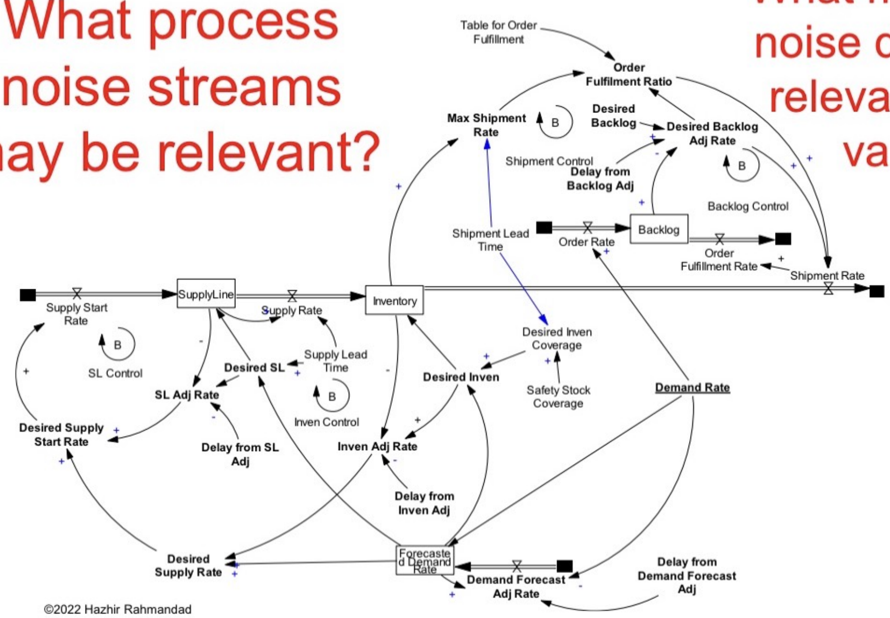
- Can significantly change the trajectory of identical model outcomes, only varied by noise





If, these were from an empirically grounded model, how useful is our theory for explaining weight dynamics?

What process noise streams may be relevant?



What measurement noise distribution is relevant for target variables?

Implement one process noise into the model

- Use a first-order autocorrelated noise formulation (BD appendix B)
  - Could you replicate Forrester's insight from Appendix K?
  - How could you tell if a model (and your parameter estimation) is potentially sensitive to process noise?
  - Any guess which models may be more influenced by process noise:
    - SEIR
    - Economic Longwave
    - Project management
    - Market Growth
    - Commodity Cycles
    - Service Quality
    - Urban Dynamics
    - World 3

John Sterman

my point about the name for pink noise is simple:

Jay Forrester knew back in the 1940s and 50s that white noise doesn't exist in reality because all real systems have some inertia that limits the power in the higher frequencies of random disturbances, whether those are in a physical servomechanism, a supply chain, the economy, or one's beliefs. In Industrial Dynamics he addressed this issue by sampling from a white noise distribution at finite intervals (rather than at every time step, "DT"). FYI, sampling at every time step in a system integrated numerically by Euler's method means that the power spectrum of the noise added to any system (process or measurement noise) will be sensitive to the time step because the highest frequency in a pseudo-white noise stream with values chosen every DT is 1/2DT). Even worse, using higher-order integration schemes such as Runge-Kutta have large impacts on the power spectrum of noise in models. The time step and numerical simulation algorithm used in any model are artifacts of the simulation method, and not a feature of reality. The results of any model, including the model's response to process and measurement noise, should not depend on the time step or simulation methods.

By sampling white noise at a finite interval, say, ST (for Sampling Time), ST>DT, Jay's early models solved the problem of having the system's response to noise be sensitive to the time step in the Euler integration, but it was rather arbitrary (the power spectrum has a sharp and unrealistic cutoff at the highest frequency allowed, 1/2ST, which is independent of DT. So in the 60s folks in system dynamics came up with the formulations for first-order autocorrelated noise. They called it pink noise to indicate that it was not white noise. Pink was used to suggest that the power spectrum was more "red" (lower frequency) than white noise, which includes "blue" and other higher frequencies in equal measure.

The problem with calling first-order autocorrelated noise (white noise put through a low-pass filter given by first-order exponential smoothing) is that, independently, the term pink noise has become associated in the world of science with 1/f noise, whereas the first-order autocorrelated noise has a power spectrum that's 1/f^2. This makes a big difference in the frequency response of systems.

