

#### Data and Uncertainty in System Dynamics

Forrester, Kalman, Markov & Bayes

Tom Fiddaman SD Seminar October 26, 2022





#### **Abstract**

Jay Forrester cautioned that "fitting curves to past system data can be misleading." Certainly that can be true, if the model is deficient. But we can have our cake and eat it too: a good model that passes traditional SD quality checks and fits the data can yield unique insights. With recent computing advances, it's practical to confront models with all available information, including time series data, to yield the best possible estimate of the state of a system and its uncertainty. That makes it possible to construct policies that are robust not just to a few indicator scenarios, but to a wide variety of plausible futures. This talk will discuss how calibration, Kalman filtering, Markov Chain Monte Carlo and sensitivity analysis work together, with particular attention to Bayesian inference. The emphasis will be on practical implementation with a few examples from real projects.

#### **Agenda**

- Introduction
- Example Chronic Wasting Disease Policy
- Methods
  - Naïve calibration
  - Maximum likelihood
  - Kalman filtering
  - Bayesian inference
  - Markov Chain Monte Carlo (MCMC)
  - Synthetic data
- References



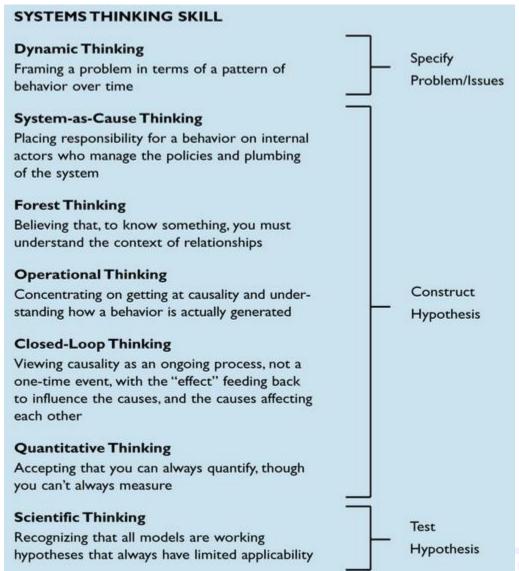
#### **A Classic SD Perspective on Data**

- fitting curves to past system data can be misleading
- given a model with enough parameters to manipulate, one can cause any model to trace a set of past data curves
- adjusting model parameters to force a fit to history may push those parameters outside of plausible values as judged by other available information.
- [tracing history] does not give greater assurance that the model contains the structure that is causing behavior in the real system
- the particular curves of past history are only a special case
- Exactly matching a historical time series is a weak indicator of model usefulness.
- We should not want the model to exactly recreate a sample of history but rather that
  it exhibit the kinds of behavior being experienced in the real system.

System Dynamics—the Next Fifty Years, Jay W. Forrester, D-4892 (2007)

#### The Seven Critical Thinking Skills ... Plus a Few

#### Barry Richmond in the Systems Thinker



#### Statistical Thinking

 Understanding how noisy measurements inform and constrain your model

#### Solution Thinking

 Focusing on helping stakeholders to understand the problem structure so they can make better decisions

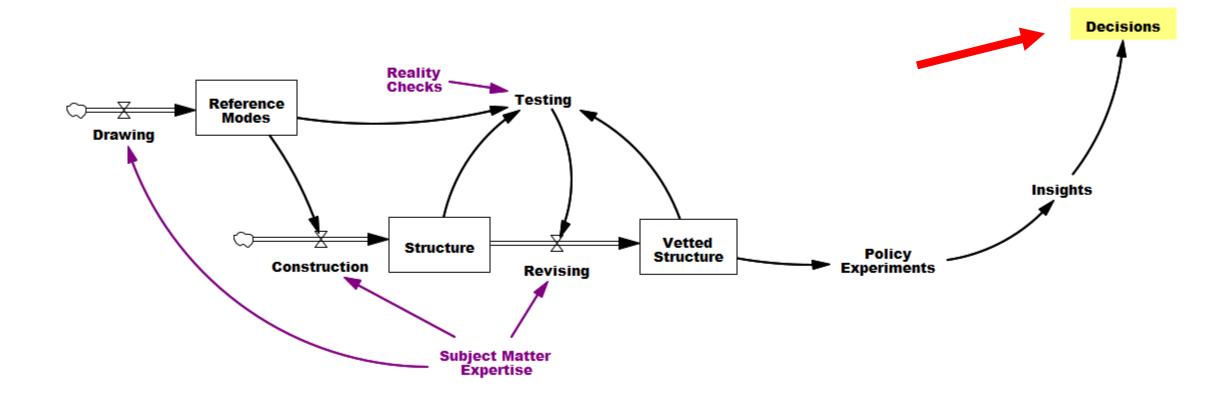
#### Behavioral Thinking

Modeling decisions with available information and limited cognition

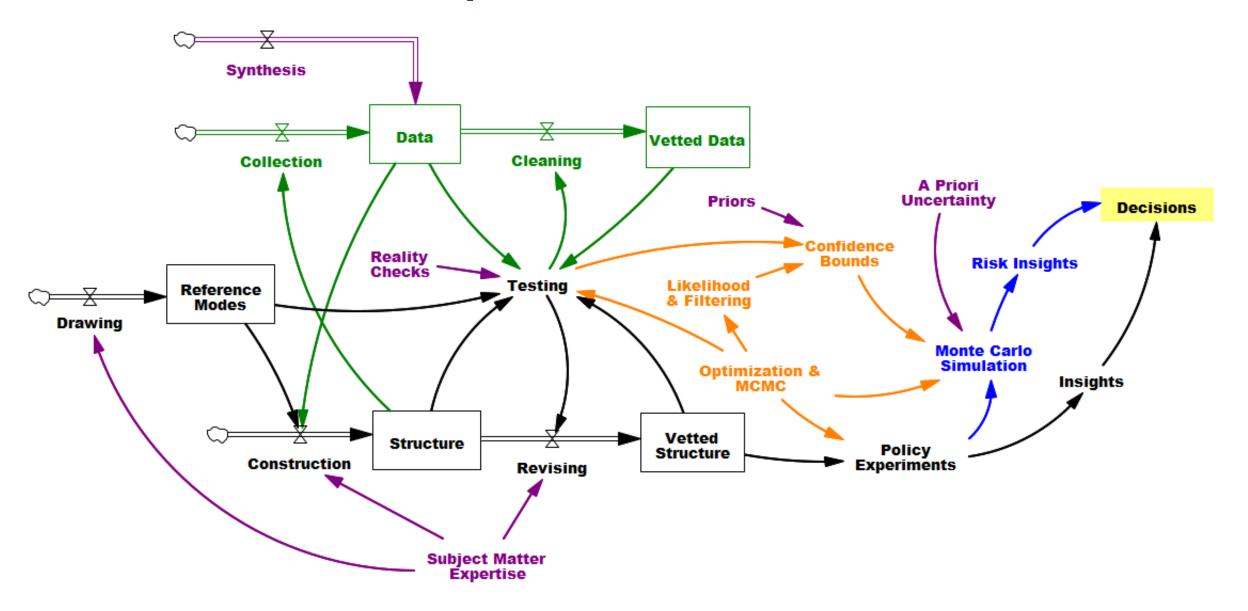
#### Complexity Thinking

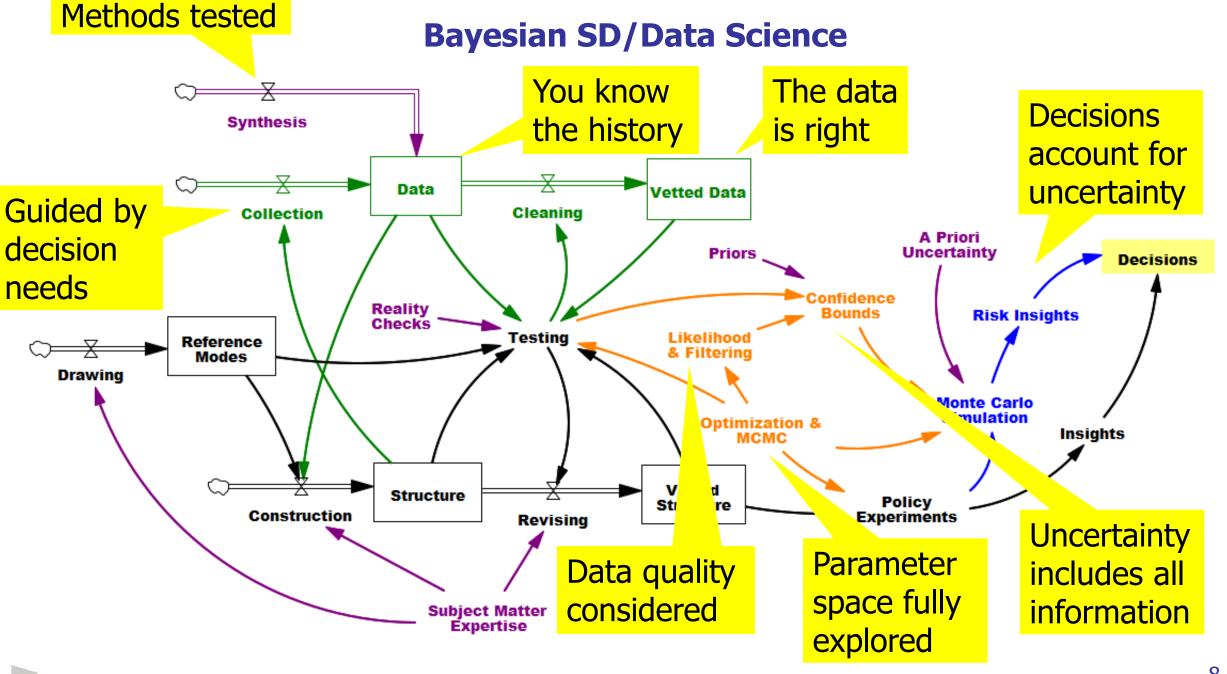
Recognizing that emergent behavior may involve granular detail

#### **Classic SD**



#### **Bayesian SD/Data Science**





#### **Uses for Data**

### Informal - "data" construed broadly

- Informs model structure directly
- Submodel calibration informs parameter choices
- Did history happen as reference modes describe?
- Do people make decisions the way they say they do?
- Do interesting features of model behavior appear in reality?
- Convincing stories demonstrating model behavior

### Formal – time series or cross sectional data

- Initialization of disaggregated states
- Verification of methods with synthetic data
- Estimation of parameters and uncertainty
- Propagation of uncertainty into outcomes
- Acceptance or rejection of candidate structures

#### What happens if you ignore the data?

- + Save lots of time on collection, preprocessing and calibration
  - + Potentially reallocate to client interaction, robustness testing and scenario experimentation
- + Less cognitive load for participants
- No learning about the data, or from the data directly
- No contribution to model quality from tests against data
- Hard to verify that asserted reference modes or decision structures match reality
- Less face validity of historical runs
- Difficulty understanding the gaps between a priori parameter values and most plausible values, given the model
- No objective basis for parameter values or confidence bounds
- Hard to understand the joint uncertainty of a parameter set

#### **Example1: Chronic Wasting Disease Policy**

- Client: WI DNR via USGS NWHC
- Question: how best to use agency resources to reduce the prevalence and geographic spread of CWD?
- Stakeholders: hunters, landowners, captive cervid farmers, wildlife NGOs, waste disposal industry, tribes, other agencies ...
- Process:
  - Structured Decision Making (essentially, stakeholder identification of metrics of interest and ranking of alternative actions' influence on each outcome)
  - Model informs action->metric mapping

### **Chronic Wasting Disease**

- Prion disease, like Mad Cow and scrapie
- Affects cervids (deer family)
- 100% fatal
- Long latent period, short clinical phase
- Environmental reservoir
- No human transmission ...yet



## Post Hunt Deer Population Data Post Hunt Deer Population Data Presumbly the projection of the project of the first of the project of the pr

