Socio-Econ. Plann. Sci. Vol. 23, No. 4, pp. 227–234, 1989
Printed in Great Britain.

0038-0121/89 $3.00 + 0.00

Univariate ARIMA Forecasts of Defined Variables: the Case of Real Economic Variables

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(Received March 1988)

Abstract—This paper provides additional evidence on whether indirect or direct autoregressive-integrated-moving average (ARIMA) model point forecasts of defined variables are better. Seven monthly real or constant-dollar economic variables for the postwar U.S. over the period 1959–1986 are examined. The variables include three real *ex-post* interest rates on government securities of varying maturity, two real monetary aggregates, a real wage, and a real foreign exchange rate. Univariate ARIMA models are identified and estimated for the within-sample period from January 1959 to December 1982 using the techniques of Box and Jenkins. Out-of-sample, updated, one-step-ahead forecasts are then made for the period January 1983 to December 1986 in order to compare the post-sample accuracy of the indirect and direct forecasts. The criteria of mean absolute error, mean squared error, and mean percentage error are used to evaluate forecast accuracy. In contrast to the findings of Kang, the results show that direct forecasts generally perform better than indirect forecasts for the set of defined variables examined.

INTRODUCTION

It is well-known that forecasting is important to informed decision-making in the areas of business, industry, and government. The ability to forecast future values of key variables accurately reduces the risk associated with decisions that are made in an uncertain world. For example, accurate forecasting of future sales at various lead times is of paramount importance to planners in business and industry who must take decisions in areas such as financial planning, inventory management, production planning, and the control and optimization of industrial processes.

Similarly, good forecasts are important to government policymakers. To cite a particularly current example, projections of the future size of the Federal budget deficit depend on forecasts of government expenditures and tax revenues, which, in turn, are related to forecasts of the growth in economic activity and interest rates. Finally, economists are interested in producing accurate macroeconomic forecasts of key variables, such as interest rates, inflation and real economic growth, that help to guide both business and government policymakers in their decision-making. (See e.g. Box and Jenkins [1] and Granger and Newbold [2] for a discussion of the importance of forecasting.)

This paper addresses an issue of current interest in forecasting. Specifically, the topic of whether indirect or direct forecasts of defined variables are better is examined. A defined variable is one which is itself not measurable, but which is instead defined in terms of other variables that are measured or measurable. Defined variables are thus variables which are constructed in terms of their constituent or component elements. Clearly, the ability to forecast defined business and economic variables with precision is important to those in business, industry, and government who must make decisions contingent on future values of such variables.

Although closely related to the forecasting of aggregated variables, the forecasting of defined variables is a more fundamental undertaking for several reasons. Among these are the fact that defined variables can be used at every level of aggregation, where aggregation can be viewed as

The views expressed are those of the author and do not reflect those of the U.S. Treasury Department. The author wishes to thank several anonymous referees and the Editor of the Journal for helpful comments which improved the paper. Any remaining errors are the responsibility of the author.

The original version of this paper was prepared for the *American Statistical Association Meeting*, San Francisco, Calif., August 1987.

a special case of the definitional relationships that exist among variables. (Kang [3] offers a more complete discussion of this issue in his study on defined variables in economics.)

There are many examples of defined variables in the areas of business and economics. For example, the real or constant-dollar money supply is a defined variable, which is constructed by deflating a nominal measure of the money supply by a general price index such as the gross national product (GNP) implicit-price deflator or the consumer price index (CPI) in order to account for movements in the price level. Other examples include real or constant-dollar interest rates, real GNP, the money multiplier, and the income velocity of money, to name just a few.† There are two distinct ways to forecast defined variables. Direct forecasts are made by forecasting the defined variables' actual values, while indirect forecasts involve forecasting the defined variables in terms of their constituent elements or component parts.