We in system dynamics therefore need to come up with a different name for the first-order autocorrelated noise and stop calling it "pink" — I suggested "brown" off the cuff when we chatted in my office about this but I'm not sure it's the best name. But we have to stop calling the 1/f^2 noise "pink" because doing so will cause a great deal of confusion.

Hazhir's def of Process noise

:= a random process (that is, there is a new draw every time period) that changes model's dynamics (typically is added to flow variables in an ODE, making it stochastic ODE) but for which we don't have direct measurements.

Appendix B

Noise

Most variables, such as industrial production (Figure 17-l), often appear to be somewhat “noisy.” We see part of the behavior as systematic—for example, the growth trend and cyclical movements in industrial production—and part as noise. What we judge to be a systematic pattern of behavior and what we judge to be meaningless random variation depends on our perspective and purpose. If the purpose of your model were to understand the determinants of long-run economic growth, movements in output other than the growth trend, including the business cycle, might be considered noise and excluded from your analysis. If your concern were the business cycle, your model would explain these cyclical movements, but you might treat the month-to-month movements around the business cycle as noise. An even more detailed model, however, might explain these rapid variations in output as part of the feedback structure. One person’s noise is another’s signal, depending on the questions in which each person is interested.

The rate equations in system dynamics models capture the decision-making processes of the agents or the physical and biological laws that cause change in system states. Because all models are approximations, the model decision rules do not capture all the sources of change in the actual flows. As explained in section 4.3.2, noise is the label we apply to that part of the actual decision stream our model cannot explain. Noise measures our ignorance.

For example, production starts in the supply chain model developed in chapter 19 depend on the firm’s labor force, the workweek, and labor productivity:

$$\text{Production Start Rate} = \text{Labor} * \text{Workweek} * \text{Labor Productivity} \quad \text{(B-1)}$$

The workweek and productivity might themselves be endogenous variables, dependent on factors such as schedule pressure, worker experience, and equipment quality. The workweek and productivity represent averages: Some workers are more productive than others; some put in more hours than others.

The actual start rate will rarely equal the average value. One worker’s baby kept him up all night, so he is less productive today. Another discovers a way to speed up her work, boosting productivity. An unexpected machine problem slashes the productivity of a third. For some purposes, including these variations adds little to the dynamics and only makes it more difficult to understand model behavior. In these cases, the deterministic dynamics are sufficient. Often, however, unpredictable variations around the average values play a critical role in the dynamics and must be modeled. As a general modeling strategy you should first understand the dynamics of your model without noise—even when noise is important. The response of the model to shocks can be assessed through idealized test inputs such as the step function, pulse, ramp, and sine wave. Once you understand how and why the system responds as it does you can consider how more realistic inputs such as noise affect the dynamics.

Variations around the average value of a variable are usually modeled as some type of random process. Noise represents those variables and states of the system we either cannot capture in the model or choose to omit. There are reasons for the variations in productivity not captured by the model, but we don't have the information needed to capture them endogenously. We don't have a way to model when a worker will lose sleep because the baby cried all night.

If the unmodeled variations in productivity were important to the model purpose the formulation for productivity could be modified to include random variations around the average, which itself could be an endogenous or exogenous variable:

$$\text{Productivity} = \text{Average Productivity} * \text{Random Effects on Productivity} \qquad \text{(B-2)}$$

How should the random effects be formulated? You graph the productivity data supplied by your client (Figure B-1) and find productivity varies randomly around a constant level. Next, you plot the distribution of productivity. The values closely approximate a normal distribution with a mean of about 0.25 widgets/person-hour and a standard deviation of 0.0123 widgets/person-hour, about 5% of the mean.

Given the data in Figure B-1 you then specify the random variations in productivity as

$$\begin{aligned}&\text{Random Effects on Productivity}\\&\quad = \text{NORMAL}(1, \text{Standard Deviation in Productivity})\end{aligned} \qquad \text{(B-3)}$$

The NORMAL(Mean, Standard Deviation) function generates, every time step, a value drawn randomly from a normal distribution with the specified mean and standard deviation.[1] You set the mean of the distribution to 1 and the standard deviation to 0.05. Since the random input has a multiplicative effect, productivity will be normally distributed with a mean of 0.25 and standard deviation of 0.0125, as observed in the data.[2]

