Statistical model: Notation

I hope we could avoid using

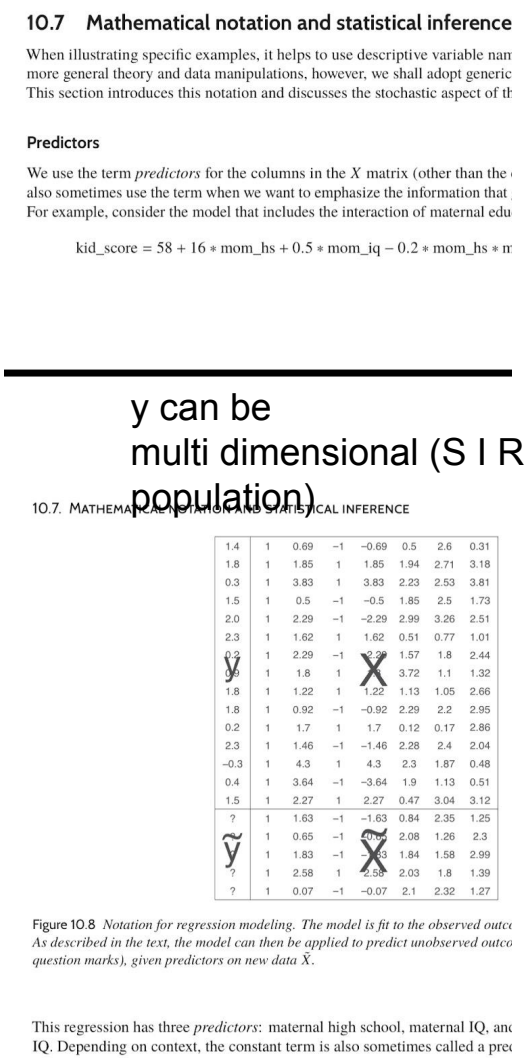
- data
- model
- variable
- parameter

at least not without any adjectives

Instead some suggestions:

- predictor for driving data/variable & assumed parameter
- state for target variable
- observed state for target variable data
- integrated result for ODE solution of target variable

Red in stan block: see 8



10.7 Mathematical notation and statistical inference

When illustrating specific examples, it helps to use descriptive variable nam
more general theory and data manipulations, however, we shall adopt generic
This section introduces this notation and discusses the stochastic aspect of th

Predictors

We use the term *predictors* for the columns in the X matrix (other than the
also sometimes use the term when we want to emphasize the information that
For example, consider the model that includes the interaction of maternal edu

$$\text{kid\_score} = 58 + 16 * \text{mom\_hs} + 0.5 * \text{mom\_iq} - 0.2 * \text{mom\_hs} * \text{m}$$

y can be
multi dimensional (S I R population)

1.4	1	0.69	−1	−0.69	0.5	2.6	0.31
1.8	1	1.85	1	1.85	1.94	2.71	3.18
0.3	1	3.83	1	3.83	2.23	2.53	3.81
1.5	1	0.5	−1	−0.5	1.85	2.5	1.73
2.0	1	2.29	−1	−2.29	2.99	3.26	2.51
2.3	1	1.62	1	1.62	0.51	0.77	1.01
0.2	1	2.29	−1	−2.29	1.57	1.8	2.44
1.9	1	1.8	1	1.8	3.72	1.1	1.32
1.8	1	1.22	1	1.22	1.13	1.05	2.66
1.8	1	0.92	−1	−0.92	2.29	2.2	2.95
0.2	1	1.7	1	1.7	0.12	0.17	2.86
2.3	1	1.46	−1	−1.46	2.28	2.4	2.04
−0.3	1	4.3	1	4.3	2.3	1.87	0.48
0.4	1	3.64	−1	−3.64	1.9	1.13	0.51
1.5	1	2.27	1	2.27	0.47	3.04	3.12
?	1	1.63	−1	−1.63	0.84	2.35	1.25
?	1	0.65	−1	−0.65	2.08	1.26	2.3
?	1	1.83	−1	−1.83	1.84	1.58	2.99
?	1	2.58	1	2.58	2.03	1.8	1.39
?	1	0.07	−1	−0.07	2.1	2.32	1.27

Figure 10.8 *Notation for regression modeling. The model is fit to the observed outco*
As described in the text, the model can then be applied to predict unobserved outco
question marks), given predictors on new data $\tilde{X}$.

