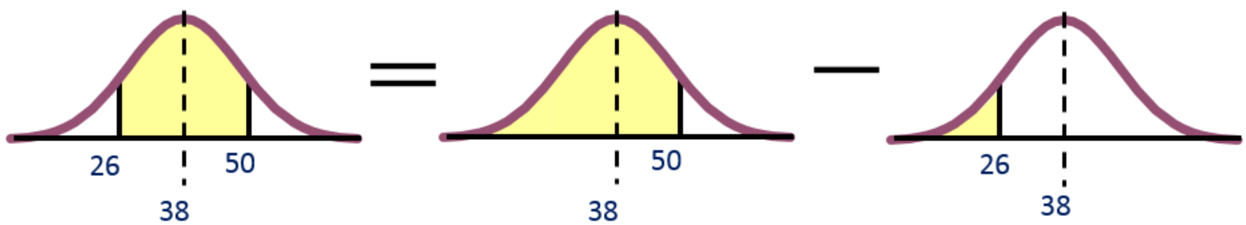
Normal Distribution: Additional Examples with Answers

1. Some numbers are not normally distributed. If the mean of the numbers is 38 and the standard deviation is 6, what proportion of values would fall between 26 and 50?



We will do this in three steps:

Step 1: Lets calculate the proportion of values that are smaller than 50.

The Z-score for 50 is (50-38)/6=2

```
> pnorm(2)
[1] 0.9772499
```

You could also call the pnorm() function directly without explicitly calculating the z-score.

```
> pnorm(50,mean=38,sd=6)
[1] 0.9772499
```

Step 2: Lets calculate the proportion of values that are smaller than 26.

The Z-score for 26 is (26-38)/6=-2

```
> pnorm(-2)
[1] 0.02275013
```

Or

```
> pnorm(26,mean=38,sd=6)
[1] 0.02275013
```

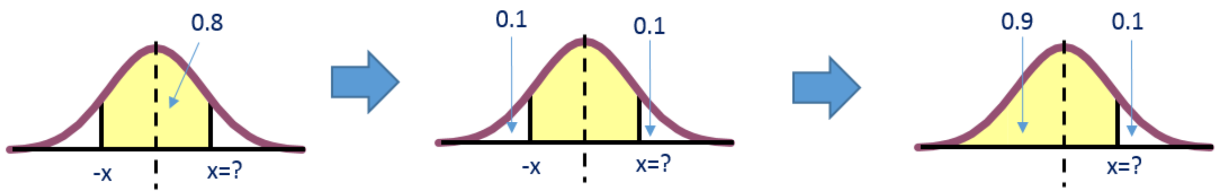
Step 3: Calculate the difference between the two values.

0.9772499-0.02275013= 0.9544998

You already knew that 95% of data is between z-score -2 and 2!

2. If the data follows a normal distribution, how many standard deviations around the mean would include at least 80% of the values?

We what to find x such that the 80% of data falls between -x and x (see left graph). We don't have a way to calculate this directly. But we can say, if 80% of the data is in the yellow area, each of the side white tails would contain 10% of values (see the middle graph). Now, we can say we want to calculate x, such that 90% of data falls below x (see right graph) and we now have a formula for it.




```
> qnorm(0.9)
[1] 1.281552
```

Remember qnorm()is inverse of pnorm(). With pnorm(), the user provides the percentage/probability and the function returns the z-score while this is the other way round for qnorm().

Now, let's check this to be sure if 80% of data lies between x=-1.281552 and x=1.281552. This is similar to question 1.

```
> pnorm(1.281552)- pnorm(-1.281552)
[1] 0.8000002
```

This confirms our answer.

3. Environmentalists are concerned about emissions of sulfur dioxide into the air. The average number of days per year in which sulfur dioxide levels exceed 150 milligrams per cubic meter in Milan, Italy, is 29. The number of days per year in which emission limits are exceeded is normally distributed with a standard deviation of 4.0 days.

A) What percentage of the years would average between 31 and 37 days of excess emissions of sulfur dioxide?

B) What percentage of the years would exceed 37 days?

Part A)

z-score for 37 is (37-29)/4=2

```
> pnorm(2)
[1] 0.9772499
```

Z-score for 31 is (31-29)/4=0.5

```
> pnorm(0.5)
[1] 0.6914625
```

Percentage of the years would average between 31 and 37 days of excess emissions of sulfur dioxide: 0.9772499-0.6914625=0.2857874. So the answer is 28.57%.

Part B)

z-score for 37 is (37-29)/4=2

```
> 1-pnorm(2)
[1] 0.02275013
```

```
Or
> pnorm(2,lower.tail = FALSE)
[1] 0.02275013
```

So 2.27% of years would exceed 37 days.

