Linear Regression

Overview

- Difference between Regression and Classification
- Linear Regression
  - Motivating Example: Predicting Housing Prices
  - Hypothesis function
  - Cost Function
  - Gradient Descent
- Applying linear regression for hand digit recognition

Supervised Learning

- Data: $D = \{ d_1, d_2, d_3, \ldots\ldots\ldots, d_n \}$ a set of n examples

  $d_i = < x_i, y_i >$

x_i

- Input vector
- Independent variables
- Explanatory variables
- Features
- Predictors

y_i

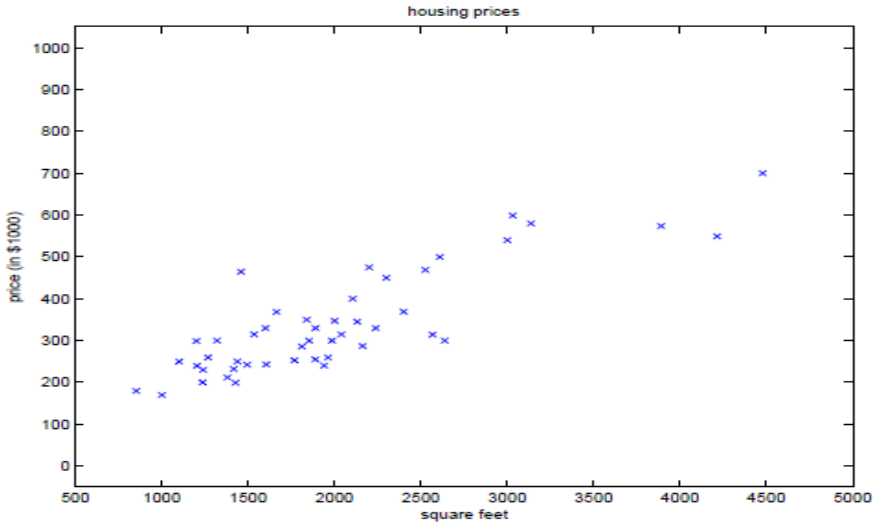
- Output scalar
- Dependent variables
- Response
- Outcome

Supervised Learning

- Regression: X discrete or continuous ⟶ Y is continuous
  - Example: Prices, Weight, Height, signal measurement, temperature etc.
- Classification : X discrete or continuous⟶Y is discrete
- Objective: learn the mapping of f : Xi ⟶ Yi

Predict Housing Prices

Living area (feet^2)	Price (1000$s)
2104	400
1600	330
2400	369
1416	232
3000	540
⋮	⋮



Can we learn to predict the price when the price of the house is not in the dataset?

Living Area = 3500 sq. ft. **Price = ?**

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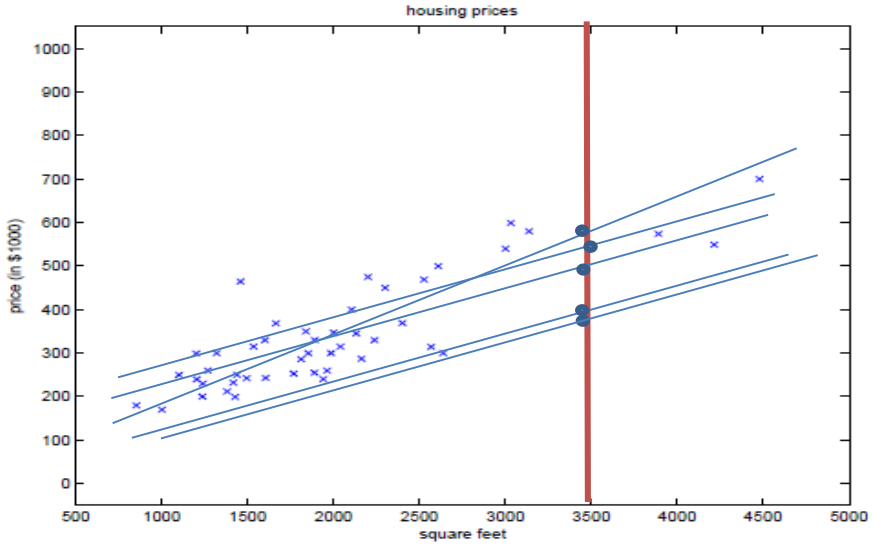
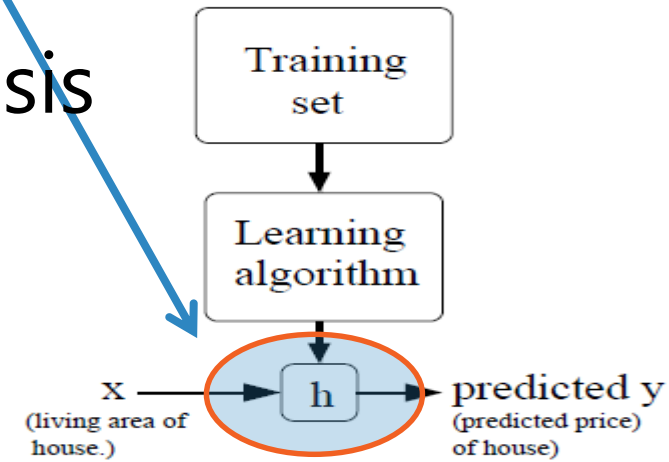
Predict Housing Prices

