FRIEDRICH-LOEFFLER-INSTITUT
FLI
Bundesforschungsinstitut für Tiergesundheit
Federal Research Institute for Animal Health

Spread of information and diseases through temporal networks

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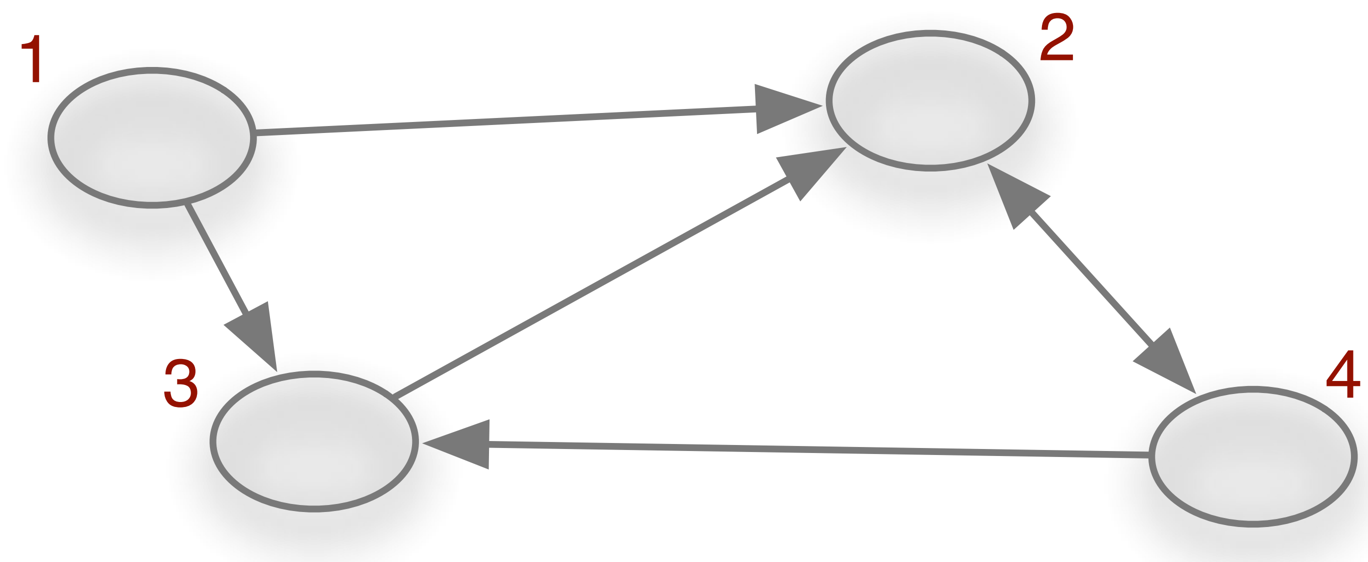
static
networks

representation: Adjacency matrix **A**

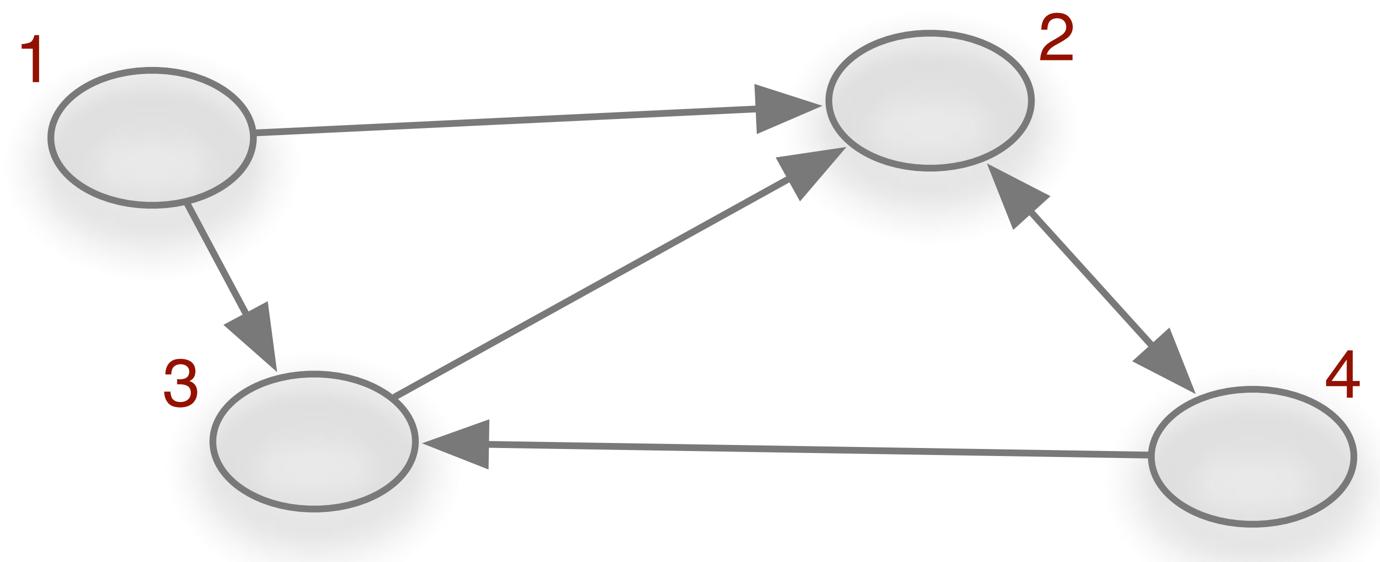


$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

static

networks

representation: Adjacency matrix **A**



$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

the big picture

typical questions

how does **X** spread through
my network?

which nodes are more important than others,
and why?

typical questions

how does **X** spread through my network?

X
information
rumors
disease
trends,…

which nodes are more important than others, and why?

typical questions

how does **X** spread through my network?

X
information
rumors
disease
trends,…

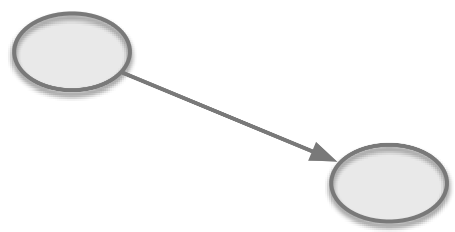
which nodes are more important than others, and why?

