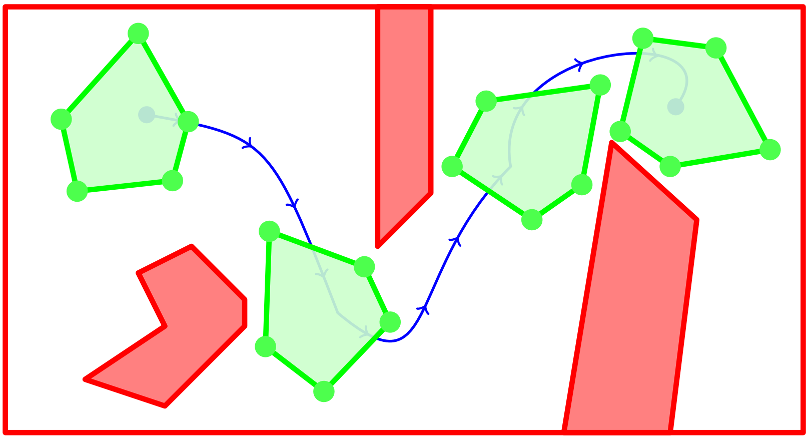
On Sampling Based Algorithms for Solving the Motion Planning Problem

Dror Atariah @ Game Duell

September 18th, 2014

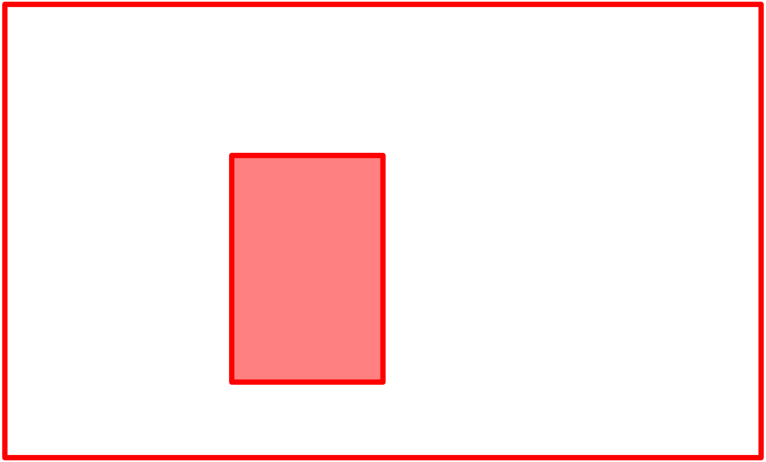
The Motion Planning Problem



From a Workspace to a Configuration Space

Simple Case

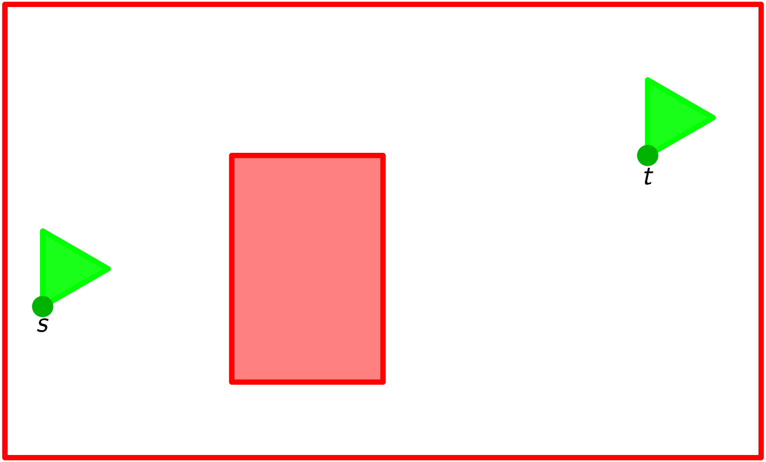
The robot can only translate (2 degrees of freedom)



From a Workspace to a Configuration Space

Simple Case

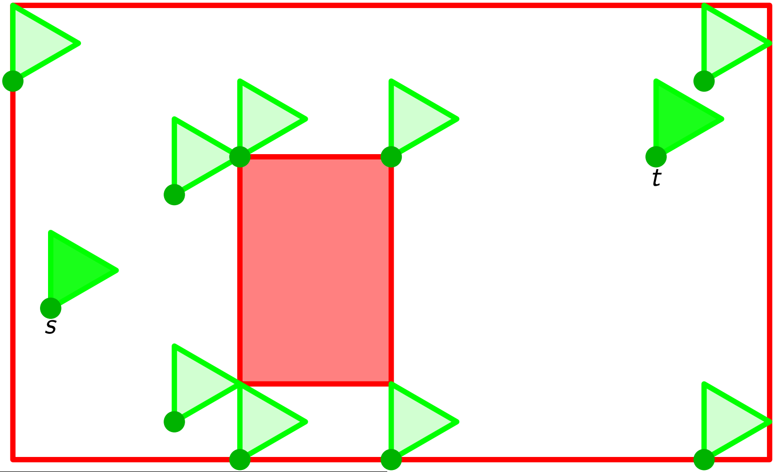
The robot can only translate (2 degrees of freedom)