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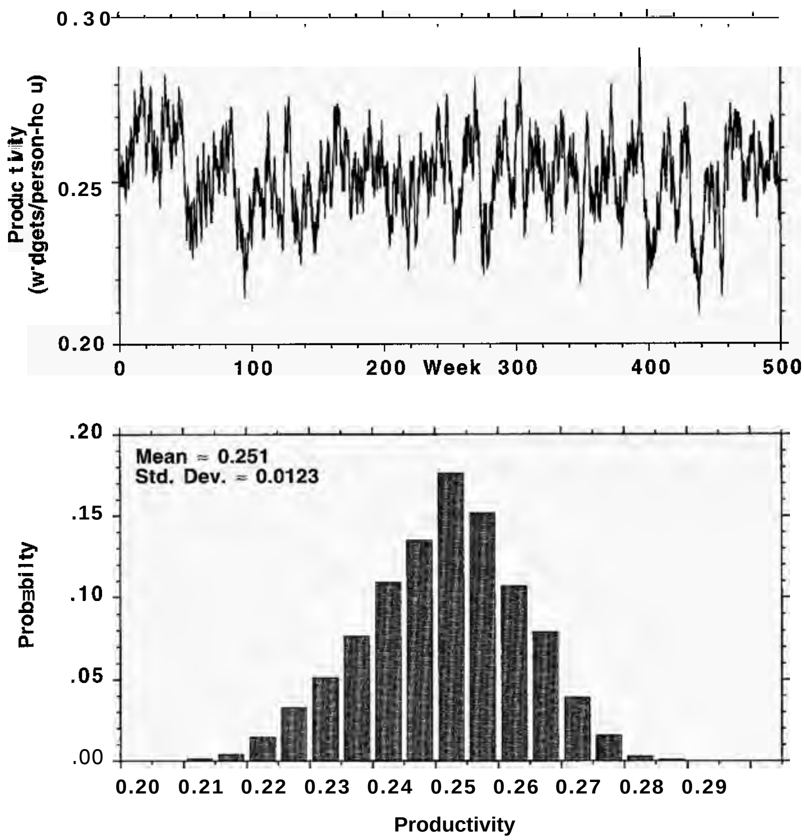
[1]All simulation software packages include built-in functions to generate random variables including the normal and uniform distributions. Because the pseudorandom numbers generated by any software package are not truly random, however, caution must be exercised to ensure that they conform to the required statistical properties. See Hellekalek (1998).

[2]The normal distribution generates values over $[-\infty, \infty]$ so occasionally productivity as specified in equation (B-3) would become negative. A robust formulation requires Random Effects on Productivity ≥ 0. This can be accomplished by truncating the distribution with a MAX function. Alternatively, the random effects could be specified as a lognormal distribution.

FIGURE B-1
Hypothetical data for labor productivity

Top: Time series. *Bottom:* Histogram showing the distribution of productivity.



To your surprise simulated productivity (Figure B-2) looks little like the actual productivity data in Figure B-1. Simulated productivity does conform to the normal distribution with the proper mean and standard deviation but it changes too fast, jumping far too quickly from value to value.

The problem is subtle. Random number generators such as the NORMAL function yield a new value every time step and successive values are independent. History doesn't matter—the values that have come before have no effect on the next value drawn from the distribution, just as the last result on a roulette wheel has no bearing on the next. The values generated by the NORMAL() function are said to be IID—independently and identically distributed. Independence means the next value of the random effect can differ by any amount from the last value. Productivity might be very high right now, but independence means one instant later it could be very low. The time step for the simulation is 0.125 weeks. A new value for productivity, completely independent of the last, is chosen eight times **a** week. It is this frequent sampling from an independent process that explains why simulated productivity in Figure B-2 jumps around far too fast.

Engineers call the random variations generated by IID processes "white noise." Imagine you have just arrived at a lively party. Each guest is speaking in a perfectly intelligible manner (at least early in the evening), but when all these sounds reach your ear at once, the result is an indecipherable cacophony. Analogously, you can think of noise as the sum of tones of all frequencies, from the

