

# Regularized Regression Models

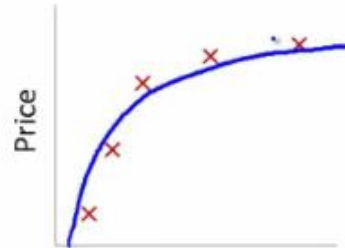
Data Science Dojo

# Linear Regression Fitting Example



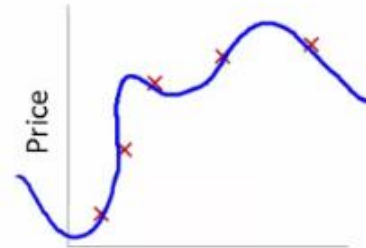
$$\theta_0 + \theta_1 x$$

High bias  
(underfit)



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

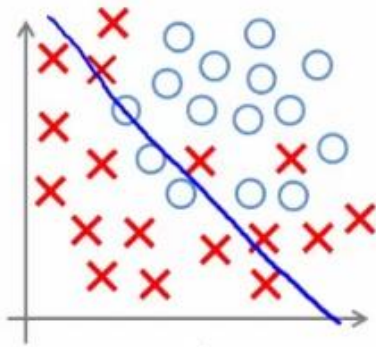
"Just right"



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

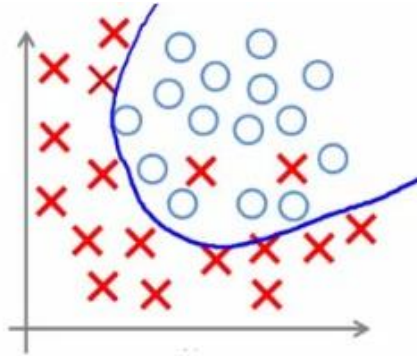
High variance  
(overfit)

# Logistic Regression Fitting Example

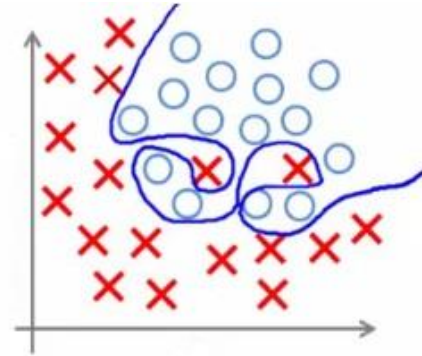


**Under-fitting**

(too simple to  
explain the  
variance)



**Appropriate-fitting**



**Over-fitting**

(forcefitting -- too  
good to be true)

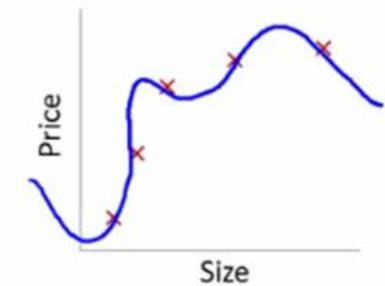
# Overfitting

- Overfitting when:
  - Complex model, too many features, not enough training samples
- How to address overfitting
  - Go through each feature to decide which to keep
  - Use model selection algorithm to automatically choose features

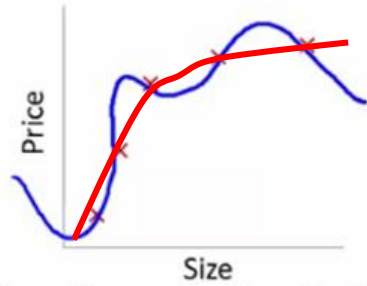
# Idea of Regularization

- Keep all the features, but reduce their magnitude of parameter effects in model
- Shrink  $\theta_j$  parameters

# Regularized regression intuition



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Close to zero

- Goal: To minimize cost function  $\theta_j$

$$\frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + 1000 \theta_3 + 1000 \theta_4$$

- Suppose we penalize and make  $\theta_3$  and  $\theta_4$  very small

# Regularization

- Two common types of regularization in linear regression

- L2 regularization (a.k.a ridge regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \theta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d \theta_i^2$$

- L1 regularization (a.k.a lasso regression)

$$\sum_{j=1}^N (y_j - \sum_{i=0}^d \theta_i \cdot x_i)^2 + \lambda \sum_{i=1}^d |\theta_i|$$

# Regularized-Ridge Regression

- Regularization by shrink  $\theta_j$  smaller values, as a result
  - “less complex” hypothesis function without eliminating features
  - More protection from overfitting

- L2: Ridge regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$
$$\min_{\theta} J(\theta)$$



# Regularized-Ridge Regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- Goal 1: find the best fit
- Goal 2: keep parameter  $\theta_j$  small
- $\lambda$  is regularization parameter to controls a trade off

# Regularized-Ridge Regression

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

- If  $\lambda$  is too large,  $\theta_j$  become too small, as if features have no effect in predicting response.
- If  $\lambda$  is too small,  $\theta_j$  are not regularized.

# QUESTIONS