

Ensemble Methods And Random Forests

Overview

■ Ensemble Methods

- Overview and rationale
- Boosting
- Bagging

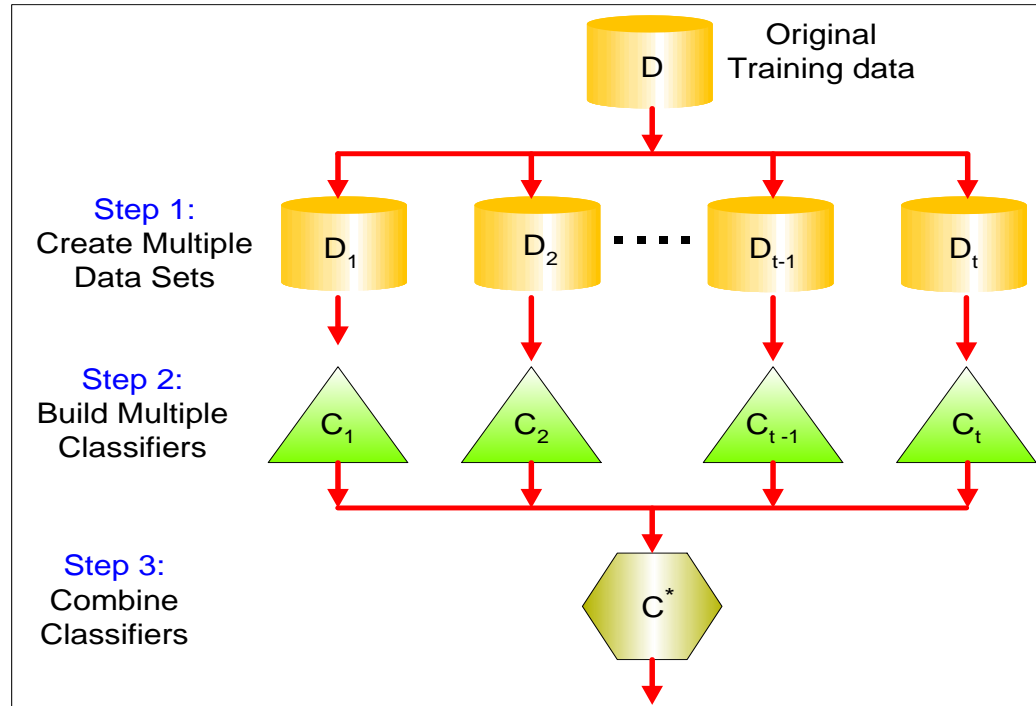
■ Random Forests

- Overview
- Some theory and mechanics
- Practical examples with R

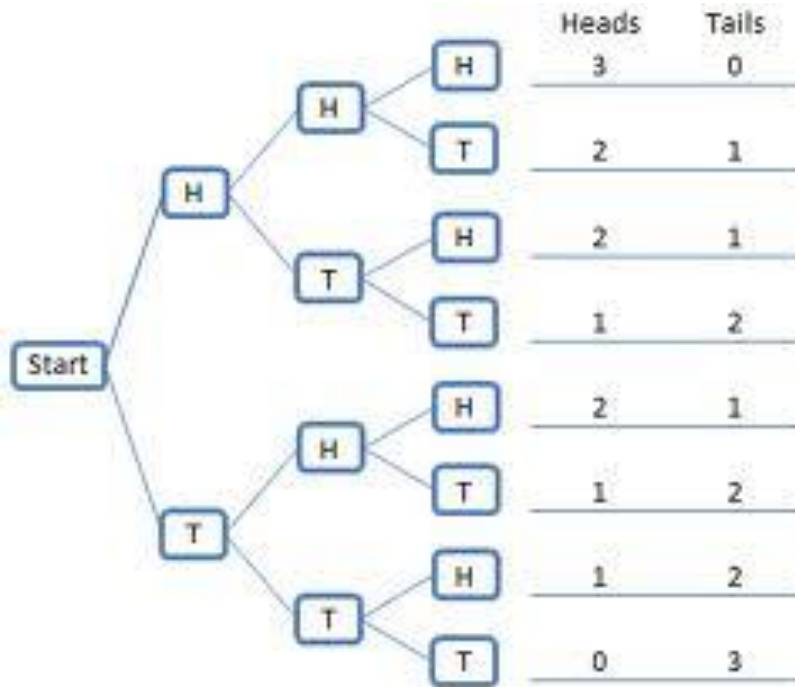
Ensemble Methods

- Improve classification accuracy by combining classifiers
- You can also have an ensemble of regression models

General Idea of Ensemble Methods



Digression – Binomial Distribution



Binomial Distribution

$$f(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Tossing A Biased Coin

What is the probability of 0,1,2,3,4,5,6 heads in six tosses of a biased coin?