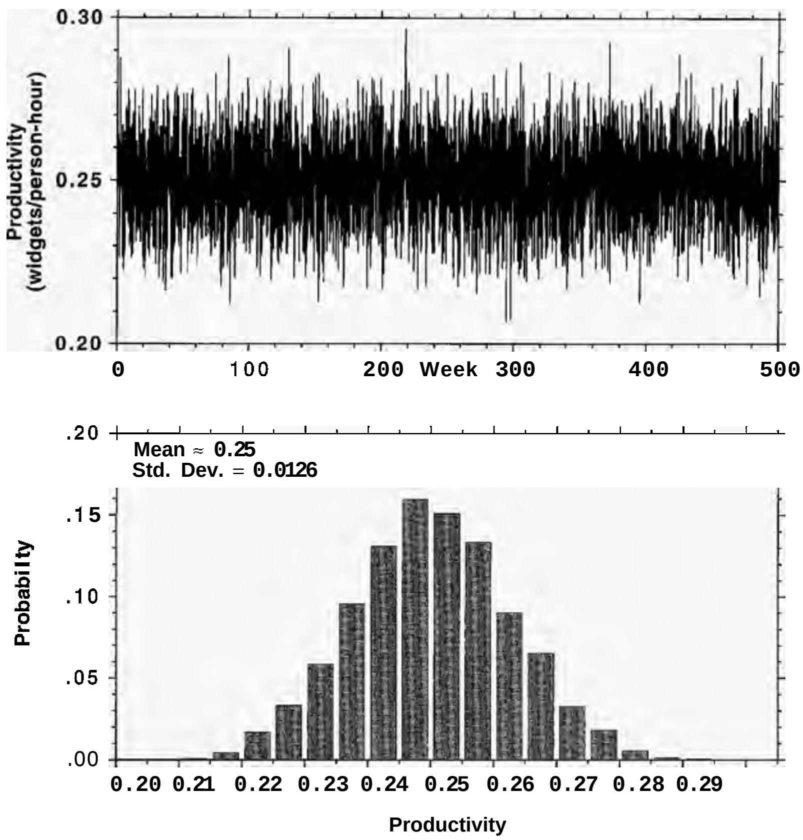


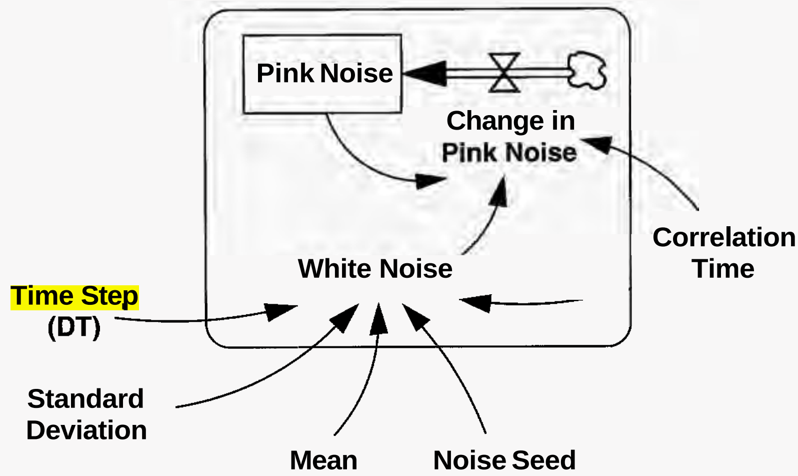
FIGURE B-2 Simulated labor productivity

Top: Time series. *Bottom:* Histogram showing the probability distribution of simulated productivity, generated by equation (B-3).

deepest bass note to the highest high C. Because white noise contains all frequencies in equal measure its current value contains no information about future values, even in the next instant.

While convenient statistically, the white noise assumption of independence does not hold in the real world. Real systems have inertia. Productivity, customer demand, the weather, and all other real quantities cannot change infinitely fast. Suppose a machine breaks down, lowering productivity. Productivity remains depressed at least until the machine can be repaired. The effects of the random shock persist for some period of time. Similar persistence applies, to varying degrees, to all causes of variations in productivity and to all variations in any quantity. The temperature in 1 hour obviously can't be too different from the temperature now. Similarly, the temperature tomorrow, the next day, and a week from now all depend, partially, on the temperature today. Temperature changes only slowly because it depends on the quantity of heat in the air, a stock that, that like all stocks, changes only gradually as it accumulates its inflows and outflows. Similarly, the stock and flow structure of all real processes gives them a certain amount of inertia. consequently, real noise processes don't contain all frequencies in equal measure. The strength of the high frequencies diminishes above some point, just as a loudspeaker cannot generate sound at frequencies higher than, say, 20,000 cycles/second (Hertz) because the inertia of the mechanical elements in the speaker limits how fast it can vibrate.