4. Shown below are the per diem business travel expenses listed by Runzheimer International for 11 selected cities around the world. Use this list to calculate the z scores for Moscow and Beijing.

City	**Per Diem Expense ($)**
Beijing	282
Hong Kong	361
London	430
Los Angeles	259
Mexico City	302
Moscow	376
New York (Manhattan)	457
Paris	305
Rio de Janeiro	343
Rome	297
Sydney	188

Calculation of z-score requires mean, Mu, and standard deviation, Sigma, first.

```
> Mu=mean(c(282,361,430,256,302,376,457,305,343,297,188))

> Mu

[1] 327

> sigma=sd(c(282,361,430,256,302,376,457,305,343,297,188))

> sigma

[1] 77.27742
```

z-score for Moscow (376-327)/77.27742 =0.6340791

z-score for Beijing (282-327)/77.27742 =-0.5823176

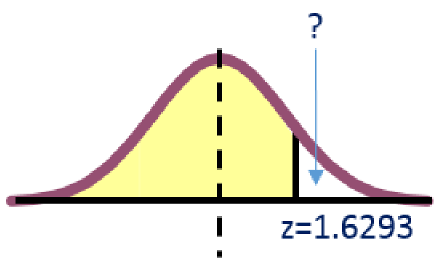
5. In a recent year, the average daily circulation of The Wall Street Journal was 1,717,000. Suppose the standard deviation is 50,940. Assume the paper's daily circulation is normally distributed.

A) On what percentage of days would circulation pass 1,800,000?

B) Suppose the paper cannot support the fixed expenses of a full-production setup if the circulation drops below 1,600,000. How often will this even happen, based on this historical information?

Part A)

The z-score of 1,800,000 is (1800000-1717000)/50940=1.629368




```
> 1-pnorm(1.629368)
[1] 0.05161757
```

OR

```
> pnorm(1.629368,lower.tail = FALSE)
[1] 0.05161757
```

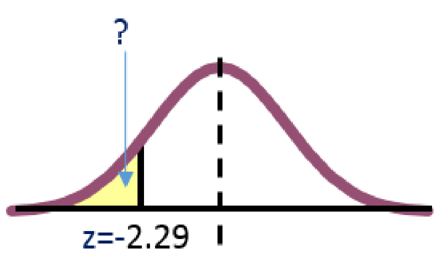
OR

```
> pnorm(1800000,mean=1717000,sd=50940,lower.tail = FALSE)
[1] 0.05161758
```

So the percentage of days would circulation pass 1,800,000 would be only around 5%.

Part B)

The z-score of 1,600,000 is (1600000-1717000)/50940=-2.29682



```
> pnorm(-2.29682)
[1] 0.01081452
```

OR

```
> pnorm(1600000,mean=1717000,sd=50940)
[1] 0.01081453
```

So the percentage of days would circulation fall below 1,600,000 would be only around 10%.

6. Mercedes and BMW have been competing head-to-head for market share in the luxury-car market for more than four decades. Even though each company produces many different models, two relatively comparable coupe automobiles are the BMW 3 Series Coupe 335i and the Mercedes CLK350 Coupe. Suppose Mercedes is concerned that dealer prices of the CLK350 Coupe are not consistent and that even though the average price is $44,520, actual prices are normally distributed with a standard deviation of $2,981. The average price for a BMW 3 Series Coupe 335i is $39,368.Suppose these prices are also normally distributed with a standard deviation of $4,367.

A) What percentage of BMW dealers are pricing the BMW 3 Series Coupe 335i at more than the average price for a Mercedes CLK350 Coupe?

B) What percentage of Mercedes dealers are pricing the CLK350 Couple at less than the average price of a MW 3 Series Coupe 335i?

Part A)

The average price of Mercedes CLK350 Coupe is $44,520. Considering the distribution of BMW 3 Series Coupe 335i prices, the z-score of this price is (44520-39368)/4367= 1.179757

```
> pnorm(1.179757)
[1] 0.8809516
```

So 88% of BWM dealerships are offering BMW 3 Series Coupe 335i with the price less than $44,520 which is the mean price of the Mercedes CLK350 Coupe.

Part B)

The average price of BMW 3 Series Coupe 335i is $39,368. . Considering the distribution of Mercedes CLK350 Coupe prices, the z-score of this price is

```
(39368-44520)/2981= -1.728279
```

```
> pnorm(-1.728279)
[1] 0.04196911
```

So only 4% of Mercedes dealers are offering CLK350 Coupe with the price lower that $39,368 that is the mean price of the BMW 3 Series Coupe 335i.