$n = 6$

$k = 0,1,2,3,4,5,6$

$P=0.3$ (prob of head)

$$\Pr(0 \text{ heads}) = f(0) = \Pr(X = 0) = \binom{6}{0} 0.3^0 (1 - 0.3)^{6-0} \approx 0.1176$$

$$\Pr(1 \text{ head}) = f(1) = \Pr(X = 1) = \binom{6}{1} 0.3^1 (1 - 0.3)^{6-1} \approx 0.3025$$

$$\Pr(2 \text{ heads}) = f(2) = \Pr(X = 2) = \binom{6}{2} 0.3^2 (1 - 0.3)^{6-2} \approx 0.3241$$

$$\Pr(3 \text{ heads}) = f(3) = \Pr(X = 3) = \binom{6}{3} 0.3^3 (1 - 0.3)^{6-3} \approx 0.1852$$

$$\Pr(4 \text{ heads}) = f(4) = \Pr(X = 4) = \binom{6}{4} 0.3^4 (1 - 0.3)^{6-4} \approx 0.0595$$

$$\Pr(5 \text{ heads}) = f(5) = \Pr(X = 5) = \binom{6}{5} 0.3^5 (1 - 0.3)^{6-5} \approx 0.0102$$

$$\Pr(6 \text{ heads}) = f(6) = \Pr(X = 6) = \binom{6}{6} 0.3^6 (1 - 0.3)^{6-6} \approx 0.0007$$

Rolling a Die

- When rolling a die 100 times, what is the probability of rolling a "4" exactly 25 times?
- Number of trials $n = 100$
- Number of successes $k = 25$
- Probability of a '4' in a single roll = $1/6$

$$\binom{100}{25} \left(\frac{1}{6}\right)^{25} \left(1 - \left(\frac{1}{6}\right)\right)^{100-25}$$

R functions:

`dbinom(25,size=100,prob=1/6) = 0.009825882`

`Pbinom` is for calculating cdfs

Multiple Choice Test

- A test consists of 10 multiple choice questions with five choices for each question. As an experiment, you **guess** on each and every answer without even reading the questions.
 - What is the probability of getting **exactly 6** questions correct on this test?
 - What is the probability of getting **at least 6** questions correct on this test?
 - What is the probability of getting **less than 6** questions correct on this test?
 - What is the probability of getting **all 10** questions correct on this test?

Applications

- Number of life insurance holders who will claim in a given period
- Number of loan holders who will default in a certain period
- Number of false starts of a car in n attempts
- Number of faulty items in n samples from a production line
- **AND Ensemble Methods**

Why Does It Work?

- Suppose there are 25 base classifiers
 - Each classifier has error rate, $\varepsilon = 0.35$
 - Assume classifiers are independent
 - Probability that the ensemble classifier makes a wrong prediction:

$$\sum_{i=13}^{25} \binom{25}{i} \varepsilon^i (1 - \varepsilon)^{25-i} = 0.06$$

Examples of Ensemble Methods

Bagging

- All classifiers are created equal

Boosting

- **Not** all classifiers are created equal

Bagging

- Sampling with replacement

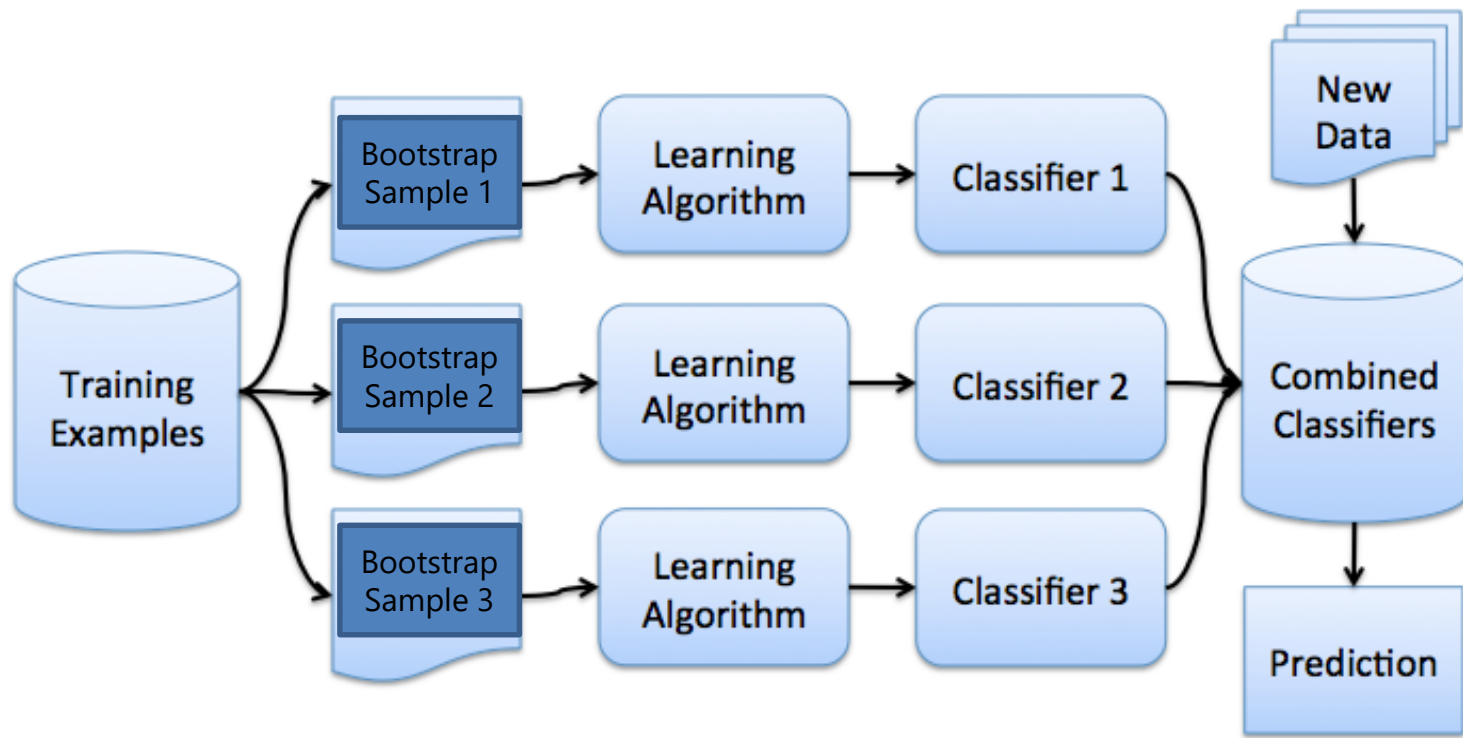
Original Data	1	2	3	4	5	6	7	8	9	10
Bagging (Round 1)	7	8	10	8	2	5	10	10	5	9
Bagging (Round 2)	1	4	9	1	2	3	2	7	3	2
Bagging (Round 3)	1	8	5	10	5	5	9	6	3	7

- Build classifier on each bootstrap sample
- For a training data of size n , each sample has probability $[1 - (1 - 1/n)^n]$ of being selected
- If n is large, this approximates to $1 - 1/e \sim 0.632$

Bagging

- Reduces variance in estimate
- Prevents overfitting
- Robust to outliers

Bagging



Boosting

- An iterative procedure to adaptively change distribution of training data by focusing more on previously misclassified records
 - Initially, all N records are assigned equal weights
 - Unlike bagging, weights may change at the end of boosting round

Boosting

- Records that are wrongly classified will have their weights increased
- Records that are classified correctly will have their weights decreased

