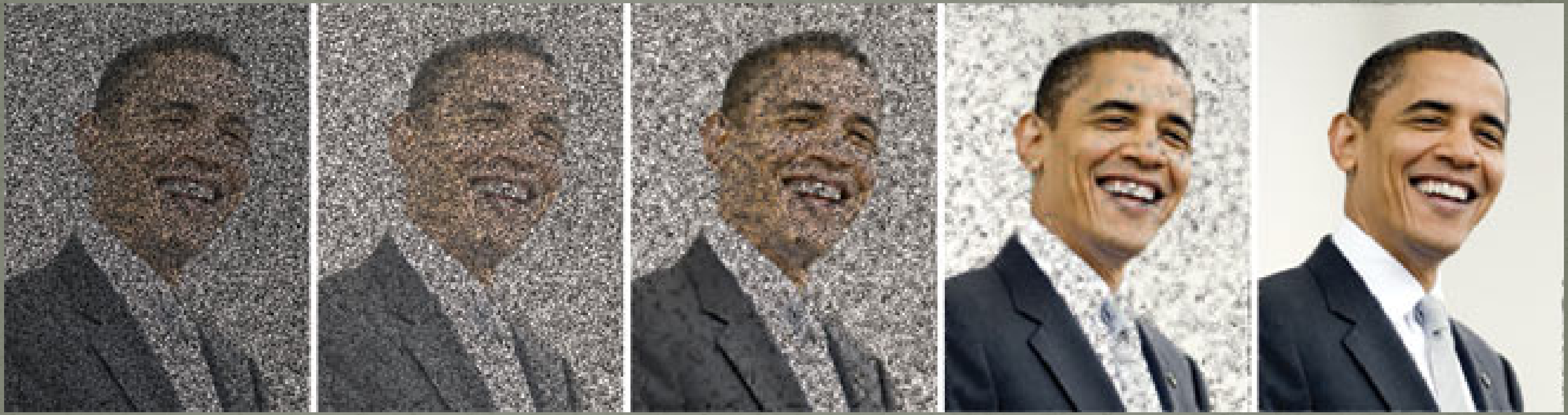
L1 Regularization for Human Beings

Why, What, and How?

Why?



Input:
20% of all pixels are set.

Output:
Full reconstruction

Source: http://www.**wired**.com/magazine/2010/02/ff_algorithm/

What?

- „Lasso“

$$\min_{x} \|y - f(x)\|_2 + \lambda \|Dx\|_1, \qquad \| x \|_1 = \sum |x_i|$$

Most of the entries in *Dx* are going to zero!

(Tibshirani, 1996), >7000 citations
http://www-stat.stanford.edu/~tibs/research.html

Example: Change Point Detection

$\min \parallel Ax - y \parallel_2^2 + \parallel Dx \parallel_1$

x contains the stepwise differences in the signal

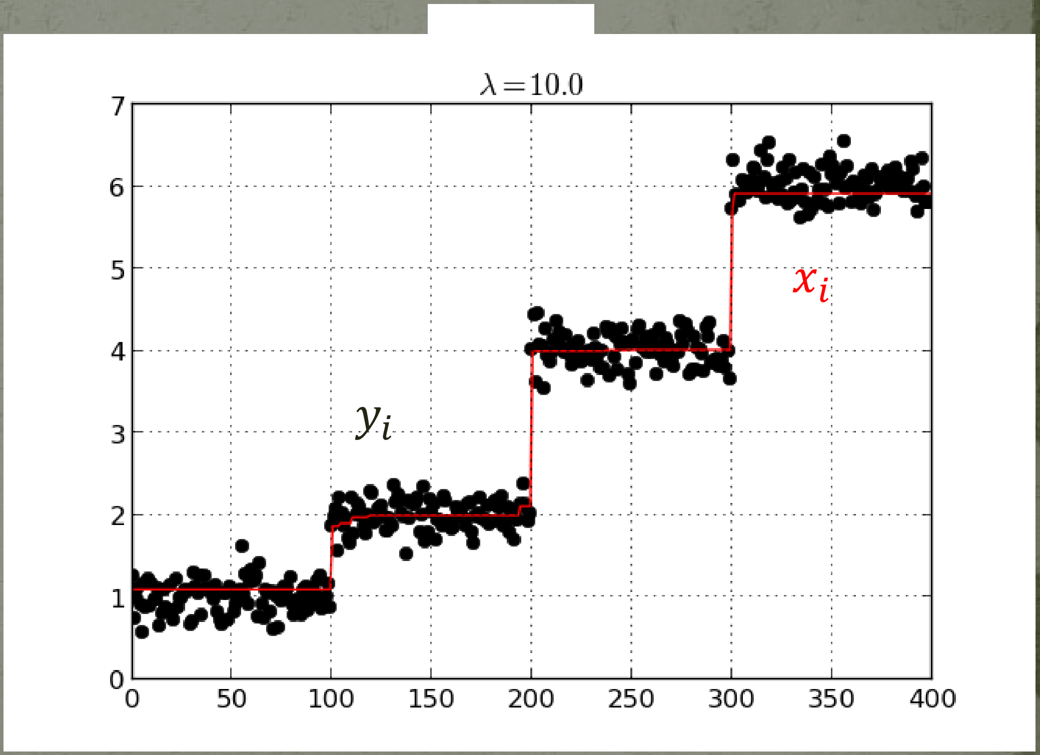
„*Differential*“ operator D:

$$D := \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$\parallel Dx \parallel$ measures the difference between two neighbouring values!