From a Workspace to a Configuration Space

Simple Case

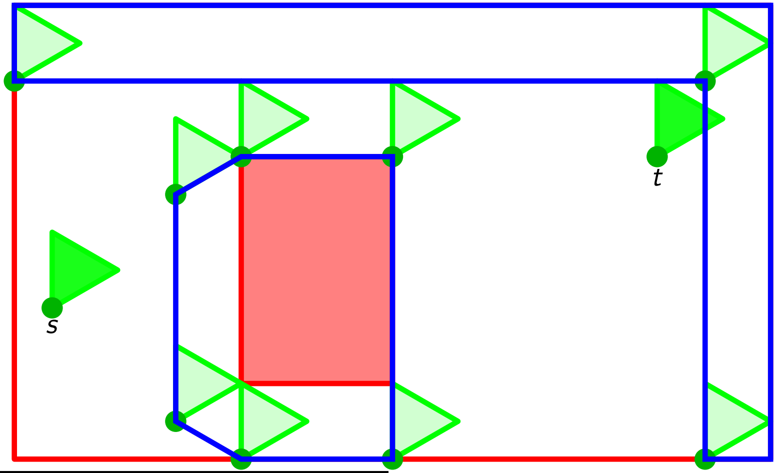
The robot can only translate (2 degrees of freedom)



From a Workspace to a Configuration Space

Simple Case

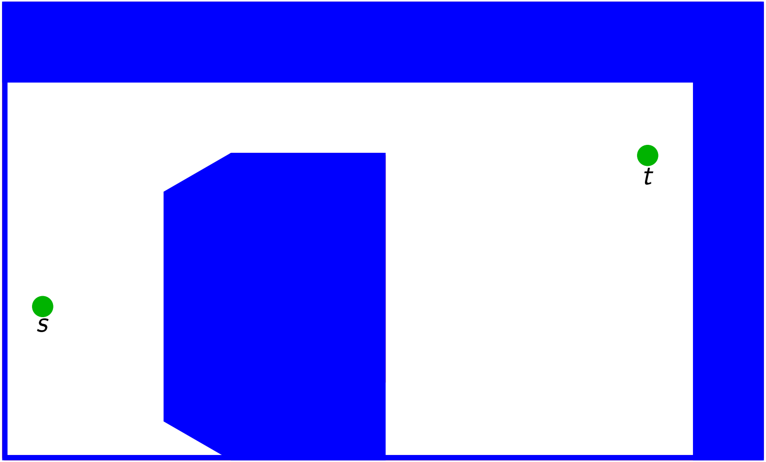
The robot can only translate (2 degrees of freedom)



From a Workspace to a Configuration Space

Simple Case

The robot can only translate (2 degrees of freedom)

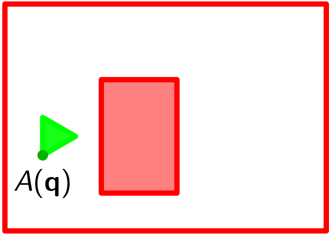


Workspace and Configuration Space

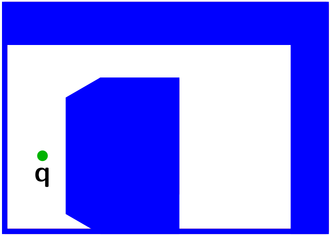
Definition

Given a *robot* A, obstacle O and a *workspace* $\mathcal{W}$, we obtain a *configuration space* denoted by $\mathcal{C}$.

- $q \in \mathcal{C} \xLeftrightarrow{\textit{bijection}} A(q)$
- $\mathcal{C} = \mathcal{C}_{\text{forb}} \cup \mathcal{C}_{\text{free}}$



