Evaluating Regression Models

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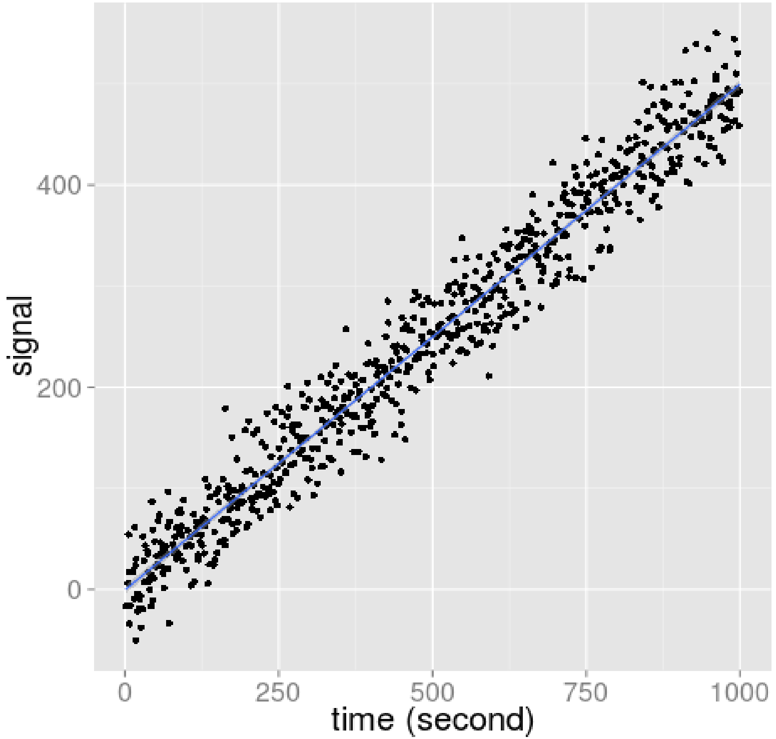
A simple example

- A **linear regression model** is built based on the training data set:

  signal =
  h(time) = -0.498 + 0.500 x time

How to evaluate this model?

What are the evaluation metrics for such regression models?



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Basic metrics

Root-mean-square deviation (RMSE)

Mean Absolute Error(MAE)

Coefficient of determination (R Squared)

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RMSE

Formula: $$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - h(\boldsymbol{x}_i))^2}{n}}$$

for data set i, $\boldsymbol{x}_{\boldsymbol{i}}$ is a vector of all the predictors,

y_i is the corresponding response;

$h(\boldsymbol{x}_i)$ is the prediction using a certain model;

$\boldsymbol{n}$ is the number of data in the test data set.

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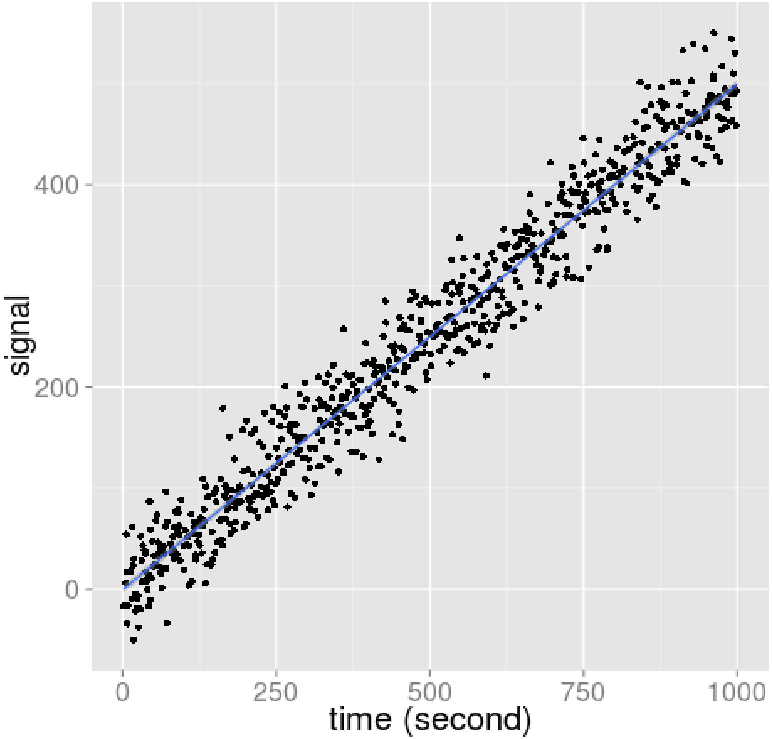
RMSE

- Back to our simple **linear regression model**:

  signal =
  h(time) = -0.498 + 0.500 x time

  700 data items are used to **train** this model;
  Another 300 data items in **testing data set** are used to measure this model.