$A = I \in R^{N \times N}$

$N = 400$



# entry in y (1..N)

Fun-Example: Ressource Allocation

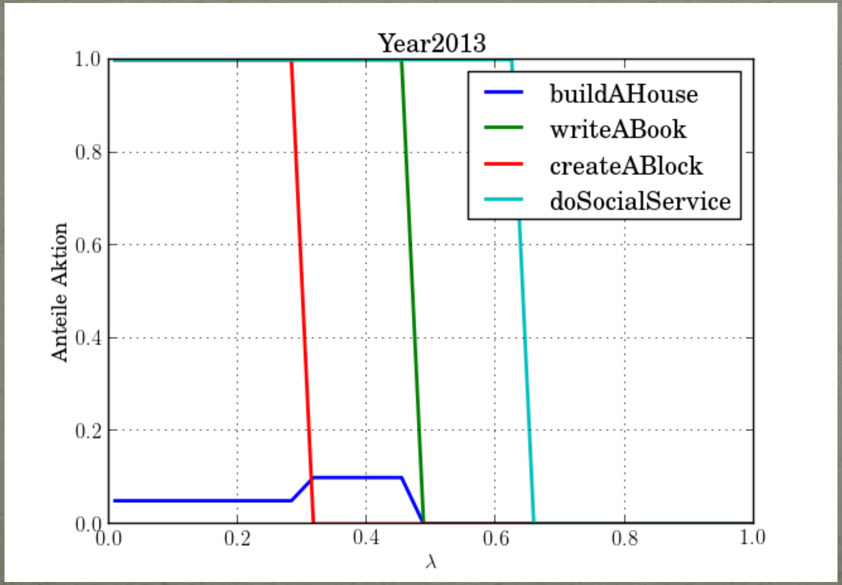
- Problem:
  - You have N tasks
  - Every task costs you c_i time to finish
  - You have c_{total} time (months?) to work
  - Every task has a coolness value u_i
  - You dont like to do many things at once!

$$\min(-u+c)^T x + \lambda \parallel x \parallel_1$$
$$subject\ to \quad c^{Tx} < c_{total}$$

With $c = (c_1, \cdots, c_n)^T$, $u = (u_1, \cdots, u_n)^T$,
And x_i from x is the „importance of the task i.

Ressource Allocation

- Tasks
  - Build a house: (coolness: 3, cost: 10)
  - Write a book: (coolness, 3, cost: 2)
  - Create a blog: (coolness, 2, cost: 0.5)
  - Do Social Service: (coolness 4, cost: 1)



Results:

1. Do the social service
2. Then write a book
3. Then create a blog **xor** build a house.

How?

1. Easiest Approach: Reformulate as a Quadratic Program:

- $\min \| Ax - y \|_2^2 + \| Dx \|_1$
- The problem is equivalent to:
- $\min \| Ax - y \|_2^2 + 1^T v$
  - s.t. $-v \leq Dx \leq v$
- And this to:
  - $\min x^T P x + q^T x$
  - s.t. $Gx < h$
- Where $x = (x, v)^T,\ P = \begin{pmatrix} A^T A & 0 \\ 0 & 0 \end{pmatrix},\ q = \begin{pmatrix} -A^T y \\ 1 \end{pmatrix},\ G = \begin{pmatrix} -D & -I \\ D & -I \end{pmatrix},$
  - $h = 0$

How2

- Use cvxopt: Convex Programming Framework (python)

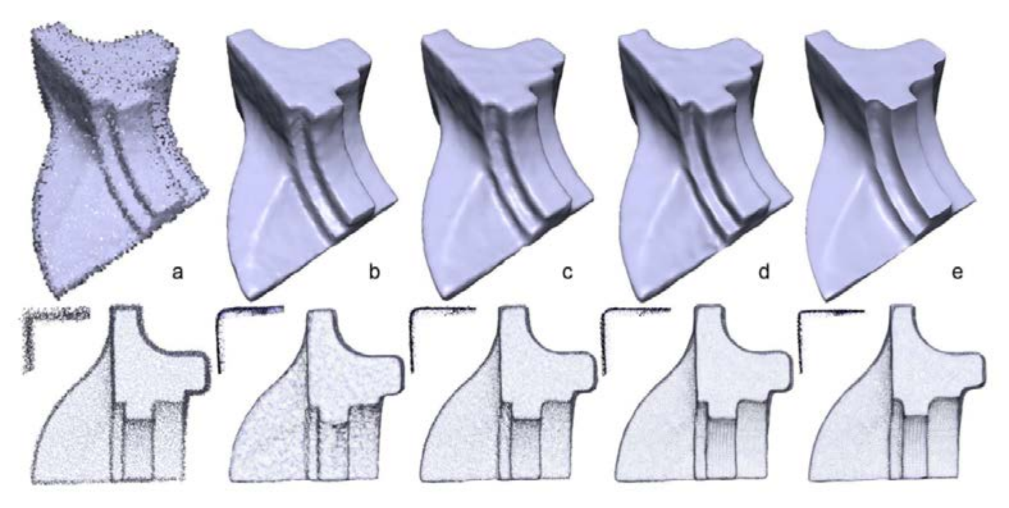
- Or Matlab implementation from http://users.ece.gatech.edu/~justin/l1magic/

