

Regularized Regression Models

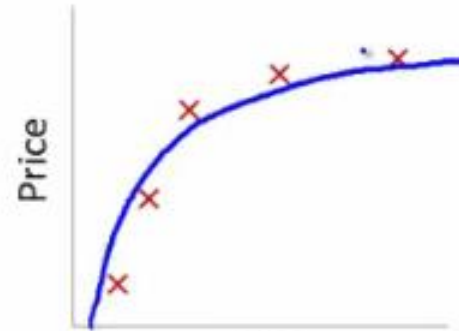
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Linear regression fitting example



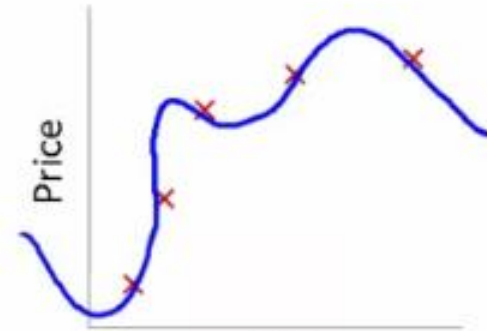
$$\theta_0 + \theta_1 x$$

High bias
(underfit)



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

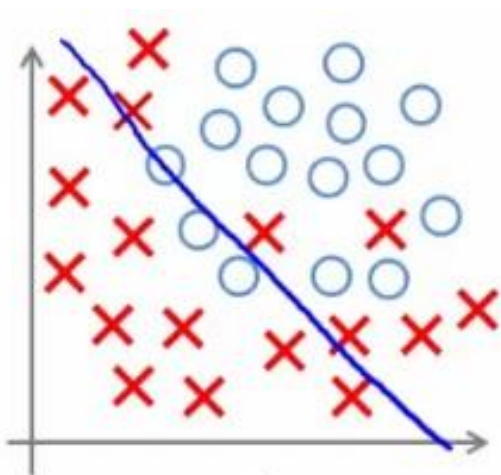
"Just right"



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

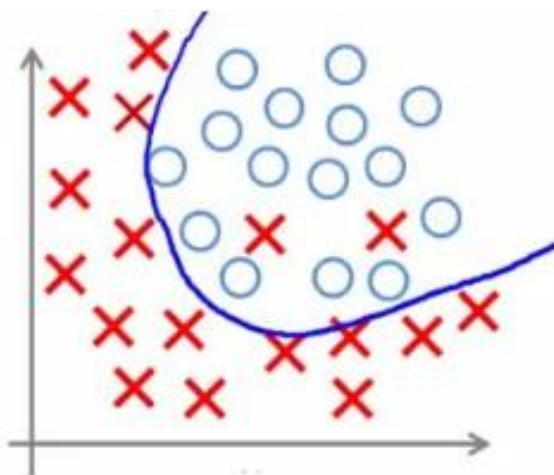
High variance
(overfit)

Logistic regression fitting example

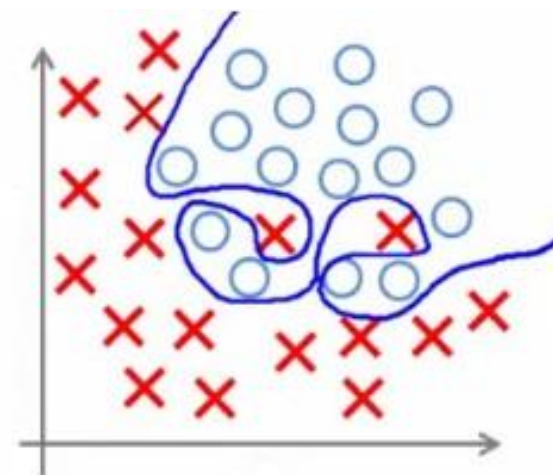


Under-fitting

(too simple to
explain the
variance)



Appropriate-fitting



Over-fitting

(forcefitting -- too
good to be true)

Overfitting

Overfitting when.....