- Can we learn to predict the price when the price of the house is not in the dataset?
- Living Area = 3500 sq. ft. Price = ?
- Use the line that is somewhere in the middle
- How do we define somewhere in the middle?



Training Process In Linear Regression

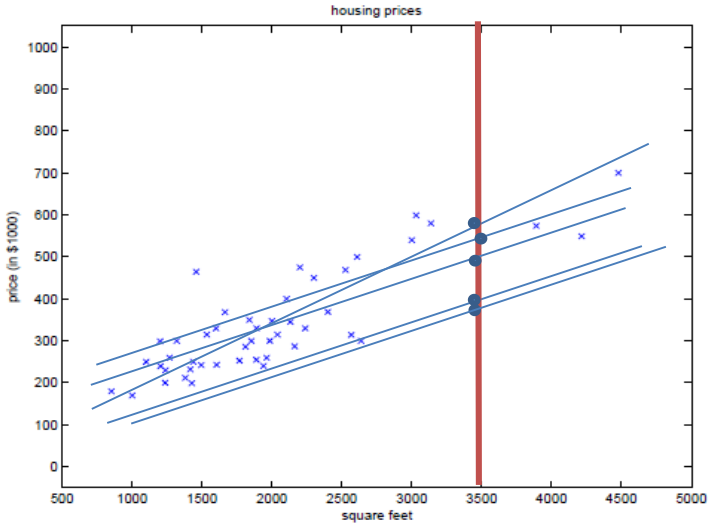
- Objective: learn the mapping of f : Xi ⟶ Yi
- Finding h is the goal here
- h is known as the hypothesis



Predict Housing Prices

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Living area (feet^2)		Price (1000$s)	
$x^{(1)}$	2104	400	$y^{(1)}$
$x^{(2)}$	1600	330	$y^{(2)}$
$x^{(3)}$	2400	369	$y^{(3)}$
$x^{(4)}$	1416	232	$y^{(4)}$
$x^{(5)}$	3000	540	$y^{(5)}$
.	⋮	⋮	.
.			.
.			.
$x^{(m)}$			$y^{(m)}$