In order to clarify these concepts, consider the following example. If M denotes the nominal or current-dollar money supply, and P is some general price index, then the real money supply, m, is defined as the ratio M/P. Both M and P are observable and represent the constituent elements in the definition of the real money supply. A direct forecast of the real money supply, m, results from using the constructed series m, i.e. M/P, as the data in the forecasting procedure. On the other hand, an indirect forecast involves forecasting M and P separately, and then dividing the forecast of M by the forecast of P, according to the definition of the real money supply, to obtain a forecast of m.‡

Makridakis and Hibon [4], Makridakis *et al.* [5], and Meese and Geweke [6] have examined the use of alternative forecasting techniques in an effort to identify those procedures that produce the most efficient forecasts for large sets of macroeconomic time series. Included among these time series were defined variables, although the studies were not specifically concerned with the forecasting of such variables.

Recently, Kang [3] offered the first investigation of whether direct or indirect forecasts perform better in forecasting defined variables. In the current study, four defined economic variables, including the monthly real interest rate on 3-month U.S. Treasury bills, the monthly money multiplier, quarterly real GNP, and quarterly money velocity, were analyzed using Box–Jenkins univariate time series analysis for the U.S. over the period January 1959 to December 1983. All variables were officially seasonally adjusted save the Treasury bill rate.

With the exception of the monetary variables examined herein (i.e. the money multiplier), Kang found that indirect forecasts generally outperformed direct forecasts.§ Noting, however, the possibility that his results were specific to the choice of defined variables, Kang stressed the importance of extending the investigation to other sets of defined variables. In particular he suggested that real defined variables such as the real wage, real foreign exchange rate, and the real money supply be studied to observe whether the same results might be obtained. The recommendation was also made that seasonally unadjusted data be used.

This paper provides further evidence as to whether direct or indirect autoregressive-integrated-moving average (ARIMA) model point, or single-valued, forecasts are better in forecasting a set of real economic variables. Seven monthly real or constant-dollar economic variables for the U.S. over the period 1959–1986 are used. The defined variables include three real interest rates, two real money supply measures, a real wage, and a real foreign exchange rate. Univariate ARIMA models are identified and estimated for the within-sample period January 1959 to December 1982. Out-of-sample updated single-step forecasts are made from January 1983 to December 1986. Forecasting accuracy is then evaluated using three metrics: mean absolute error, mean squared error, and mean absolute percentage error.

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†The money multiplier is defined as the ratio of the money supply to the monetary base or high-powered money The income velocity of money is computed by dividing nominal GNP by the money supply

‡A simple example can be used to illustrate how aggregated and defined variables are related. Total consumption expenditure of households is derived by simply aggregating the consumption expenditures of individual households. Real consumption expenditure, a defined variable which is computed by deflating nominal consumption expenditure with a price index, can be computed at the aggregate level as well as at the individual household level

§In his study, Kang provides direct and indirect forecasts for all the defined variables as well as for all the measured constituent nominal variables. Since we are specifically interested in defined variables, only direct and indirect forecasts for these variables are reported in the current paper The value of forecasting observed nominal variables, as in [3], remains unclear

FORECASTING METHODOLOGY AND DATA

Lutkepohl [7] has demonstrated that indirect forecasts have a mean squared error that does not exceed that of the direct univariate forecasts of aggregate time series variables under the restrictive condition that the underlying ARIMA processes generating both the aggregate and component variables are known. However, Lutkepohl also noted that when the underlying processes are unknown, which is esentially always the case since the true model is not usually known, the mean squared error of direct forecasts can be smaller than the mean squared error associated with indirect forecasts. This suggests that the issue of whether direct or indirect forecasts are better is essentially an empirical question.

In addition to the work of Kang [3], this view is supported by others. For example, Dunn *et al* [8], Rose [9], Tiao and Guttman [10], Wei and Abraham [11] and Kohn [12] all report mixed results regarding the superiority of direct vs indirect forecasts.