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RMSE

Using the testing data set with n = 300 data items:

$$y_i: \; -5.905, \; 48.261, \; 4.115, \; -8.370, \; 42.222, \; 10.320, \; \ldots$$

$$h_{(x_i)}: \; 0.502, \; 1.003, \; 1.503, \; 4.505, \; 5.005, \; 6.507, \; \ldots$$

$$h_{(x_i)} - y_i: \; 6.408, \; -47.259, \; -2.612, \; 12.875, \; -37.217, \; -3.814, \; \ldots$$

$$\left(h_{(x_i)} - y_i\right)^2: \; 41.059, \; 2233.367, \; 6.823, \; 165.766, \; 1385.102, \; 14.546, \; \ldots$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(y_i - h(\boldsymbol{x}_i)\right)^2}{n}} = 29.17852$$

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MAE

Formula: $$MAE = \frac{\sum_{i=1}^{n} |residual_i|}{n} = \frac{\sum_{i=1}^{n} |y_i - h(\boldsymbol{x}_i)|}{n}$$

for data set i, $\boldsymbol{x}_i$ is a vector of all the predictors,
y_i is the corresponding response;
$h(\boldsymbol{x}_i)$ is the prediction using a certain model;
n is the number of data in the test data set.

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MAE

Using the testing data set with n = 300 data items:

$$y_i:\ -5.905,\ 48.261,\ ,\ 4.115,\ -8.370,\ 42.222,\ 10.320,\ \ldots$$

$$h_{(x_i)}:\ 0.502,\ 1.003,\ 1.503,\ 4.505,\ 5.005,\ 6.507,\ \ldots$$

$$h_{(x_i)}-y_i:\ 6.408,\ -47.259,\ -2.612,\ 12.875,\ -37.217,\ -3.814,\ \ldots$$

$$|h_{(x_i)}-y_i|:\ 6.408,\ 47.259,\ 2.612,\ 12.875,\ 37.217,\ 3.814,\ \ldots$$

$$MAE = \frac{\sum_{i=1}^{n}|y_i - h(\boldsymbol{x}_i)|}{n} = 23.02094$$

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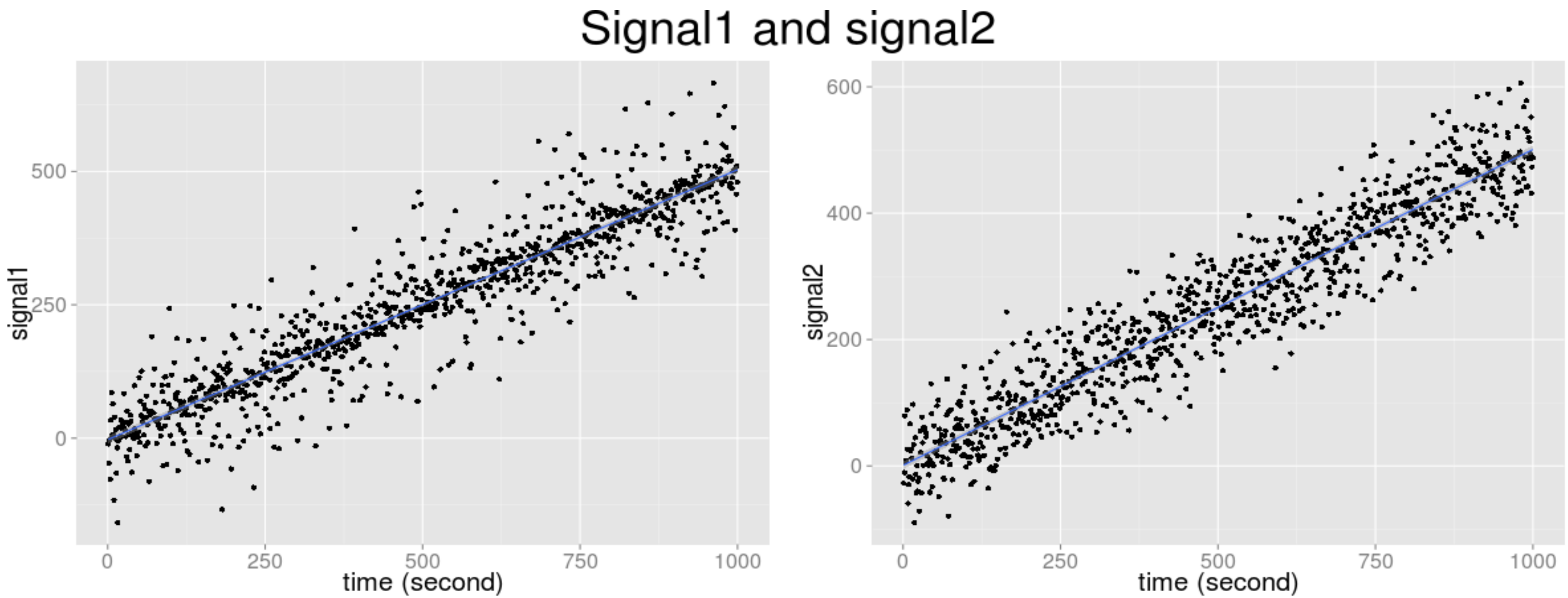
MAE *vs.* RMSE