- Complex model, too many features, not enough training samples.

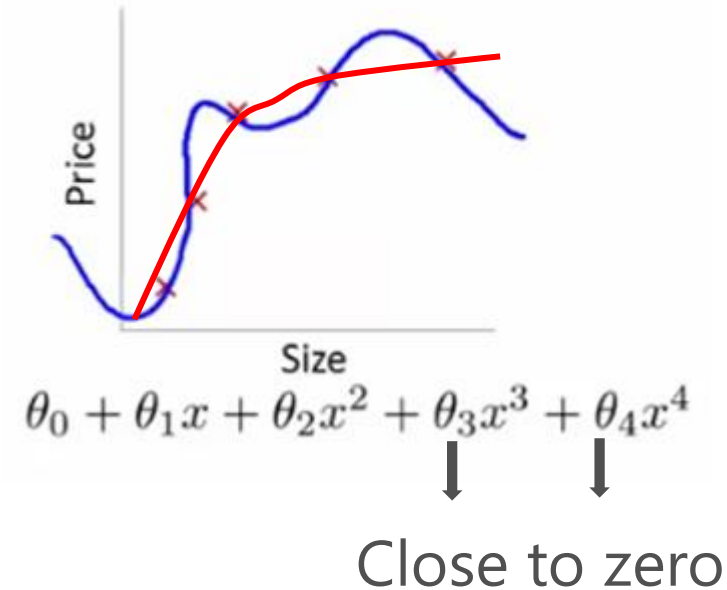
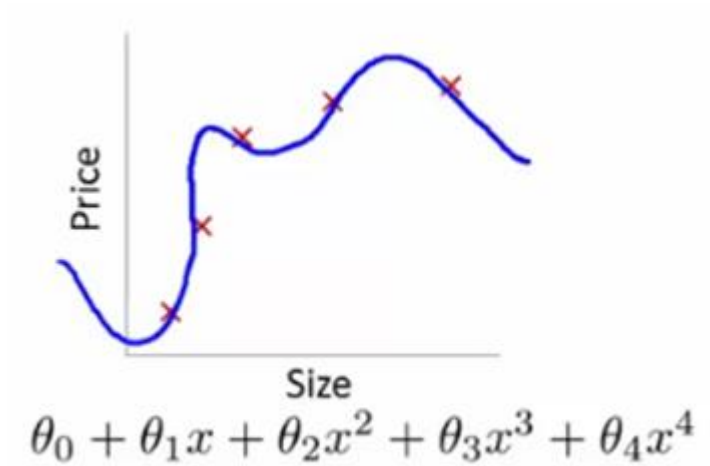
How to address overfitting??

- Go through each features to decide which to keep.
- Use model selection algorithm to automatically choose features.

Idea of Regularization

- Keep all the features, but reducing their magnitude of parameter effects in model.
- Shrink θ_j parameters

Regularized regression intuition



- Goal: To minimize cost function θ_j

$$\frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + 1000 \theta_3 + 1000 \theta_4$$

- Suppose we penalize and make θ_3 and θ_4 very small

Regularization

- Two common types of regularization in linear regression
- L2 regularization (a.k.a ridge regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \beta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d \beta_i^2$$

- L1 regularization (a.k.a lasso regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \beta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d |\beta_i|$$

Regularized-Ridge regression

Regularization by shrink θ_j smaller values, as a result

- “less complex” hypothesis function without eliminating features
- More protection from overfitting.

L2: Ridge regression

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

Regularized-Ridge regression

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- Goal 1: find the best fit
- Goal 2: keep parameter θ_j small
- λ is regularization parameter to controls a trade off

Regularized-Ridge regression

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- If λ is too large, θ_j become too small, as if features have no effect in predicting response.
- If λ is too small, θ_j are not regularized.

Questions?