ANOVA

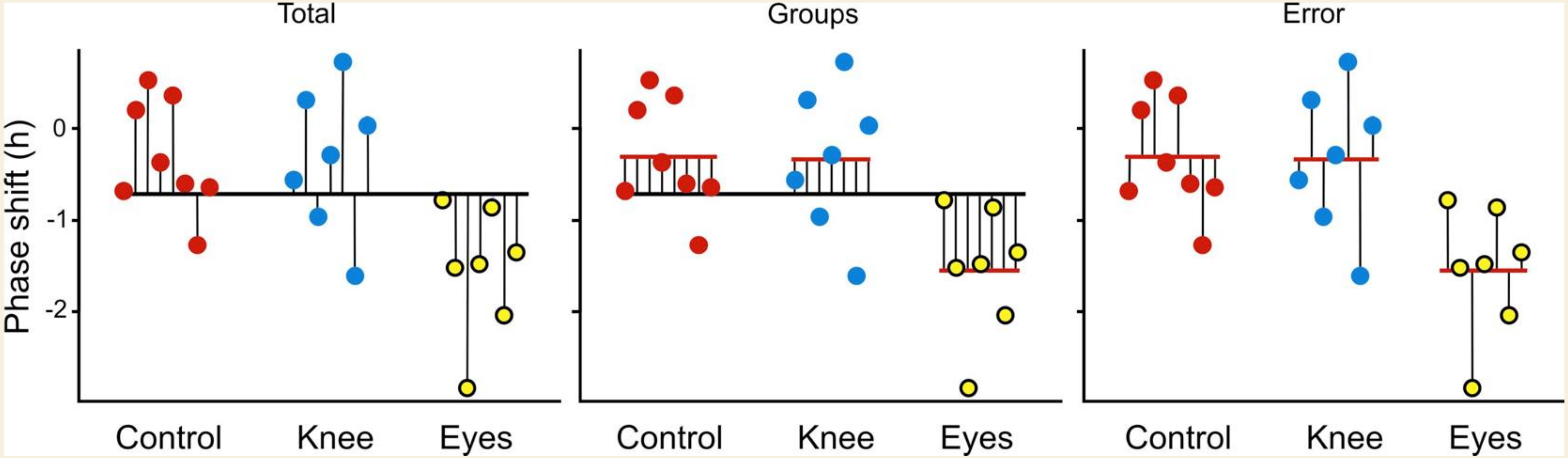
COMPARING MEANS OF **MORE THAN 2 GROUPS**

BRIAN STOCK

03.26.17

ANOVA = ANALYSIS OF VARIANCE

How much of the total variance is from differences in **group means**?