Approach

In order to evaluate the performance of the direct and indirect forecasting procedures, the approach used by Kang was followed to facilitate comparison of the results. Although there are several alternative univariate forecasting techniques that can be used, e.g. simple regression on a time trend, stepwise autoregression, traditional structural econometric models, and exponential smoothing appoaches, the use of Box–Jenkins time series techniques has been shown to produce forecasts of sufficient accuracy for the sample sizes used in this paper. For example, Granger and Newbold [2] present evidence that shows the success and versatility of univariate ARIMA techniques relative to alternative techniques in modelling time series with at least 40–50 observations. In addition, Nelson [13] finds that ARIMA models perform better in post-sample forecasting competitions than do large-scale structural econometric models.

The following approach was employed to generate the direct and indirect point forecasts. First, univariate seasonal ARIMA models were fitted to each of the defined variables, as well as to each of the measured constituent variables over the period January 1959 to December 1982. All variables used are seasonally unadjusted to begin with in the analysis; any seasonality that is present in the time series is modeled directly in the process of fitting the ARIMA model. Model-based seasonal adjustment, such as the case here, has received support in the literature e.g. [14].

Second, after adequate ARIMA models were identified and estimated, out-of-sample, updated, single-step (i.e. one-period-ahead) forecasts were computed for the post-sample period that spans January 1983 to December 1986†. This means that 48 updated one-step-ahead direct and indirect forecasts were made for each defined variable in the analysis

Third, the criteria of mean absolute error (MAE), mean squared error (MSE), and mean absolute percentage error (MAPE) were used to compare the accuracy of the direct and indirect forecasts. The 48 one-period-ahead updated forecast errors, which are computed as the difference between the actual and forecasted value of the series, from the out-of-sample period, are used to calculate the three measures.

The three criteria used to evaluate forecast accuracy have the following formulas:

$$\mathrm{MAE} = N^{-1} \sum_{i=1}^{N} |X_{T+i} - {}_{T+i-1}\hat{X}_{T+i}| \tag{1}$$

$$\mathrm{MSE} = N^{-1} \sum_{i=1}^{N} \{X_{T+i} - {}_{T+i-1}\hat{X}_{T+i}\}^2 \tag{2}$$

$$\mathrm{MAPE} = N^{-1} \sum_{i=1}^{N} \frac{|X_{T+i} - {}_{T+i-1}\hat{X}_{T+i}|}{X_{T+i}} \tag{3}$$

†Let $\hat{z}_{t+l}$, $l \geqslant 1$, be the forecast of some time series, z_t, l-periods in the future, where t denotes the forecast origin. For a single-step forecast, $l = 1$, which means that we are forecasting z_t one period ahead. Updating refers to the fact that actual values of the time series z_t are added to the out-of-sample forecasting period 1 month at a time, which means that the forecast origin is continually moving one-period ahead or is updated. For example, the actual value for z_{t+1} is used as the starting point for a one-step-ahead forecast of z_{t+1} at a time $t+2$. Thus updating continues until the end of the out-of-sample period is reached.

where X_{T+i} denotes the actual value of the time series at time $T+i$, ${}_{T+i-1}\hat{X}_{T+i}$ denotes the single-step forecast of X made at time $T+i-1$, N is the number of forecasts made in the out-of-sample period, and T denotes the length of the within-sample period used to identify and estimate the ARIMA models.

Data

Before presenting the ARIMA models that were fitted to each of the economic time series, a discussion of the data is warranted. Three *ex-post* real interest rates are used. These include rates on the 3-month U.S. Treasury bill, and 10-year and 20-year government bonds. By definition, the *ex-post* real interest rate is computed by subtracting the actual inflation rate from the nominal interest rate. The monthly percentage change in the CPI (all urban, base year = 1980) served as the measure of price inflation and was computed as the unweighted first difference of the natural logarithm of the CPI in successive time periods.