FIGURE B-3
Pink noise: structure



It is therefore necessary to model noise as a process with inertia, or memory—as a process in which the next value is not independent of the last but depends in some fashion on history. Realistic noise processes with persistence are termed "pink noise." Compared to white noise, which contains all frequencies in equal measure, pink noise filters out the high frequencies at the blue end of the spectrum, leaving more of the reddish frequencies. The challenge is to formulate a simple model of randomness that allows the modeler to specify the degree of persistence, or equivalently, the *power spectrum* of the noise (roughly, the amplitude or strength of each frequency).[3] The statistical properties of the resulting noise should also be insensitive to the choice of the time step (within broad limits).

The inertia in real variables implies the existence of at least one stock in any noise generating process. **A** simple formulation for pink noise begins with white noise, then smooths it using some type of information delay. The information delay represents the sources of inertia in the noise generating process. The simplest formulation is first-order exponential smoothing. As described in chapter 11, first-order exponential smoothing means the current value is the exponentially weighted sum of all past values of the input. Figure B-3 shows the structure for first-order pink noise, also known as first-order autocorrelated noise.

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[3]The power of any signal is the energy it contains per time interval and is the integral of the squared signal. The power spectrum is the distribution of the total power by frequency. Pure white noise contains constant power in all frequency ranges. The power contained in the range from 1 Hz to 2 Hz is the same as that in the range from 1001 to 1002 Hz. Since white noise spans all frequencies from 0 to ∞, it contains infinite power, an impossibility. All real processes have finite power, meaning the power per frequency interval must eventually fall to zero **as** the frequency rises. A ***frequency*** *domain* analysis decomposes a time series into sine waves of different frequencies and phases. Frequency domain tools such as Fourier analysis measure the power in each frequency in any time series; other methods such as autocorrelation and cross-correlation functions, ARIMA models, and vector autoregressive models help modelers understand the way current values of a time series depend on its own past values and possibly the past values of other variables. Warner (1998) provides an elementary treatment of spectral analysis; Granger and Newbold (1977) provide a more mathematical approach covering frequency and time domain methods. See also Franses (1998).

Pink noise is formed by first-order exponential smoothing of a white noise input. The delay time is the correlation time constant:

Pink Noise = INTEGRAL(Change in Pink Noise, Mean) (B-4)

Change in Pink Noise = (White Noise − Pink Noise)/Correlation Time (B-5)

The white noise input is constructed from a uniform distribution on the interval [−0.5, 0.5], sampled every time step of length dt.[4] The user specifies the mean and standard deviation of the pink noise process, which determines the mean and standard deviation of the white noise input:

White Noise = Mean + Standard Deviation * [(24 * Correlation Time/dt)$^{0.5}$] * UNIFORM(−0.5, 0.5, Noise Seed) (B-6)

where the UNIFORM(Min, Max, Seed) function generates a sequence of values drawn from a uniform distribution on the interval [Min, Max]. The user also specifies the noise seed. By fixing the noise seed, every simulation will generate exactly the same sequence of random values, facilitating comparison of simulations with different policies and parameters. Changing the noise seed changes the realizations of the random process but not its statistical properties. The scaling factor (24 * Correlation Time/dt)$^{0.5}$ adjusts the amplitude of the white noise so that the standard deviation of the pink noise output equals the specified value.[5]

In continuous time, noise can include all frequencies. Since simulations proceed by discrete time steps, the highest frequency in any model variable is twice the time step dt (to complete one cycle of ***up-down-up*** requires a minimum of two time steps). Exponential smoothing attenuates high frequencies. It is known as a low pass filter because it lets low frequencies pass essentially full strength but progressively attenuates cycle periods near or shorter than the time constant. The

? longer the correlation time, the greater the attenuation at any frequency. Thus, the longer the correlation time constant, the larger the amplitude of the white noise must be. The less frequently random values are sampled (the larger dt), the smaller

[4]This formulation for pink noise assumes the model is solved by Euler integration (appendix A). Higher-order integration methods such **as** Runge-Kutta change the power spectrum and other properties of noise and should generally be avoided in models with random disturbances.

[5]The pink noise formulation given here smooths **a** *uniformly* distributed white noise stream. Nevertheless, the distribution of the resulting pink noise is asymptotically *normal.* An alternate formulation smooths a normally distributed white noise signal to yield pink noise that is always Gaussian:

$$\text{White Noise} = M + \left[S^2 * \frac{(2 - (dt/T_c))}{(dt/T_c)}\right]^{0.5} * \text{NORMAL}(0,\ 1, \text{Noise Seed}) \qquad \text{(B-6')}$$

is approximated as {(2)/(dt/T)}^.5

where M is the mean, S is the standard deviation, T_c is the correlation time constant, and NORMAL(0, 1, Noise Seed) generates **a** Gaussian distribution with mean 0 and variance 1.