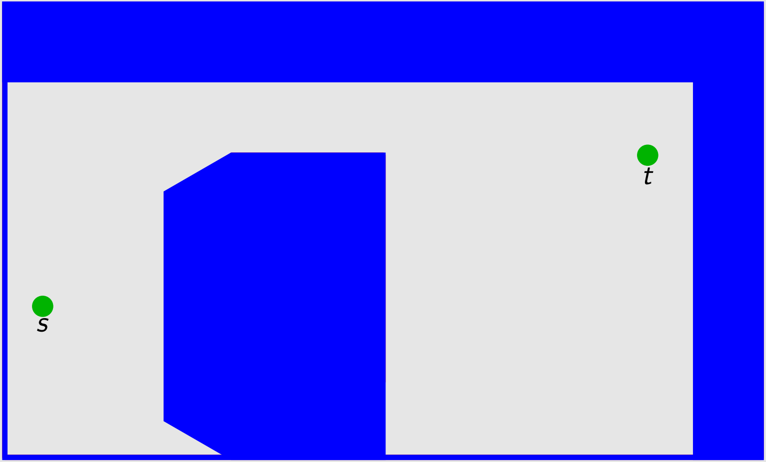
$\mathcal{W}$



$\mathcal{C}$

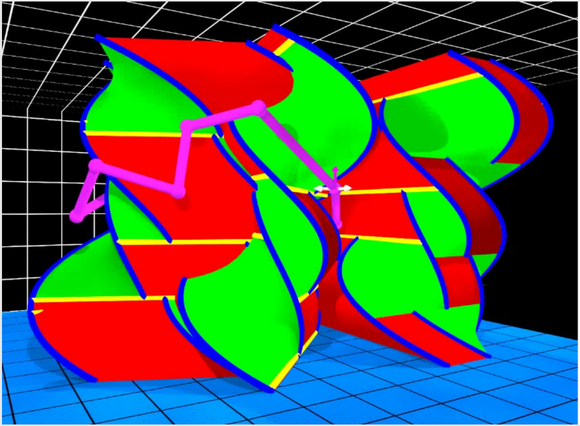
Configuration Space

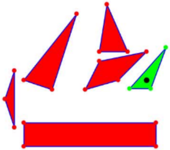
Example



Configuration Space

Example





Problem Statement

Formal Problem

Navigate a given robot A in a workspace $\mathcal{W}$ that is scattered with obstacles O from a source placement $A(\mathbf{s})$ to a target one $A(\mathbf{t})$.
Equivalently, find a *free path* in $\mathcal{C}$ from $\mathbf{s}$ to $\mathbf{t}$.

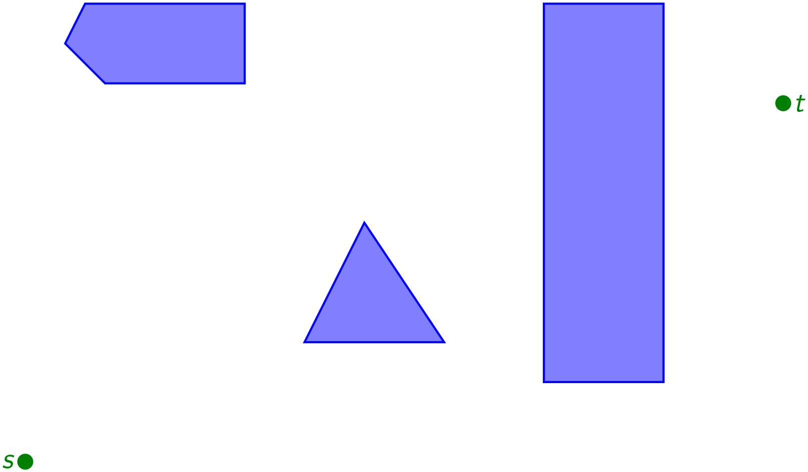
Outline

1. Probabilistic Roadmap (L. Kavraki et al. 1996)
2. Motion Planning via Manifold Samples (Salzman et al. 2013)

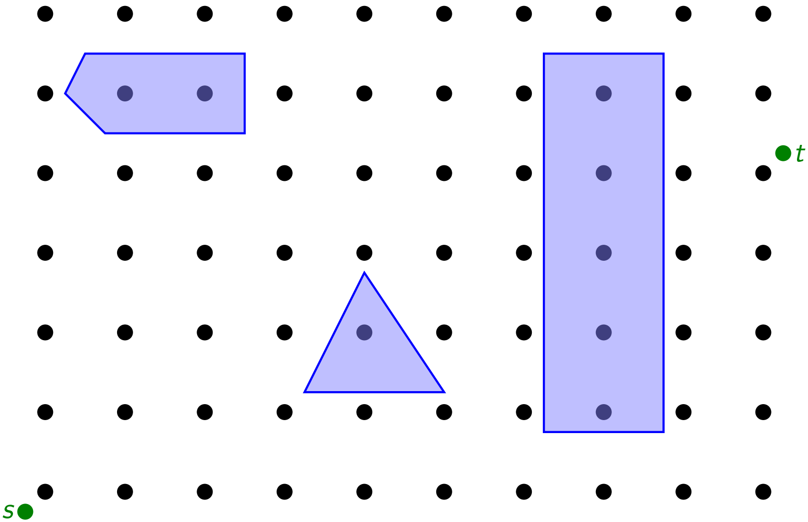
In a nut shell

- A and $\mathcal{W}$ yield $\mathcal{C}$
- Sample $\mathcal{C}_{\text{free}}$
- Build a *roadmap graph* $\mathcal{G} = (V, E)$
  - V - sample points in $\mathcal{C}_{\text{free}}$
  - E - free (local) motions
- Connect $\mathbf{s}$ and $\mathbf{t}$ to $\mathcal{G}$ and find free path