m = Total number of training examples

θ_j = Parameters

Hypothesis Function-Linear Regression

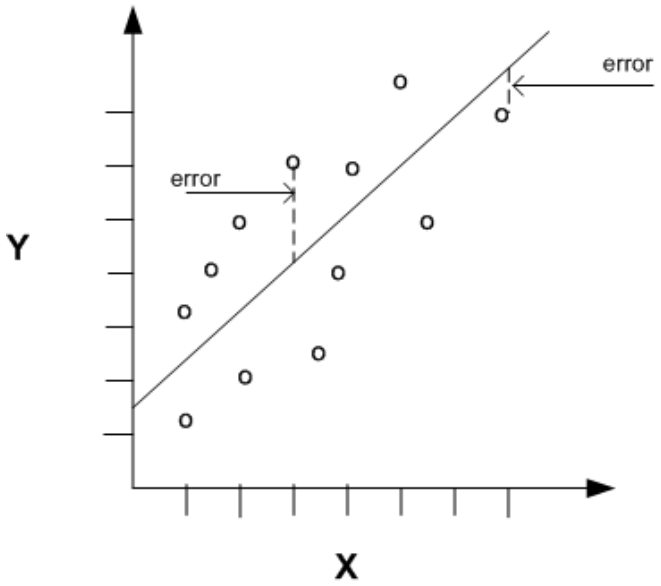
$$h_\theta(x) = \theta_0 + \theta_1 x$$

$h_\theta(x^i)$ is the prediction of the hypothesis h_θ for given input x^i
y^i is the target values (what we would call labels in a classification problem)

We want the prediction $h_\theta(x^i)$ to be as close to the true label y^i as possible.
How do we do that?

Cost Function

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$



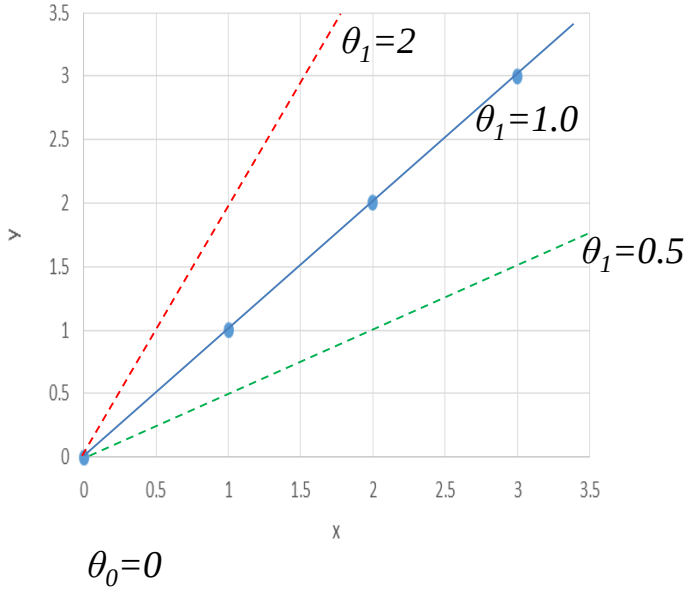
- In Plain English
  - find a linear line that is as close to the actual outputs as possible.
- Mathematically
  - Find linear function of X that minimizes the sum of squared residuals from Y.
  - a.k.a loss function

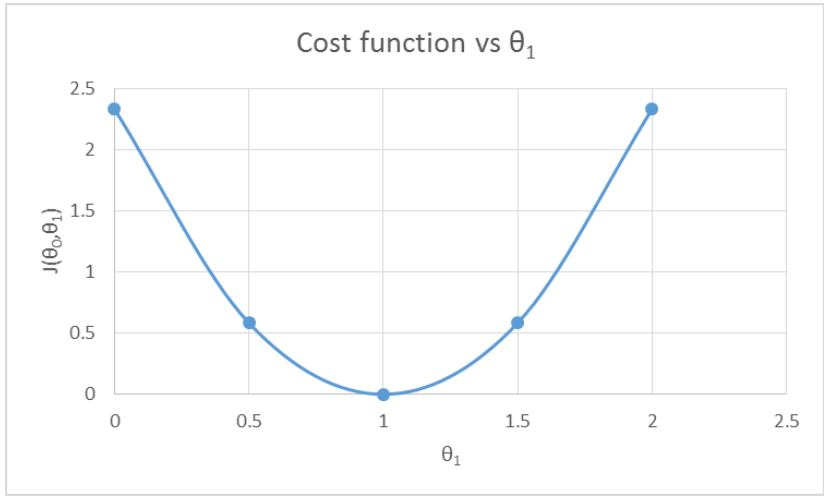
Hypothesis and Cost Function

Supposed:

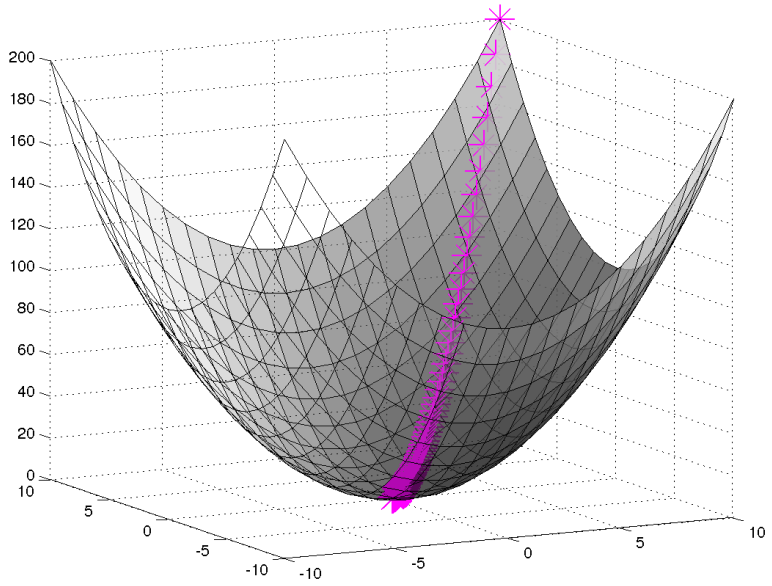
$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$





The Cost Function



Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

Parameters: θ_0, θ_1

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Gradient Descent Algorithm

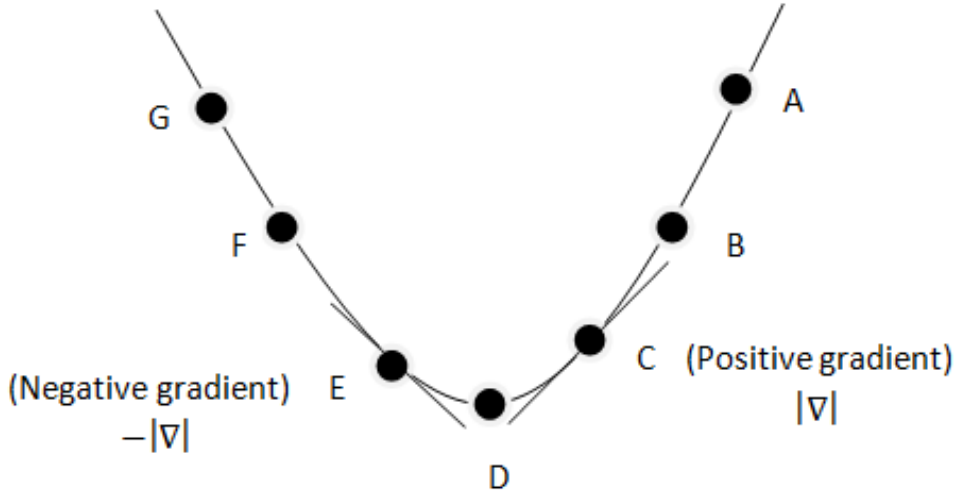
- Goal : $\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0,\theta_1)$
- Gradient descent starts with some initial θ and then performs an update for each value θ_j