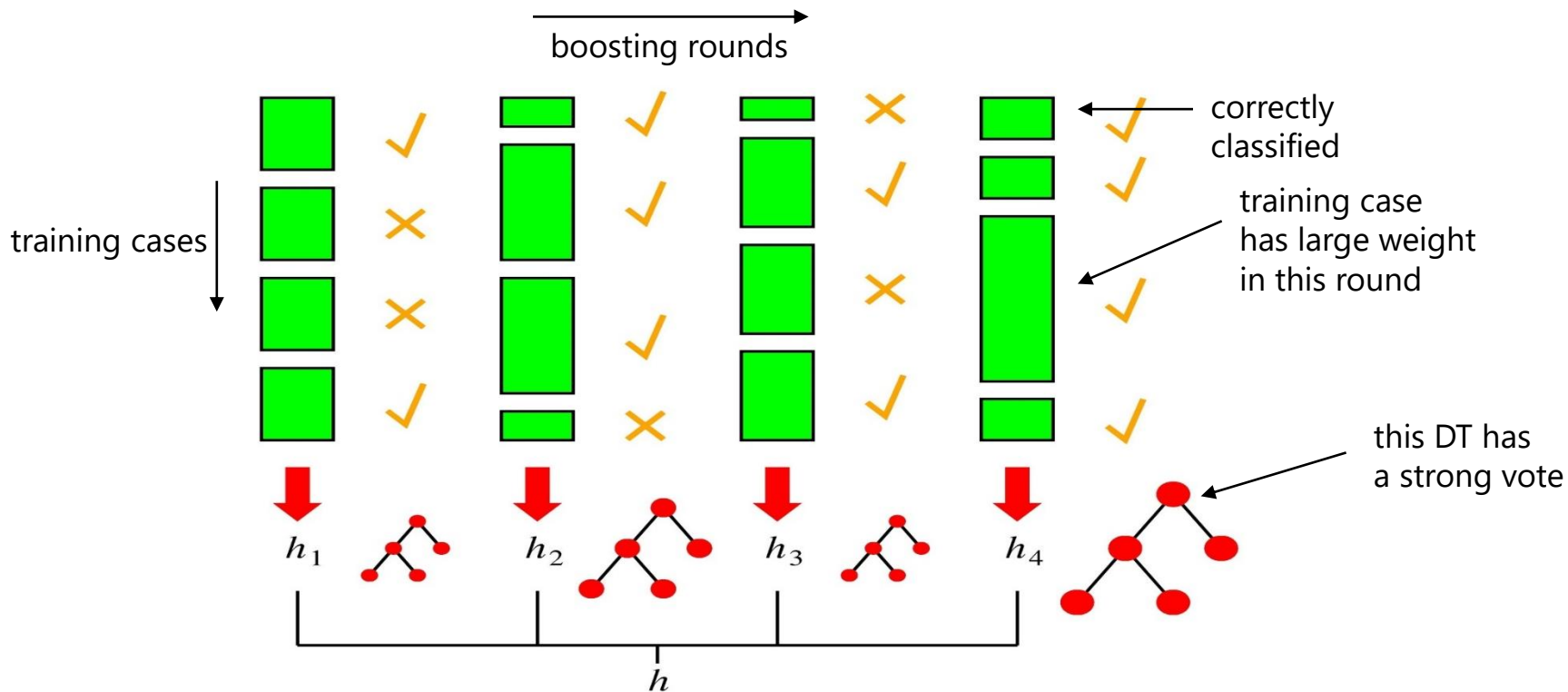
Original Data	1	2	3	4	5	6	7	8	9	10
Boosting (Round 1)	7	3	2	8	7	9	4	10	6	3
Boosting (Round 2)	5	4	9	4	2	5	1	7	4	2
Boosting (Round 3)	4	4	8	10	4	5	4	6	3	4

- Example 4 is hard to classify
- Its weight is increased, therefore it is more likely to be chosen again in subsequent rounds

Boosting Intuition

- We adaptively weigh each data case.
- Data cases which are wrongly classified get high weight (the algorithm will focus on them).
- Each boosting round learns a new (simple) classifier on the weighed dataset.
- These classifiers are weighed to combine them into a single powerful classifier.
- Classifiers that obtain low training error rate have high weight.
- We stop by monitoring a hold out set.