targeted
vaccination
ads
(mis)information

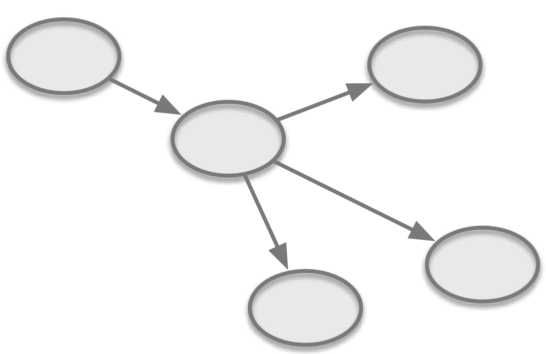
network **analysis**

local scale

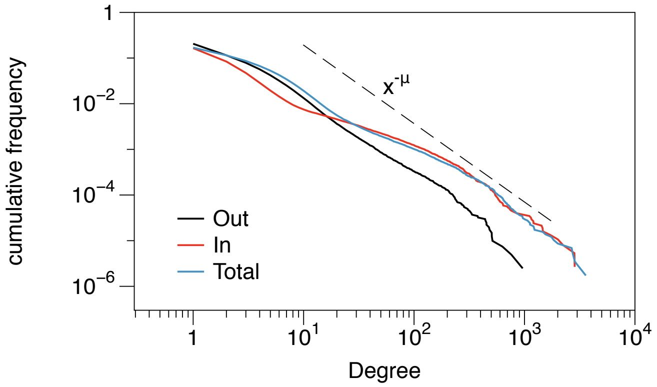
node to node connections

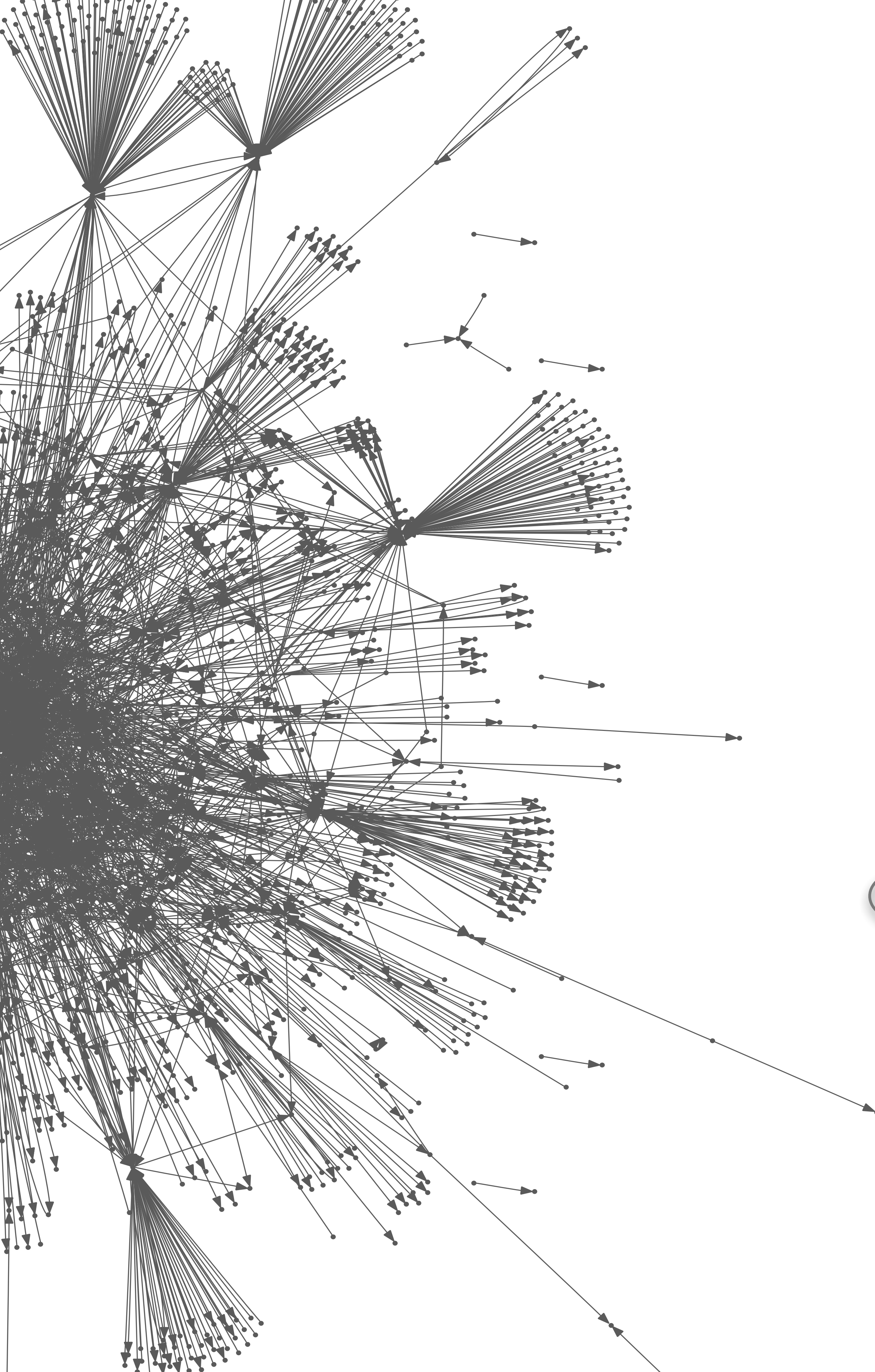


nearest neighbors



Degree statistics



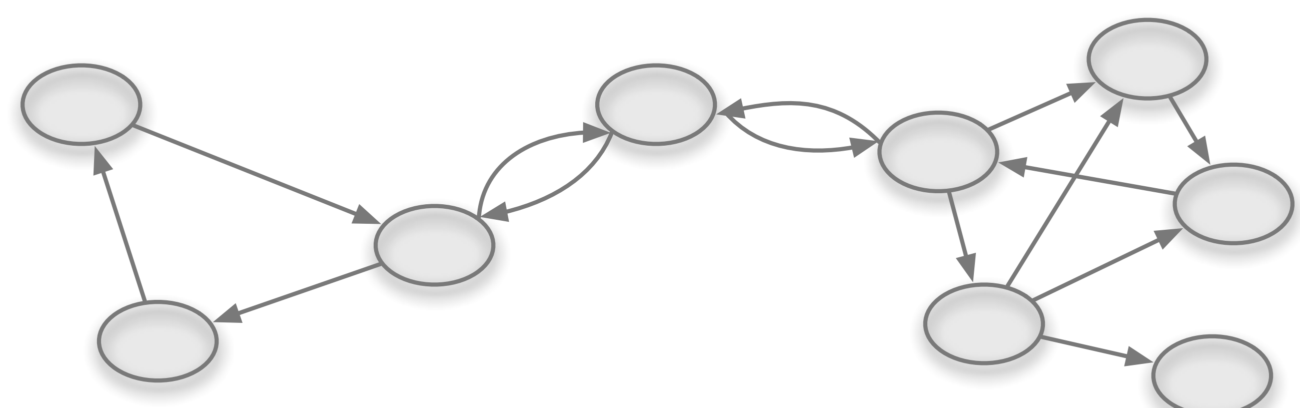


network analysis
global scale

paths between nodes

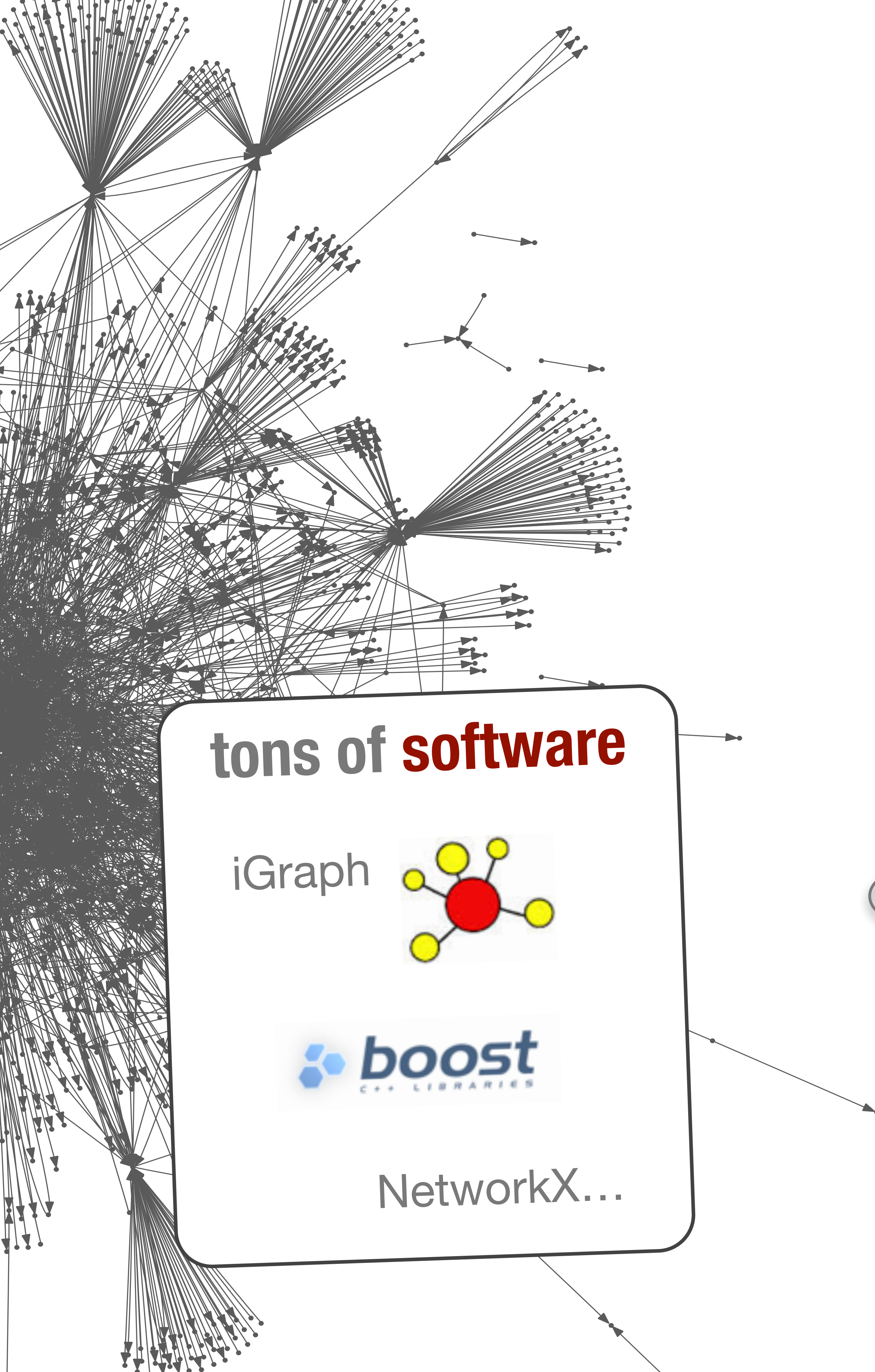
components

node centrality



network **analysis**

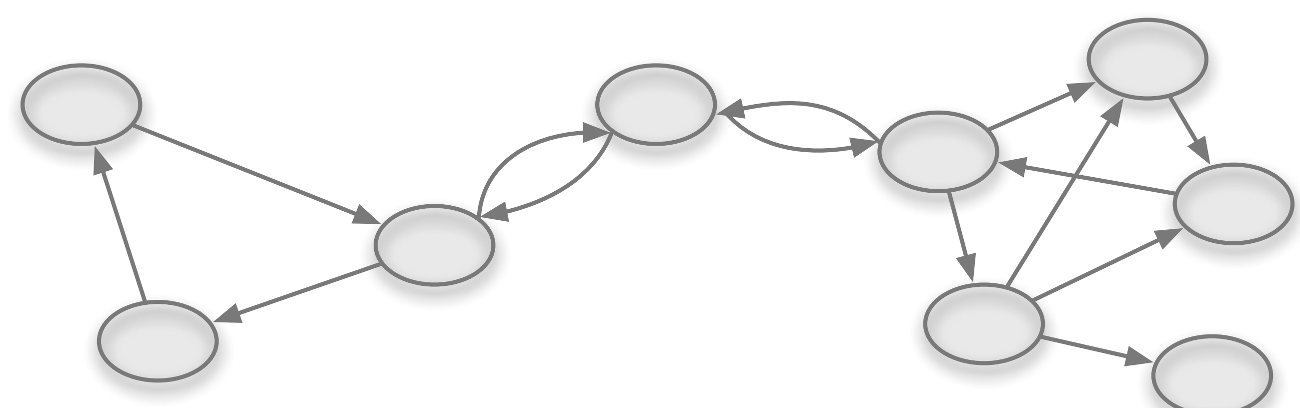
global scale



paths between nodes

components

node centrality



tons of **software**

iGraph

boost

NetworkX…

a temporal network data-set

livestock pig trade

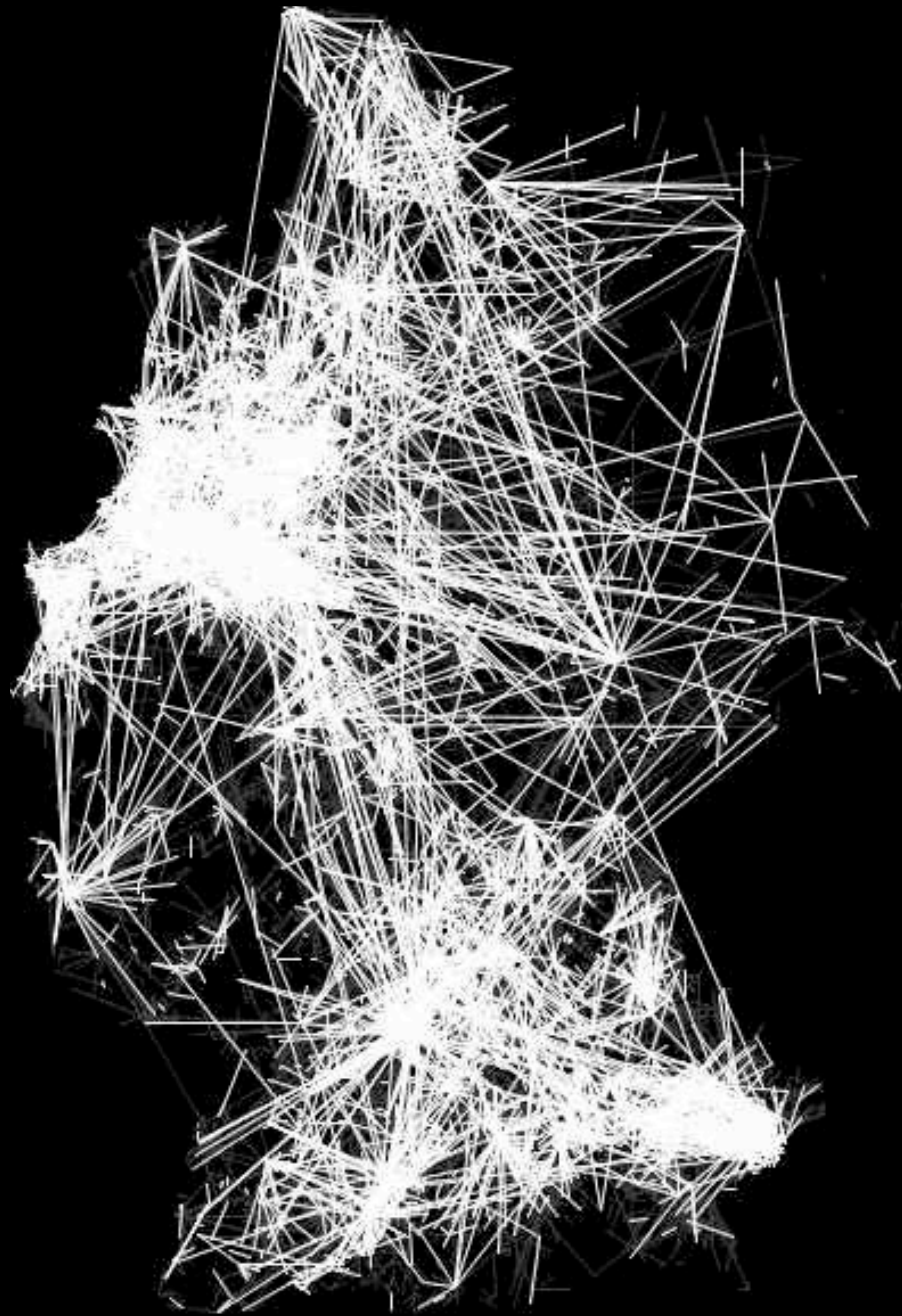
farm to farm

a temporal network data-set

livestock pig trade

farm to farm

a temporal
network data-set



livestock pig trade

farm to farm

temporal networks

$$\mathbf{A}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_2 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_1 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