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$

- Repeat until θ converges

Gradient Descent Algorithm

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$



Gradient Descent Algorithm

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$ *For j=0 and j= 1*

Repeat until converge {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

}

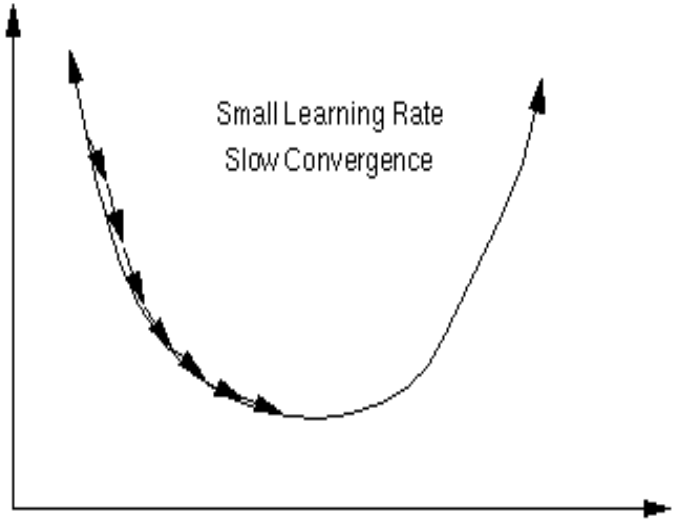
Gradient Descent Algorithm

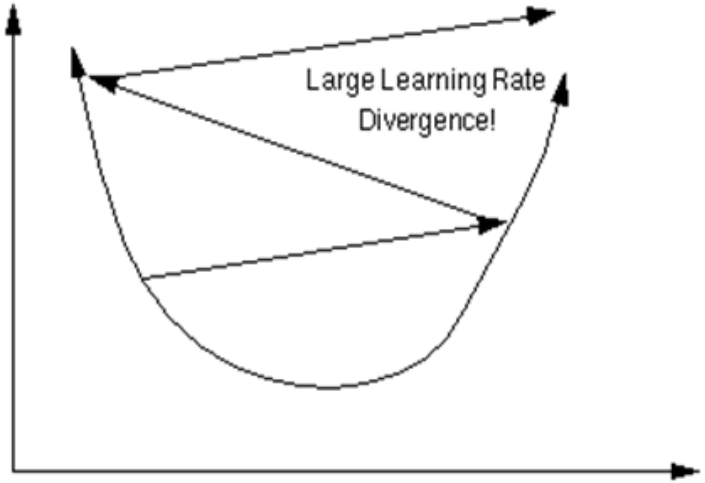
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$

- α is known as the learning rate
- Each time the algorithm takes a step in the direction of the steepest, $J(\theta)$ decreases.
- α determines how quickly or slowly the algorithm will converge to a solution