Total = Group + Error

No **variance between groups** = all group means are the same

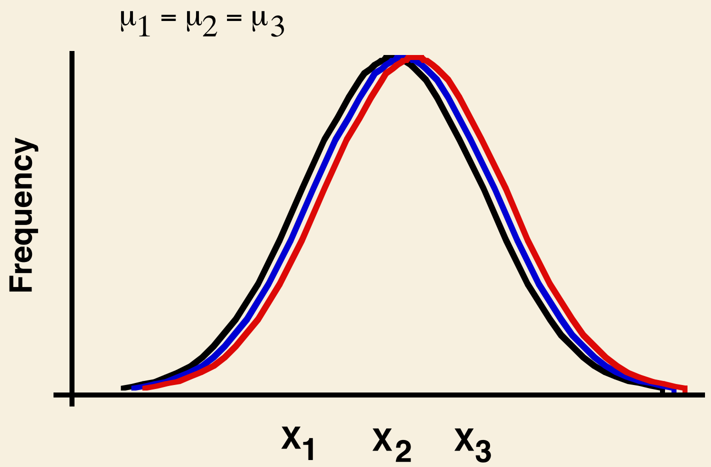
ANOVA HYPOTHESES

H_0 : Variance among groups = 0

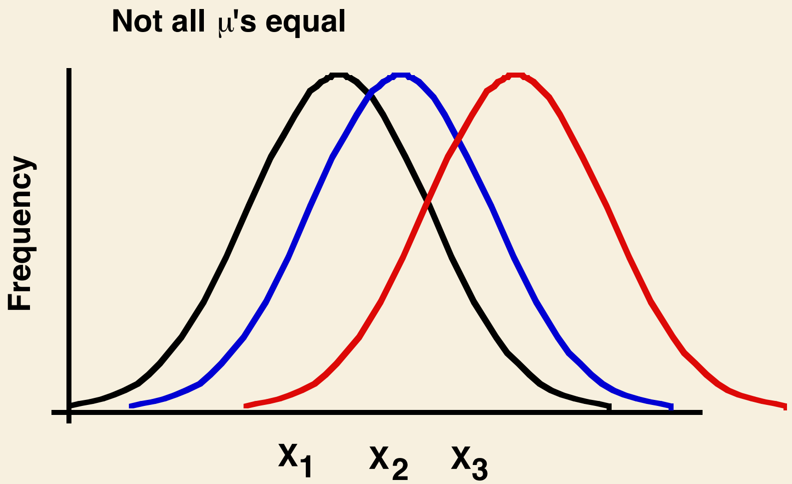
$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \ldots \mu_k$

H_A: Variance among groups is not 0

H_A: at least one population mean is different



H_0: all populations have equal means



H_A: at least one population mean is different.

EX: CATCH/DAY AT 4 SITES

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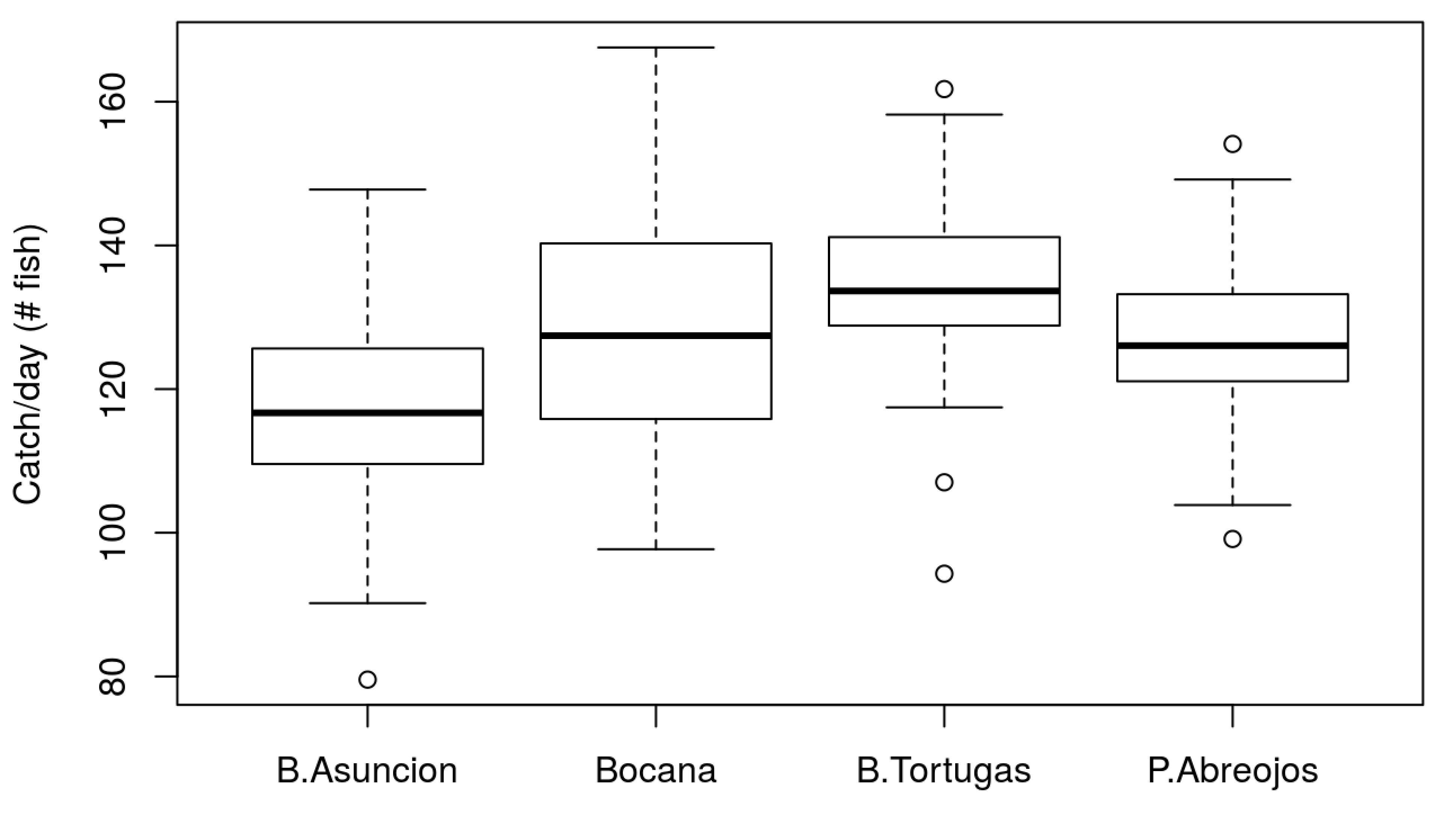