The structure of this alternate pink noise formulation is the same as the one above; the only difference is the distribution of the white noise and the resulting scaling factor to ensure the pink noise has the specified standard deviation. (You can easily derive the scaling factors for each formulation by expanding the Euler integration for the pink noise variable **as** the weighted sum of the white noise at time t, t − dt, t − 2dt, etc.). If you had strong data showing the unconditional distribution of noise in **a** process to be Gaussian, the formulation in equation (B-6') would be preferable. In practice, it is unlikely the data would allow you to discriminate between the two forms.

the amplitude of the white noise must be, because less frequent sampling means the power in the white noise is concentrated in lower frequencies that are not attenuated by the smoothing process.

The correlation time constant captures the degree of inertia in the noise process. In first-order pink noise the correlation between current and past values decays exponentially with a time constant equal to the correlation time.[6]

The data shown in Figure B-1 were generated by the pink noise structure with a standard deviation of 5%, correlation time constant of 4 weeks, and time step of 0.125 weeks.

Comparing Figures B-1 and B-2 it is clear that the (unconditional) distributions of the two noise inputs are about the same. The pink noise structure generates a distribution that is essentially normal, with the specified mean and standard deviation (the reported mean and standard deviation differ slightly from the specified values because the sample of data is finite). However, successive values of pink noise are correlated with past values, so the pink noise process does not change as rapidly as the independent, uncorrelated values generated by the normal distribution in equation (B-3).

Does it matter? Yes. Noise consists of signals of various frequencies and amplitudes. Dynamic systems act as filters, selectively attenuating some frequencies while amplifying others. Many systems resonate strongly at certain frequencies. If the noise contains significant power near the resonant frequency, the system will fluctuate, sometimes violently. The Tacoma Narrows bridge, in the state of Washington, vividly demonstrated the power of a system to amplify random noise. Built in 1940, people immediately noticed its tendency to swing in even light winds. On November 7, the bridge, driven by modest winds of about 40 miles per hour, began to oscillate through huge swings. A few hours later it collapsed. Unknown to the designers, the suspension span had a strong resonance near the frequencies in the vortices created as the wind passed around it. A new span was built on the same towers, but this time stiffened so its resonance peak was smaller and far from the frequencies created by the wind. It still stands today. Similar resonance phenomena arise in social and economic systems, as illustrated by the supply chain models in chapters 17–20: Small random variations in customer orders induce large fluctuations in production near the natural frequency of each system.

To illustrate, Figure B-4 and Table B-1 compare simulations of the inventory–workforce model developed in chapter 19 with the two noise inputs in Figures B-1 and B-2.[7] In both simulations, customer orders are constant and the random

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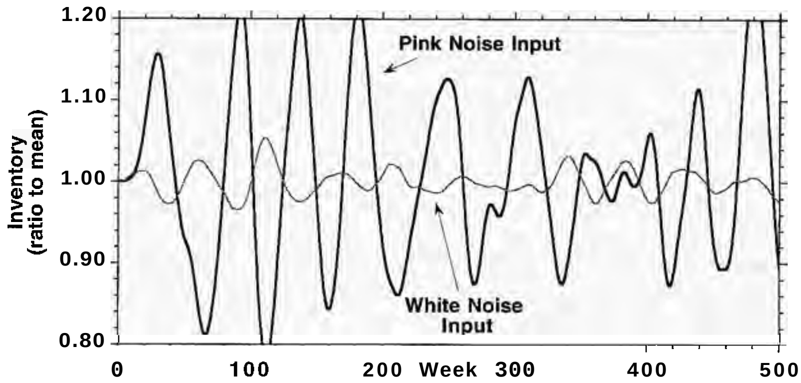
[6] Occasionally data analysis will show that the autocorrelation function is not well approximated by exponential decay. In these cases, higher-order pink noise formulations may be used, formed by cascading several first-order pink noise delays in series (see chapter 11). The standard deviations of each stage must be scaled appropriately so the output has the proper standard deviation. In practice, it is rarely necessary to use higher-order noise processes. Complex autocorrelations and cross-correlations among the exogenous random effects in your model suggest there is significant feedback and stock and flow structure you should probably be modeling explicitly and endogenously.