Boosting in a Picture



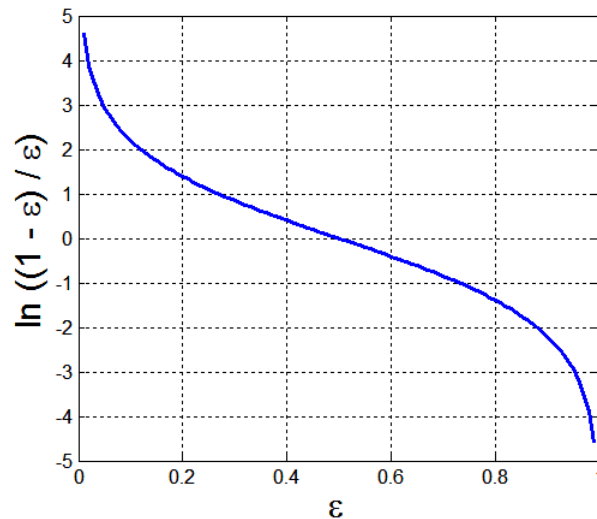
AdaBoost (Adaptive Boosting)

- Base classifiers: C_1, C_2, \dots, C_T
- Error rate [Weighted loss function]:

$$\varepsilon_i = \frac{1}{N} \sum_{j=1}^N w_j \delta(C_i(x_j) \neq y_j)$$

- Importance of a classifier:

$$\alpha_i = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_i}{\varepsilon_i} \right)$$



AdaBoost

- Weight update:
$$w_i^{(j+1)} = \frac{w_i^{(j)}}{Z_j} \begin{cases} \exp^{-\alpha_j} & \text{if } C_j(x_i) = y_i \\ \exp^{\alpha_j} & \text{if } C_j(x_i) \neq y_i \end{cases}$$

where Z_j is the normalization factor

- If any intermediate rounds produce error rate higher than 50%, the weights are reverted back to $1/n$ and the resampling procedure is repeated.
- Classification:
$$C^*(x) = \arg \max_y \sum_{j=1}^T \alpha_j \delta(C_j(x) = y)$$

Random Forests Classifier

- *Random forests are a **combination of tree predictors** such that each tree depends on the values of a random vector sampled independently and with same distribution for all trees in the forest.*

- Leo Breiman

What Is A Random Forest?

- An ensemble classifier using many decision tree models
- Can be used for classification or regression
- Accuracy and variable importance information is built-in

How Do Random Forests Work?

- A different subset of the training data are selected ($\sim 2/3$), with replacement, to train each tree
- Remaining training data (aka out-of-bag data or simply OOB) is used to estimate error and variable importance
- Class assignment is made by the number of votes from all of the trees, and for regression, the average of the results is used

Which Features Are Used For Learning?

- A randomly selected subset of variables is used to split each node
- The number of variables used is decided by the user (mtry parameter in R)
- A smaller subset produces less correlation (lower error rate)