ASSUMPTIONS

1. Random samples

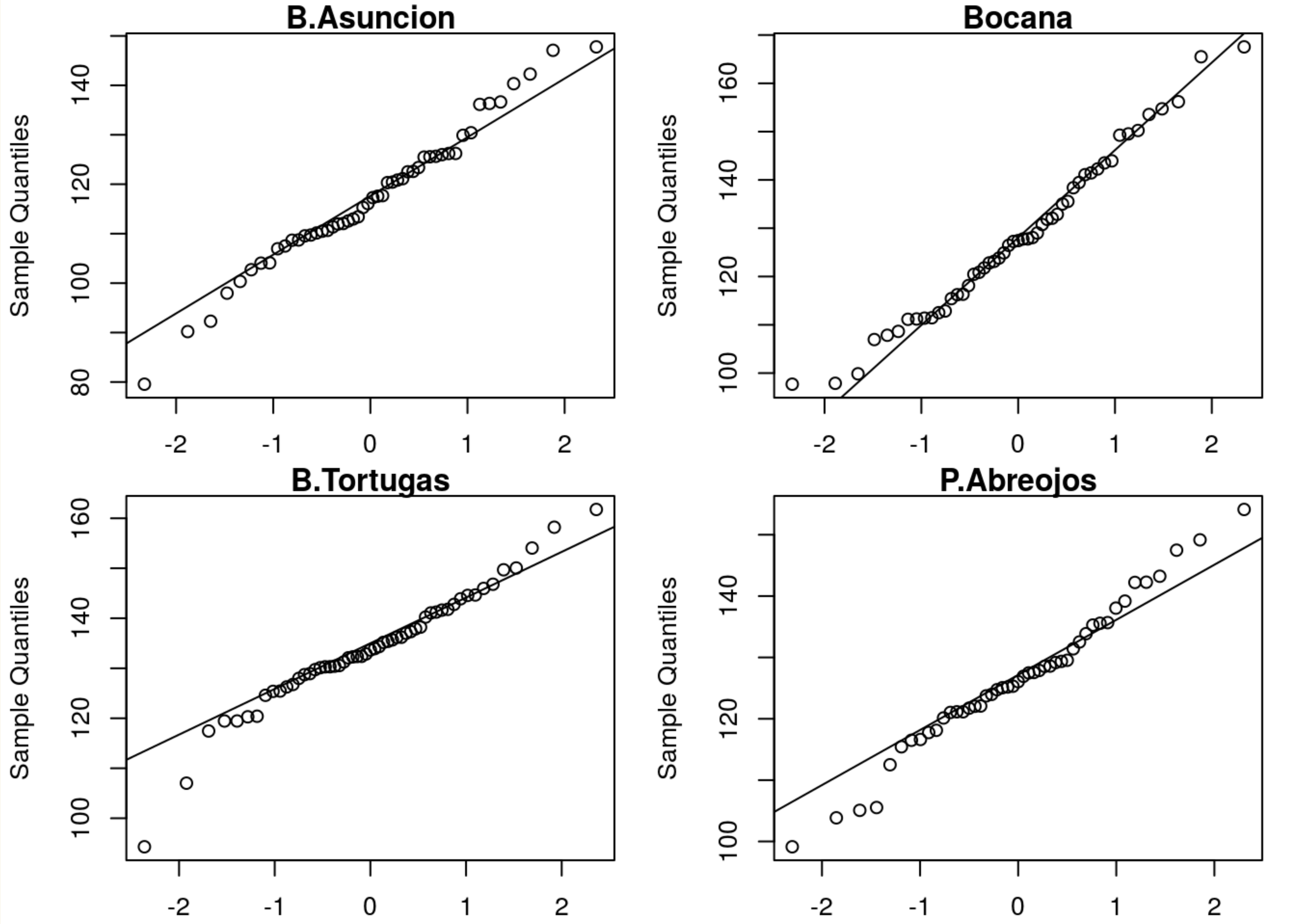
2. Each group is normally distributed

But "surprisingly robust" – Central Limit Theorem

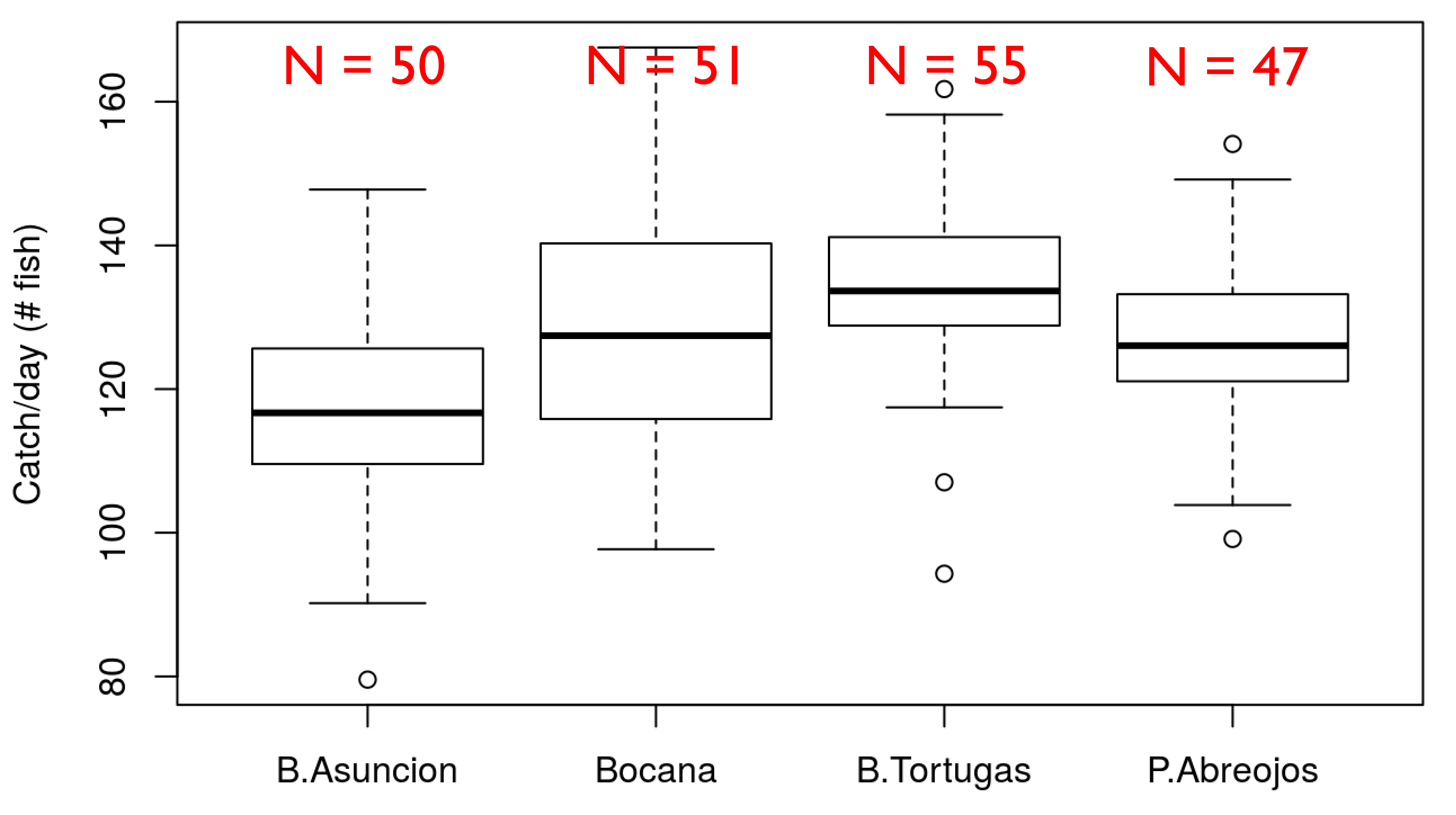
3. Each group has equal variance (*Homoscedasticity*)

Robust if sample sizes are large and equal, *up to 10-fold difference in variances*

IS EACH GROUP NORMAL-DISTRIBUTED?



ARE VARIANCES ROUGHLY EQUAL?