time

adjacency matrix sequence

temporal
networks

adjacency matrix sequence

$$\mathbf{A}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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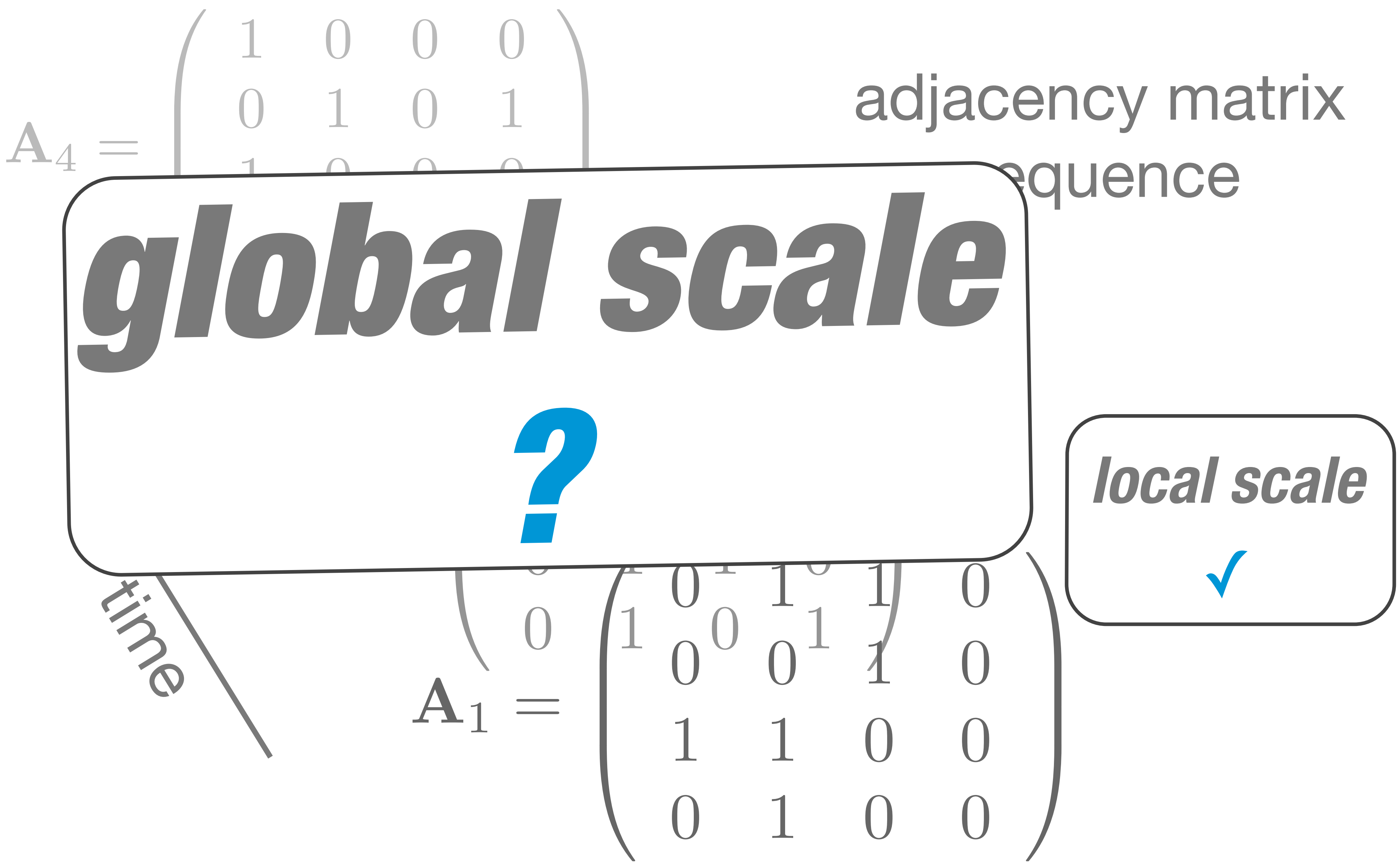
time

local scale

✓

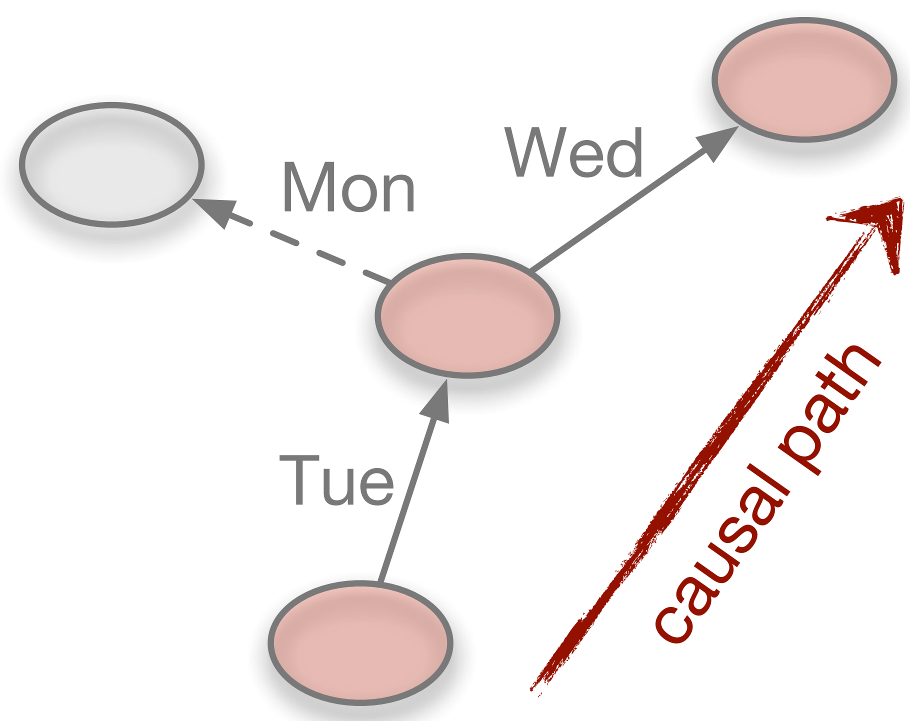
temporal
networks



role of causality in
paths

chronologic sequence of edges

fundamental difference to static: **causal** paths



questions

questions

can I approximate my network as a static one?

questions

can I approximate my network as a static one**?**

how long does a path take**?**

questions

can I approximate my network as a static one?

how long does a path take?

can I reach node x at all?

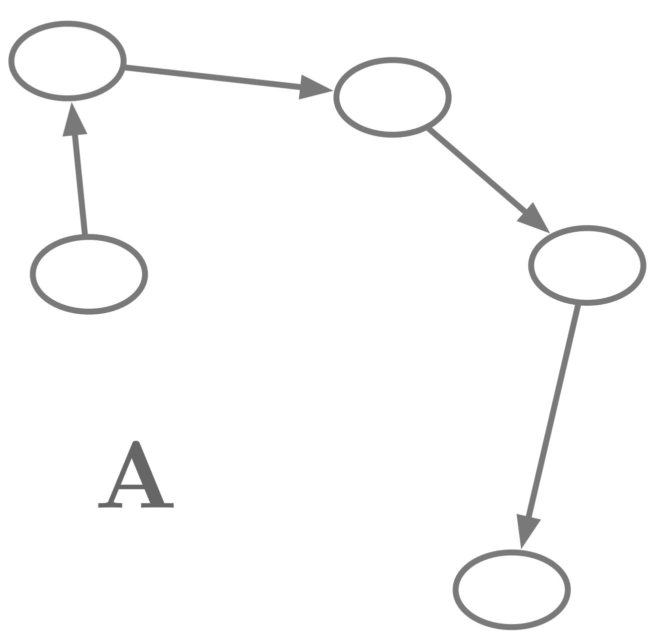
accessibility matrices &

accessibility graphs

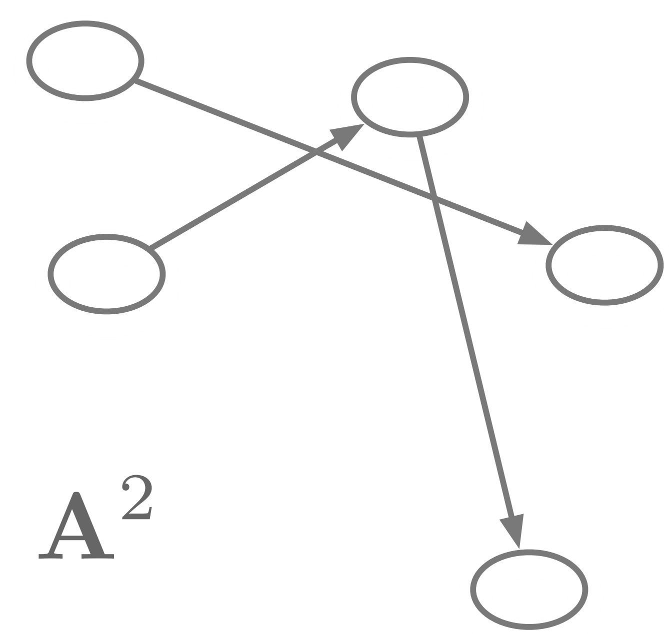
adjacency matrix **A**:
number of paths of length 1

square of adjacency matrix **A**^{2}:
number of paths of length 2

Example (static):



$\mathbf{A}$



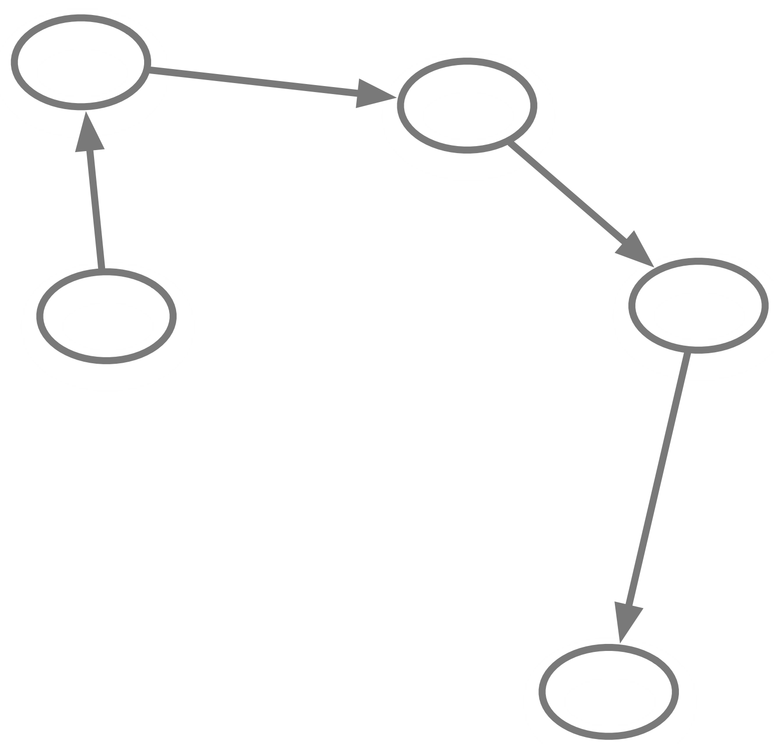
$\mathbf{A}^2$

accessibility matrices &

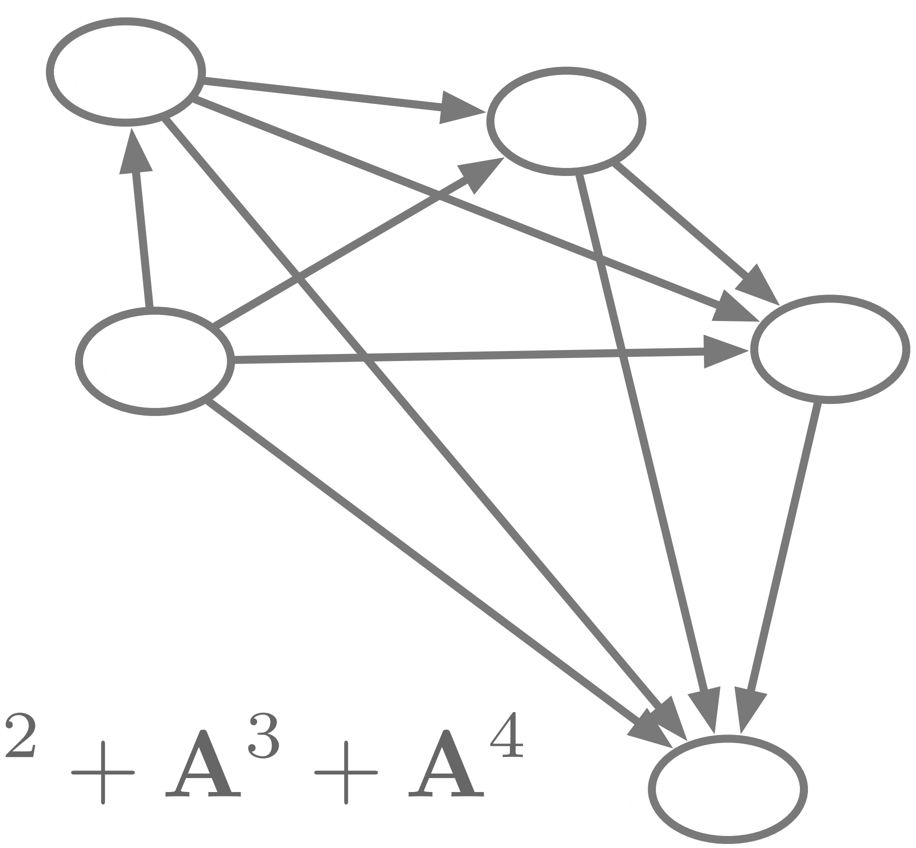
accessibility graphs

initial graph

$\mathbf{A}$



its accessibility-graph



$$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4$$

accessibility matrices of

static networks

example:
Random DiGraph
1000 nodes
2000 edges

$$\mathbf{P} = \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots + \mathbf{A}^n$$