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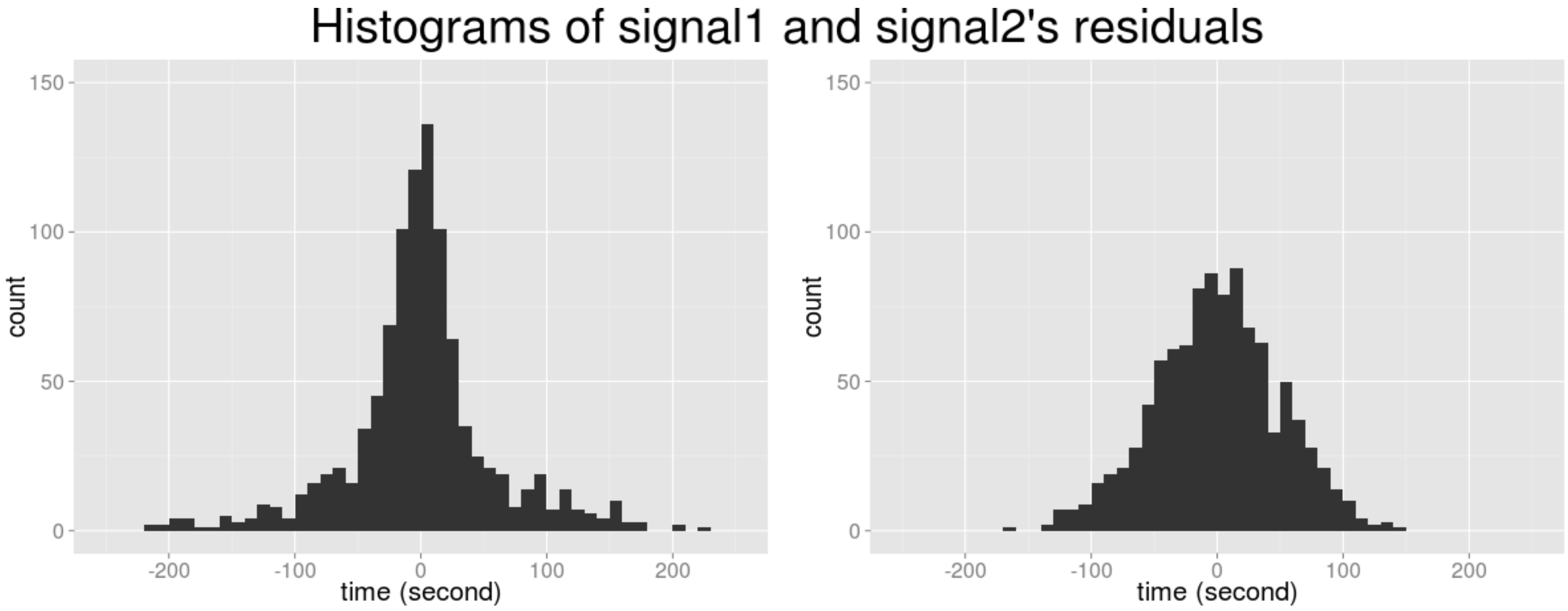
MAE *vs.* RMSE



