

Mathematical models describing the effects of different tax evasion behaviors – An understanding

Final Report

Overview

“This publication aims at deriving and explaining the emergence of a population's aggregate feature like the income distribution, as a result of the whole of economic exchanges and interactions which take place between the individuals. More specifically, our focus here is on the effects that the heterogeneity of taxpayer behaviours has on the income distribution profiles of the categories of individuals evading to different degrees.”

We see that the authors develop a probabilistic model based on a list of assumptions to describe economic activity (read: exchange of money, taxation – evasion and compliance, and redistribution) in an observable, closed population. This is done systematically in three major steps:

- i. Restating dynamics of a model developed in previous publications describing the basic economic parameters of the population and their transactions
- ii. Defining a probability-based payer-payee model for all transactions
- iii. Defining customizable tax evasion behavior for the population

It is then illustrated how these modules assimilate into a fully-functioning mathematical system to meet the objective. Many phenomena have been generalized to suit the model's integrity, and many others have been used to fill loopholes.

The conclusion is preceded by application of the developed model to a hypothetical population. The results are illustrated for a variety of cases.

Fundamentals and foundations on which the model is based

The model assumes the presence of a tax system and a redistribution process as well as the occurrence of tax evasion. The focus or the fundamental aim of the model is to analyze the effects of heterogeneity of taxpayer behavior on the income distribution profiles of the categories of individuals evading to various degrees.

Assumptions in the model

The totality of individuals remains constant for the time period under consideration.

The individuals are divided into n number of classes according to their income level and m number of sectors according to their evasion behavior. The total number of groups is $n*m$.

The frequency of payment of individuals of one income class to the individuals of another income class is a fixed one.

The amount involved in a transaction, S is much smaller than the size, r of an income class.

We don't know who exactly is going to interact with whom, we only know at a probabilistic level how often individuals in a group interact with individuals of another group.

Analysis of employed methods

The population is broadly classified by average income into classes. Further on, individuals are tagged with parameters that describe their evasion behavior, probability to pay in a transaction with another individual and so on.

Transactions are assumed to take place between individuals belonging to defined income classes – individual behavior is not taken into account unless required, and generalization persists.

Though the paper begins by assuming nonlinear differential equations to fit the model, it evolves to be basically probabilistic - all transactions are defined in terms of probabilities of their occurrence (or vice versa) using simple rules of probability theory. The authors suggest, and demonstrate, that numerical solutions be used to employ the model instead of finding analytical solutions to the differential equations owing to their complexity.

No purely statistical methods were found.

The exploration of aggregate effects in the asymptotic stationary income distribution of the population and the comparison of different situations of tax compliance and tax evasion, we find that evasion leads to an increment in the number of individuals in the poorest and in the richest classes, at the detriment of the middle classes.

Evasion grows in response to an increase in tax rates.

The introduction of behavioral sectors into the model allows us to observe another important effect, namely the appearance of a clear difference between the average incomes of honest taxpayers and tax evaders.

Applications

Alpha (α), the index used to describe evasion behavior, is assumed to be constant irrespective of the outcome of the transactions. This limits the direct application of the model to a real-world scenario where such behavior might not be found.

In addition, we read that “higher tax evasion behaviors is seen in higher income groups”. By extension, if an individual moves to a higher income group as a result of a transaction, he will show modified (read: higher) tax evasion behavior. This leads us to speculate about the employment of the model over an extended period of time to any idealized population also.

In the real world, probabilities of transactions are not identical for all members of a given income class – bias naturally exist – in contrast to a fundamental assumption of the model. Members of an income class may have typical repetitive transactions that are unique and easily differentiated from other members of the same income class; a bank, for example, may be the payee of regular transactions of predictable value between other income classes.

Why is this mathematical model a kinetic one?

This mathematical model of tax evasion is inspired by the Boltzmann approach to statistical mechanics. In the Boltzmann approach, particles with the same velocity are grouped together. In our socio-economic example we group people of the same income class together. Integrals are replaced by sums wherever possible to ensure accuracy in the model. Also the model is based on the Boltzmann approach because of the advantage of closed mathematical formulation.

Criticism

The graphs provided are unclear. The parameters are not mentioned.

The phenomenon of partial migration is unclear.

The function named ‘T’ is vaguely described as “variation” of an unnamed quality.

The propensity of individuals to be compliant or non-compliant does not always remain constant in time.

Incorporation of audit actions and punishments into the model is required.

Future scope of the model

This model of tax compliance or the lack thereof will play an important role towards overcoming economic inequality. This model will help public policy makers and RS agents worldwide by describing the nuances of tax compliance and evasion. How evaders gain systematically through direct interactions? How the number of people in higher classes and lower classes of income increase significantly due to evasion? How the Gini index alone is not sufficient to give the true standing of tax redistribution and evasion in an economy? These questions and many more will be answered by research into this model and similar ones.

Bibliography

Modanese, G., & Bertotti, M. L. (n.d.). *Mathematical models describing the effects of different tax evasion behaviours*. Italy.

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