#### **Parameters**

CWD transmission

Environmental prions <</li>

Deer fertility & mortality

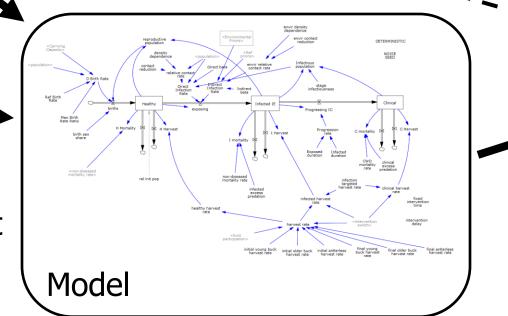
#### **Decisions**

- Hunting
- Baiting & feeding
- Surveillance
- Carcass management
- Safe practices
- Timing

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### Comparisons

- Prevalence
- Fraction positive
- Population
- Age, sex structure
- Harvest
- Surveillance results
- Hunt effort
- Human exposure



#### **Phase 1 Approach**

#### Situation

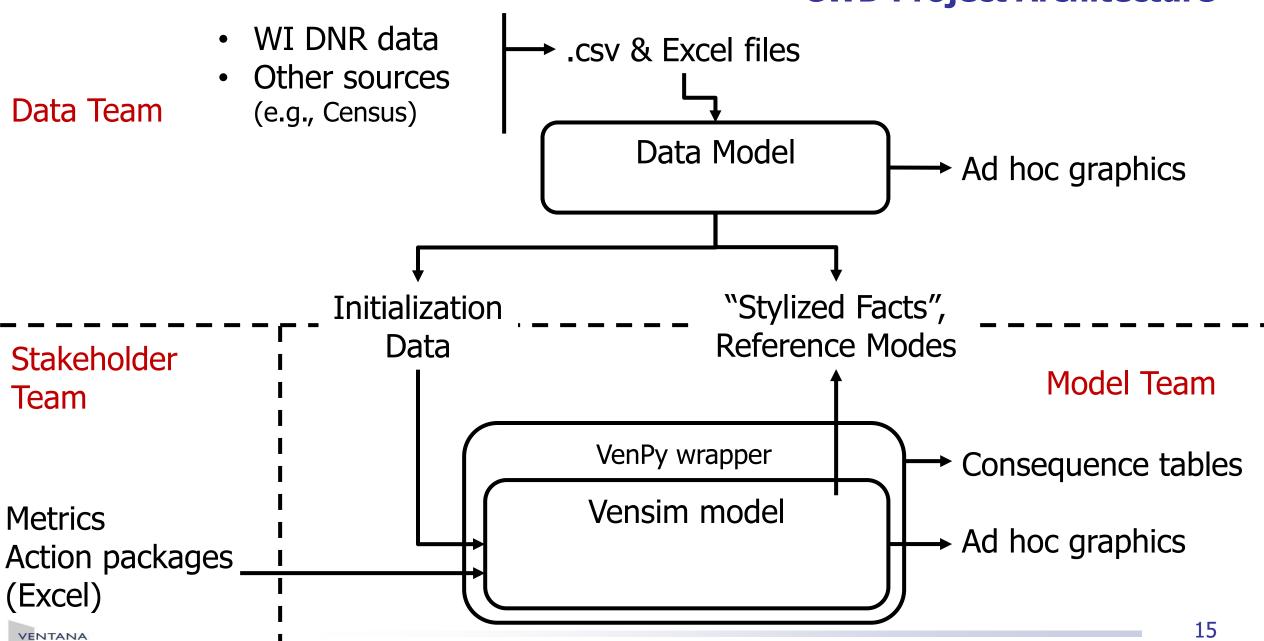
- Limited time and fixed schedule for stakeholder interactions
- Heavy demands for scenario evaluation
- Good precedent models in the literature
- But large uncertainty about some features

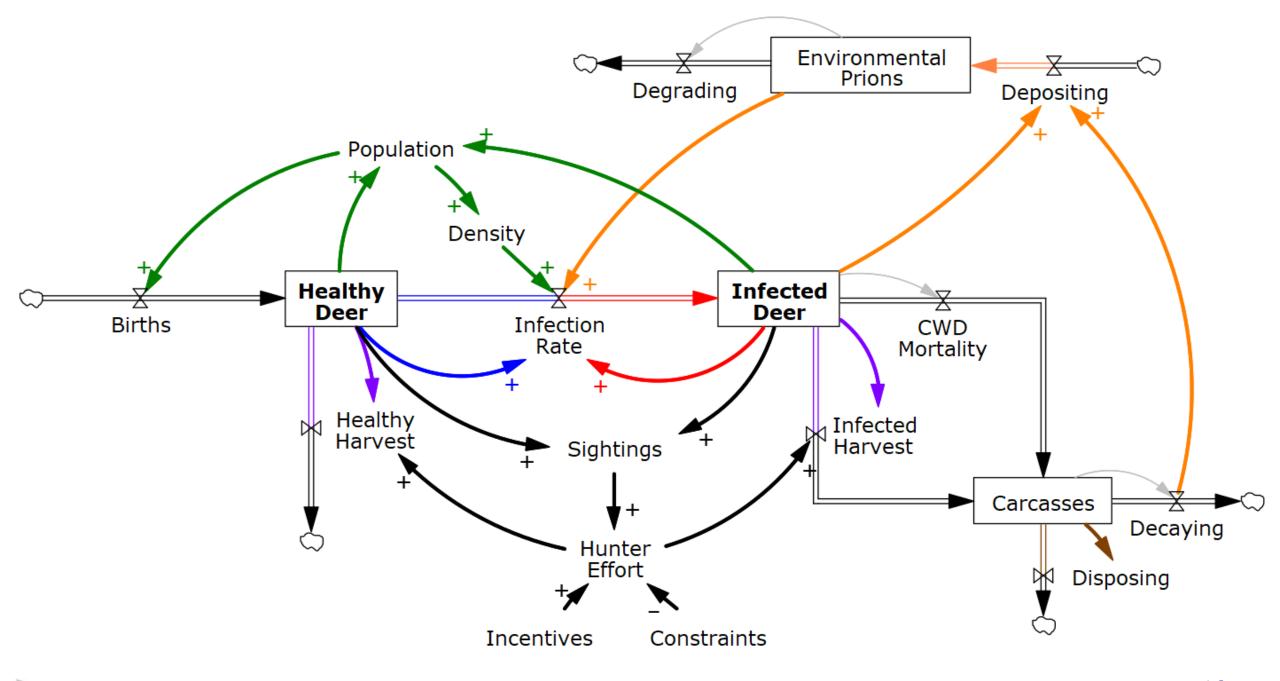
#### Strategy

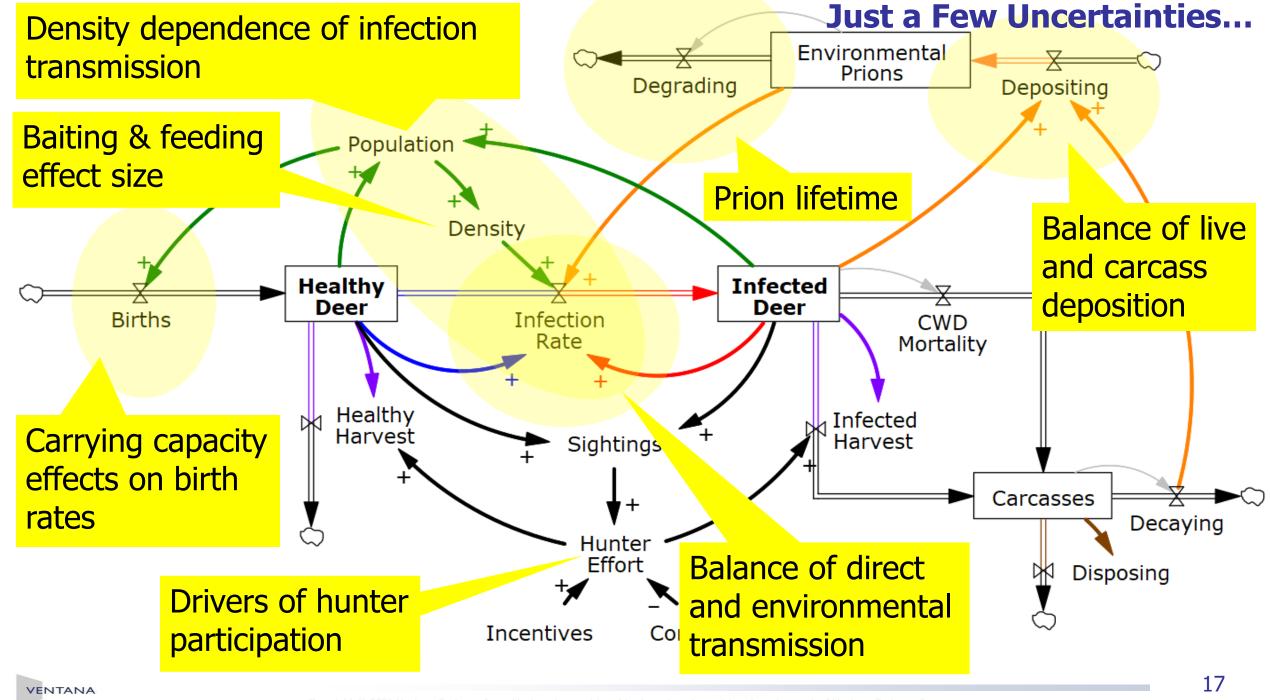
- No formal maximum likelihood calibration
- Calibrate very loosely to replicate the range of disease prevalence growth rates observed in minimally-controlled situations
- Establish parameters primarily from literature and subject matter experts
- Develop notional uncertainties from:
  - Subject matter experts
  - Disagreement in the literature
  - Some experimentation with face validity of model results
- Considerable use of data to describe behaviors seen in the model