GREAT, LET'S DO AN ANOVA!

```
##               Df Sum Sq Mean Sq F value   Pr(>F)
## site           3   7423  2474.5   13.18 6.95e-08 ***
## Residuals    199  37363   187.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Df = degrees of freedom

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```

Sum sq = Sum of squares = **variance**

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```

Mean Sq = Mean Squares = **Sum Sq / Df**

GREAT, LET'S DO AN ANOVA!

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```

F = F-statistic = **Mean Sq Group / Mean Sq Residuals**

If there is no difference, F <= 1

GREAT, LET'S DO AN ANOVA!

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```

Pr(>F) = ***p*-value**

If there IS a difference, $p < 0.05$

SO WHAT?

All an ANOVA tells us is that *one* of the group means is different from the rest…

What else might we want to know?

MULTIPLE COMPARISONS

Probability of a Type I error in *N* tests =
$1\text{-}(1\text{-}\alpha)^{N}$

For 20 tests, the probability of
at least one Type I error is
~65%

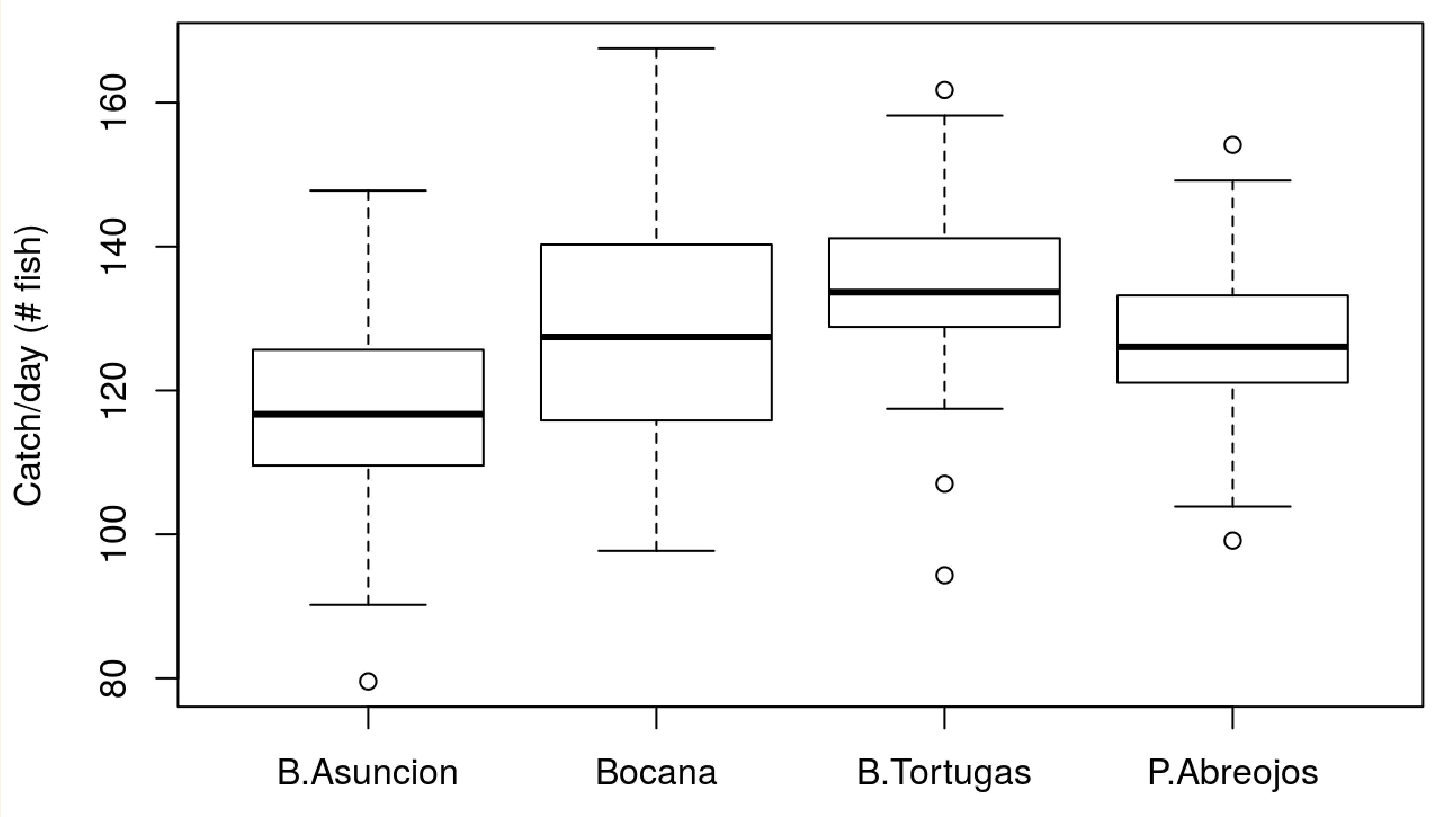
Yikes!

"BONFERRONI CORRECTION" FOR MULTIPLE COMPARISONS

Uses a smaller α value:

$$\alpha' = \frac{\alpha}{\text{number of tests}}$$

WHICH GROUPS ARE DIFFERENT?



TUKEY-KRAMER TEST

Done after finding variation among groups with 1-factor ANOVA

Compares ***all*** group means to ***all other*** group means

- "pairwise"

TUKEY NULL HYPOTHESES

$$H_0: \ \mu_1 = \mu_2$$
$$H_0: \ \mu_1 = \mu_3$$
$$H_0: \ \mu_2 = \mu_3$$

…

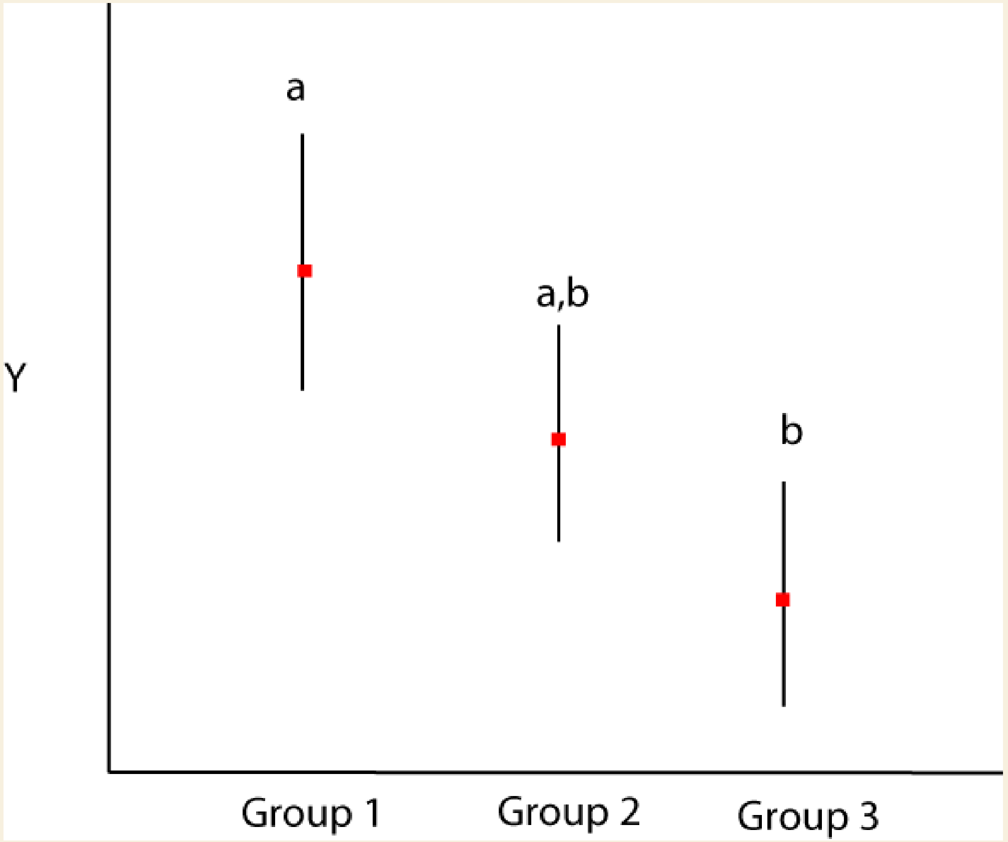
WHY NOT USE A SERIES OF TWO-SAMPLE *T*-TESTS?

Multiple comparisons would cause the *t*-tests to reject too many true null hypotheses.

Tukey-Kramer adjusts for the number of tests.

Tukey-Kramer also uses information about the variance within groups from all the data, so it has ***more power*** *than a t-test with a Bonferroni correction*.

LETTERS SHOW TUKEY RESULTS



$H_0: \; m_1 = m_2$ Cannot reject

$H_0: \; m_1 = m_3$ Reject

$H_0: \; m_2 = m_3$ Cannot reject

LETTERS SHOW TUKEY RESULTS