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Data assimilation practicals

Recapitulation of Bayes' Theorem

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Bayes' Theorem

Likelihood. Pdf of the observations given a value of the state variable.

Prior pdf. Pdf of the state variables coming from the model.

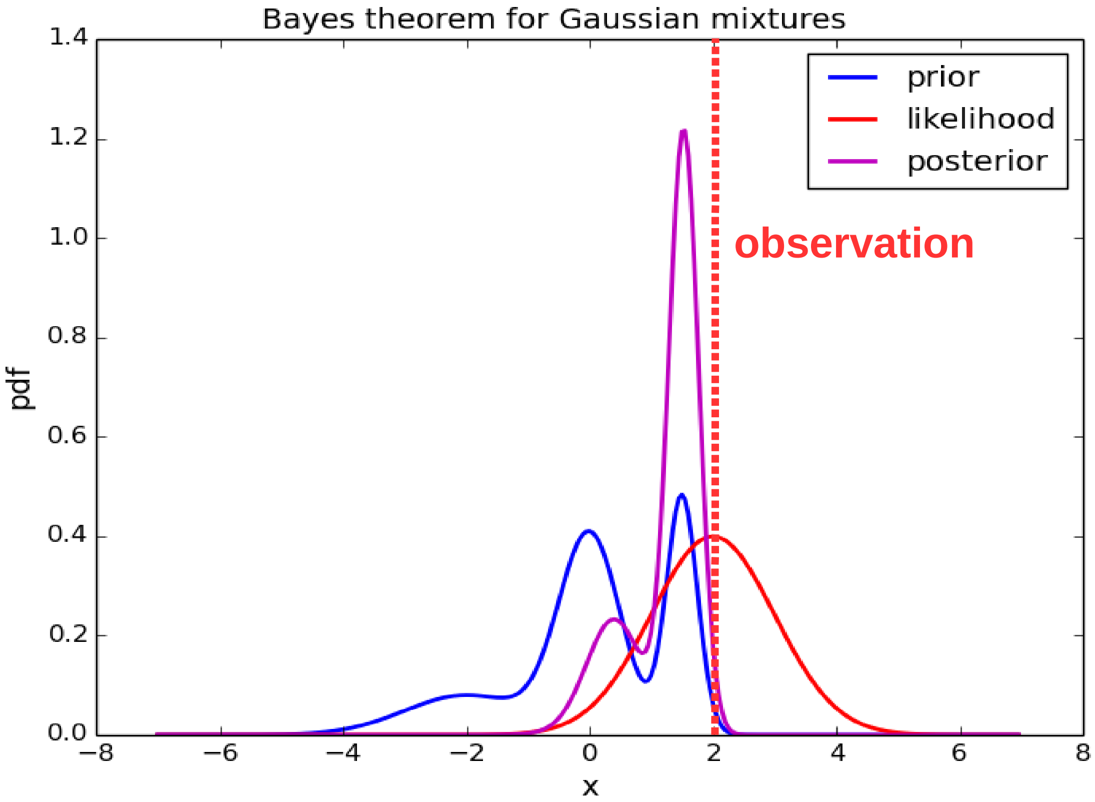
$$p\left(\mathbf{x}|\mathbf{y}\right) = \frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})}$$

Posterior pdf. Pdf of the state variables given the observations.

Marginal pdf of the observations. It acts as a normalisation constant. Usually not necessary to compute explicitly.

$$p(\mathbf{y}) = \int_{\mathcal{X}} p(\mathbf{y}|\mathbf{x})p(\mathbf{x})d\mathbf{x}$$

Bayes' Theorem: illustration in 1D



Gaussian mixtures

Gaussian (or normal) pdf:

$$\phi\left(x;\mu,\sigma\right)=\frac{1}{\sqrt{2\pi}\sigma}exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Gaussian mixture (convex linear combination of Gaussian pdf's):

$$p(x)=\sum_{n=1}^{N}\alpha_n\phi\left(x;\mu_n,\sigma_n\right)$$