The two measures of the real money supply utilized were the real money supply narrowly defined, $M1$, and the broader measure of money supply, $M2$. By definition, $M1$ includes coinage, cash, and demand deposits, or checking accounts, held by the non-banking public. The broader measure, $M2$, consists of $M1$ plus time deposits or savings accounts. The real wage variable is defined as the ratio of the nominal wage divided by a price index; it was constructed by deflating the nominal wage rate for total manufacturing in the U.S. by the CPI. Finally, the real exchange rate is defined as the nominal U.S. dollar price of a British pound divided by the ratio of the U.S. to the U.K. consumer price index. It should be noted that observations on the exchange rate correspond to the period of floating or flexible exchange rates and cover the period January 1974 to December 1986

Data sources

The interest rate data were taken from issues of the Salomon Brothers publication, *An Analytic Record*. Raw data for the two measures of the money supply, the wage rate, the exchange rate, and the consumer price index for both the U.S. and the U.K. were taken from various issues of the International Monetary Fund's publication, *International Financial Statistics*.

UNIVARIATE SEASONAL ARIMA MODELS

The basic goal of the time series modeling techniques of Box and Jenkins [1] is to construct a stochastic model that approximates the generating mechanism for a given time series realization It is well known that an iterative process of model identification, parameter estimation, and diagnostic checking for model adequacy is followed until a time series model is obtained which transforms each of the time series into a white-noise process. By definition, a white-noise process is purely random.

Several diagnostic procedures, including the use of portmanteau Q-statistics and comparison of the individual autocorrelation coefficients to their asymptotic standard errors, can be used to determine whether a particular ARIMA model adequately captures the autocorrelation structure of the time series that are being examined. An adequate model, once it is identified and estimated, can then be used to produce forecasts of the time series at various lead times.†

Since the economic variables examined here are seasonally unadjusted, multiplicative seasonal ARIMA models were specified for each of the seventeen time series involved in producing the direct and indirect forecasts. The univariate multiplicative seasonal ARIMA model for a zero mean discrete-time stochastic process z_t is given by:

$$\phi_p(B)\,\Phi_P(B^s)\,\nabla^d\nabla_s^D z_t = \theta_q(B)\,\Theta_Q(B^s)a_t, \quad a_t^{iid} \sim (0, \sigma_a^2)\,\forall t, \sigma_a^2 < \infty \quad t = 1, \ldots, T \tag{4}$$

where $\phi_p(B)$ and $\theta_q(B)$ are regular autoregressive and moving average lag polynomials of orders p and q, respectively; $\Phi_P(B^s)$ and $\Phi_Q(B^s)$ are seasonal autoregressive and moving average lag polynomials of orders P and Q, respectively, where s denotes the seasonal periodicity, or the number of observations per seasonal period. B denotes the backward shift operator, where $Bz_t = z_{t-1}$, a_t denotes a zero mean white-noise innovation process with finite variance, and ∇^d and

†The forecast lead time refers to the number of periods into the future that the forecast is made

Table 1. ARIMA models for interest rates and inflation

Nominal 3 month T-bill	$(1-B)^1\,3\text{MB} = (1 - \underset{(4.49)}{0.265}B^6 - \underset{(2.96)}{0.178}B^7)(1 + \underset{(3.62)}{0.225}B^{10})(1 + \underset{(2.38)}{0.152}B^{11})a_t$	$Q = 40.7\ (32)$
Real 3 month T-bill	$(1-B)^1\,\text{R3MB} = (1 - \underset{(23.6)}{0.826}B - \underset{(1.12)}{0.050}B^6 + \underset{(3.22)}{0.145}B^9)(1 + \underset{(0.79)}{0.55}B^{10})a_t$	$Q = 40.3\ (32)$
Nominal 10-year bond	$(1-B)^1\,10\text{YB} = (1 + \underset{(1.75)}{0.159}B)(1 - \underset{(3.06)}{0.189}B^7)(1 + \underset{(2.96)}{0.183}B^{11})a_t$	$Q = 33.1\ (33)$
Real 10-year bond	$(1-B)^1\,\text{R10YB} = (1 - \underset{(21.18)}{0.784}B)(1 + \underset{(2.32)}{0.144}B^9)a_t$	$Q = 44.2\ (34)$
Nominal 20-year bond	$(1-B)^1\,20\text{YB} = (1 + \underset{(3.08)}{0.182}B - \underset{(2.50)}{0.149}B^3)(1 - \underset{(2.68)}{0.169}B^7)(1 + \underset{(3.88)}{0.241}B^{11})a_t$	$Q = 30.0\ (32)$
Real 20-year bond	$(1-B)^1\,\text{R20YB} = (1 - \underset{(22.46)}{0.802}B)(1 + \underset{(2.70)}{0.167}B^9)(1 + \underset{(2.59)}{0.161}B^{10})a_t$	$Q = 43.4\ (33)$
Inflation (% change in CPI)	$(1-B)^1\,\%\,\text{CPI} = (1 - \underset{(23.02)}{0.806}B)(1 + \underset{(2.51)}{0.155}B^9)(1 + \underset{(2.46)}{0.153}B^{10})a_t$	$Q = 43.4\ (33)$