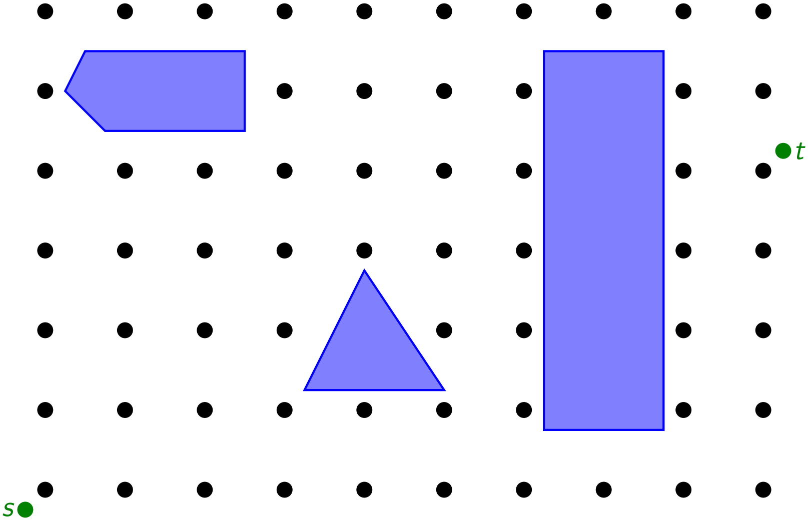
PRM Demo



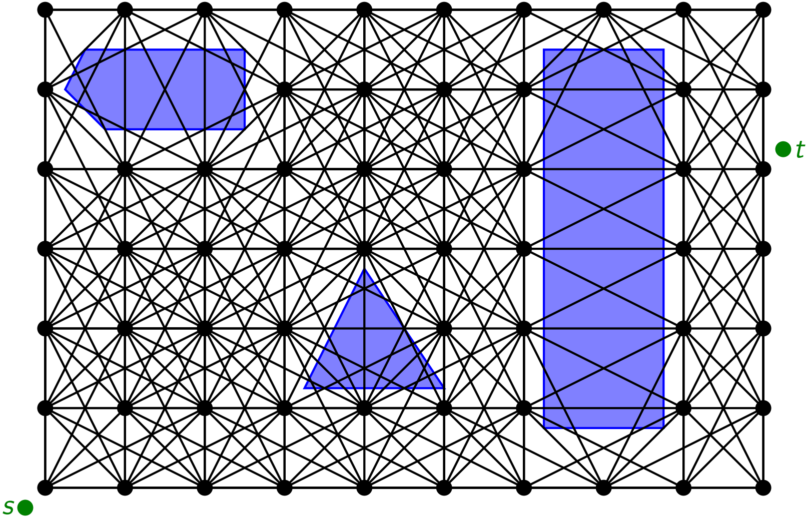
PRM Demo



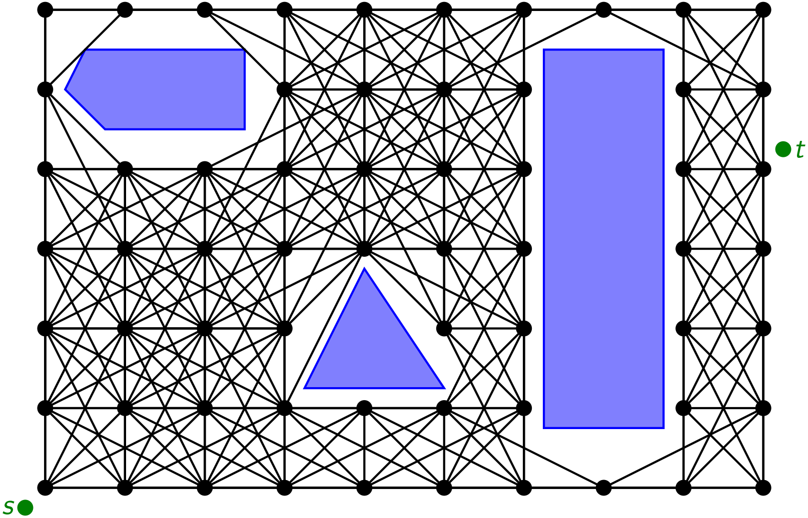
PRM Demo



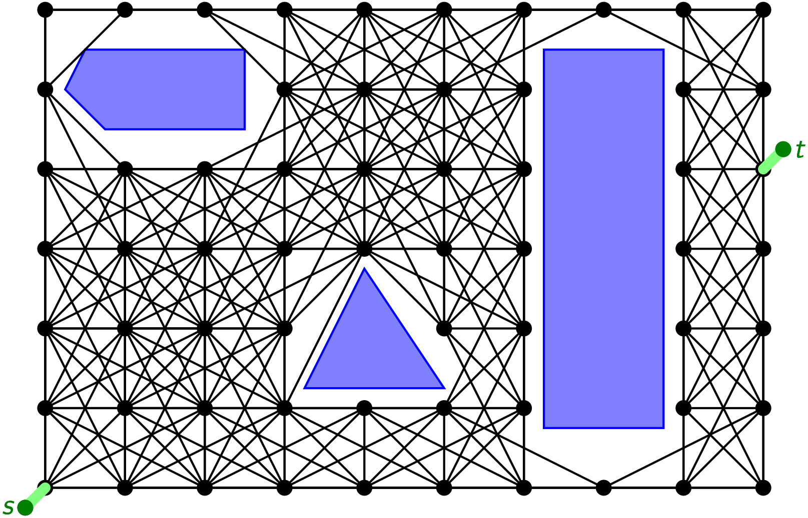
PRM Demo



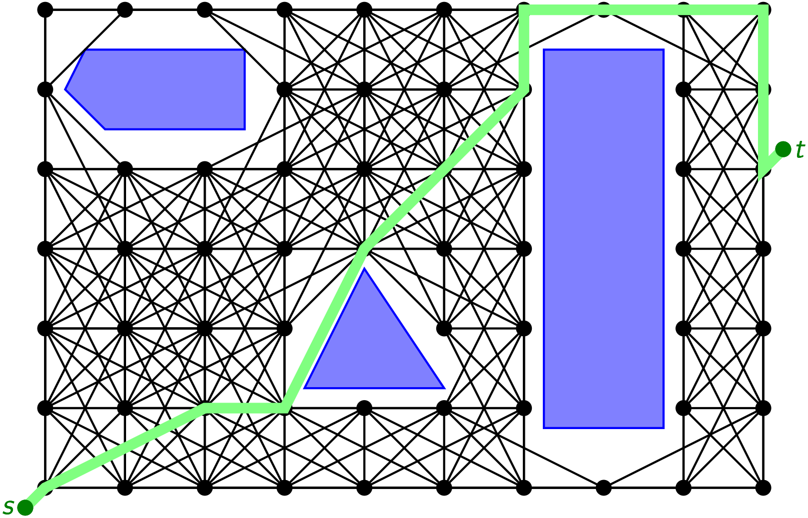
PRM Demo



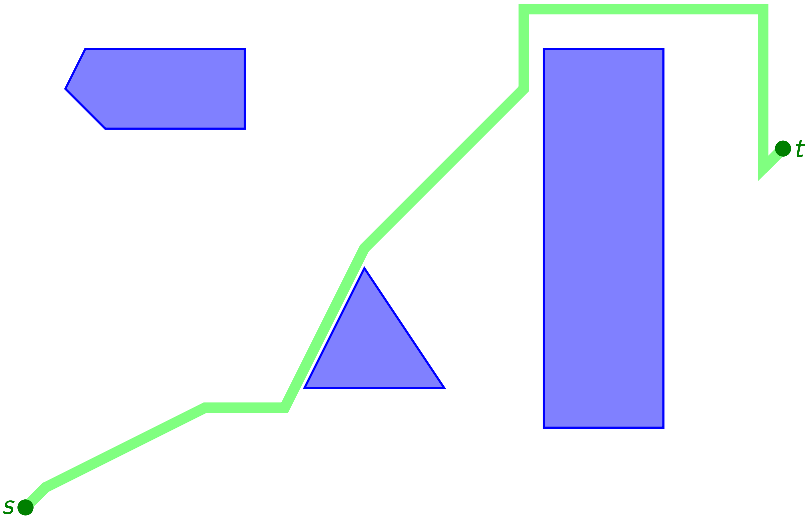
PRM Demo



PRM Demo



PRM Demo



Classical PRM L. Kavraki et al. 1996

Algorithm PRMBuildGraph

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
loop
    $\mathbf{q} \leftarrow$ a point in $\mathcal{C}_{\text{free}}$
    $V \leftarrow V \cup \{\mathbf{q}\}$
    $N_{\mathbf{q}} \leftarrow$ useful neighbors of $\mathbf{q}$
    for all $\mathbf{q}' \in N_{\mathbf{q}}$ **do**
        if $\mathbf{q}'$ and $\mathbf{q}$ are not connected in $\mathcal{G}$ **then**
            if the local planner finds a path between $\mathbf{q}'$ and $\mathbf{q}$ **then**
                $E \leftarrow E \cup \overline{\mathbf{q}\mathbf{q}'}$

Classical PRM L. Kavraki et al. 1996

Algorithm PRMBuildGraph

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
loop
  $\mathbf{q} \leftarrow$ a point in $\mathcal{C}_{\text{free}}$
  $V \leftarrow V \cup \{\mathbf{q}\}$
  $N_{\mathbf{q}} \leftarrow$ useful neighbors of $\mathbf{q}$
  for all $\mathbf{q}' \in N_{\mathbf{q}}$ **do**
    if $\mathbf{q}'$ and $\mathbf{q}$ are not connected in $\mathcal{G}$ **then**
      if the local planner finds a path between $\mathbf{q}'$ and $\mathbf{q}$ **then**
        $E \leftarrow E \cup \overline{\mathbf{q}\mathbf{q}'}$

Classical PRM L. Kavraki et al. 1996

Algorithm PRMBuildGraph