#### **CWD Project Architecture**

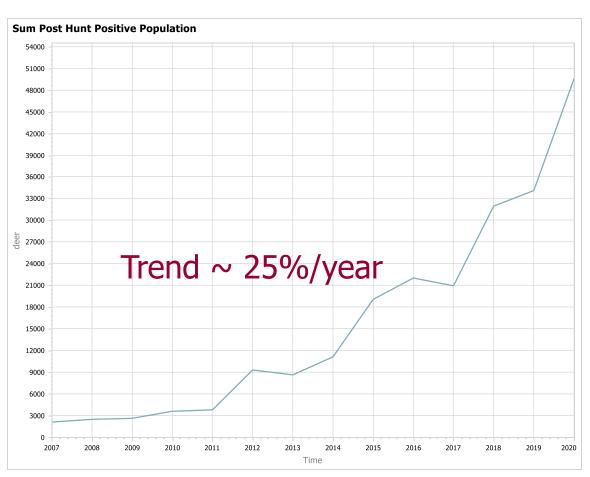


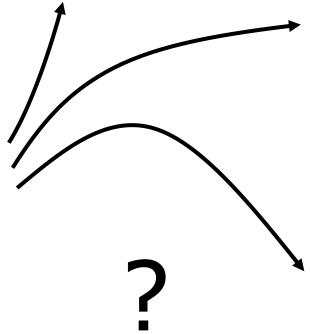


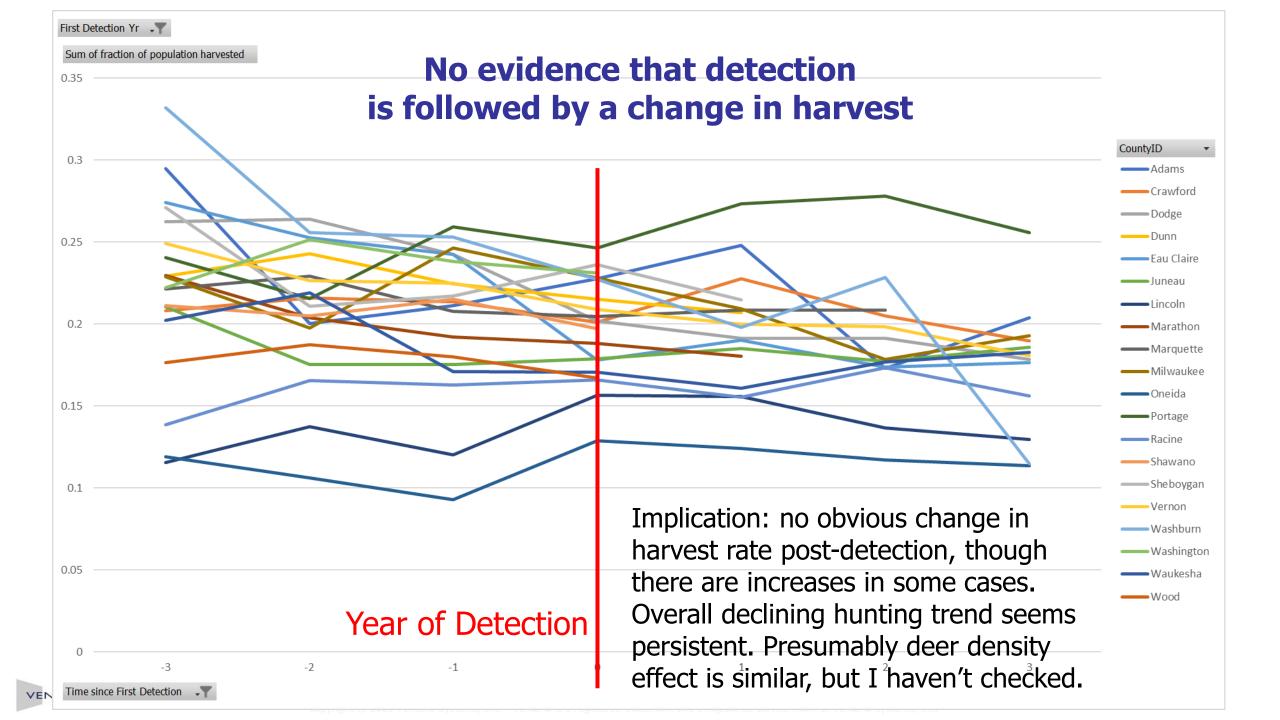


# **Statewide Positive Population**

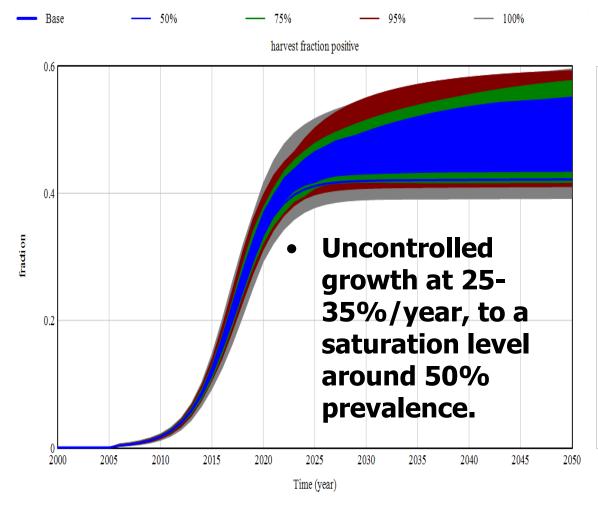
= Fraction Positive x
Post Hunt Population,
summed over
counties

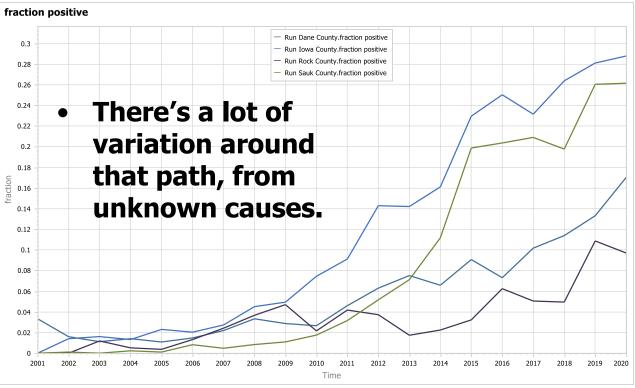






### Status Quo = Growth Highest prevalence areas are approaching saturation





#### **SDM Consequence Table: Mean Outcomes**

**Automated with VenPy + Vensim DLL** 

	Harvest Action					
Metric	Base	Uniform	Antlerless	Older Bucks	All Bucks	Perfect Targeting
population	871	376	379	875	879	778
older buck population	154	53	86	117	98	118
healthy population	456	269	214	505	554	569
prevalence	0.48	0.29	0.44	0.42	0.37	0.27
harvest fraction positive	0.46	0.28	0.39	0.44	0.38	0.37
positive harvest consumed	74	31	36	76	74	61
clinical prevalence	0.02	0.02	0.02	0.02	0.02	0.02
total harvest	255	185	150	280	314	271
trophy harvest	46	26	26	58	49	47
relative harvest effort	0.96	1.60	1.38	1.09	1.24	1.19
Vegetation Index	1.02	1.09	1.08	1.02	1.02	1.04



#### Phase 1 – Good Outcomes

- Complex discussions with stakeholders, organized around the model
- Systematic evaluation of policies in combination, not in isolation
- More policy experiments than discussion could possibly support
- Low level of conflict

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- Significant "discoveries" for modelers and stakeholders
  - Difficulty of achieving control with existing policies
  - Reasons for testing to overstate and understate true prevalence
  - Advantages to early intervention
  - Critical importance to follow-up on surveillance

#### Phase 1 – Not so good?

 Difficult to assess: possible reticence of stakeholders who didn't buy into the modeling process

#### Missed opportunities:

- Characterizing the historical trajectory what did policies achieve?
- Exploring counterfactual histories where would we be with no control effort?
- Challenging past mistakes
- Density-dependent transmission and bathtub dynamics

#### Probably negative:

- The baseline, frequency dependent simulation makes control too hard
- Little characterization of uncertainty of results



#### **2012 Deer Trustee Report**

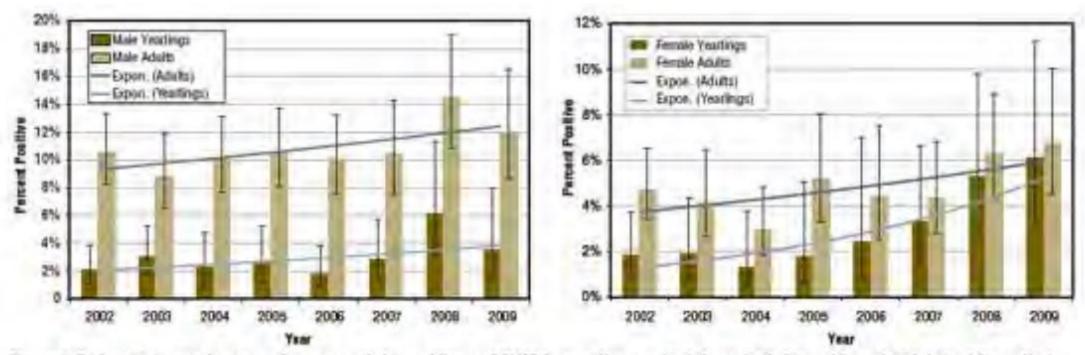
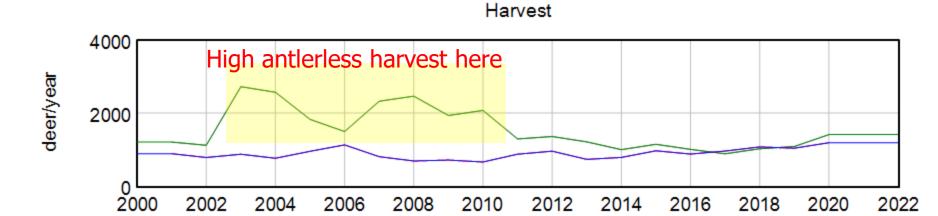


Figure 4. Estimated prevalence and exponential trend lines of CWD in yearling and adult male (left) and female (right) white-tailed deer from the western core monitoring area, 2002-2009. Vertical lines are 95% confidence intervals.

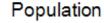
"Figure 14 presents graphs used in the planning document. The graphs imply (using fitted exponential trend lines) an upward trend in infection rates, even for yearlings. Yet, the graphs also present 95% confidence limits for each year; and, in every case these limits overlap. From a statistical standpoint, this means there were no significant differences between years! Wrong!

Also, illogical: if there's no evidence of exponential growth of a disease, that's a win for control!

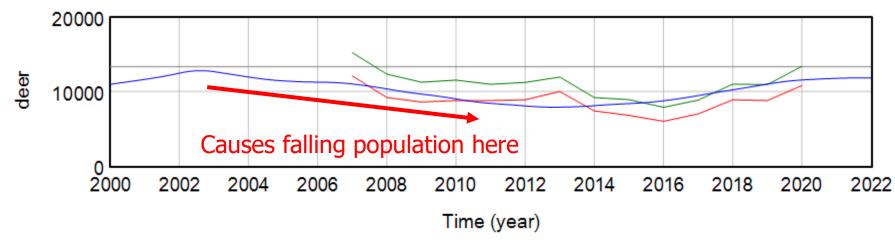
### A possible high DD history



buck harvest[Rock] : v30 priors high nod
 harvest bucks[Rock] : v30 priors high nod
 antlerless harvest[Rock] : v30 priors high nod



Time (year)

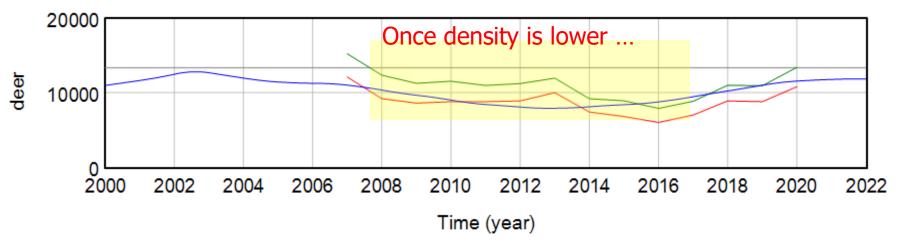


population[Rock]: v30 priors high nod

Post Hunt Deer Population Data[Rock]: v30 priors high nod
Pre Hunt Deer Population Data[Rock]: v30 priors high nod

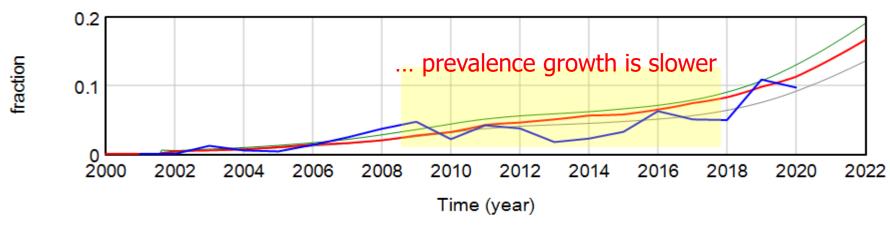
### A possible high DD history

#### Population



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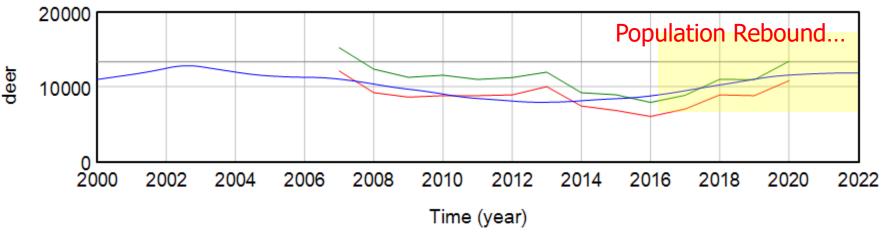
#### Fraction Positive



- fraction positive[Rock]: v30 priors high nod
- curr Fraction Positive[Rock]: v30 priors high nod
  - true prevalence[Rock]: v30 priors high nod
  - apparent prevalence[Rock]: v30 priors high nod