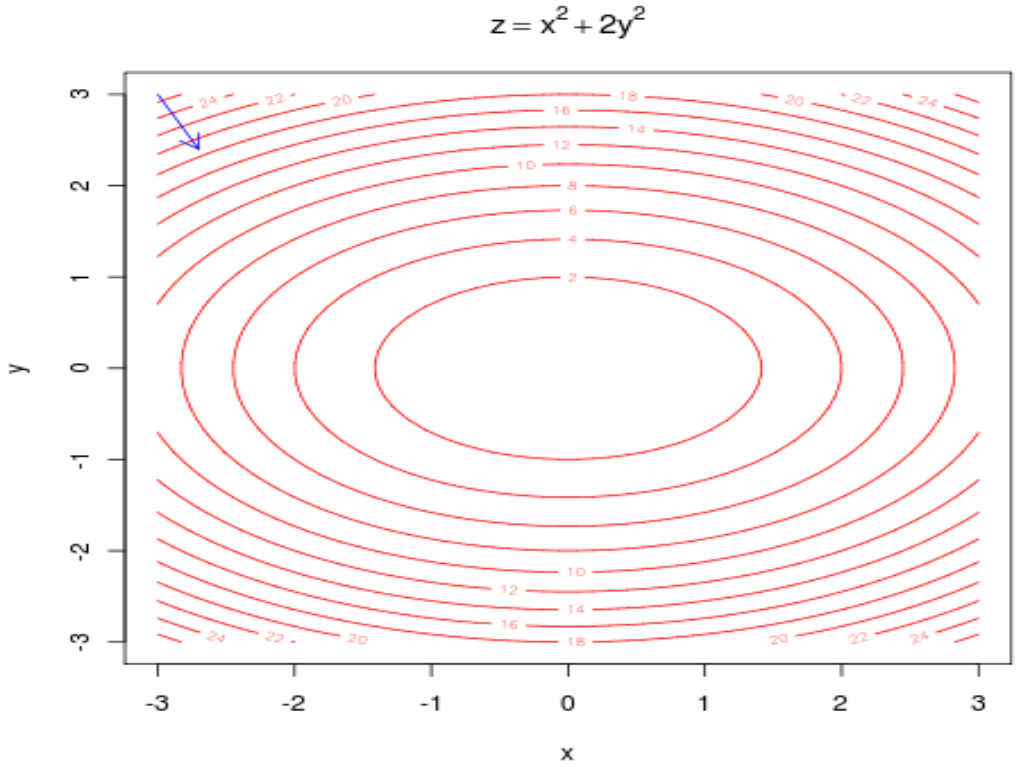
Gradient Descent Algorithm

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta).$$





Digression - Gradient Descent Intuition



Visualize
animation in R

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Multivariate Linear Regression

Predict Housing Prices – With Two Features

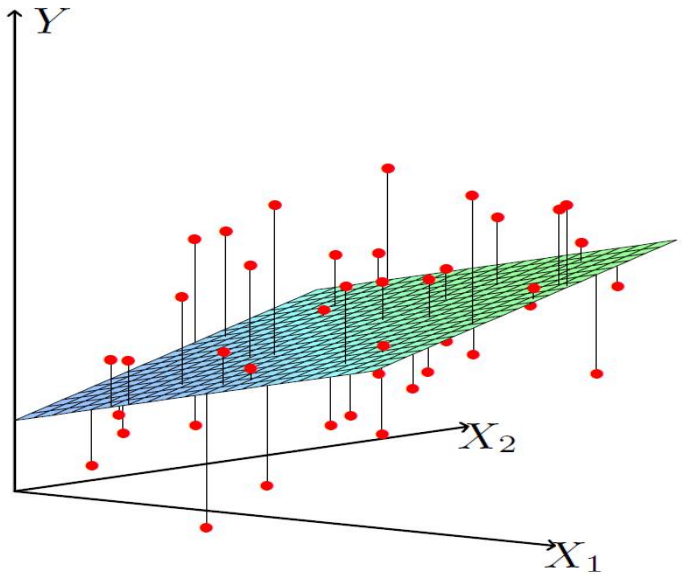
$x^i{}_1$ Living area (feet^2)	$x^i{}_2$ #bedrooms	y^i Price (1000$s)
2104	3	400
1600	3	330
2400	3	369
1416	2	232
3000	4	540
⋮	⋮	⋮

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

A linear function is just one of the choices to approximate the target variable.

θ is the space of linear functions mapping the space of input variables to the output/target variables

Predict Housing Prices – With Two Features



When we have two features, the linearity is in plane instead of line.

Algebraic Notation

The function approximating the target variable *y* is given by:

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\theta^T = [\theta_0\, \theta_1 \theta_2]$$

as

$$h(x) = \sum_{i=0}^{n} \theta_i x_i = \theta^T x,$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$$

$$\theta^T x = [\theta_0\, \theta_1 \theta_2] \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \sum_{i=0}^{n} \theta_i x_i$$

Batch Gradient Descent

For ONE training examples, we get the following update rule:

$$\theta_j := \theta_j + \alpha \left(y^{(i)} - h_\theta(x^{(i)})\right) x_j^{(i)}.$$

What do we observe here about the magnitude of the update?

For ALL training examples, we get the following update rule:

Repeat until convergence {

$$\theta_j := \theta_j + \alpha \sum_{i=1}^{m} \left(y^{(i)} - h_\theta(x^{(i)})\right) x_j^{(i)} \qquad \text{(for every } j\text{)}.$$

}

Stochastic Gradient Descent

- Consider the following algorithm:

Loop {

for i=1 to m, {

$$\theta_j := \theta_j + \alpha \left(y^{(i)} - h_\theta(x^{(i)})\right) x_j^{(i)} \qquad \text{(for every } j\text{)}.$$