Note critical values for X_K^2 at 5% significance level and K degrees of freedom are $X_{32}^2 = 46.42$, $X_{33}^2 = 47.43$, $X_{34}^2 = 48.64$

∇_s^D denote regular and seasonal difference factors With regard to the difference factors, they can be rewritten using the backward shift operator, B, as $\nabla^d = (1-B)^d$ and $\nabla_s^D = (1-B^s)^D$. All lag polynomials are assumed to be stable with roots outside the unit circle, which satisfies the conditions necessary for stationarity and invertibility of the ARIMA model.

Estimated models

Tables 1–3 report the estimated ARIMA models for each of the series. In total, seventeen ARIMA models were identified and estimated for the seven defined variables and their respective constitutent elements. In addition to parameter estimates, the absolute value of computed t-statistics for the coefficient estimates are provided in parentheses, as well as Box–Pierce portmanteau Q-statistics. The portmanteau statistic is used to test the null hypothesis of white-noise model innovations and has a χ^2 distribution. Degrees of freedom for the Q-statistics are reported in parentheses and critical χ^2 values are provided in the footnotes to the tables.

Table 2 ARIMA models for money, wage rate, and CPI

Nominal $M1$	$(1-B)^1(1-B^{12})^1 M1 = (1 - \underset{(1.62)}{0.103}B^4)(1 + \underset{(2.41)}{0.152}B^9)(1 - \underset{(8.65)}{0.507}B^{12})a_t$	$Q = 38.6\ (33)$
Real $M1$	$(1 - \underset{(74.16)}{0.979}B)(1-B^{12})^1\text{R}\,M1 = (1 + \underset{(1.36)}{0.087}B^3 + \underset{(2.26)}{0.145}B^6)(1 - \underset{(12.13)}{0.631}B^{12})a_t$	$Q = 37.6\ (32)$
Nominal $M2$	$(1 - \underset{(6.29)}{0.369}B)(1-B)(1-B^{12})\,M2 = \underset{(2.87)}{0.352} + (1 + \underset{(3.62)}{0.222}B^9)(1 - \underset{(10.31)}{0.562}B^{12})a_t$	$Q = 31.8\ (32)$
Real $M2$	$(1 - \underset{(8.31)}{0.507}B - \underset{(2.15)}{0.133}B^2)(1-B)^1(1-B^{12})^1\text{R}\,M2 = (1 + \underset{(2.71)}{0.170}B^9)(1 - \underset{(17.68)}{0.757}B^{12})a_t$	$Q = 40.8\ (32)$
Nominal wage	$(1-B)^1(1-B^{12})^1\text{W} = \underset{(3.33)}{0.001} + (1 - \underset{(15.28)}{0.732}B^{12})a_t$	$Q = 35.6\ (34)$
Real wage	$(1-B)^1(1-B^{12})^1\,\text{RW} = (1 + \underset{(2.70)}{0.168}B^{11})(1 - \underset{(18.50)}{0.755}B^{12})a_t$	$Q - 36.4\ (34)$
CPI	$(1 - 0.436B - 0.176B^2)(1-B)^1(1-B^{12})^1\,\text{CPI} = (1 + 0.396B^9 + 0.259B^{10})^{(1 + 0.037B^8 + 0.307B^{13})} \times (1 - 0.803B^{12})a_t$	$Q = 41.8\ (29)$