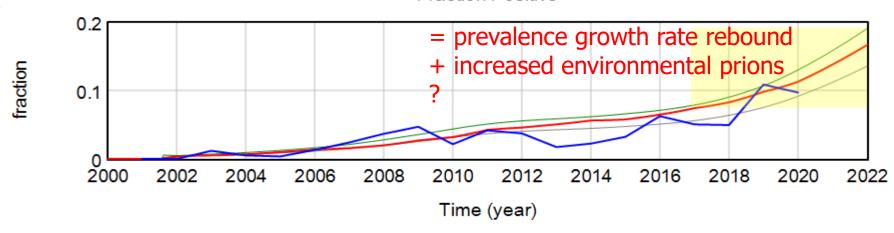
### A possible high DD history

#### Population



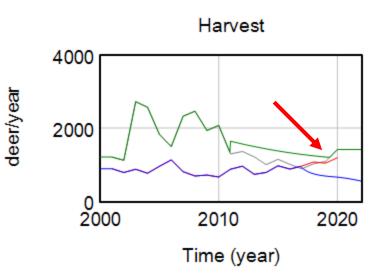
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#### Fraction Positive

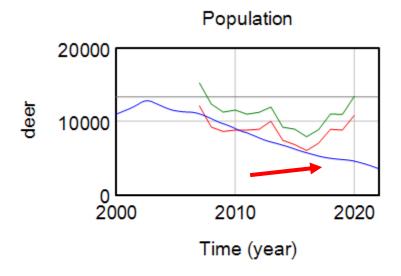


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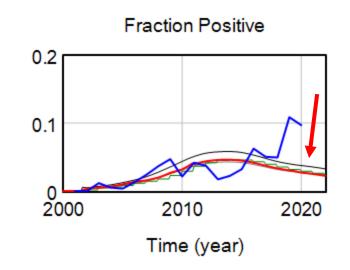
### What if the high antlerless harvests in 2003-2010 had been sustained?



buck harvest[Rock]: v30 priors high cou harvest bucks[Rock]: v30 priors high cou antlerless harvest[Rock]: v30 priors high harvest antlerless[Rock]: v30 priors high



population[Rock]: v30 priors high counte Post Hunt Deer Population Data[Rock]: v Pre Hunt Deer Population Data[Rock]: v Ref population[Rock]: v30 priors high co



fraction

fraction positive[Rock]: v30 priors high c curr Fraction Positive[Rock]: v30 priors h Reported Fraction Positive[Rock]: v30 priors h apparent prevalence[Rock]: v30 priors high c

#### A good decision...

- Maximizes outcomes of interest...
- ... from the perspective of multiple stakeholders ...
- ... and is robust to uncertainty about the future and system structure

#### Not forecasting:

"If we know what will happen, we'll know what to do."

#### Rather:

- Anticipate the inevitable
- Control the controllable
- Hedge the rest

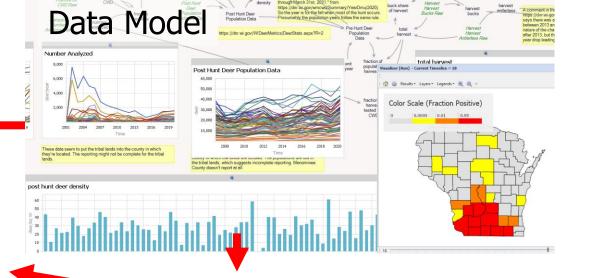
#### **Driving Data**

#### Parameters + Uncertainty

- **CWD** transmission
- Environmental prions
- Deer fertility & mortality

#### **Decisions**

- Baiting & feeding
- Carcass management
- Timing



Comparisons

#### **Process**

#### Outcomes

- + Uncertainty
- Prevalence
- Fraction positive
- Population
- Age, sex structure
- Harvest
- Surveillance results
- Hunt effort
- Human exposure

- Hunting
- Surveillance
- Safe practices

Model

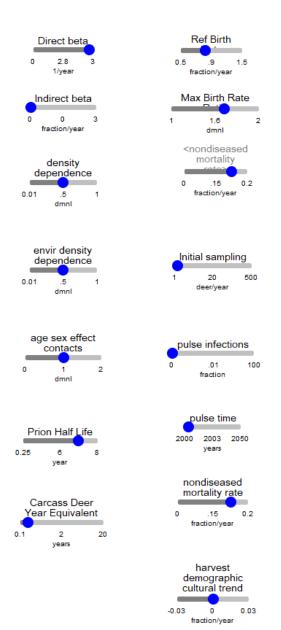
#### **Naïve Calibration**

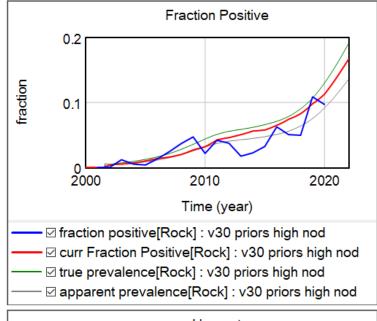
- Build a control panel that compares model and data series
- Do some hand calibration to discover what parameters matter, and how much they can plausibly vary
- This may be a big share of the value!
- Then:
  - Automate using the optimizer
  - Initially, don't worry too much about the details
  - Essence:
    - Create a payoff or objective function that characterizes the goodness of fit
    - Use some algorithm to iterate over a list of parameters to maximize the payoff

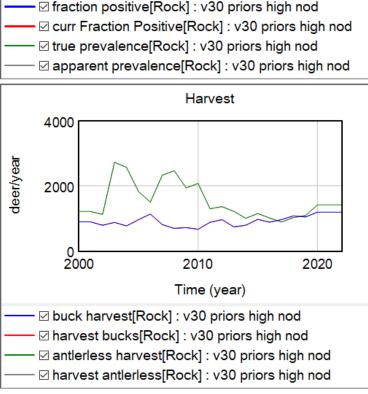


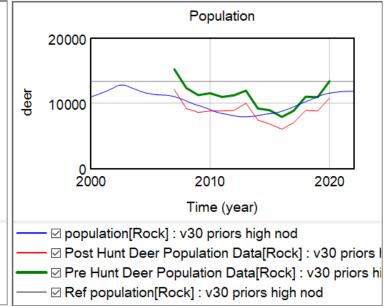
#### **Hand Calibration Infrastructure**

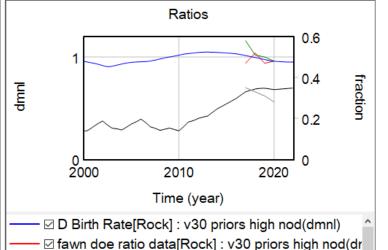
#### **Assumptions**









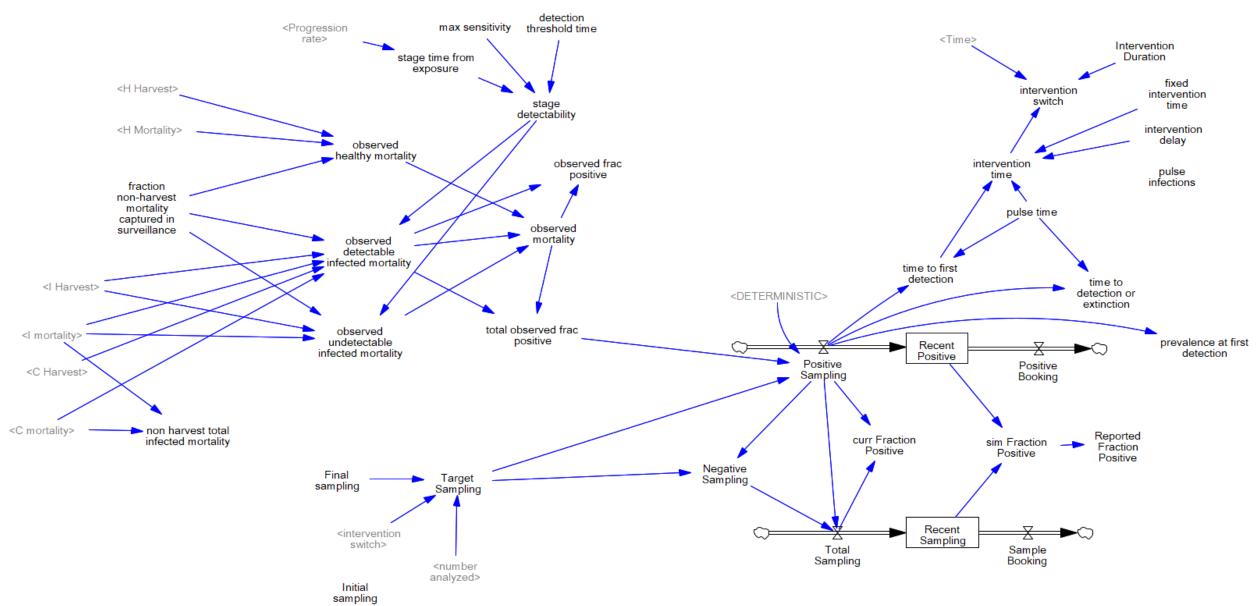


☑ yearling bucks share of all bucks[Rock] : v30 prio

☑ yearling does share of all does[Rock]: v30 priors

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#### **Modeling the Reporting Process**



### Formal Calibration Fundamentally, what are we doing?

- Create a model of the process that generated the data
  - Dynamic structure of the underlying system
  - Distribution of errors, lags and other features of the measurement process
  - Distribution of disturbances to the system state
  - Priors for unknown parameters or informally characterized behaviors
- Assuming the model is right, what parameters are most likely to have generated the data, and (maybe) are most consistent with our priors?

#### Output

- Frequentist: if I keep repeating this experiment, the parameter will be in my confidence interval 95% of the time
- Bayesian: starting from my prior knowledge, after seeing the data I'm 95% certain the parameter lies within my uncertainty interval

# **Example2: State COVID19 Modeling**

#### Context

- Early days of the pandemic April-June 2020
- Red state with a tech-savvy governor

#### Questions

- Tactical interpretation of new data (almost daily)
- What emergency medical resources will be needed (i.e. when will hospitalization peak, at what level)?
- How many tests are needed?
- What are the consequences of reopening (thereby cancelling many NPIs)?
- No data = no project

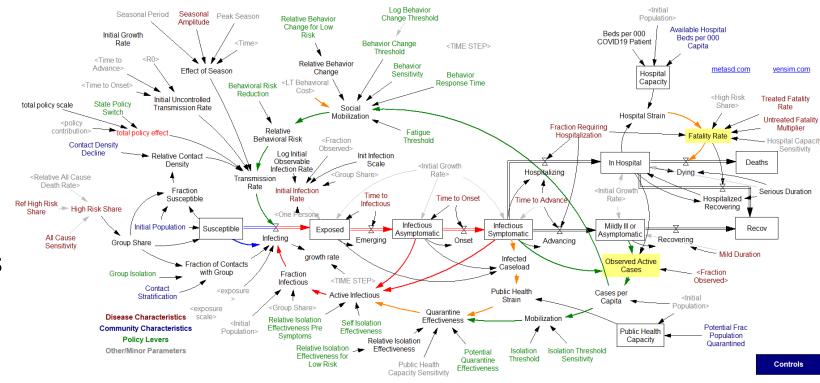
#### The Model

#### Enhanced SEIR

- Higher-order delay structure
- Test coverage effects
- NPI policies
- Behavior, compliance
- Detailed hospital sector