}

}

- This algorithm updates the parameters θ_j using each training example instead of all training examples.
- If the training set is big i.e., m is large, this technique converges quicker than batch gradient descent.
- Stochastic gradient descent may oscillate around the minimum of $J(\theta)$ and may not completely converge

Batch vs. Stochastic Gradient Descent

Batch Gradient Descent

Repeat until convergence {

$$\theta_j := \theta_j + \alpha \sum_{i=1}^{m} \left(y^{(i)} - h_\theta(x^{(i)})\right) x_j^{(i)} \qquad \text{(for every } j\text{)}.$$

}

- To update each parameter value, scan through the whole training data
- Converges to the minimum value slowly
- Preferred for small datasets

Stochastic Gradient Descent

Loop {

for i=1 to m, {

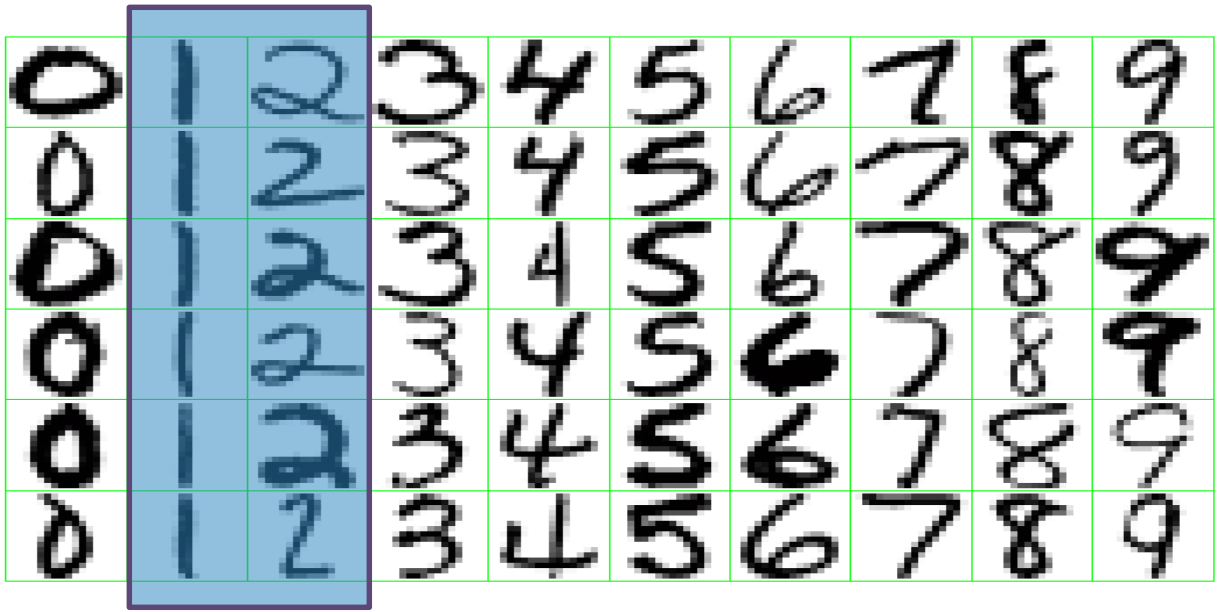
$$\theta_j := \theta_j + \alpha \left(y^{(i)} - h_\theta(x^{(i)})\right) x_j^{(i)} \qquad \text{(for every } j\text{)}.$$

}

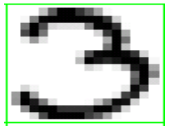
}

- Update the parameter values with one training example at a time
- Converges to the 'proximity' of minimum value fast but may keep oscillating near the minimum
- Preferred for large datasets

Example: Handwritten Digit Recognition



Extracting Features For Learning



$\{x_1, x_2, x_3 \ldots\ldots\ldots X_{256}, y = \text{'three'}\}$

- Each xi corresponds to a feature value in the image
- y is a label of the training data; can be numeric or categorical, '3' or 'three'
- Each image is converted to row vectors and the appropriate learning algorithm is used
- Convention
  - x_i represents the ith feature in a training sample
  - y represents the label for the training sample

QUESTIONS