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
loop
  $\mathbf{q} \leftarrow$ a point in $\mathcal{C}_{\text{free}}$
  $V \leftarrow V \cup \{\mathbf{q}\}$
  $N_{\mathbf{q}} \leftarrow$ useful neighbors of $\mathbf{q}$
  for all $\mathbf{q}' \in N_{\mathbf{q}}$ **do**
    if $\mathbf{q}'$ and $\mathbf{q}$ are not connected in $\mathcal{G}$ **then**
      if the local planner finds a path between $\mathbf{q}'$ and $\mathbf{q}$ **then**
        $E \leftarrow E \cup \overline{\mathbf{q}\mathbf{q}'}$

PRM Implementation Details

- How to sample?

Sample the Configuration Space

Objective

Obtain a (uniform) sample of $\mathcal{C}_{\text{free}}$

Steps

- (Randomly) pick $\mathbf{q} \in \mathcal{C}$
- *Collision detection* for $A(\mathbf{q})$

Configuration Points Picking

- Random
- Grid
- Cell-Based
- Halton points (pseudo random and deterministic!) more. . .
- Random-Halton

Classical PRM L. Kavraki et al. 1996

Algorithm PRMBuildGraph

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
loop
  $\mathbf{q} \leftarrow$ a point in $\mathcal{C}_{\text{free}}$
  $V \leftarrow V \cup \{\mathbf{q}\}$
  $N_{\mathbf{q}} \leftarrow$ useful neighbors of $\mathbf{q}$
  for all $\mathbf{q}' \in N_{\mathbf{q}}$ **do**
    if $\mathbf{q}'$ and $\mathbf{q}$ are not connected in $\mathcal{G}$ **then**
      if the local planner finds a path between $\mathbf{q}'$ and $\mathbf{q}$ **then**
        $E \leftarrow E \cup \overline{\mathbf{q}\mathbf{q}'}$

PRM Implementation Details

- How to sample?
- Determine whether a sample point is free?