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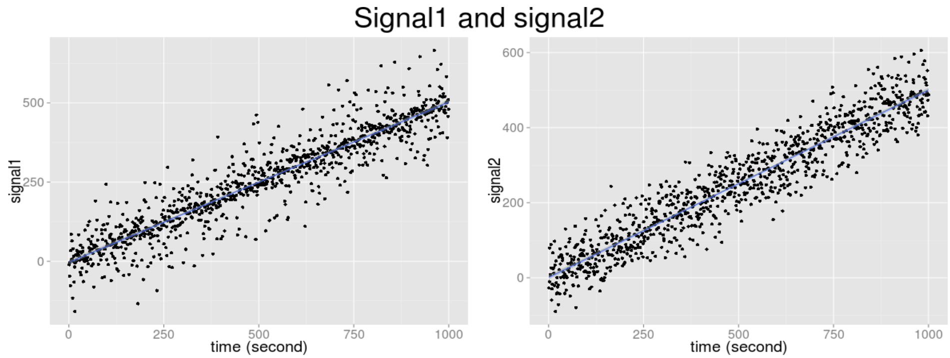
MAE *vs.* RMSE



MAE: 41.926 (better) < 43.199

RMSE: 61.458 > 54.516 (better)

In RMSE, larger deviation cost more

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Coefficient of Determination (R squared)

Formula: $$R^2 = 1 - \frac{VAR_{res}}{VAR_{tot}}$$

(the proportion of variance explained by the model!)

where $$VAR_{res} = \sum_{i=1}^{n} (y_i - h(\boldsymbol{x}_i))^2 / n, \quad VAR_{tot} = \sum_{i=1}^{n} (y_i - \bar{y})^2 / n$$

for data set i, $\boldsymbol{x}_i$ is a vector of all the predictors,

y_i is the corresponding response;

$h(\boldsymbol{x}_i)$ is the prediction using a certain model.

$\bar{y}$ is the mean of all responses;

n is the number of data in the test data set.

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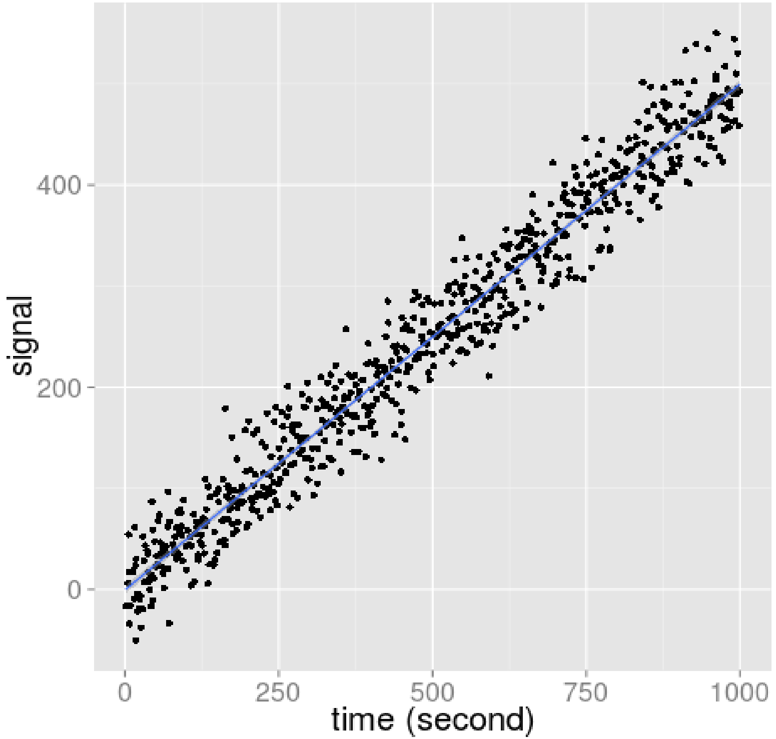
The simple example

$$R^2 = 0.958$$

The model built by real-world data is usually not that good.

$$R^2 = 0.6$$

can be a good model.



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Adjusted R squared

$$R^2_{adj} = 1 - (1 - R^2)\frac{n-1}{n-p-1}$$

Where p is the total number of predictors in the model (not counting the constant term),

n is the sample size.

Penalized by the more predictors

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QUESTIONS