†Note critical values for Y_K^2 at 5% significance level and K degrees of freedom are $Y_{29}^2 = 42.55$, $Y_{32}^2 = 46.42$, $Y_{33}^2 = 47.43$, $Y_{34}^2 = 48.64$

Table 3. ARIMA models for exchange rate and U.S./U.K. price ratio

Nominal exchange rate	$(1 - \underset{(31.37)}{0.960}B)\text{ER} = \underset{(8.27)}{1.894} + a_t$	$Q = 16.23\ (34)$
Real exchange rate	$(1-B)^1\,\text{RER} = \underset{(0.16)}{0.001} + (1 - \underset{(2.27)}{0.218}B)a_t$	$Q = 11.30\ (34)$
U.S./U.K. price ratio	$(1-B)^1 \text{P}_{\text{US}}/\text{P}_{\text{UK}} = \underset{(4.00)}{-0.004} + (1 + \underset{(2.14)}{0.210}B^{10})a_t$	$Q = 44.17\ (34)$

Note critical values for X_K^2 at 5% significance level and K degrees of freedom are $X_{34}^2 = 48.64$

The period January 1959 to December 1982 was used to identify and estimate ARIMA models for all series except those for the nominal and real exchange rates. These were modeled over the post-float period from January 1974 to December 1982 during which exchange rates were flexible. January 1983 to December 1986 represents the out-of-sample period which is used to produce the updated forecasts. The Box–Pierce Q-statistics are computed based on 36 autocorrelations.

Table 1 presents the estimated ARIMA models for the nominal and real interest rates, as well as the percentage change in the CPI. Inspection of the autocorrelation function for the original series shows that all seven series were nonstationary and required some degree of regular differencing. First differencing was sufficient to induce stationarity in each of the series.

All the interest rates as well as the inflation rate were modeled as seasonal moving average processes, since there was no indication of an autoregressive component. Besides the regular moving average components, the three nominal and real interest rates generally displayed seasonal moving average components at 7, 9, 10 and 11 month lags. (The existence of seasonality in interest rates for postwar U.S. has been widely documented in the literature, e.g. [15].) Similarly, there was a seasonal moving average component in the CPI series. Comparison of the computed Q-values with the critical values shows that the estimated ARIMA models are all adequate at the 5% level

Table 2 presents the estimated ARIMA models for the nominal and real money supply measures, the nominal and real wage rates, and levels of the CPI. All the series except for real $M1$ required first differencing to induce stationarity,† A first-degree seasonal difference, $(1 - B^{12})^1$ was used to adjust each of the series in Table 2 for significant autoregressive seasonality. Each series also revealed a 12th-order moving average seasonal component, which was adjusted for in the respective ARIMA models. Both the nominal wage rate and nominal $M2$ displayed drift, as shown by the significant intercept terms in their respective ARIMA models. Real $M1$, nominal $M2$ and the CPI were modeled with both regular autoregressive and moving average lag polynomials. All the other time series variables were modeled as moving average processes only. The Q-values show that all models were adequate at the 5% level.

Finally, Table 3 presents the estimated models for the nominal and real exchange rates, as well as for the U.S./U.K. ratio of consumer prices. The nominal exchange rate was modeled as a first-order autoregressive process.‡ Both the real exchange rate and the ratio of consumer prices required first-differencing to induce stationarity. Moving average models proved to be adequate for these series. In addition, both were modeled as having drift, although the drift term for the real exchange rate was not significant. The Q-values were all less than their critical values at the 5% level and thus revealed no model inadequacy.§¶

FORECASTING RESULTS

Table 4 presents results on the accuracy of the direct and indirect ARIMA forecasts for the three real interest rates. the two real money measures, the real wage, and the real foreign exchange rate.