- More Infos: http://dsp.rice.edu/cs

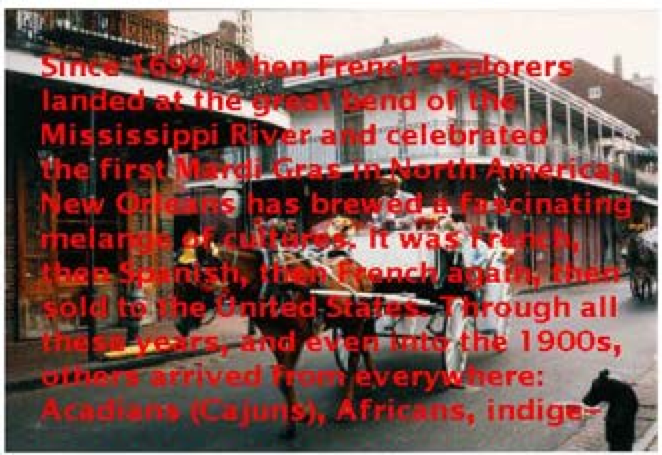
More Magic

Common methods (L2)

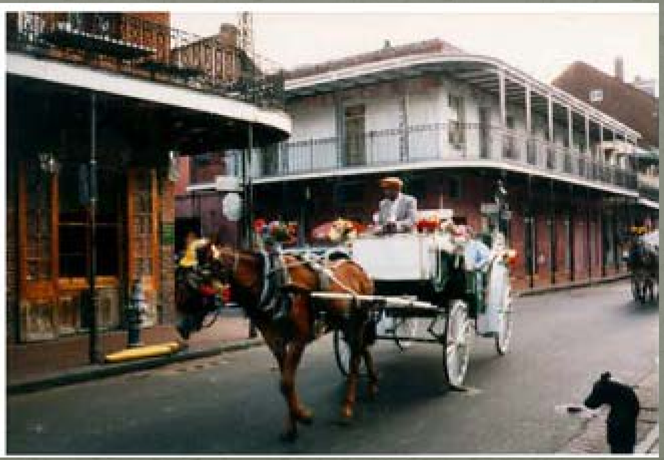
L1-Sparse errors



Inpainting



Since 1699, when French explorers landed at the great bend of the Mississippi River and celebrated the first Mardi Gras in North America, New Orleans has brewed a fascinating melange of cultures. It was French, then Spanish, then French again, then sold to the United States. Through all these years, and even into the 1900s, others arrived from everywhere: Acadians (Cajuns), Africans, indige-

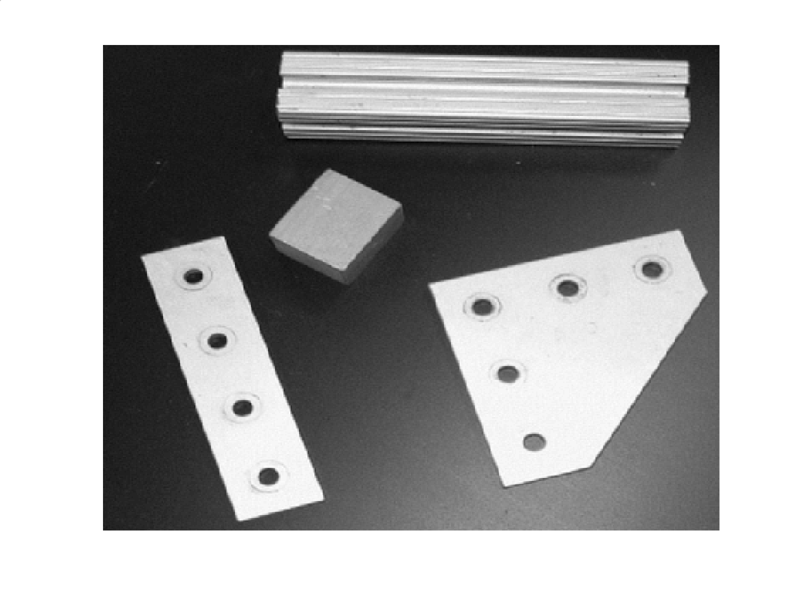


Result

Last Demo…

Input

Result



Code

- https://github.com/eugenfunk/compressive_sensing