[7] The model used in this appendix is identical to the inventory–workforce model developed in chapter 19 except that expected productivity, used in the determination of desired labor, is modeled as a first-order information delay of actual productivity with a time constant of 13 weeks.

FIGURE B-4 Response of inventory–workforce model to white and pink noise

The system resonates near its natural frequency when driven by noise. The noise input in both cases has a standard deviation of 5%. Most of the power in white noise is contained in the high frequencies, which are attenuated by the system. Pink noise contains more power near the system's natural frequency, causing large cycles.



variations in productivity are the only perturbations disturbing the system. Both noise streams have the same standard deviation and distribution. The only difference is the degree of autocorrelation. The pink noise signal has a correlation time constant of 4 weeks, while the white noise signal is independent (no autocorrelation).

The response of the production system to the two noise streams is very different. Very little happens when the system is perturbed by white noise. There are small variations in inventories, the workforce, and other variables, but the system attenuates nearly all of the noise because most of the power is concentrated in the high frequencies. The independent variations in productivity cause large changes in production starts from dt to dt. Since dt is 1/8 week, this means production starts vary widely from day to day. Low output today is more than likely to be offset by higher than normal values within a few days. Inventory absorbs most of the short-term variations and there is little need for the firm to alter its workforce.

Pink noise, however, means that when output is low today it is likely to remain low for a few weeks. During this time, inventories can fall significantly, forcing the firm to hire workers to expand output and triggering the system's latent oscillatory response to shocks. The variations in productivity are too slow to be absorbed by inventory or filtered out by the hiring delays. The noise contains significant power near the system's natural frequency of about 1 year. Consequently, the system resonates strongly—the standard deviation of production starts is 7.6% of the mean, even though the standard deviation of the noise input is only 5%.

Table B-1 shows the standard deviation as a fraction of the mean value for key variables in both the white noise case and the pink noise case. Under white noise, the system strongly attenuates the random shocks. The standard deviation of inventory is less than 2%, showing how inventory buffers the system from the high frequency variations in production starts caused by the white noise. The standard deviations of production, inventory, and labor are all much less than that of productivity. Only the hiring rate amplifies the white noise input.

Under correlated noise, however, inventory, workforce, and other key variables all fluctuate by more than the variation in productivity. The standard deviation of hiring is six times greater than when the system is driven by white noise and 13 times greater than the standard deviation in productivity.

TABLE B-1 Autocorrelated noise alters the amplification generated by a system.

	Standard Deviation/Mean Pink Noise	Standard Deviation/Mean White Noise	Ratio
Production Starts	0.0760	0.0511	1.49
Production	0.0547	0.0087	6.30
inventory	0.1055	0.0160	6.59
Labor	0.0591	0.0085	6.99
Hiring Rate	0.6539	0.1097	5.96

Various statistical tools can help you estimate the autocorrelation time constant, if sufficient data are available. Most regression and times series software packages readily compute the autocorrelation function, showing the correlation between the current value of the variable and its values at each interval in the past. From the autocorrelation function you can estimate the time constant for the pink noise function. If numerical data are unavailable, use your best judgment and conduct extensive sensitivity tests.

To illustrate, Oliva (1996) developed a model of a bank's retail loan operation to explore the determinants of service quality (chapters 14 and 21). Customer demand and worker absenteeism, two important inputs to the model, both exhibited small variations around their averages (the standard deviations were less than 4% of the means). To model these random variations Oliva estimated the autocorrelation functions for each, finding a correlation time constant of about 2 weeks for absenteeism and about 1 week for orders. That is, customer orders this week were weakly dependent on orders last week, but absenteeism tended to persist for longer periods. Oliva also found that the random variations in orders and absenteeism were independent of each other, so each could be modeled as a separate pink noise process.* Oliva was then able to simulate the effects of various policies affecting service quality while the model system was perturbed with realistic patterns of orders and absenteeism.