Collision Detection

Definition (Predicate)

$$\phi\colon \mathcal{C} \to \{\mathrm{TRUE}, \mathrm{FALSE}\}, \text{ s.t.:}$$

$$\phi(\mathbf{q}) = \begin{cases} \mathrm{TRUE} & \text{if } \mathbf{q} \in \mathcal{C}_{\text{forb}} \\ \mathrm{FALSE} & \text{otherwise} \end{cases}$$

Definition (Distance Function)

The distance in $\mathcal{W}$ between $A(\mathbf{q})$ and the obstacles is measured by $d\colon \mathcal{C} \to [0, \infty)$.

Two-phase Collision Detection

Fact

Collision detection is an expensive task that is repeatedly invoked!

Break-up the Task

- Broad Phase (e.g. using bounding boxes)
- Narrow Phase (e.g. using hierarchical methods; See LaValle 2006, § 5.3.2)

Classical PRM L. Kavraki et al. 1996

Algorithm PRMBuildGraph

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
loop
  $\mathbf{q} \leftarrow$ a point in $\mathcal{C}_{\text{free}}$
  $V \leftarrow V \cup \{\mathbf{q}\}$
  $N_{\mathbf{q}} \leftarrow$ useful neighbors of $\mathbf{q}$
  for all $\mathbf{q}' \in N_{\mathbf{q}}$ **do**
    if $\mathbf{q}'$ and $\mathbf{q}$ are not connected in $\mathcal{G}$ **then**
      if the local planner finds a path between $\mathbf{q}'$ and $\mathbf{q}$ **then**
        $E \leftarrow E \cup \overline{\mathbf{q}\mathbf{q}'}$

PRM Implementation Details

- How to sample?
- Determine whether a sample point is free?
- How to determine whether an *edge* is free or not?

Local Planner

Objective

Given $\mathbf{q}, \mathbf{q}' \in \mathcal{C}_{\text{free}}$ find a path in $\mathcal{C}_{\text{free}}$ connecting them.

Trade-off

- Accurate and slow
- Inaccurate and quick

Local Planner

Objective

Given $\mathbf{q}, \mathbf{q}' \in \mathcal{C}_{\text{free}}$ find a path in $\mathcal{C}_{\text{free}}$ connecting them.

Trade-off

- Accurate and slow
- Inaccurate and quick

General Local Planner L. Kavraki et al. 1996

- Given $\mathbf{q}, \mathbf{q}' \in \mathcal{C}_{\text{free}}$
- Let $\{\mathbf{c}_i\}$ be points on the *line* connecting $\mathbf{q}$ and $\mathbf{q}'$, s.t. $\mathrm{d}(A(\mathbf{c}_i), A(\mathbf{c}_{i+1})) \leq \epsilon$
- $\forall i$: collision detection for $A_\epsilon(\mathbf{c}_i)$

Binary Local Planner Geraerts and Overmars 2004

- Given $\mathbf{q}, \mathbf{q}' \in \mathcal{C}_{\text{free}}$
- Let $\mathbf{c}$ be the midpoint of the *line* connecting $\mathbf{q}$ and $\mathbf{q}'$
  - Collision detection for $A(\mathbf{c})$
  - Repeat on two sides of $\mathbf{c}$

Completeness of the Method

Completeness of the Method

Definition

A method is *probabilistically complete* if the probability of answering a query *incorrectly* after building a roadmap tends to zero as the number of samples goes to *infinity*.

Influencing Parameters

- N - number of samples
- $\gamma\colon [0, L] \to \mathcal{C}_{\text{free}}$ - free path from $\gamma(0) = \mathbf{s}$ to $\gamma(L) = \mathbf{t}$.
- $R = \inf_{0 \le t \le L} \inf_{y \in O} |\gamma(t) - y|$

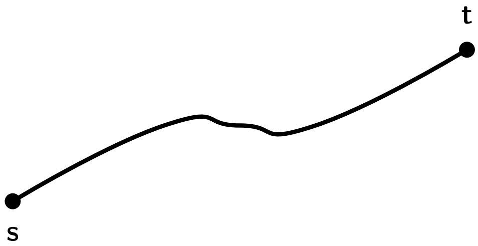
Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \le \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

PRM is Probabilistic Complete

Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \leq \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

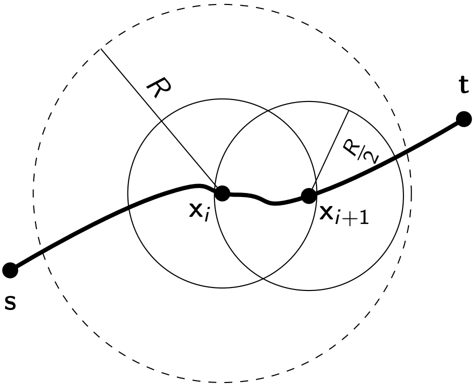


PRM is Probabilistic Complete

Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \leq \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