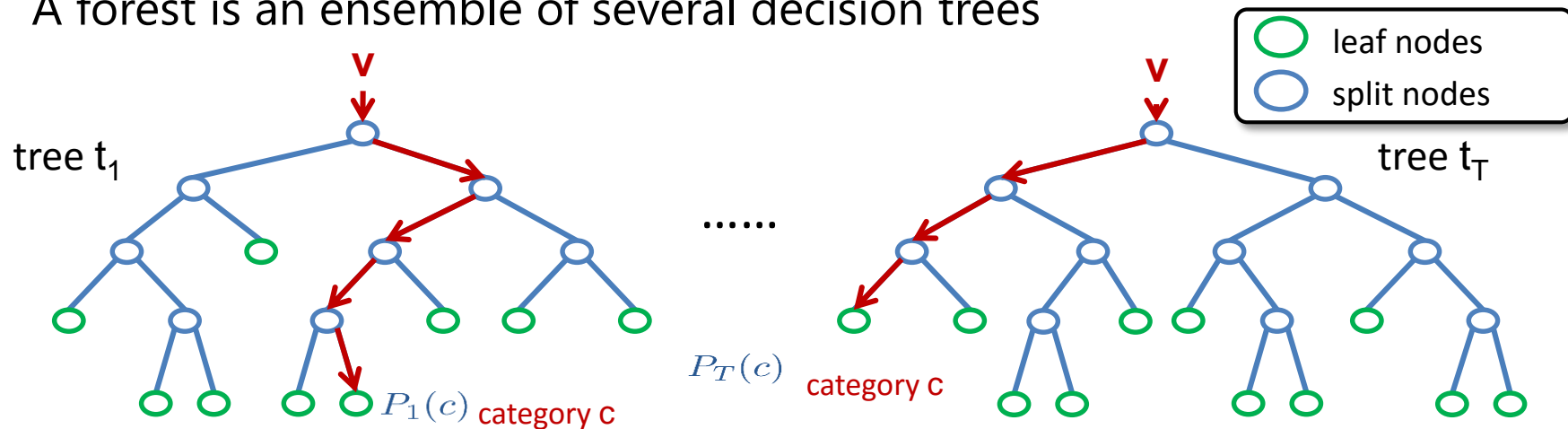
Rules of Thumb

■ Given:

- **N:** Total number of training data points
 - **M:** Number of features in training data
 - **m:** Number of features randomly selected for training each node
- Sample the data with replacement N times for building the training data for each tree.
 - $m \ll M$
 - Classification: $m = \sqrt{M}$
 - Regression: $m = M/3$

A Forest of Trees

A forest is an ensemble of several decision trees



Classification is
$$P(c|\mathbf{v}) = \frac{1}{T} \sum_{t=1}^T P_t(c|\mathbf{v})$$

[Amit & Geman 97]
[Breiman 01]
[Lepetit *et al.* 06]

Learning a Forest

- Dividing training examples into T subsets **improves generalization**
 - Reduces **memory requirements & training time**
- Train each decision tree t on subset I_t
 - Same decision tree learning as before
- Multi-core friendly (GPU implementation)

Implementation Details

- How many features and thresholds to try?
 - Just one = “extremely randomized” [Geurts *et al.* 06]
 - Few → fast training, may under-fit, may be too deep
 - Many → slower training, may over-fit
- When to stop growing the tree?
 - Maximum depth
 - Minimum entropy gain
 - Delta class distribution
 - Pruning

Common Pitfall

A Random Forest and a Boosted Decision Tree
are not the same

randomForest

R package for Random Forest

Setting up randomForest

Installation

```
> install.packages('randomForest')
```

Loading in R environment

```
> library(randomForest)
```

Documentation:

<http://cran.r-project.org/web/packages/randomForest/randomForest.pdf>

Obtaining the model

```
> iris.rf <- randomForest(  
  Species ~ .,  
  data=iris,  
  importance=TRUE,  
  proximity=TRUE  
)
```

Printing the model

```
> print(iris.rf)
```

```
# Type of random forest: classification
```

Number of trees: 500

```
# No. of variables tried at each split: 2
```

```
# OOB estimate of error rate: 4.67%
```

```
# Confusion matrix: setosa versicolor virginica class.error
```

```
# setosa      50      0      0      0.00
```

```
# versicolor  0      47      3      0.06
```

```
# virginica   0      4      46      0.08
```