#### Integrate data streams

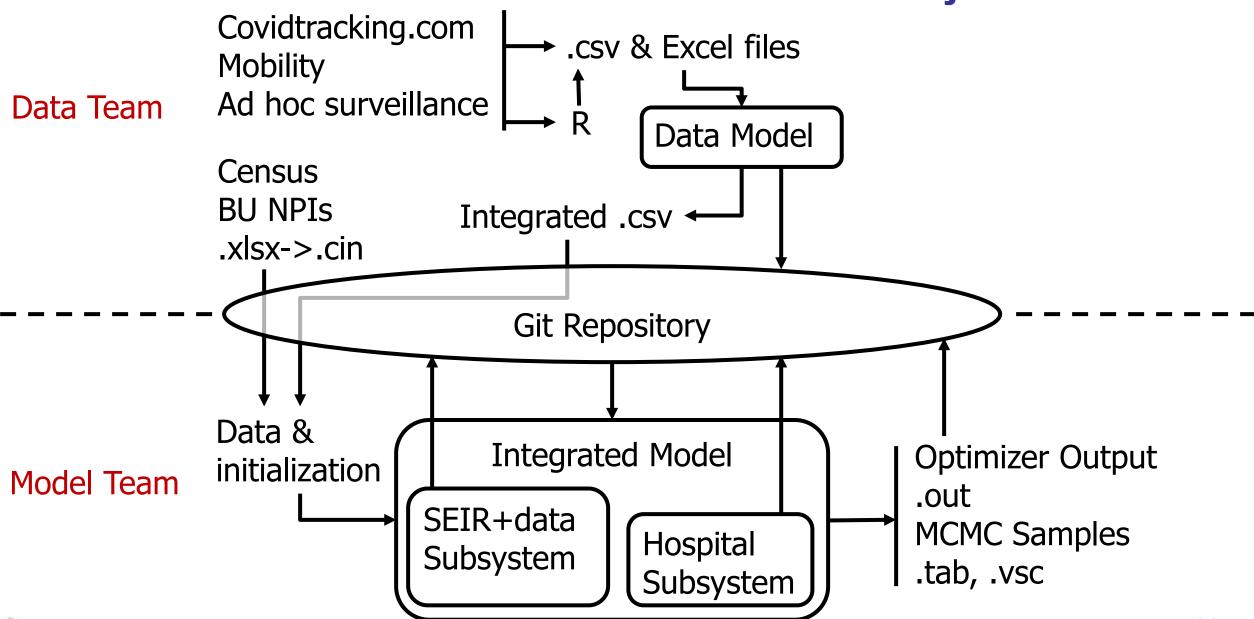
- Cases, hospitalization, deaths
- Test composition
- Mobility (cell phones)
- Weather, population, etc.



#### **Detail**

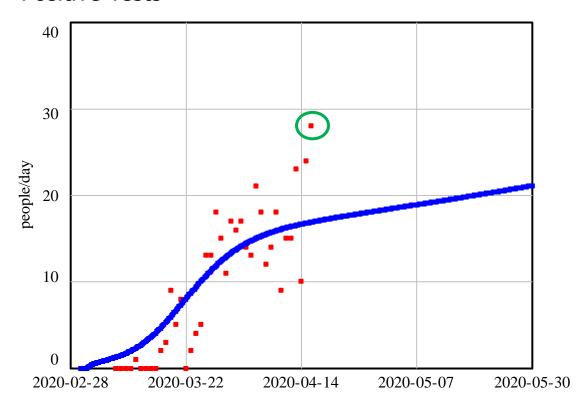
- 2 risk groups
- 50 states
- 50 x 50 interstate transmission (reduced by adjacency)
- 45 NPIs (essentially exogenous step functions)
- 5 hospital resource classes
- 7353 stocks
- 13003 constants (about 300 estimated)
- 2351 time series data (about 400 used for estimation)

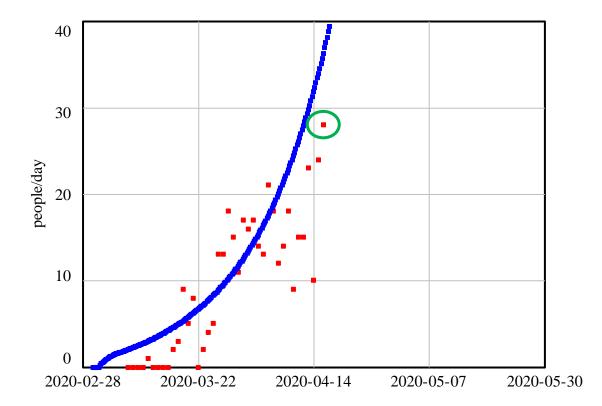
#### **Project Architecture**



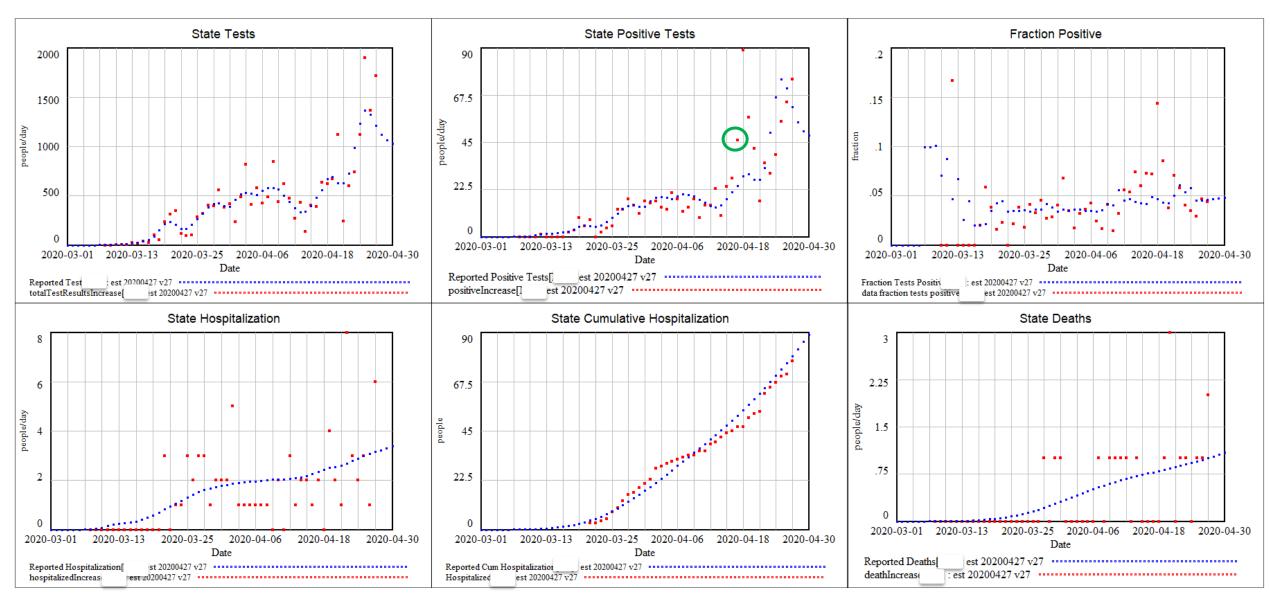
# Tax Day, 2020: Reason for concern?

#### **Positive Tests**

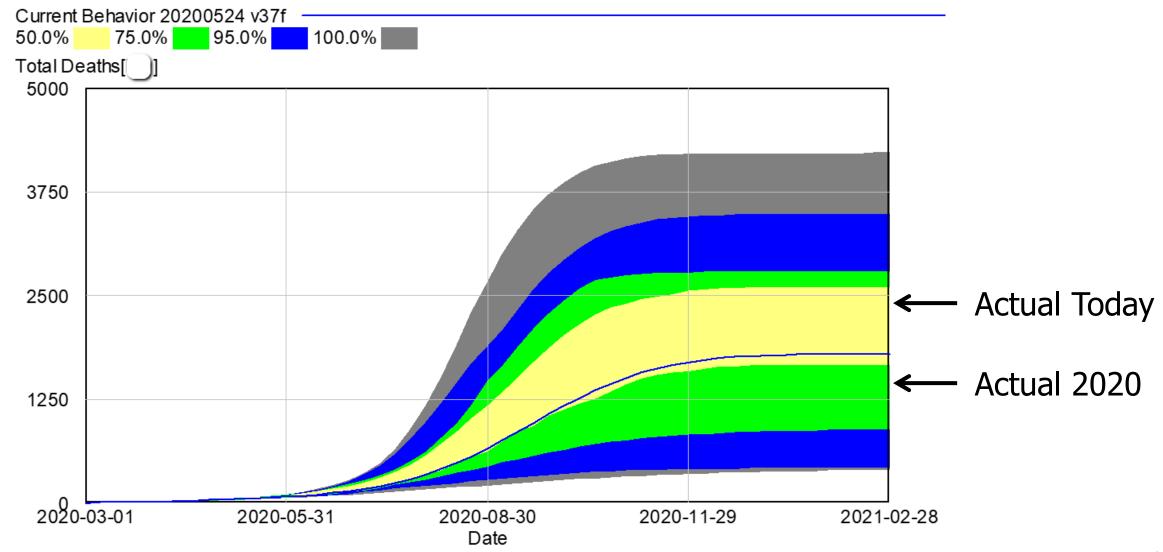




### **10 Days Later**



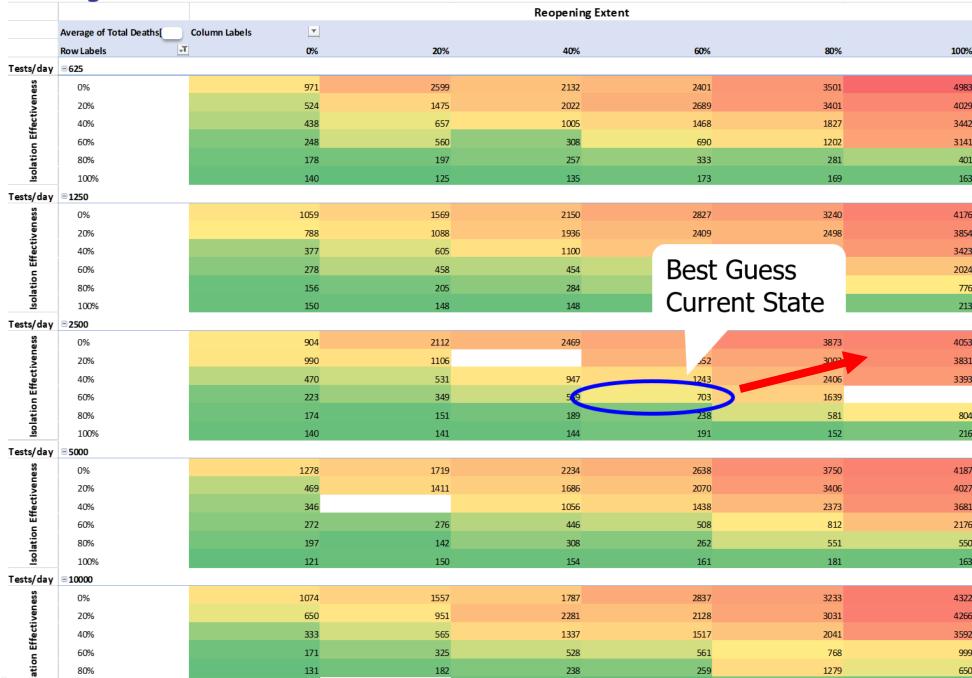
# **Outputs**



# Contingency Tables for Uncertain Outcomes

#### Average Deaths across Uncertain Scenarios

135



157

131

217

100%

# You're already a Bayesian

#### SD uses lots of a priori information

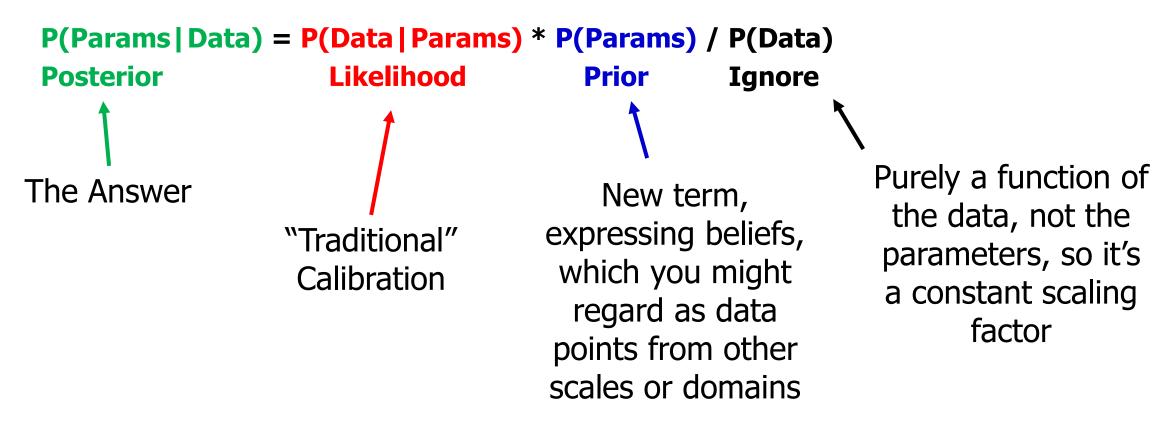
- Model structure
- Reference modes
- Dynamic hypotheses
- CLDs
- Parameters sourced from SMEs, literature, other models

#### You probably use Bayesian updates

- Adaptive expectations or smoothing
- Kalman filtering
- If you have lots of data, the answer is probably the same!