- $n = \lceil 2L/R \rceil$ points on γ
- $d(\mathbf{x}_i, \mathbf{x}_{i+1}) \leq R/2$ and $B_{R/2}(\mathbf{x}_{i+1}) \subset B_R(\mathbf{x}_i)$

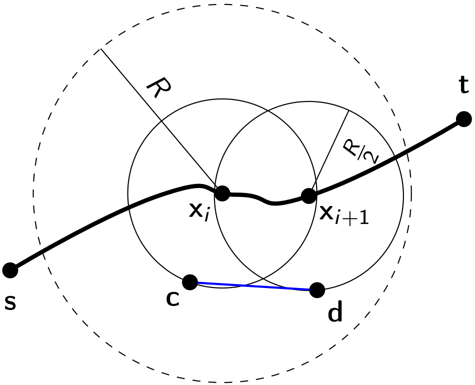


PRM is Probabilistic Complete

Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \leq \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

- $n = \lceil 2L/R \rceil$ points on γ
- $d(\mathbf{x}_i, \mathbf{x}_{i+1}) \leq R/2$ and $B_{R/2}(\mathbf{x}_{i+1}) \subset B_R(\mathbf{x}_i)$
- $\Rightarrow \overline{\mathbf{cd}}$ is free

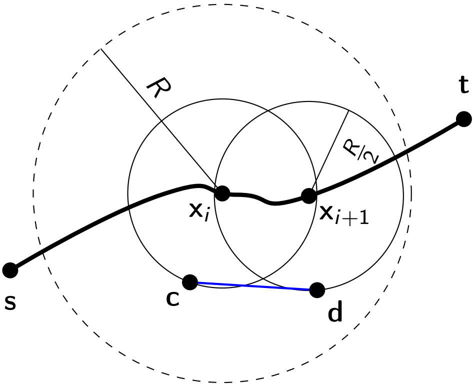


PRM is Probabilistic Complete

Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \leq \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

- $n = \lceil 2L/R \rceil$ points on γ
- $d(\mathbf{x}_i, \mathbf{x}_{i+1}) \leq R/2$ and $B_{R/2}(\mathbf{x}_{i+1}) \subset B_R(\mathbf{x}_i)$
- $\Rightarrow \overline{\mathbf{cd}}$ is free
- PRM succeeds if each $B_{R/2}(\mathbf{x}_j)$ contains a sample
- $P(B_{R/2}(\mathbf{x}_j)$ is empty$) = \left(1 - \frac{|B_{R/2}|}{|\mathcal{C}_{\text{free}}|}\right)^N$

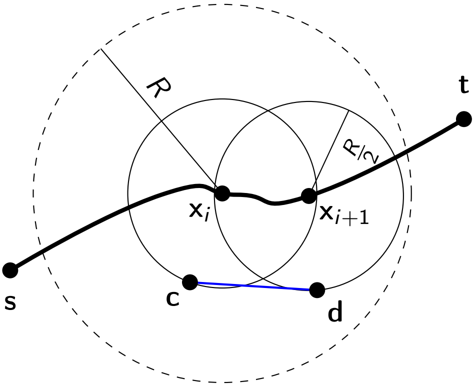


PRM is Probabilistic Complete

Theorem (L. E. Kavraki et al. 1998)

$$P(\text{FAILURE}) \leq \frac{2L}{R}\left(1 - \frac{\pi R^2}{4|\mathcal{C}_{free}|}\right)^N$$

$$\begin{aligned}
&P(\text{FAILURE}) \\
&\leq P(\text{some ball is empty}) \\
&\leq \sum_{j=1}^{n-1} P(j\text{-th ball is empty}) \\
&= \left(\left\lceil \frac{2L}{R} \right\rceil - 1\right)\left(1 - \frac{|B_{R/2}|}{|\mathcal{C}_{\text{free}}|}\right)^N
\end{aligned}$$



In a nut shell

- A and $\mathcal{W}$ yield $\mathcal{C}$
- Sample $\mathcal{C}_{\text{free}}$
- Build a *roadmap graph* $\mathcal{G} = (V, E)$
    - V - sample points in $\mathcal{C}_{\text{free}}$
    - E - free (local) motions
- Connect $\mathbf{s}$ and $\mathbf{t}$ to $\mathcal{G}$ and find free path

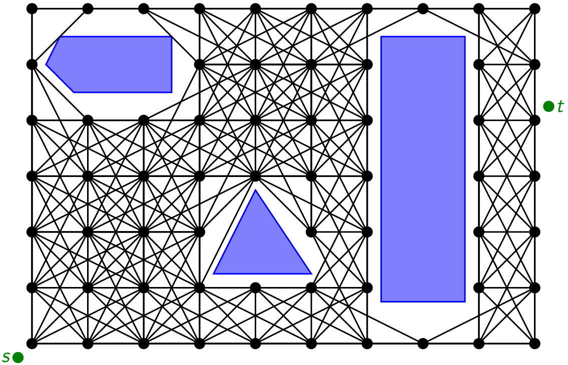
Outline

1. Probabilistic Roadmap (L. Kavraki et al. 1996)
2. Motion Planning via Manifold Samples (Salzman et al. 2013)

Motion Planning via Manifold Samples Salzman et al.