Restricting Tree Size

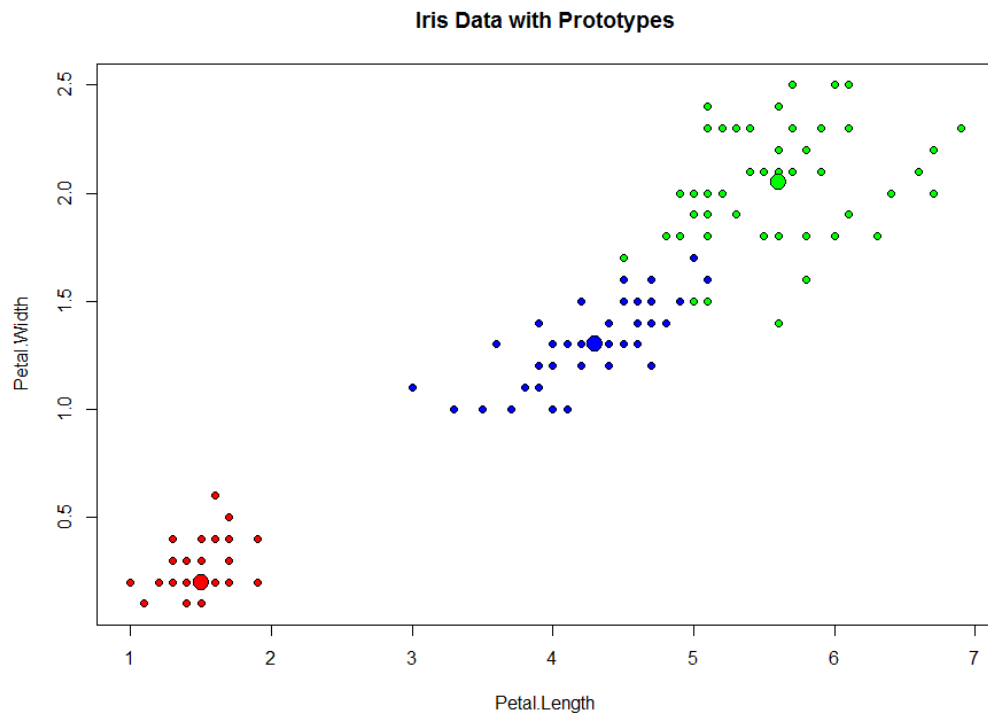
```
(treesize(randomForest(Species ~ .,  
  data=iris,  
  maxnodes=4,  
  ntree=30))))
```

Finding Class Centers

classCenter function

```
> data(iris)
> iris.rf <- randomForest(iris[,-5], iris[,5], prox=TRUE)
> iris.p <- classCenter(iris[,-5], iris[,5], iris.rf$prox)
> plot(iris[,3], iris[,4],
      pch=21, xlab=names(iris)[3], ylab=names(iris)[4],
      bg=c("red", "blue", "green")[as.numeric(factor(iris$Species))],
      main="Iris Data with Prototypes")
> points(iris.p[,3], iris.p[,4], pch=21, cex=2,
      bg=c("red", "blue", "green"))
```

Finding Class Centers



Combining Trees

You can combine two or more random forests into one.

The confusion, err.rate, mse, and rsq components will be NULL

```
> data(iris)
> rf1 <- randomForest(Species ~ ., iris, ntree=50,
norm.votes=FALSE)
> rf2 <- randomForest(Species ~ ., iris, ntree=100,
norm.votes=FALSE)
> rf3 <- randomForest(Species ~ ., iris, ntree=150, mtry=3,
norm.votes=FALSE)
> rf.all <- combine(rf1, rf2, rf3)
> print(rf.all)
```

Combining Trees

```
> print(rf.all)
```

Call:

```
randomForest(formula = Species ~ ., data = iris,  
ntree = 50, norm.votes = FALSE)
```

Type of random forest:

classification Number of trees: 150

No. of variables tried at each split: 2

Variable Importance and Gini

```
> data(iris)
> iris.rf <- randomForest(Species ~., data=iris, importance=TRUE,
  proximity=TRUE)
> round(importance(iris.rf), 2)
```

	setosa	versicolor	virginica	MeanDecreaseAccuracy	MeanDecreaseGini
Sepal.Length	6.28	8.88	7.11	10.48	9.26
Sepal.Width	4.87	0.77	4.85	5.17	2.33
Petal.Length	21.48	33.88	28.44	33.95	42.97
Petal.Width	22.96	32.47	32.10	34.60	44.65