# **Bayesian System Dynamics**

- Bayes Rule: P(A|B) = P(B|A)\*P(A)/P(B)
- Conjunction Rule: P(A and B) = P(A)\*P(B), assuming A, B independent

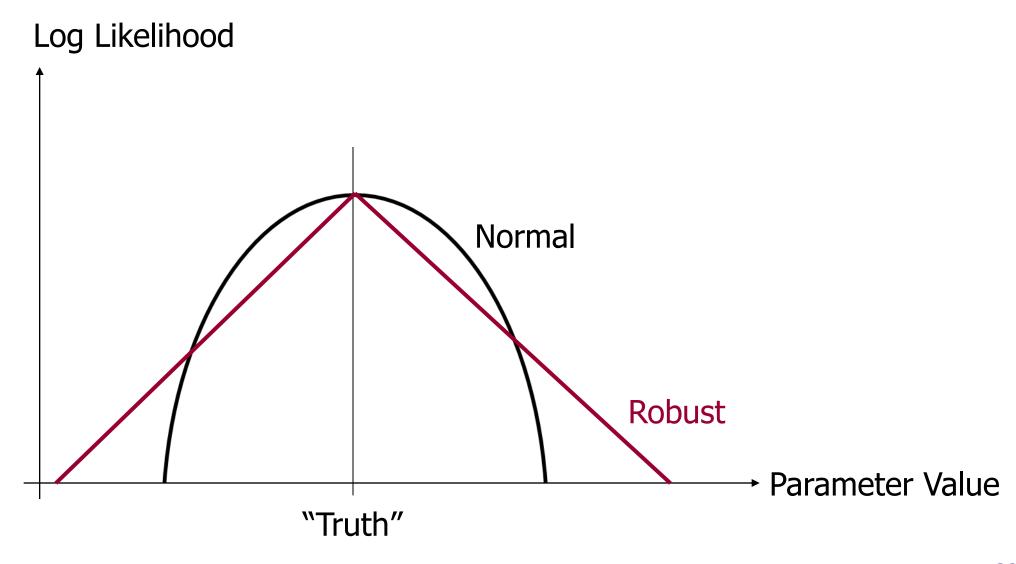


# **Log-Likelihood Gaussian errors**

• Likelihood = 
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\frac{model-data}{\sigma})^2}{2}}$$

- This is the PDF of the Gaussian (Normal) distribution
- σ represents the scale of the error associated with a data point
- σ might vary with time, or with the scale of the data
- You can estimate σ
- Maximizing Log(Likelihood) is the same as maximizing Likelihood, but more convenient because Log() turns multiplication into addition
- Log(Likelihood) =
  - LN( $\sigma$ ) the bigger the  $\sigma$ , the lower the likelihood, as it's spread thinner
  - $LN(\sqrt{2\pi})$  this is a constant we can ignore
  - $\frac{\left(\frac{model-data}{\sigma}\right)^2}{2}$  the weighted sum of squares, adjusted by the divisor /2

#### What does the likelihood look like?



#### **Priors**

- A prior expresses our belief about a parameter before we see the data
- No priors = uniform priors
  - It's not always a good choice, \*but\* if you have lots of data, it might not matter.
- Non-informative or Maximum Entropy priors
  - Contribute as little information as possible, i.e. assume maximum ignorance a priori
  - For a scale parameter like a time constant, this may be simple, like -LN(param) for positive parameters
  - This can be tricky to construct, and you're rarely \*completely\* ignorant about a parameter – after all, you thought to put it in the model to begin with

#### Informative priors

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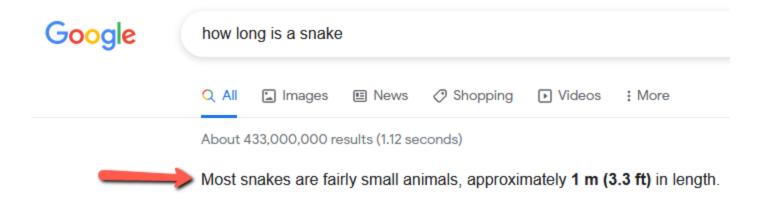
- If you or experts or literature have some opinion about a parameter, you can
  use a subjective probability distribution to characterize that
- This can also be tricky if multivariate correlations are important, but we can avoid that to some extent by choosing orthogonal parameters
- You can also use priors to characterize informal knowledge about expected behavior of the model

# **Expressing Priors**

- A prior is a lot like a data point!
- If our belief is Normal (Gaussian):

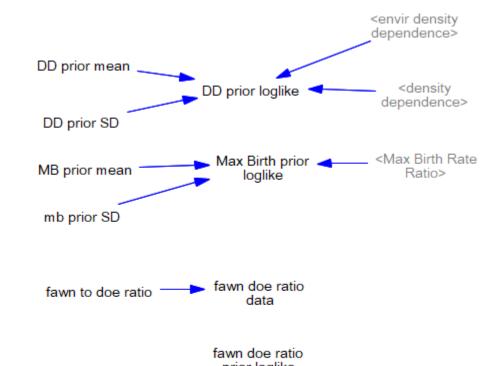
• Likelihood = 
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\frac{param-\mu}{\sigma})^2}{2}}$$

- For an MCMC log likelihood ratio, we only need the last term
- μ represents our belief about the mean value of the parameter, i.e. best guess
- $-\sigma$  represents our belief about the plausible variation; high uncertainty = large  $\sigma$



# **Example CWD**

- Our CWD model has a view dedicated to priors
- Most are simple (geometric mean & standard deviation)
- A few operate on composites, like generation time (combination of several delays)
- A few express hierarchy: how much variety of behavior is plausible across counties?
- The prior likelihoods are additional terms in the payoff

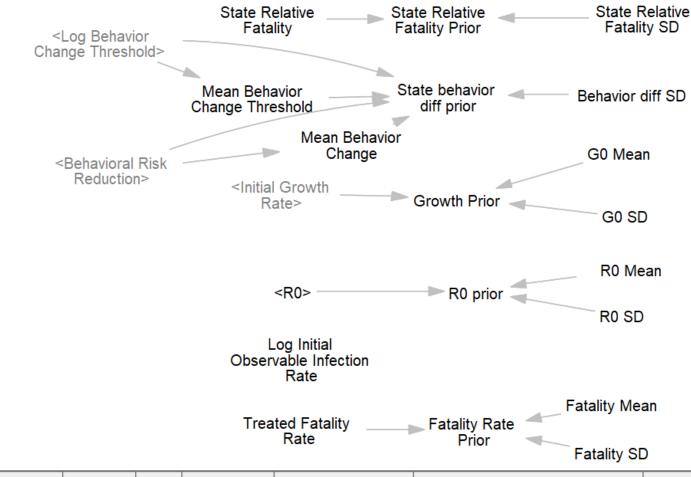




Policy		Timing	Transformation		Compare To	Weight
Calibration			Log	antlerless harvest[County]	harvest antierless[County]	0.1
Policy		Always	None	fawn doe ratio prior loglike[County]		1
Calibration	Normal		Log	yearling harvest share[County,buck]	yearling bucks share of all bucks[County]	0.1
Calibration	Normal		Log	yearling harvest share[County,doe]	yearling does share of all does[County]	0.1
Policy		Initial	None	DD prior loglike[County]		1
Policy		Initial	None	Max Birth prior loglike[County]		1

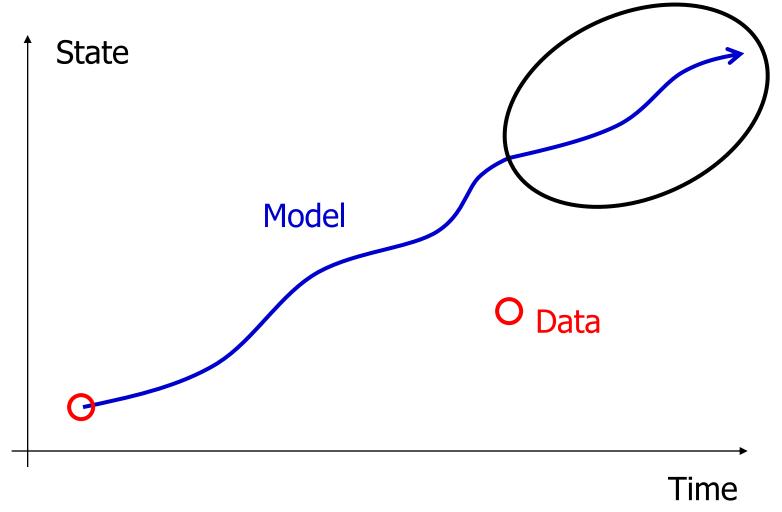
### **Example Covid**

- Our model has a view dedicated to priors
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- A few express hierarchy: how much variety of behavior is plausible across states?
- The prior likelihoods are additional terms in the payoff



Enabled	Policy	Distribution	Timing	Transformation	Variable name	Compare To	Weight	
<b>✓</b>	Calibration	OD Poisson		None	Reported Deaths[zone]	data deaths incr[zone]	death data	w
~	Calibration	OD Poisson		None	Reported Hospitalization[z	data Hospitalized incr[zone]	hospitaliza	tic
~	Calibration	OD Poisson		None	Reported Positive Tests[zor	data test positive Incr[zone]	case data	νe
~	Calibration	Gaussian		None	observed mobility effect[zo	data Google mobility[zone]	Mobility E	ffe
~	Policy		Initial	None	R0 prior		1	
~	Policy		Initial	None	Fatality Rate Prior		1	
~	Policy		Initial	None	Frac Hosp Prior		1	
~	Policy		Initial	None	High Symptom Prior		1	
	Policy		Initial	None	Generation Time Prior		1	`
<								>

# If the model is in the wrong part of the state space, its response may be wrong

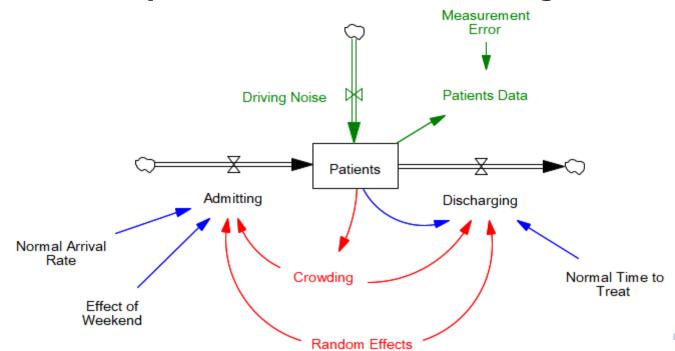


If you're estimating a parameter that affects this part of the trajectory, the response may be particularly wrong