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†Although real $M1$ did not appear to be nonstationary on the basis of both visual inspection of a plot of the series over the estimation period as well as examination of the autocorrelation function, a t-test showed that the coefficient of 0.979 in the regular autoreggressive lag polynomial was not less than 1. This means that an alternative model for real $M1$ would involve first differencing However, the diagnostics that were done for the ARIMA model employed showed the chosen model specification without first-differencing to be adequate.

‡The nominal exchange rate could also be modeled as a random walk with drift The autocorrelation function did not reveal nonstationarity in the series, but the autoregressive coefficient of 0 960 was close to 1. By definition, a random walk with drift is an integrated stochastic process which requires first-differencing to make is stationary It is given by $z_t = x_{t-1} + m + a_t$, where a_t is white-noise and m is a constant Diagnostics based on the portmanteau statistic and the individual sample autocorrelations showed that the model specification used in the paper was adequate

§The Box–Pierce Q-stastistic is given by:

$$Q = T \sum_{i=1}^{K} r_i^2 \sim \chi^2_{(K)}$$

where T is the sample size, r_i^2 is the squared autocorrelation coefficient at lag i, and K denotes the number of autocorrelations. The asymptotic standard error of the autocorrelation coefficient is $\sqrt{T^{-1}}$, where T is the sample size.

¶In addition to the Box-Pierce Q-statistic, the individual sample autocorrelations for the innovations, a_t, from each ARIMA model were compared to their asymptotic standard errors to detect possible model inadequacy. No indication of model misspecification was found, with none of the sample autocorrelations being larger than twice its standard error

Table 4 Accuracy of direct and indirect forecasts

Variable	MAE Direct	MAE Indirect	MSE Direct	MSE Indirect	MAPE Direct	MAPE Indirect
Real 3MB	2.15	2.83	7.92	7.55	—	—
Real 10YB	1.95	2.03	6.97	7.21	—	—
Real 20YB	2.19	2.10	7.86	7.67	—	—
Real $M1$	3.17	2.88	17.83	12.27	0.0067	0.2911
Real $M2$	6.63	7.31	77.25	82.20	0.0034	0.0038
Real wage	0.0179	0.0231	0.0006	0.0009	0.0023	0.0031
Real exchange rate	0.0375	0.0404	0.0022	0.0025	0.0254	0.0276

†Note MAPE is not calculated for the real interest rates since these rates can take on negative values

Forecasting with ARIMA models involves predicting future values of an observed time series conditional on an information set that contains the current and past values of the time series.

The three measures of forecast accuracy, MAE, MSE and MAPE, were constructed with the 48 updated single-step forecast errors from the out-of-sample period January 1983 to December 1986. These out-of-sample forecast errors were computed as the difference between the actual value of the time series one-period-ahead, z_{t+1}, and the *ex-post* direct or indirect forecast of the series one-period-ahead, $\hat{z}_{t+1}$. That is, the one-period-ahead forecast error is computed as $e_{t+1} = z_{t+1} - \hat{z}_{t+1}$.

The results presented in Table 4 differ markedly from those reported by Kang [3]. Recall that Kang found, on average, that indirect forecasts perform better than direct forecasts. We, however, found the opposite to be the case here. For the set of real economic variables examined, our evidence shows that direct forecasts generally outperform indirect forecasts when evaluated using the MAE, MSE and MAPE as criteria.

In regard to the three *ex-post* real interest rates, the results are mixed. For the 3-month U.S. Treasury bill rate, direct forecasts are better than indirect forecasts based on the MAE. However, just the opposite holds for the MSE.† The results are divided for the rates on government securities with longer maturities. For the rate on 10-year government bonds, direct forecasts perform better according to both the MAE and MSE. In sharp contrast, indirect forecasts outperform direct forecasts according to the same criteria for the rate on 20-year government bonds.