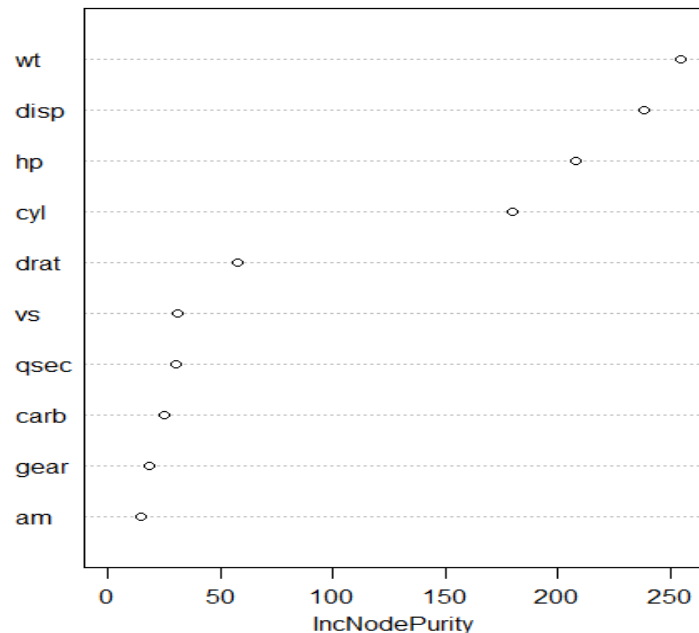
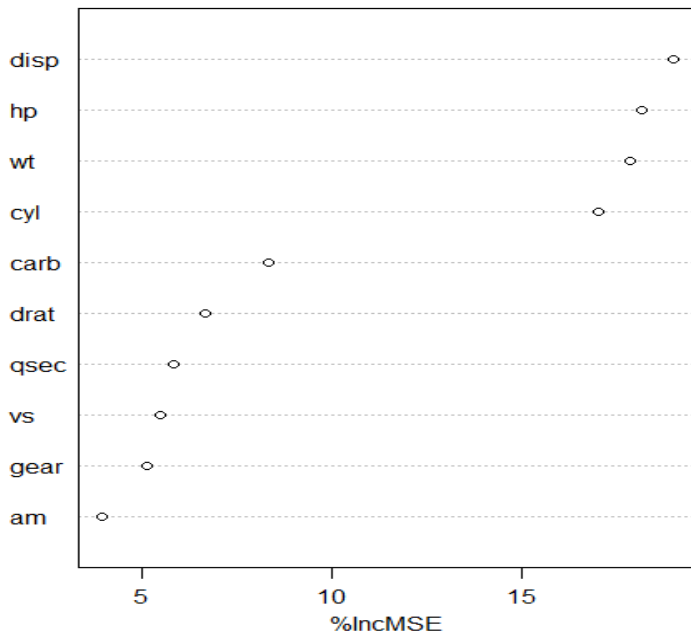
Important Factors In Determining Car Mileage

```
> set.seed(4543)
> data(mtcars)
> mtcars.rf <- randomForest(
  mpg ~ .,
  data=mtcars,
  ntree=1000,
  keep.forest=FALSE,
  importance=TRUE
)
> varImpPlot(mtcars.rf)
```

Index	Attribute Name	Description
[1]	mpg	Miles/(US) gallon
[2]	cyl	Number of cylinders
[3]	disp	Displacement (cu.in.)
[4]	hp	Gross horsepower
[5]	drat	Rear axle ratio
[6]	wt	Weight (lb/1000)
[7]	qsec	1/4 mile time
[8]	vs	V/S
[9]	am	Transmission (0 = automatic, 1 = manual)
[10]	gear	Number of forward gears
[11]	carb	Number of carburetors

Important Factors In Determining Car Mileage

mtcars.rf



Regression With Random Forests

```
> set.seed(131)
> ozone.rf <-
  randomForest(Ozone ~
    .,
    data=airquality,
    mtry=3,
    importance=TRUE,
    na.action=na.omit)
> print(ozone.rf)
#Impute Missing Values by median/mode.
> ozone.rf <-
  na.roughfix(ozone.rf
)
```

	Ozone	Solar.R	Wind	Temp	Month	Day
1	41	190	7.4	67	5	1
2	36	118	8	72	5	2
3	12	149	12.6	74	5	3
4	18	313	11.5	62	5	4
5	NA	NA	14.3	56	5	5
6	28	NA	14.9	66	5	6
7	23	299	8.6	65	5	7
8	19	99	13.8	59	5	8
9	8	19	20.1	61	5	9
10	NA	194	8.6	69	5	10
11	7	NA	6.9	74	5	11
12	16	256	9.7	69	5	12
13	11	290	9.2	66	5	13
14	14	274	10.9	68	5	14
15	18	65	13.2	58	5	15

Prediction

>

```
predict(ozone.rf, data=airquality  
)
```

Resources

Random Forests Homepage

http://www.stat.berkeley.edu/~breiman/RandomForests/cc_home.htm

IRIS Data

IRIS data ships with R. You can learn more about IRIS data here:

<http://archive.ics.uci.edu/ml/datasets/Iris>

QUESTIONS