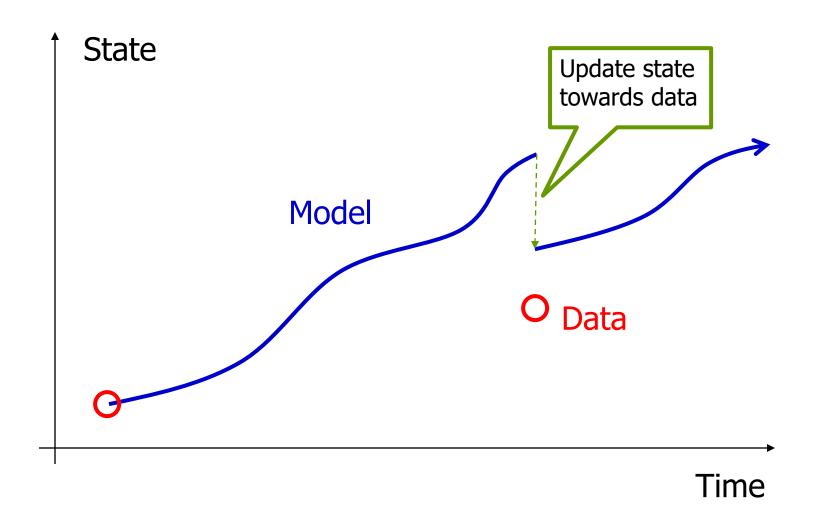
# **Kalman filtering**

#### Motivation

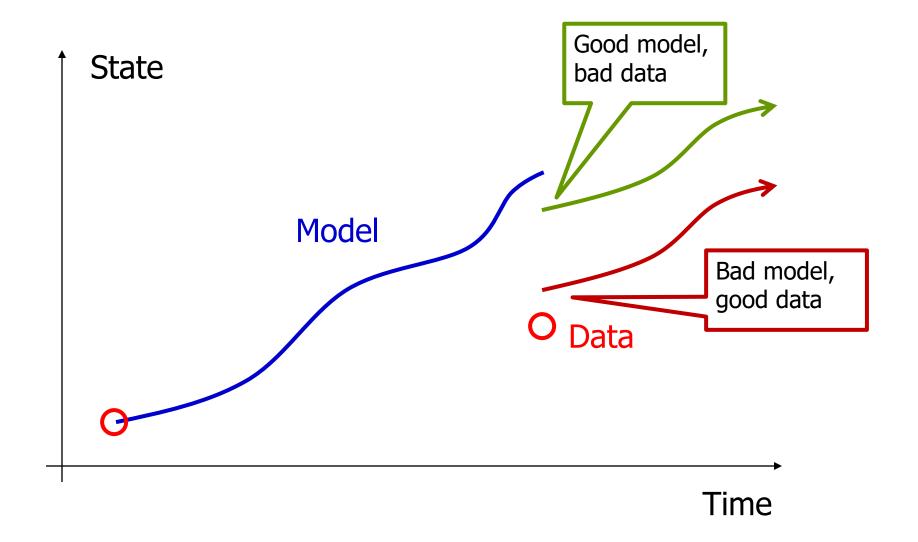
- The model may not reflect everything that affects the system state
  - Random noise from events (e.g., Poisson arrivals)
  - Structure we don't know about
- Over time, the model state drifts away from reality
- The Kalman filter is a special case of Particle Filtering with Gaussian noise



# The Kalman filter updates the model trajectory towards the data



# How far? Depends on the estimated quality of the model and data



# **Confidence Bounds & Uncertainty Intervals**

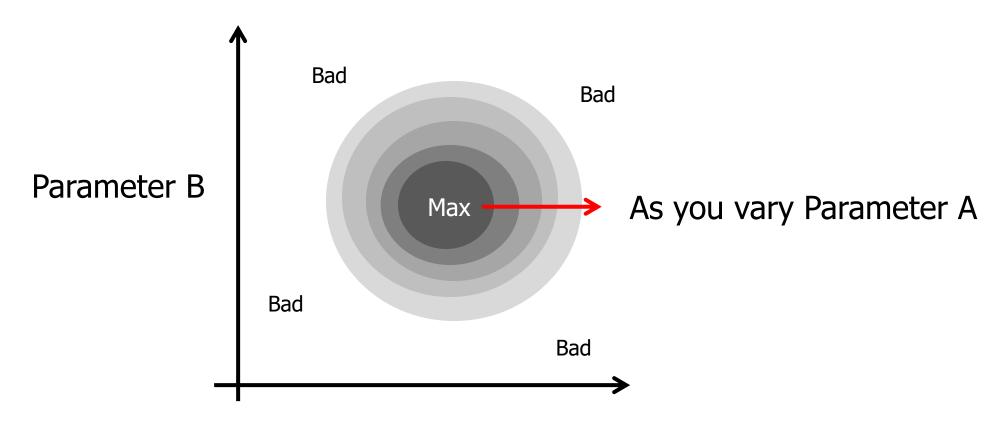
#### Motivation

- Statistical
  - Is an effect significantly different from zero?
  - After seeing the data, what do we believe about a parameter?
- Practical
  - What does uncertainty imply for policy?
  - What data might narrow the bounds?

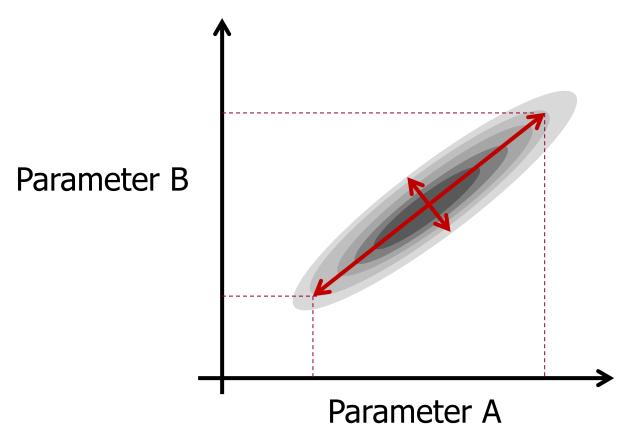
#### Computation

- Old way
  - Optimize to find the best fit to data
  - Explore the payoff surface around the maximum
- New ways
  - Bootstrapping (draw samples from the data)
  - Markov Chain Monte Carlo (MCMC)

#### **Multidimensional Likelihood**



#### **Traditional Method: Measure the Ellipse**



- This may be hard if the likelihood surface is shaped like a banana, or a hedgehog, or a bag of 10-dimensional jellybeans...
- One-dimensional confidence bounds omit information about the joint distribution of parameters

### **Using MCMC to Reveal the Posterior**

Posterior

P(Params | Data) = P(Data | Params) \* P(Params) / P(Data)

Likelihood Prior Ignore

We'd like to

know this

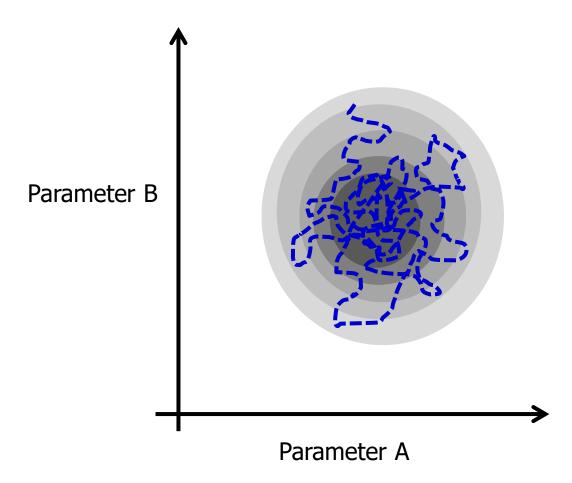
Without this torm, the pos

Without this term, the posterior isn't a properly-scaled probability distribution, i.e. it doesn't integrate to 1. So, we need a way to sample the posterior that only cares about the *relative* likelihoods of the data & priors.

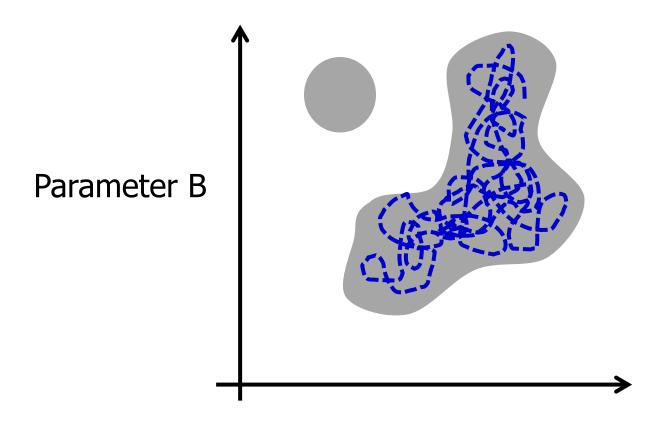
distribution.

#### **MCMC**

- Basic idea: unleash a random walk on the likelihood surface
- Probability of accepting a proposed step is proportional to likelihood
- Density of the resulting path converges to the underlying likelihood



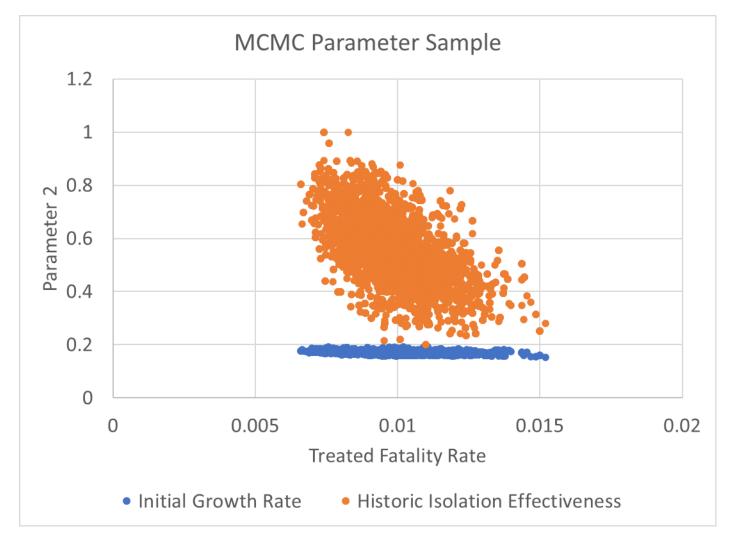
# **MCMC**



Parameter A

- A sample of points describing the joint distribution of parameters
- Diagnostics
- You can then use this to generate sensitivity runs reflecting the sampled parameters

# **MCMC Output**

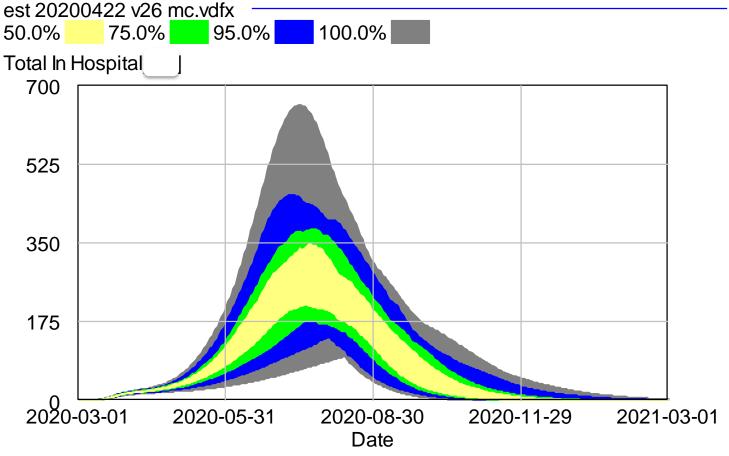


# **Using the MCMC Sample for Sensitivity Runs**

- What does the parameter distribution imply for the distribution of behavior?
- Does the data lie within the uncertainty interval?