accessibility matrices of

static networks

example:
Random DiGraph
1000 nodes
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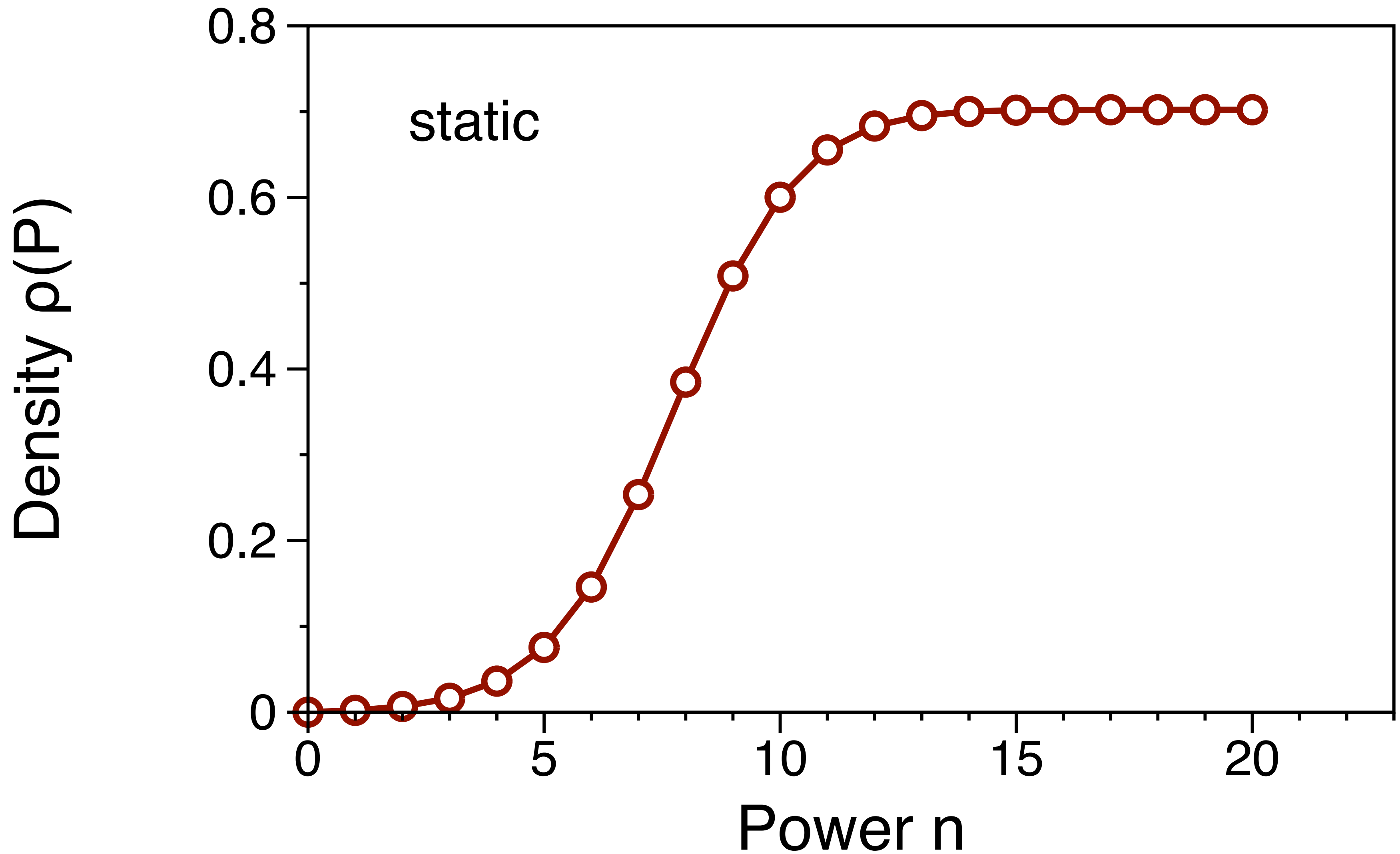

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Random DiGraph
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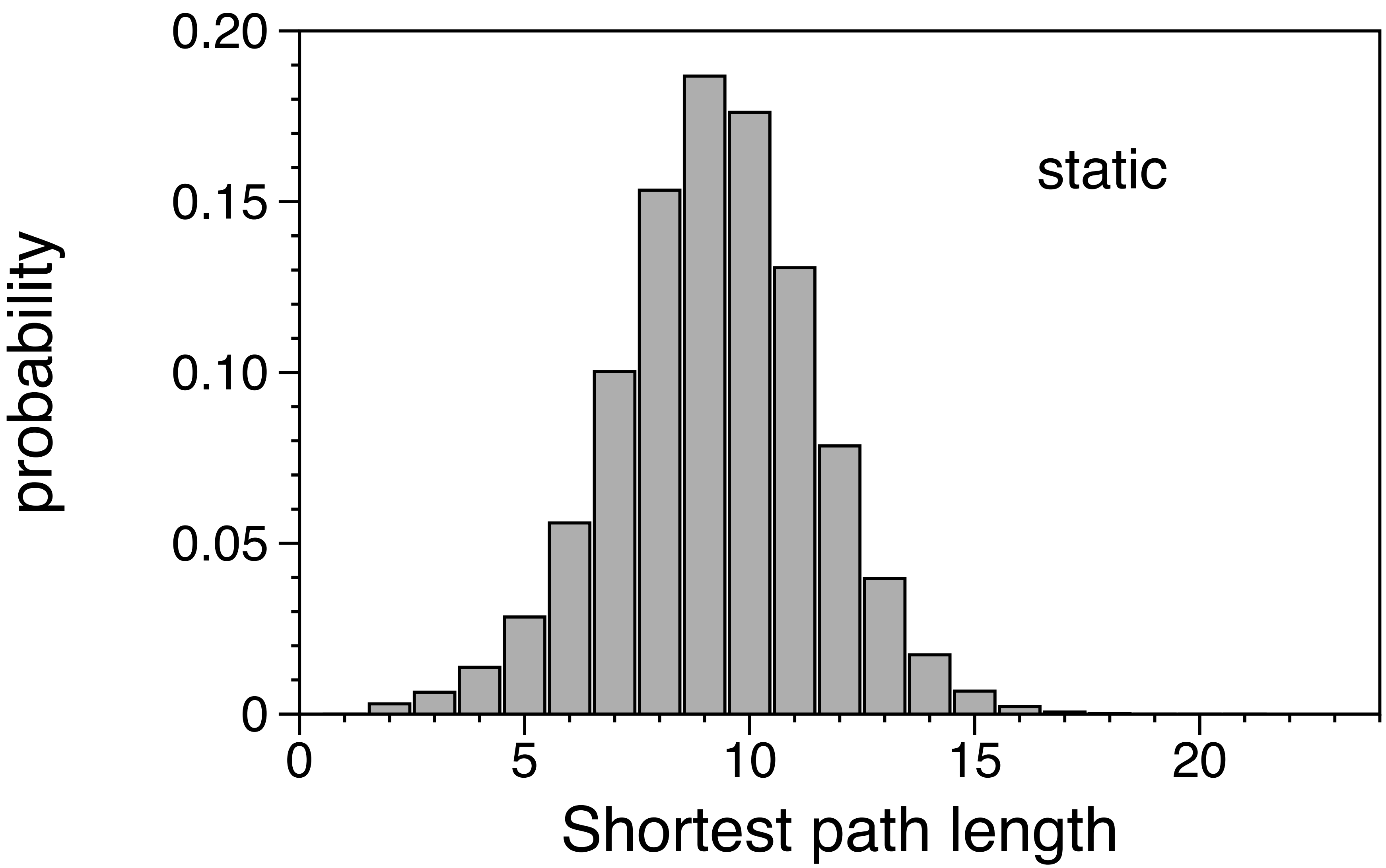
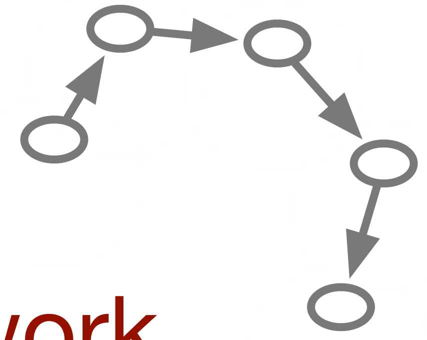


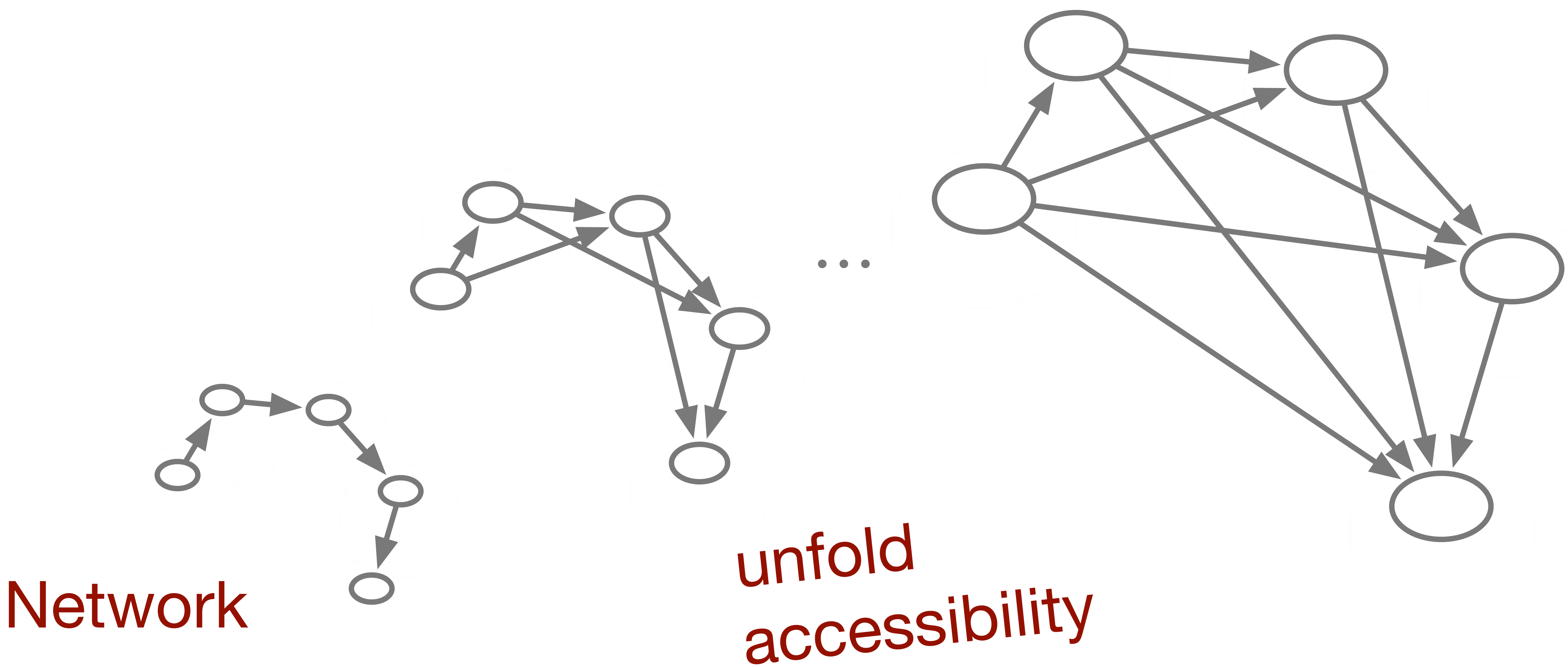
recap:
the idea



Network

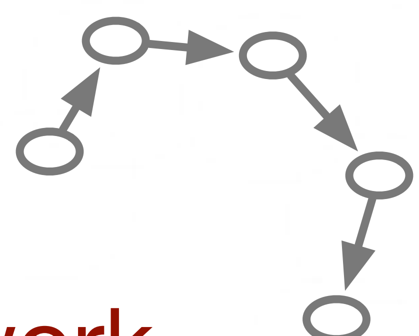
recap:

the idea

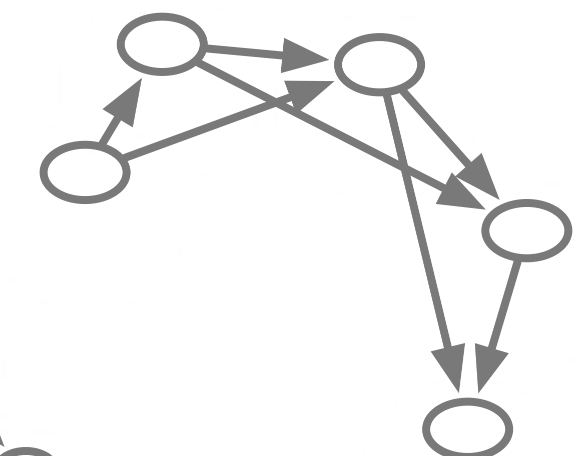


recap:

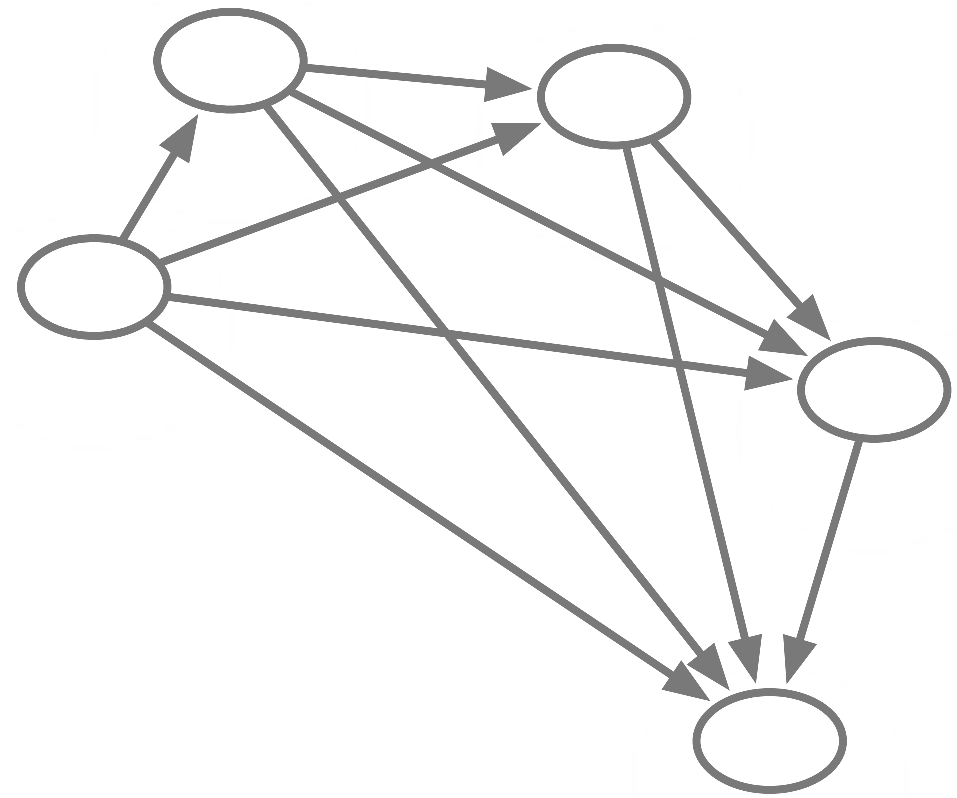
the idea



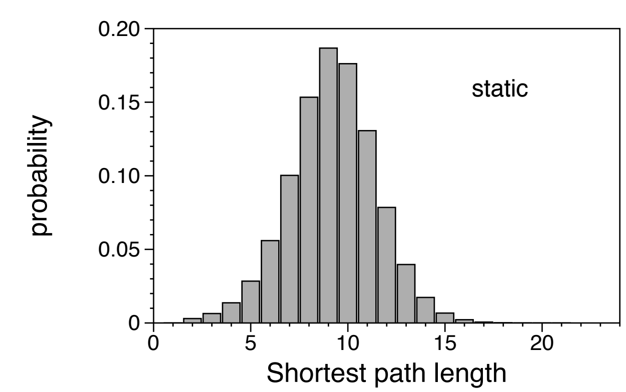
Network



...



unfold accessibility



accessibility matrices of **temporal networks**

$$\mathcal{P}_n = \mathbf{I} + \underbrace{\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \cdots + \mathbf{A}_n}_{\mathbf{A}} +$$

$$+ \underbrace{\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_3 + \mathbf{A}_1\mathbf{A}_2\mathbf{A}_3 + \cdots}_{\mathcal{O}(\mathbf{A}_i^2)}$$

$$= \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

Lentz et al., Phys. Rev. Lett, 2013
Grindrod et al., Phys. Rev. E, 2011

accessibility matrices of **temporal networks**

$$\mathcal{P}_n = \mathbf{I} + \underbrace{\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3 + \cdots + \mathbf{A}_n +}_{\mathbf{A}}$$

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$$= \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$

easy implementation!

Lentz et al., Phys. Rev. Lett, 2013
Grindrod et al., Phys. Rev. E, 2011

implementation.

implementation.

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} (\mathbf{I} \vee \mathbf{A}_i)$$

Boolean matrices

implementation.

Sparsity

`dense matrix` `sparse matrix`

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} (\mathbf{I} \vee \mathbf{A}_i)$$

Boolean matrices

implementation.

Partitioning. For each node

$$i(t) = \left[\prod_{t'=0}^{t} (\mathbf{I} + \mathbf{A}_{t'})\right] i(0)$$

Sparsity

`dense matrix` `sparse matrix`

$$\mathcal{P}_n = \bigwedge_{i=1}^{n} (\mathbf{I} \vee \mathbf{A}_i)$$

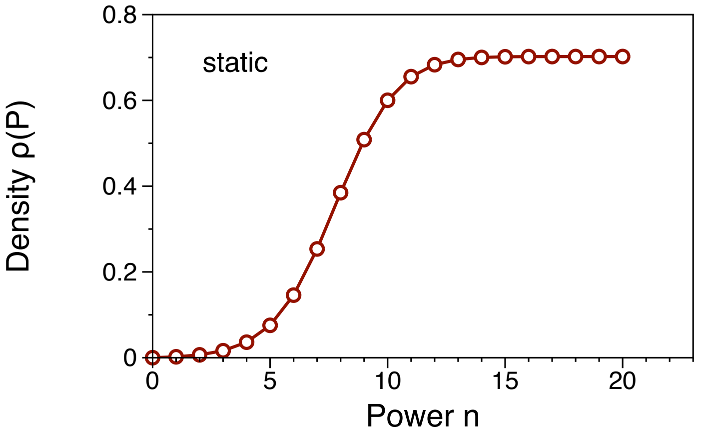
Boolean matrices

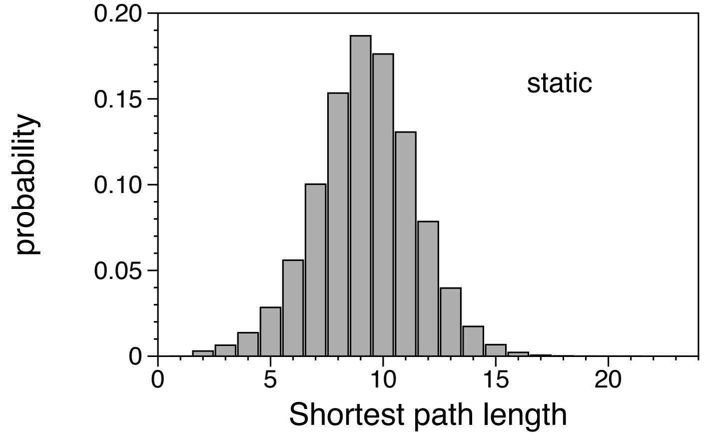
application.