In forecasting the real money supply measures, real $M1$ is forecasted more accurately using indirect forecasts when evaluated with the MAE and MSE but does not hold in terms of the MAPE. In contrast, direct forecasts of real $M2$ perform better than indirect forecasts according to all three accuracy criteria. Turning our attention to the real wage and real exchange rate, direct forecasts also outperform indirect forecasts for all three measures of forecast accuracy.

While the results of the forecast comparison are split for the interest rates, this, in general, is not the case for the two real money measures—the real wage and the real exchange rate. For these latter four time series, direct forecasts perform better than indirect forecasts according to all three metrics. The exception to this is real $M1$, where direct forecasts performed better based on the MAPE, but not on the MAE or MSE. Of the eighteen paired comparisons between direct and indirect forecasts examined in Table 4, direct forecasts are shown to be better than their indirect counterparts in thirteen of the eighteen cases. This evidence supports the conclusion that direct forecasts generally outperform indirect forecasts for the particular set of defined variables examined.

The substantial difference between the results reported by Kang [3] and those reported here may be attributed to several factors.‡ First, the set of real variables examined here is quite different from the set examined by Kang. Thus, while Kang looked at the same short-term interest rate, he also examined real GNP, the money multiplier, and money velocity. In this paper, real interest rates on government securities with much longer maturities, as well as real money aggregates, wage rate, and the exchange rate were analyzed. In addition, all the variables used here were measured

†Kang [3] also examined the real rate on 3-month U.S. Treasury bills but found different results He found indirect forecasts performed better for both the MAE and MSE There are two possible explanations for this First, the estimation and forecast periods used by Kang are different from those used in this paper Second, since the identification of ARIMA models is more art than science, it is possible for two different ARIMA models to adequately reflect the autocorrelation structure of a time series Kang's model is different from the one used here, although both are judged to be adequate

‡The comparison with Kang is necessary and useful since his study is the only other one which deals specifically with the forecasting of defined real economic variables.

monthly, while half the variables examined by Kang were monthly with the other half measured quarterly.

Second, Kang used officially seasonally adjusted data, while the raw data used in this paper were, to start, seasonally unadjusted; they were seasonally adjusted later, in the process of fitting seasonal ARIMA models. It is well known that the use of officially adjusted data can result in biased findings when the relationships between seasonally adjusted data are examined [16]. This occurs primarily because the techniques used by government agencies to seasonally adjust data often distort the informational content of that data.

Continuing, we find, third, that the periods used to identify and estimate the ARIMA models and to generate the out-of-sample forecasts are different for the two studies.

Finally, the out-of-sample period used in this paper corresponds to a period of both rapid and dramatic change in the U.S. economy. This alone might very well account for the discrepancy in the results. More specifically, the late 1970s and early to mid-1980s witnessed several significant changes in the economic environment. First, the Federal Reserve shifted from a federal funds rate instrument to a monetary instrument in conducting monetary policy in October 1979. Second, there was a significant decrease in the power of labor to negotiate wages with management. Finally, there was an orchestrated depreciation of the U.S. dollar in an attempt to restrain the growth of an alarmingly large Federal budget deficit and a worsening trade deficit.

SUMMARY AND CONCLUSION

This paper provides additional evidence on the accuracy of indirect and direct ARIMA forecasts of defined variables. Seven monthly real economic variables for the U.S., including three *ex-post* real interest rates, two real money supply measures, a real wage, and a real foreign exchange rate were examined. In contrast to the results reported earlier by Kang [3], direct forecasts generally outperformed indirect forecasts for the set of defined variables examined.

Based on our findings, it appears that the accuracy of indirect and direct ARIMA forecasts of defined variables is sensitive to the choice of defined variables used in the analysis Thus, it seems that the following may have an impact: the choice between using financial or non-financial variables, whether or not the raw data are officially seasonally adjusted, and whether the data are measured monthly or quarterly. In addition, the choice of historical period for the analysis no doubt also exerts an influence. The results reported here corroborate the view that the overall accuracy of direct and indirect forecasting techniques is an empirical issue. Additional studies that look at varying sets of defined variables are therefore needed before any general conclusions can be drawn.

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