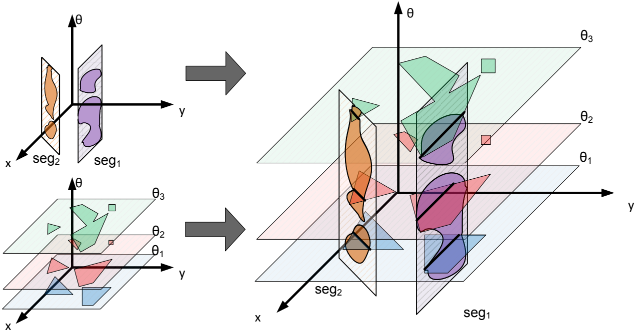
2013

- In PRM connectivity of $\mathcal{C}$ is captured via point samples
- Try to obtain "bigger" samples and improve the capturing



Outline of MMS

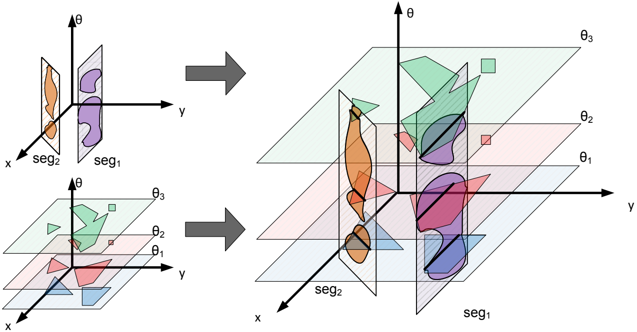
- A and $\mathcal{W}$ yield $\mathcal{C}$
- Sample $\mathcal{C}$
- Build a *connectivity graph* $\mathcal{G} = (V, E)$
  - V - Free Space Cells (FSC)
  - E - Between intersecting FSC's
- Connect **s** and **t** to $\mathcal{G}$ and find free path



Courtesy of Oren Salzman

Preprocessing

- Family of constraints $\Psi \Rightarrow$ manifolds in $\mathcal{C}$
  Example: Translating and rotating planar robot
  - Horizontal planes
  - Vertical slabs
- Decompose the manifolds into *Free Space Cells*
- Connect, in $\mathcal{G}$, intersecting FSC's



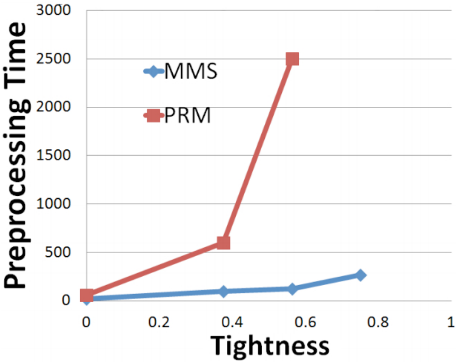
Courtesy of Oren Salzman

Discussion

- MMS is also probabilistic complete

Discussion

- MMS is also probabilistic complete
- No significant improvements in simple cases
- 20-fold speedup in a coordination tight setting
- Significant improvement in tight cases



Courtesy of Oren Salzman

Summary

- Introduced the motion planning problem, and
- Sample based methods to solve it:
  - Probabilistic Roadmap Method (PRM)
  - Motion Planning via Manifold Samples (MMS)
  - Rapidly-exploring Random Trees (RRT)

    due to Lavalle and Kuffner 2000
  - PRM*, RRT* and RRG (=Rapidly-exploring Random Graph)

    due to Karaman and Frazzoli 2011

Thank you for your attention!

- Twitter: @drorata
- LinkedIn: `www.linkedin.com/in/atariah`

Halton Points

- $k \in \mathbb{Z}$ and a prime $p \Rightarrow k = \sum_{i=0}^{r} a_i p^i$ s.t. $0 \leq a_i < p$
- Let $\Phi_p(k) = \sum_{i=0}^{r} \frac{a_i}{p^{i+1}}$
- For primes $p_1 < p_2 < \ldots < p_{d-1}$, then the k-th d-dimensional *Halton point* is

$$\left(\frac{k}{n}, \Phi_{p_1}(k), \Phi_{p_2}(k), \ldots, \Phi_{p_{d-1}}(k)\right) \in [0,1]^d,$$

  where $k = 0, 1, \ldots, n-1$
- For further details see Wong et al. 1997; Chazelle 1998 and LaValle 2006, § 5.2

Back

References I

Chazelle, B. (1998). "The Discrepancy Method."

Geraerts, R. and M. Overmars (2004). "A Comparative Study of Probabilistic Roadmap Planners".

Karaman, S. and E. Frazzoli (2011). "Sampling-based Algorithms for Optimal Motion Planning".

Kavraki, L. E. et al. (1998). "Analysis of probabilistic roadmaps for path planning."

Kavraki, L. et al. (1996). "Probabilistic roadmaps for path planning in high-dimensional configuration spaces".

LaValle, S. M. (2006). *Planning Algorithms*.

Lavalle, S. M. and J. J. Kuffner (2000). "Rapidly-Exploring Random Trees: Progress and Prospects".

Salzman, O. et al. (2013). "Motion Planning via Manifold Samples".

Siciliano, B. and O. Khatib, eds. (2008). *Springer Handbook of Robotics*.

Wong, T.-T. et al. (1997). "Sampling with Hammersley and Halton Points".