characteristic time scales

results I

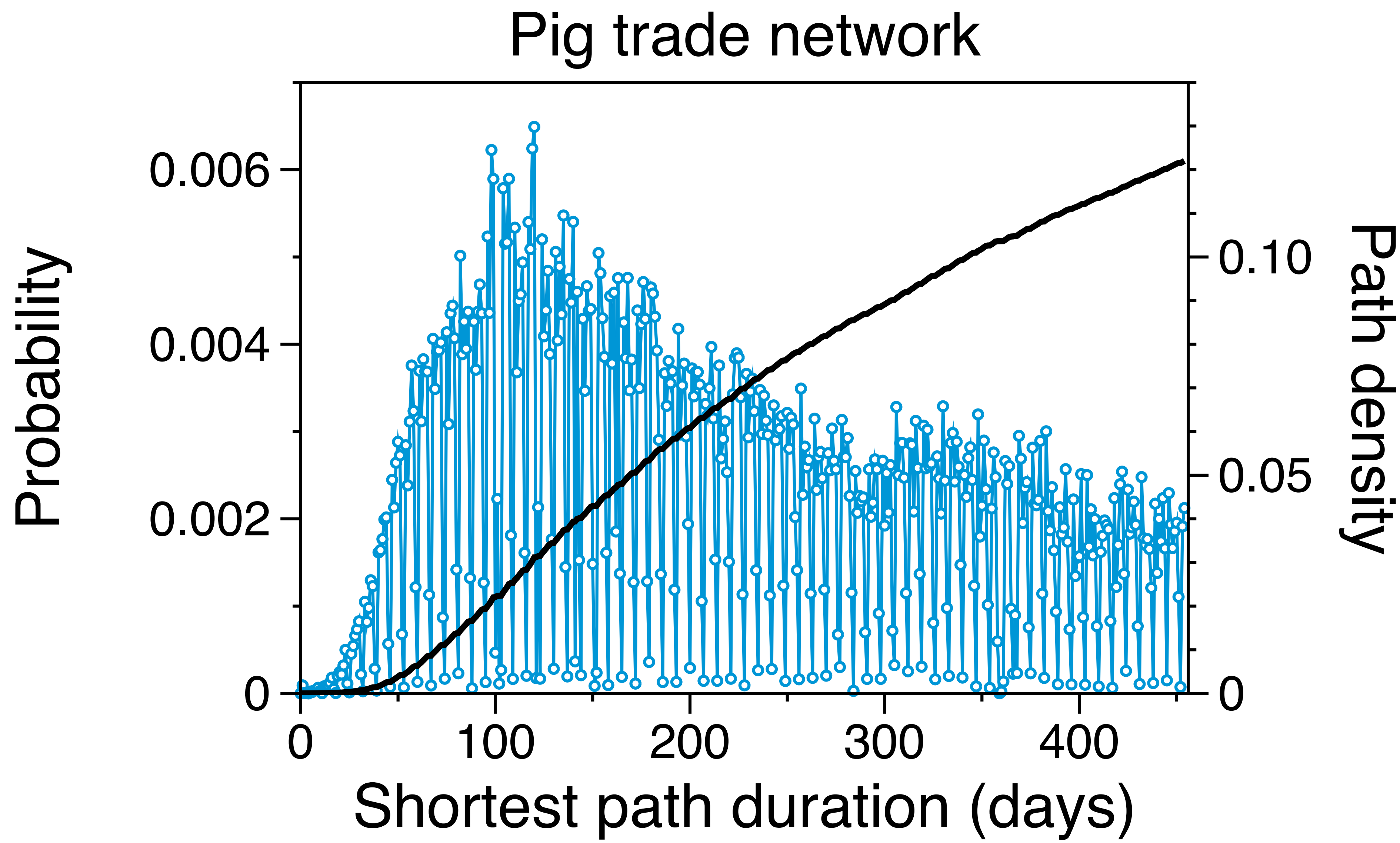
static random network



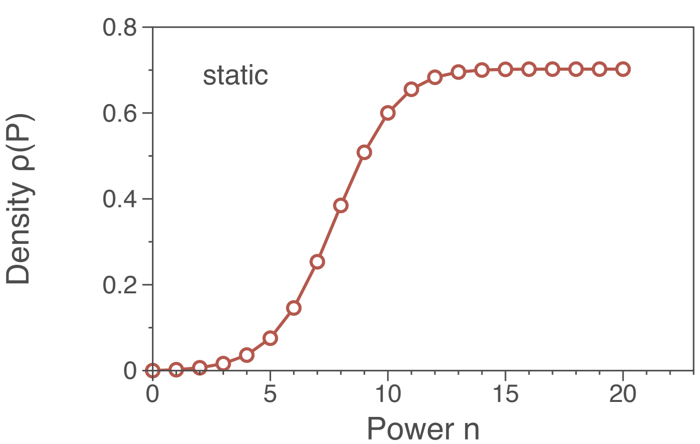


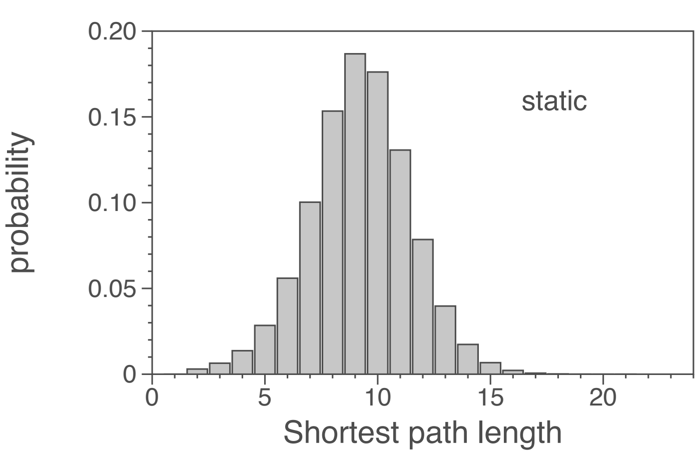
characteristic time scales

results I

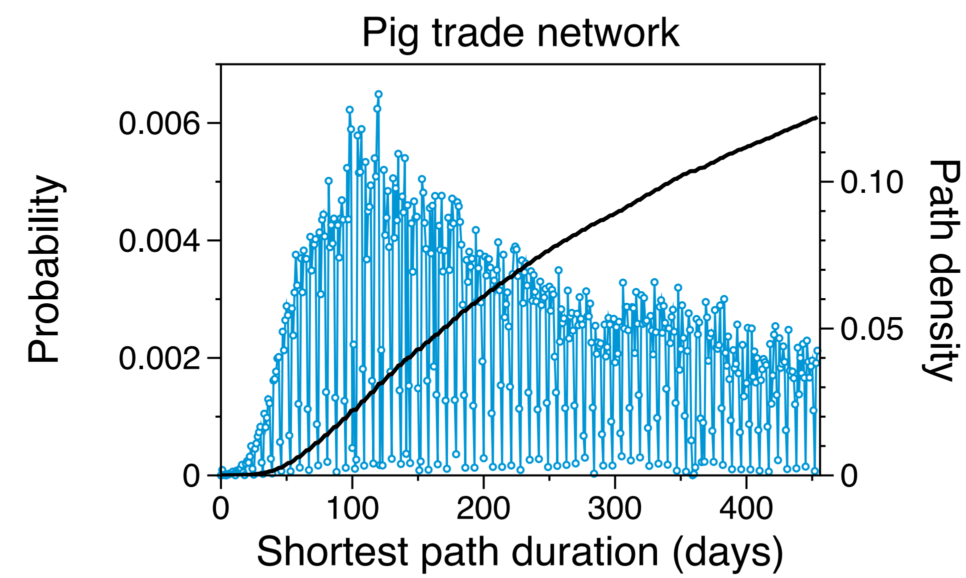


static random network

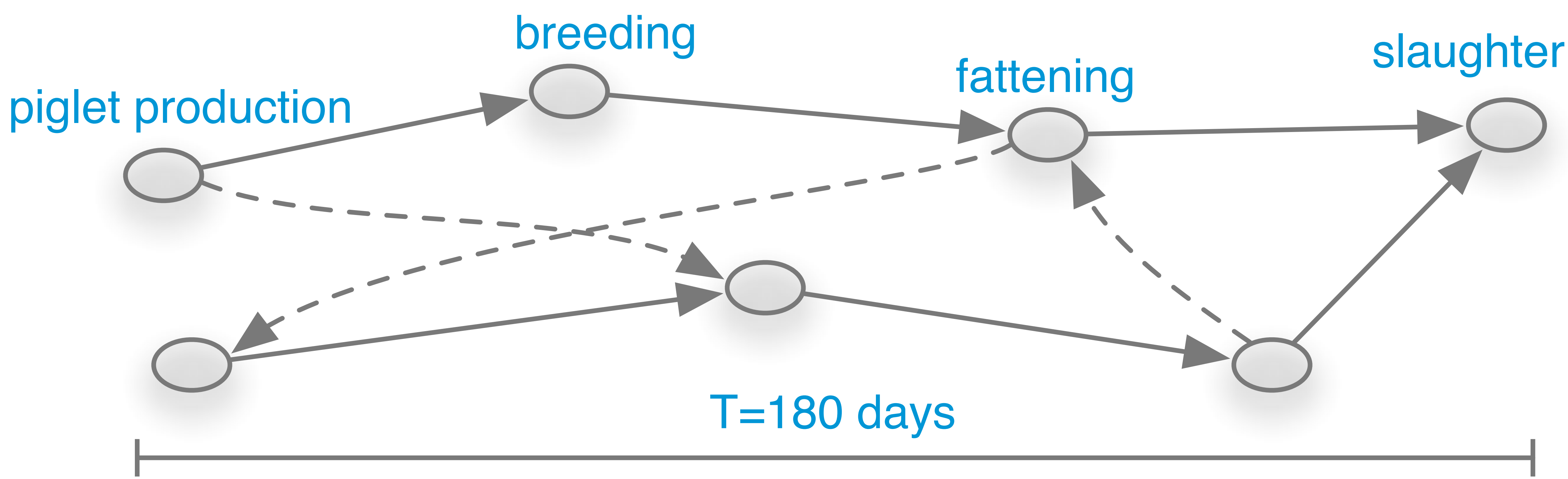




characteristic time scales
results I



Pork production chain



causal fidelity

results II

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

number of paths:

static representation:	1.4 billion
temporal representation:	1.0 billion

causal fidelity

results II

comparison:
static vs. temporal accessibility-graph

number of paths:

static representation:	1.4 billion
temporal representation:	1.0 billion

28 % of all paths in the aggregated network do actually not exist!

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

temporal (real) path density

static path density

causal fidelity

results II

comparison:

static vs. temporal accessibility-graph

Definition: causal fidelity

$$\boxed{c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}}$$

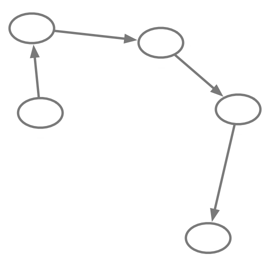
temporal (real) path density

static path density

pig trade net

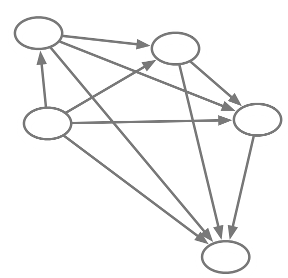
$c = 0.72$

recap.

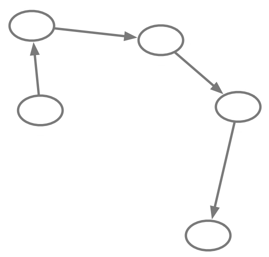


network

perspective:
accessibility

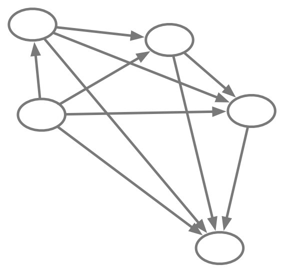


recap.



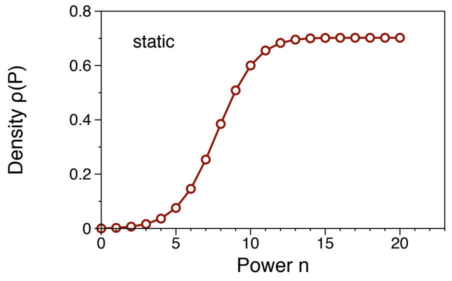
network

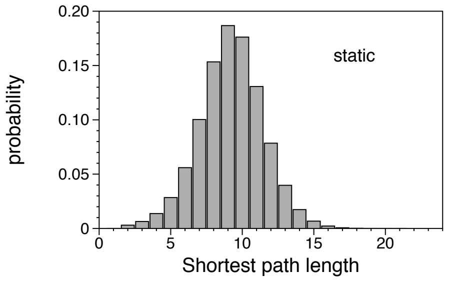
perspective:
accessibility



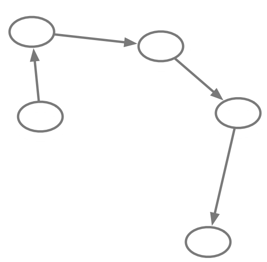
new

path-lengths



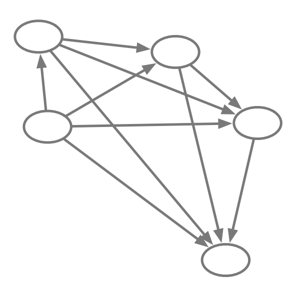


recap.



network

perspective:
accessibility

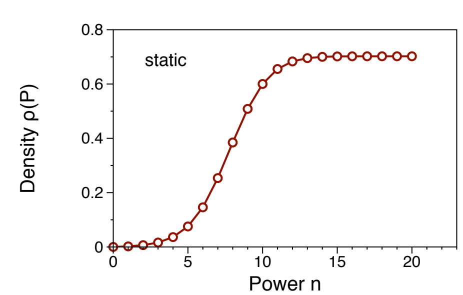


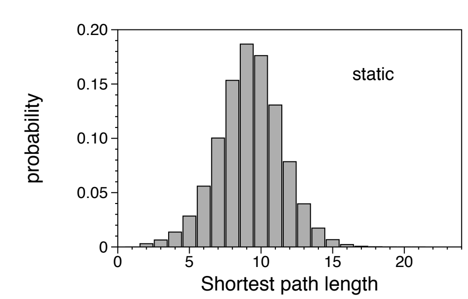
new

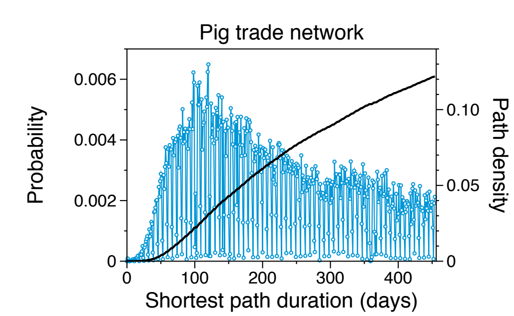
path-lengths

new:
temporal accessibility

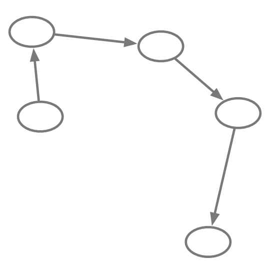
$$\mathcal{P}_n = \prod_{i=1}^{n} (\mathbf{I} + \mathbf{A}_i)$$





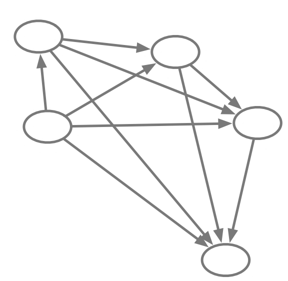


recap.



