605_HW14.Rmd

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TAYLOR SERIES

Problem Set 1 :

work out some Taylor Series expansions of popular functions.

$$f(x) = \frac{1}{(1-x)}$$

$$f(x) \quad = \quad e^x$$

$$f(x) \quad = \quad \ln(1 \quad + \quad x)$$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion.

Solution :

Taylor Series Approximation is used to represent functions as an infinite sum of polynomial terms that are calculated using a function's derivatives evaluated at a single point.

Taylor's Theorem states that any function that is infinitely differentiable can be repre- sented as a polynomial of the following form of

$$f(x) \quad = \quad \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

which equals ,

$$f(a) \quad + \quad f^{(1)}(a)(x-a) \quad + \quad \frac{f^{(2)}}{2!}(a)(x-a) \quad + \quad ...$$

where $f^{(n)(a)}$ represents the nth derivative of $f(x)$ evaluated at $x = a$.

For

$$f(x) = \frac{1}{(1-x)}$$

Derivatives are,

$f(x) \quad = \quad (1-x)^{-1} \quad ; \quad f(0) \quad = \quad 1$

$f^{(1)}(x) \quad = \quad 1(1-x)^{-2} \quad ; \quad f^{(1)}(0) \quad = \quad 1$

$f^{(2)}(x) \quad = \quad (-2)(1-x)^{-3} \quad ; \quad f^{(2)}(0) \quad = \quad 2$

$f^{(3)}(x) \quad = \quad (3*2)(1-x)^{-4} \quad ; \quad f^{(3)}(0) \quad = \quad (3*2)$

$f^{(4)}(x) \quad = \quad -(4*3*2)(1-x)^{-5} \quad ; \quad f^{(4)}(0) \quad = \quad (4*3*2)$

Therefore, plugging in Taylor Theorem Polynomial,

$$= 1 \quad + \quad x \quad + \quad x^2 \quad + \quad x^3 \quad + \quad x^4 \quad + \quad ..$$

which reduces to

$$\sum_{n=0}^{\infty} x^n$$

For

$$f\,(x) \quad = \quad e^x$$

Derivatives for function,

$f(x) \quad = \quad e^x \quad ; \quad f(0) \quad = \quad 1$

$f^{(1)}(x) \quad = \quad e^x \quad ; \quad f^{(1)}(0) \quad = \quad 1$

$f^{(2)}(x) \quad = \quad e^x \quad ; \quad f^{(2)}(0) \quad = \quad 1$

$f^{(3)}(x) \quad = \quad e^x \quad ; \quad f^{(3)}(0) \quad = \quad 1$

$f^{(4)}(x) \quad = \quad e^x \quad ; \quad f^{(4)}(0) \quad = \quad 1$

Plugging in Taylor theorem polynomial,

Therefore,

$$= 1 \quad + \quad x \quad + \quad \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 ...$$

$=$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

For,

$$f\,(x) \quad = \quad \ln{(1 \quad + \quad x)}$$

Calculating derivatives of the functions, we have,

$f(x) \quad = \quad ln(1 \quad + \quad x); \quad f(0) \quad = \quad 0$

$f^{(1)}(x) \quad = \quad (1+x)^{-1}; \quad f^{(1)}(0) \quad = \quad 1$

$f^{(2)}(x) \quad = \quad -1(1+x)^{-2}; \quad f^{(2)}(0) \quad = \quad -1$

$f^{(3)}(x) \quad = \quad 2(1+x)^{-3}; \quad f^{(3)}(0) \quad = \quad 2$

$f^{(4)}(x) \quad = \quad -(3 \quad * \quad 2)(1+x)^{-4}; \quad f^{(4)}(0) \quad = \quad -(3 \quad * \quad 2)$

$f^{(5)}(x) \quad = \quad (4 \quad * \quad 3 \quad * \quad 2)(1+x)^{-5}; \quad f^{(5)}(0) \quad = \quad (4 \quad * \quad 3 \quad * \quad 2)$

Plugging in Taylor theorem polynomial,

$$0 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6 + ...$$

equals,

$$= \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$