This regression has three *predictors*: maternal high school, maternal IQ, and
IQ. Depending on context, the constant term is also sometimes called a pred

Stan syntax: Block

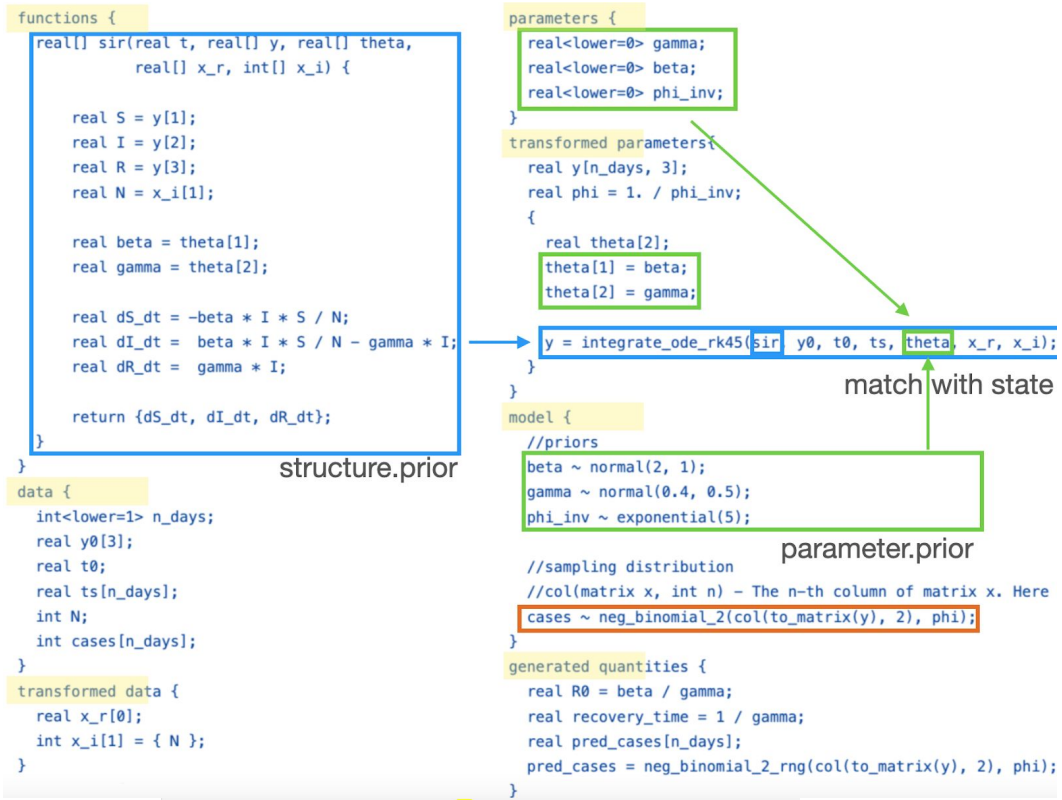
data:
- init_outcomes (y0)
- predictors (N)
- observed outcome (cases)
- time index (t0, ts)

tf data: predictors

param:
- coefficient parameter (beta, gamma)
- prior parameter (=hyper parameter)
- measurement scale parameter (phi)

tf param:
- outcome_dydt = f(outcome, t, theta, predictor)
- coded as "integrated_result = rk45(f, outcome, init_outcome, t, theta, predictor)"

model:
- coefficient + prior parameter
- observed outcome ~ integrated result




```
y = integrate_ode_rk45(sir, y0, t0, ts, theta, x_r, x_i);
```

with

- `sir`, the name of the function that returns the derivatives, f;
- `y0`, the initial condition;
- `t0`, the time of the initial condition;
- `ts`, the times at which we require the solution to be evaluated;
- `theta`, `x_r`, `x_i`, arguments to be passed to f.

GQ block for synthesis

- GQ block can be used to generate outcome variable which can be used as "observed outcome" in the following estimation step. The left is generation and the right os estimation.
- But here, the variability of assumed parameter is not considered as both use the same X [N]

```
data {
  int<lower=1> N;
  real X[N];
}

generated quantities {
  real beta;
  real alpha;
  real y[N];

  beta = normal_rng(0, 10);
  alpha = normal_rng(0, 10);

  for (n in 1:N)
    y[n] = normal_rng(X[n] * beta
+ alpha, 1.2);
}
```

```
data {
  int<lower=1> N;
  vector[N] X;
  vector[N] y;
}

parameters {
  real beta;
  real alpha;
}

model {
  beta ~ normal(0, 1);
  alpha ~ normal(0, 10);

  y ~ normal(X * beta + alpha, 1.
2);
}
```

Prior calibration overview