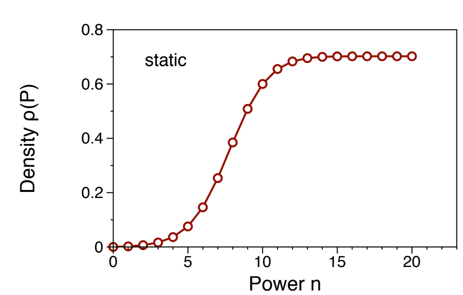
network

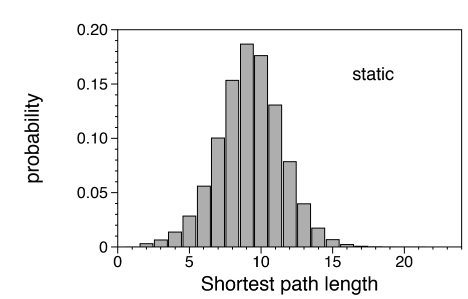
perspective:
accessibility

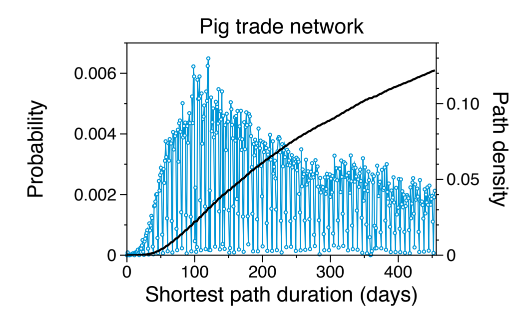


new

path-lengths







new:
causal fidelity

$$c = \frac{\rho(\mathcal{P}_T)}{\rho(\mathbf{P}_T)}$$

new:
temporal accessibility

$$\mathcal{P}_n = \prod_{i=1}^{n}(\mathbf{I} + \mathbf{A}_i)$$

announcement

BMBF research project,
starts early 2016

small companies

funding is 50/50,
BMBF/company

Bundesministerium
für Bildung
und Forschung

FRIEDRICH-LOEFFLER-INSTITUT
FLI
Bundesforschungsinstitut für Tiergesundheit
Federal Research Institute for Animal Health

announcement

BMBF research pro
starts early 2016

small companies

funding is 50/50,
BMBF/company

scenario
bio terroristic attack

objective
implementation of early-warning platform

funding
50%

Bundesministerium für Bildung und Forschung

FRIEDRICH-LOEFFLER-INSTITUT
FLI
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Federal Research Institute for Animal Health

appendix

further datasets.