# **Total Hospitalized**

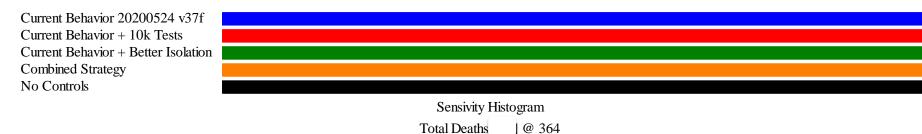
\*includes excess demand from unconfirmed cases

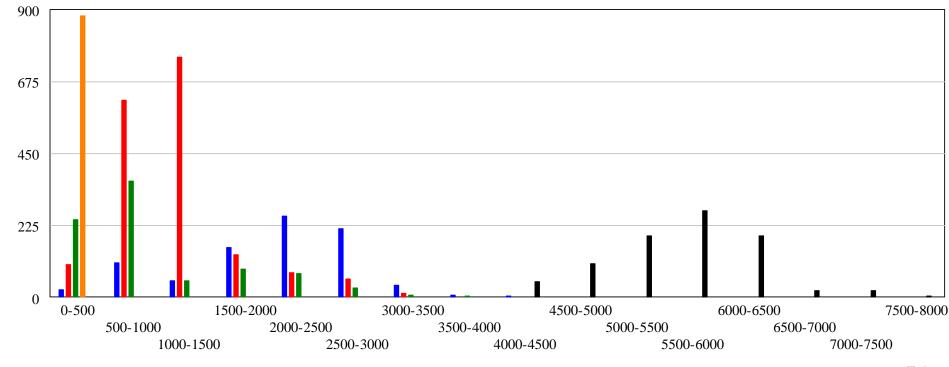


## What policy performs best under combined uncertainties?

- Simple: every run is a sensitivity run
- Fancy: stochastic policy optimization

#### **Distribution of Outcomes - Deaths**





# **Synthetic data**

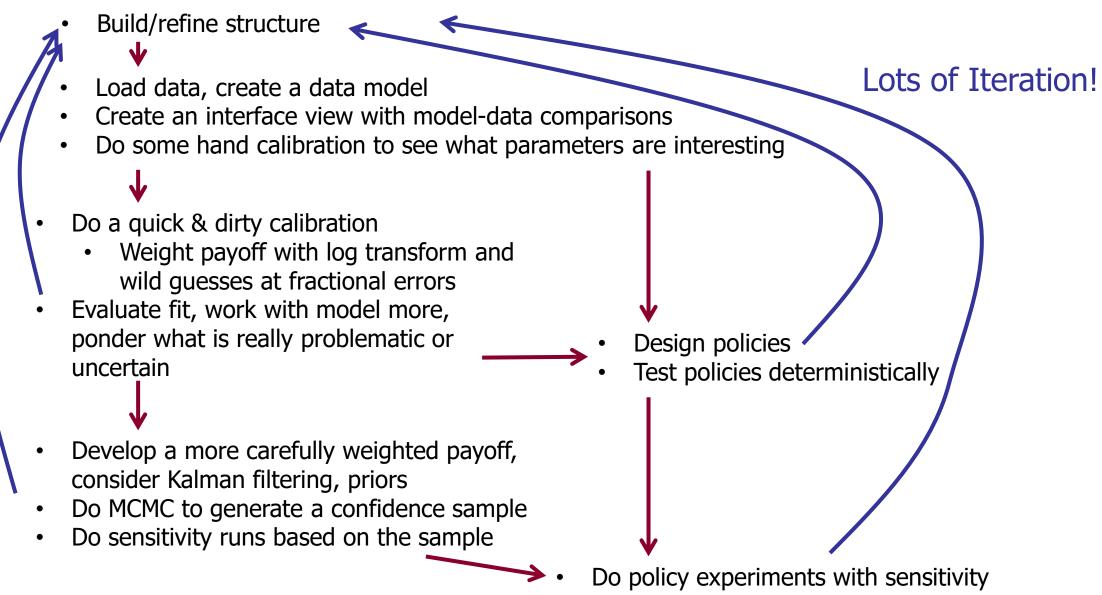
#### Purpose:

- Test your procedures end to end
- Can you get useful parameter estimates from limited data?
- How important are sources of noise or other features of the data?
- What if the model structure is a simplified version of reality?

#### Procedure

- Interpret your model as the truth
- Change some parameters to make things harder
- Add noise to some model outputs and/or states
  - Measured cases = RANDOM FUNCTION( true cases )
  - Patients = INTEG( admitting discharging + NOISE, initial patients )
- Truncate the frequency and duration of the measurements
- Use the synthetic data to see if you can recover parameters and distributions
- More fun if you have an adversary!

### **My Typical Playbook**

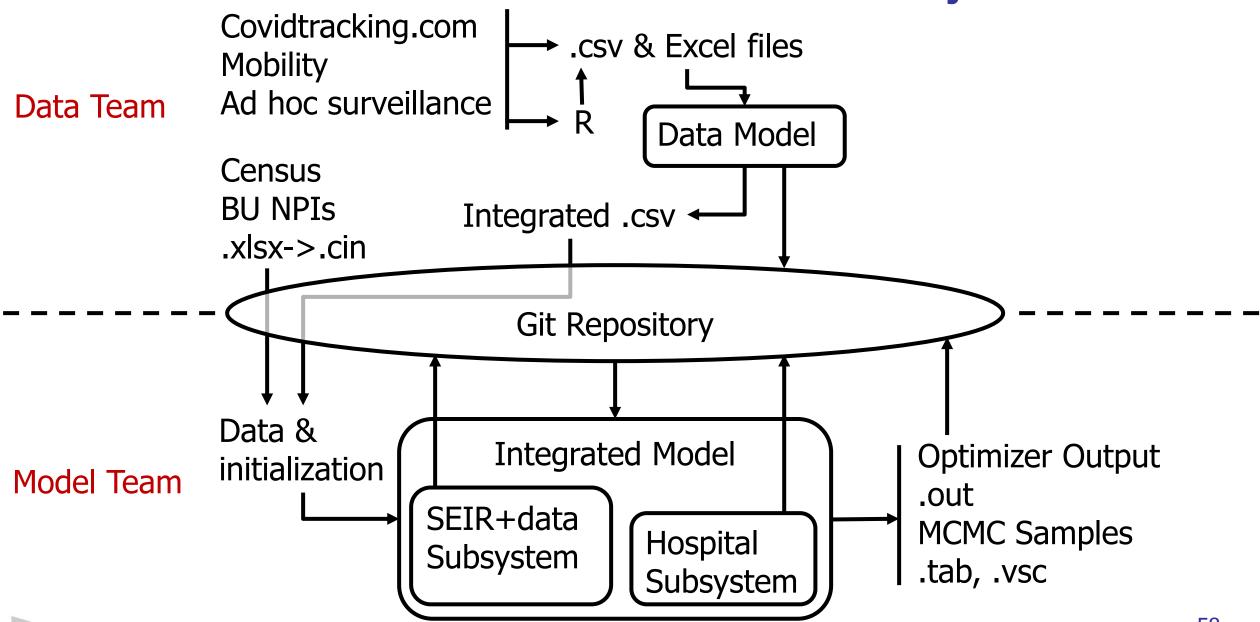


#### **Some Useful Habits**

- Dynamics first no point devoting statistical horsepower to a dumb model
- Eyeballs first learn about the model response before you automate fitting
- Consider the data, and data model, an investment
- Use source control and scripting to make things replicable
- Model the data reporting system
- Seek balance between dynamics, statistics and process
- Bring the clients along
- Guide data collection with decision needs
- Remember that this is analysis it's just a tool, not a religion
- Always be done! Jim Hines



# **Contrast: State COVID Project Architecture**



#### **Bottom Line**

#### Jay was wrong!

- Fitting to data doesn't make the model worse
- It's hard to make a sensible model fit arbitrary data
- If you can't reproduce history, you have some explaining to do
- Data is an important information source (not sufficient but necessary)
- Intuitive characterizations of system behavior or decision rules may be wrong

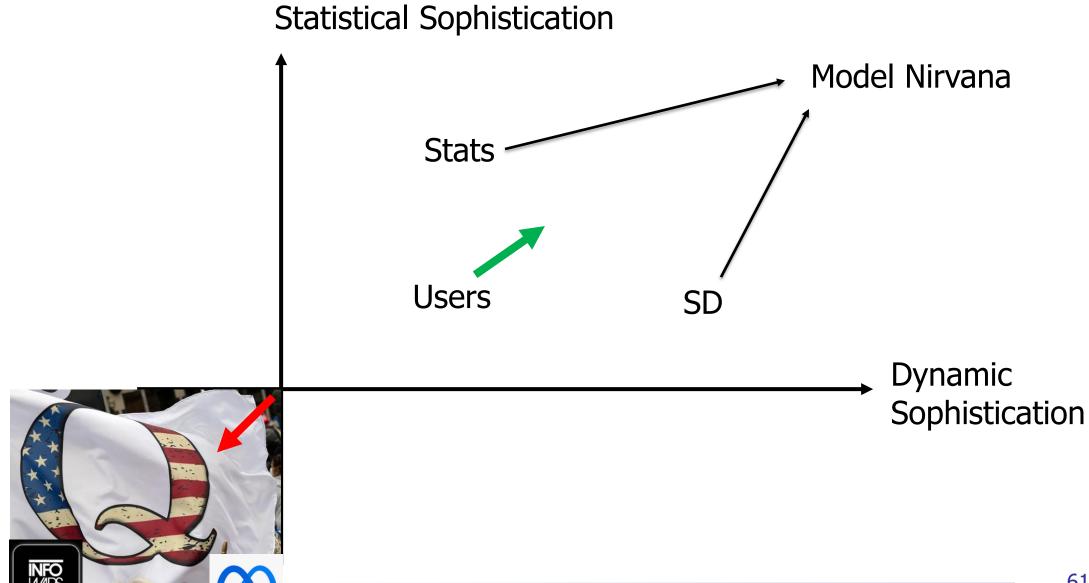
#### Jay was right!

- A good model has to come first
  - Appropriate stocks, flows, feedback, nonlinearity
  - Dimensional consistency and material conservation
  - Decisions use available information
  - Robustness to extreme conditions
- Models should reproduce all possible realizations of the data and test policies outside the historical range
- There are opportunity costs to intensive use of data

# **Challenges for the Future**

- What's the right way to mix data, calibration and dynamics?
- How do we make calibration and data easier and cheaper?
- What new skills do we need on modeling teams?
- How do we work with other communities, like machine learning?
- How can we bring users along without overwhelming them?

# **A New Challenge**



#### **Selected References & Resources**

- Vensim Data & Calibration workshops (ISDC 2022) <a href="https://vensim.com/conference/#using-data-in-vensim">https://vensim.com/conference/#using-data-in-vensim</a> (\*)
- Vensim manuals and sample models (\*)
- VenPy <a href="https://github.com/VensimOfficial/venpy">https://github.com/VensimOfficial/venpy</a>
- Gelman, Carlin, Stern & Rubin (1995-2020) Bayesian Data Analysis, <a href="http://www.stat.columbia.edu/~gelman/book/">http://www.stat.columbia.edu/~gelman/book/</a> (\*)
- Nathaniel Osgood & Juxin Liu (2015) Combining Markov Chain Monte Carlo Approaches and Dynamic Modeling, in Rahmandad & Oliva, Analytical Methods for Dynamic Modelers, MIT Press
- Nathaniel Osgood (2022) Using Particle Filtering with Dynamic Models in Health: Overview & Intuition, <a href="https://youtu.be/dHf-MM9WIIg">https://youtu.be/dHf-MM9WIIg</a> (\*)
- Jair Andrade and Jim Duggan (2021) A Bayesian approach to calibrate system dynamics models using Hamiltonian Monte Carlo, SDR <a href="https://onlinelibrary.wiley.com/doi/pdf/10.1002/sdr.1693">https://onlinelibrary.wiley.com/doi/pdf/10.1002/sdr.1693</a> (\*)
- "Behavioral dynamics of COVID-19: estimating underreporting, multiple waves, and adherence fatigue across 92 nations." Rahmandad H, Lim T, Sterman J (2021) System Dynamics Review 37(1):5-31.
- "Simulation-based estimation of the early spread of COVID-19 in Iran: actual versus confirmed cases." Ghaffarzadegan, N., Rahmandad, H. (2020) System Dynamics Review, 36(1):101-129
- (\*) at least these items are open access last time I checked

# Thanks!