Without random noise the loan center remained in equilibrium with demand and capacity in balance and constant service quality. However, when realistic random variations in demand and the workforce were added to the model, quality standards tended to erode over time, even when capacity was sufficient to meet demand on average and even though the random shocks were small. The random

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[8]Sometimes different noise processes are not independent but are cross-correlated. For example, unpredictable short-run variations in interest rates might be correlated with random shocks in commodity prices. It is a simple matter to incorporate such cross-correlations (e.g., the shocks perturbing a variable can be modeled as an appropriately weighted sum of variable-specific noise and the noise with which it is correlated). Multivariate time series tools such as vector autoregressive models and cross-spectral analysis can help you identify the correlational structure among the variables and their histories (see the references in note 3). However, a complicated set of cross-correlations suggests there is important feedback structure you should probably be modeling explicitly. **A** good model generates the variances, autocorrelations, and cross-correlations observed in the real system without being forced by too many exogenous inputs.

variations in demand and capacity meant the bank occasionally found itself short of capacity. Loan center personnel responded by spending less time with each customer so they could clear the backlog of work each day. These reductions in time per customer gradually became embedded in worker norms. Management interpreted the reduction in time per customer as improvements in productivity caused by their get-tough management policies, unaware that spending less time with customers reduced service quality, eventually feeding back through customer defections to other banks. Oliva found that reducing the time spent per customer caused a significant reduction in the value of loans issued, directly reducing bank revenue. Lower revenues then fed back to financial pressure leading to staff reductions and still more pressure to spend less time on each customer. The resulting positive feedback, if unchecked, could act as a death spiral for the organization. Small, random variations in capacity and orders elicited the latent self-reinforcing quality erosion created by the policies of the bank and the behavior of its workers and managers.

CHALLENGE

Exploring Noise

Use the inventory–workforce model and pink noise structure to explore the sensitivity of model behavior to the noise correlation time and simulation time step.

1. Figure B-4 and Table B-1 show that increasing the noise correlation time from 0 (the white noise case) to 4 weeks increases the oscillatory response of the inventory–workforce model. Explore how the system responds to even longer correlation times. Does the amplitude of the production cycle continue to increase? What happens if the correlation time is very long? Why?
2. In all the simulations above the time step was 0.125 weeks. Explore how the behavior of pink noise and of the inventory–workforce model depend on the time step. Can you correct the problems caused by rapid changes in white noise by using a longer time step? Why/why not?
3. So far only one source of noise has been considered. Does the behavior of the model change if you also assume customer orders vary randomly? Assume customer orders vary randomly with a small standard deviation around a constant average. Also assume productivity and orders are independent of one another. Assume to start that the correlation time for orders is 13 weeks (one-quarter year) but conduct sensitivity tests. How does the inclusion of multiple sources of noise alter the behavior of the model? Does it alter any of the policy conclusions you reached in chapter 19 regarding ways to improve the stability of the system? Explain.

SUMMARY

Guidelines for the Use of Noise

- It is usually best to omit noise until you understand the dynamics generated by the feedback structure of your model. Use idealized test inputs such as the step, pulse, ramp, and sine wave to develop your understanding of the system's response to shocks, then add more realistic inputs such as noise or historical data as necessary.
- After you understand the dynamics of your model, ask yourself whether random variations in the environment are likely to be important. If so, you must add random variations at key points. You should test the effects of random variations on your conclusions and policy recommendations even if you think they probably won't matter. If they don't matter, you can omit them.
- In principle, effects outside the boundary of your model can perturb every variable. All parameters are candidates for the inclusion of some type of noise, and the reported values of all state variables include some measurement error. You do not need to include noise in every variable and parameter. Decide which are the most important sources of random variation to include. Important sources will be those parameters that are *both* variable in the real system and whose variation matters to the dynamics. There is no point in including noise in a parameter that has little impact on the behavior of the model.
- All real noise sources have inertia and attenuate high frequencies. Never use white noise in your models. Use the pink noise structure to model random shocks. Estimate the distribution, standard deviation, and correlation time constant from the data. Use your best judgment when numerical data are not available.
- If you have multiple sources of noise in your model, consider whether they are independent or correlated. Independence is convenient but not always accurate. Use the available data to estimate the cross-correlations of multiple noise inputs.
- Be sure your results are not sensitive to the choice of time step. As with any structure, the time step should be small relative to the smallest time constant in the model, including the correlation time for any pink noise processes (see appendix A).
- You can compare different sensitivity and policy tests in models with random effects by using the same noise seed in each simulation. Since the

sequence of random shocks will be exactly the same in each simulation with the same noise seeds, any differences among them must be due to your policy or parameter changes.

- When using random inputs be sure to run your model long enough, or enough times, to ensure your results are not contingent on the particular realizations of the random processes. Calculate the distributions of the variables over long time periods or over a large sample of simulations. Don't assume any one simulation is representative of all.
- As with any model, conduct extensive sensitivity tests to be sure you assess the robustness of your results to plausible variations in assumptions, including assumptions about the statistical properties of any noise inputs.