sexual contact network

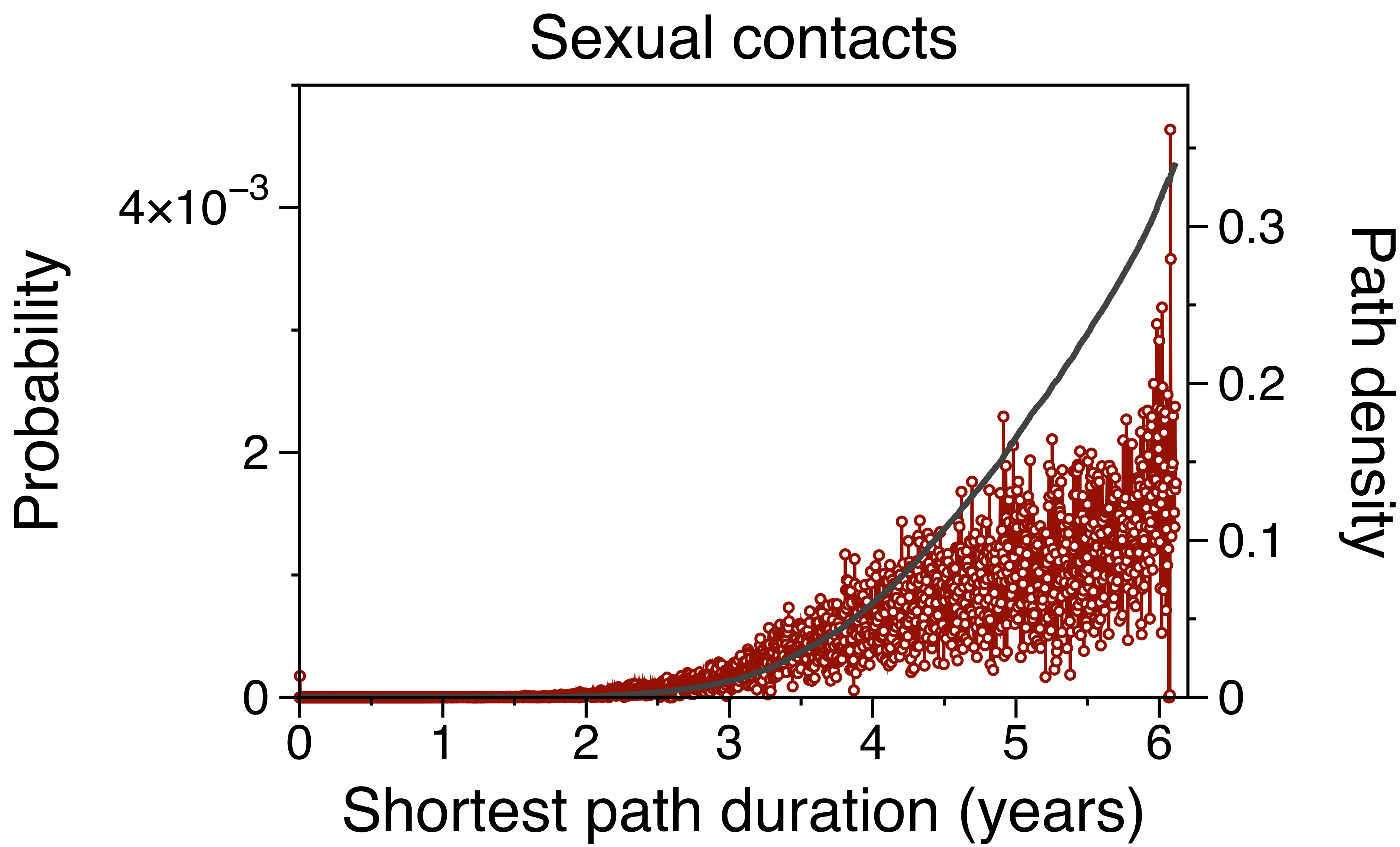
Rocha et al., PNAS 2010

face-to-face conference contacts

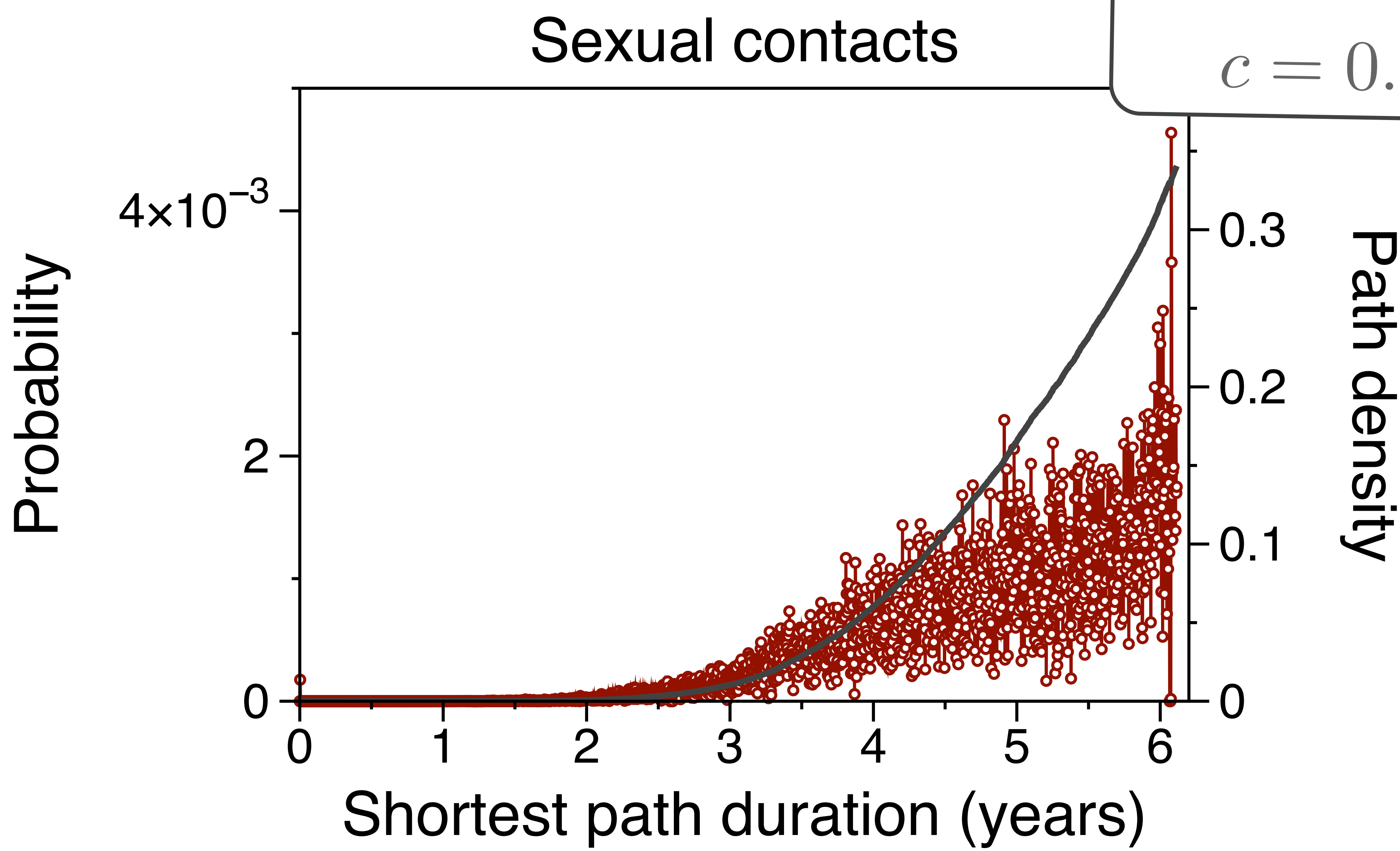
sociopatterns.org

sexual contact network

results III



sexual contact network
results III

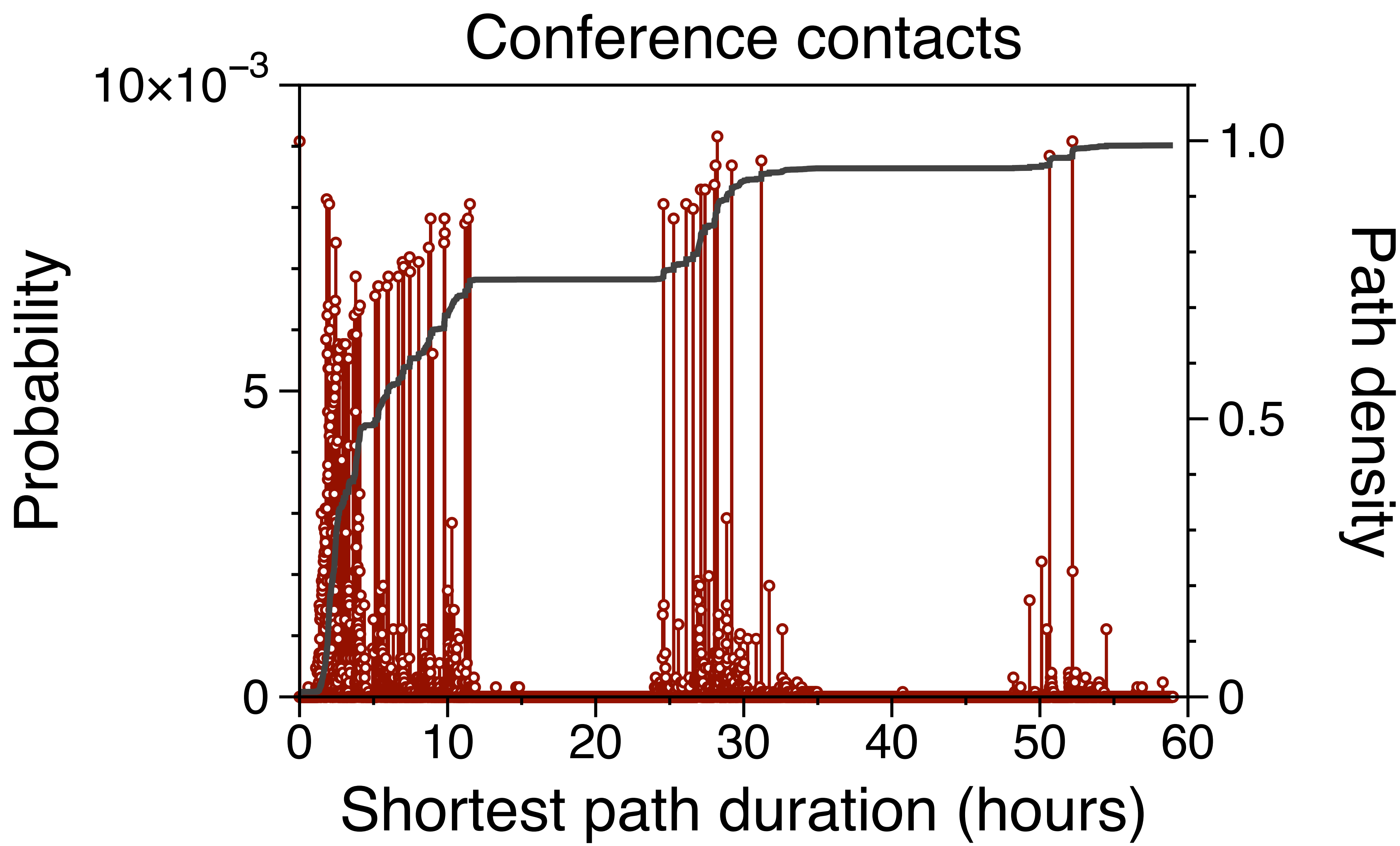


causal fidelity

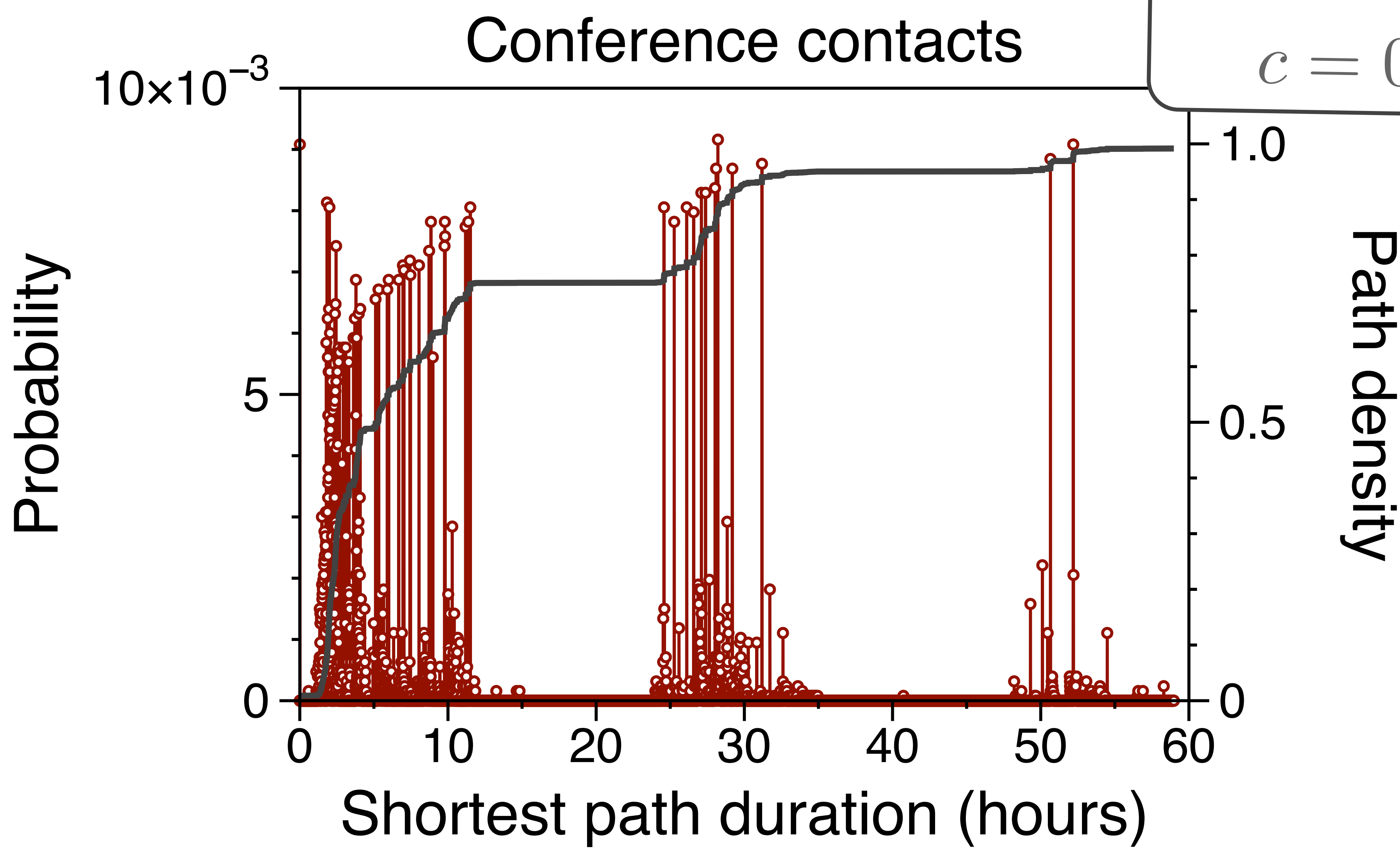
$c = 0.38$

conference contacts

results III



conference contacts
results III

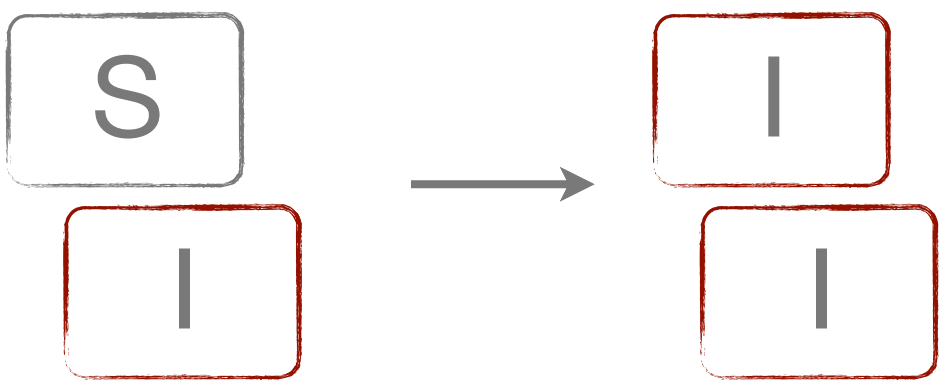


causal fidelity

$c = 0.99$

finite memory

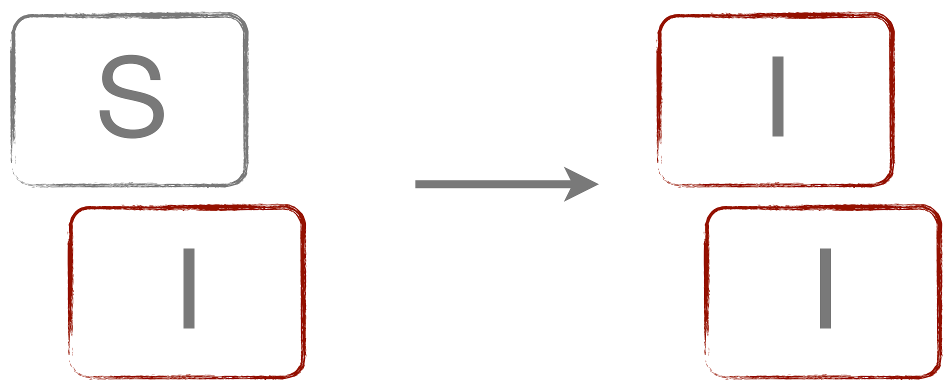
so far: SI-type process



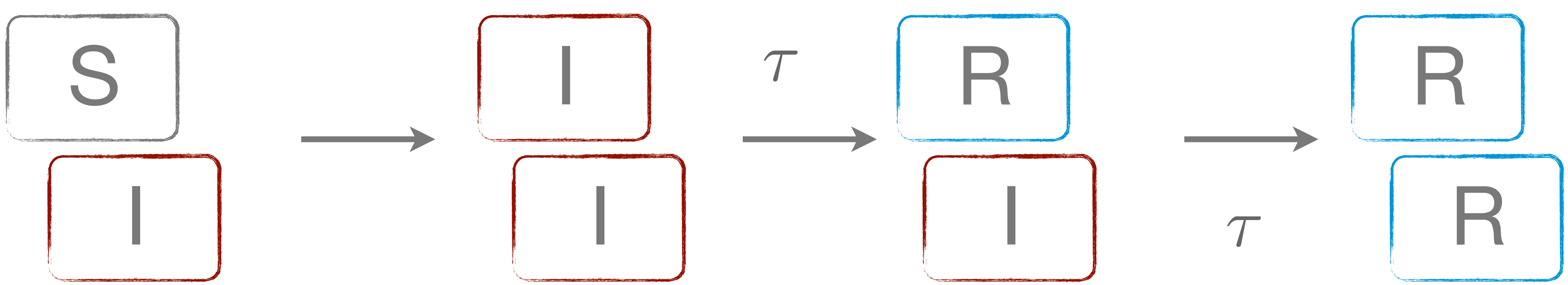
finite

memory

so far: SI-type process



finite memory: SIR-type process



finite
memory

rewrite equation for accessibility matrix using incidence matrices

$$\mathcal{I}(t) = \mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t') \circ \neg \left[\sum_{t'=1}^{t} \mathcal{I}(t-t') \right]$$

Koher, Lentz, 2015 (in preparation)

finite

memory

rewrite equation for accessibility matrix using incidence matrices

$$\underbrace{\mathcal{I}(t)} = \underbrace{\mathbf{A}(t) \sum_{t'=1}^{\tau} \mathcal{I}(t-t')} \circ \underbrace{\neg \left[\sum_{t'=1}^{t} \mathcal{I}(t-t') \right]}$$

new infectious paths

infectious contacts within memory time

path was not infective before

Koher, Lentz, 2015 (in preparation)

impact of memory