‘Toy’ models

Introduction to Data Assimilation
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Data Assimilation Research Centre

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Toy models

- We will use two ‘simple’ models to test our data assimilation techniques.

- Both were designed by Edward Lorenz. Under certain choices of parameters, they exhibit chaotic behavior and present a challenge for data assimilation and predictability.

- Their size allows us to visualize results in a simple manner and explore different settings: observation operators, magnitude of background and observational error, time frequency of observations, etc.

1. Lorenz 1963 3-variable model

This model comes from a simplified description of the Rayleigh-Benard convection. It has 3 variables.

The time evolution of the model is described by:

$$\dot{x}^{(1)} = \sigma\left(x^{(2)} - x^{(1)}\right)$$
$$\dot{x}^{(2)} = x^{(1)}\left(r - x^{(3)}\right) - x^{(2)}$$
$$\dot{x}^{(3)} = x^{(1)}\,x^{(2)} - b x^{(3)}$$

parameters

$$\sigma = 10$$
$$r = 8/3$$
$$b = 28$$

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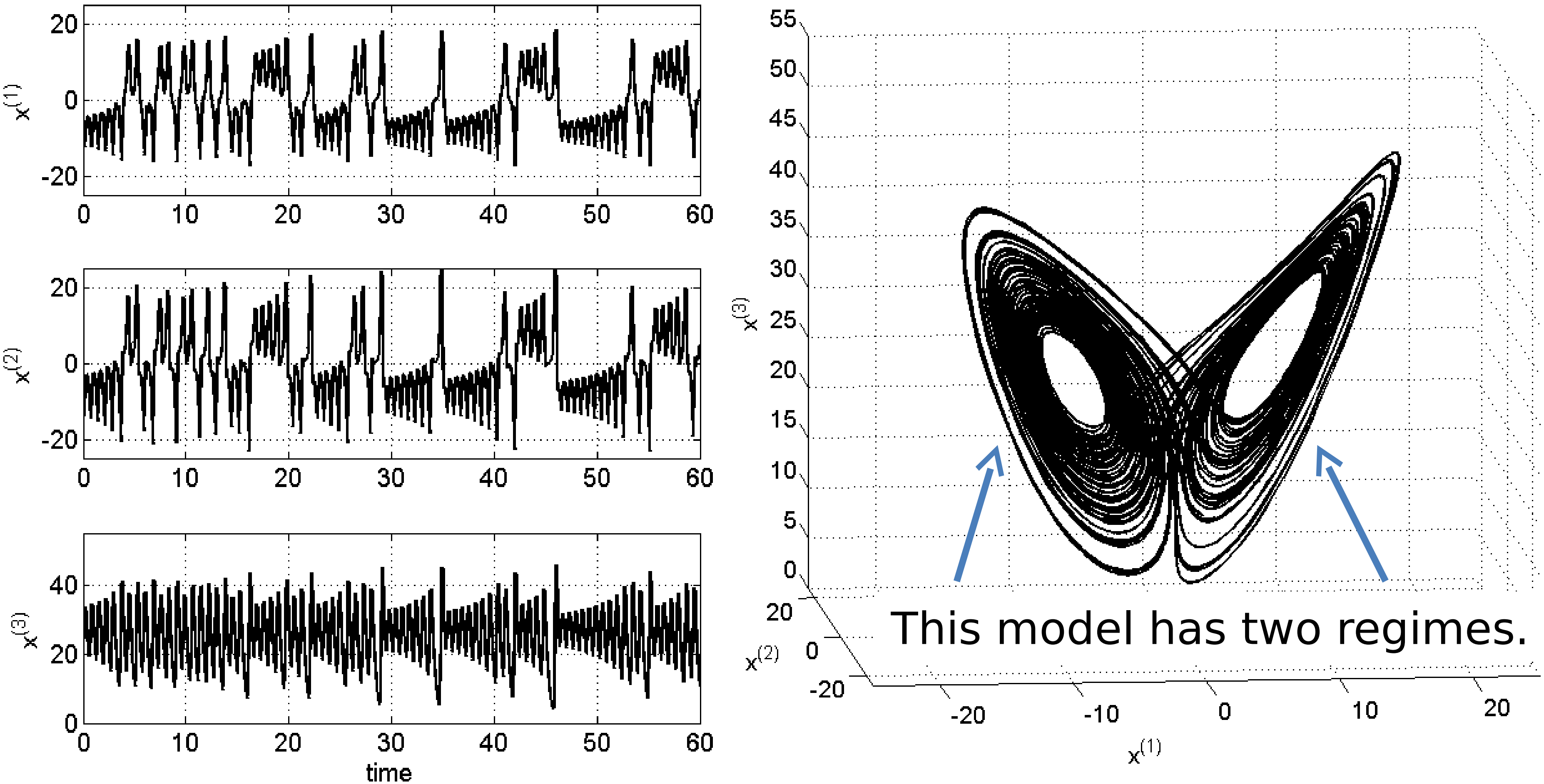
This model presents strong non-linearity and a regime transition.

1. Lorenz 1963 3-variable model



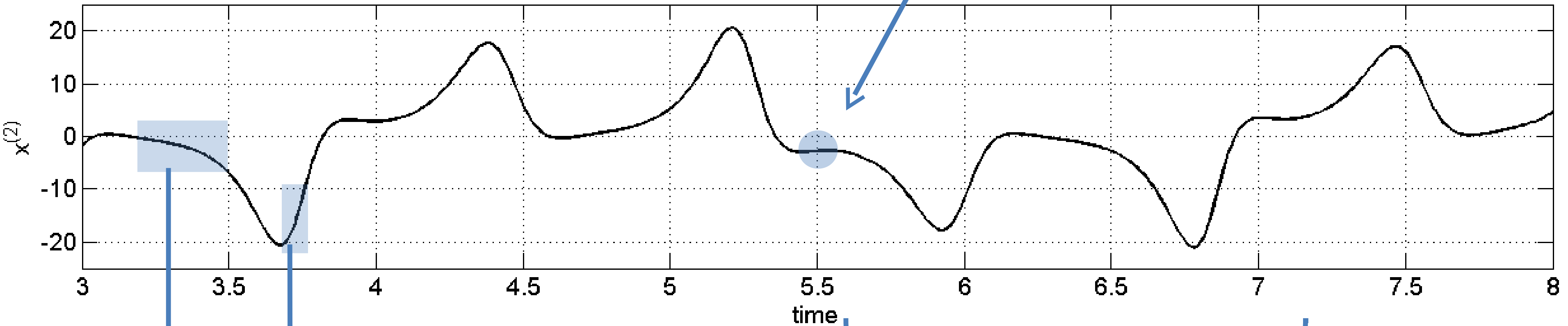
Time evolution (right) and representation of the attractor (left) for the model. Figures obtained integrating with RK4 and a time step 0.01.

1. Lorenz 1963 3-variable model

Evolution of $x^{(2)}$

'warm' regime

Regime transition

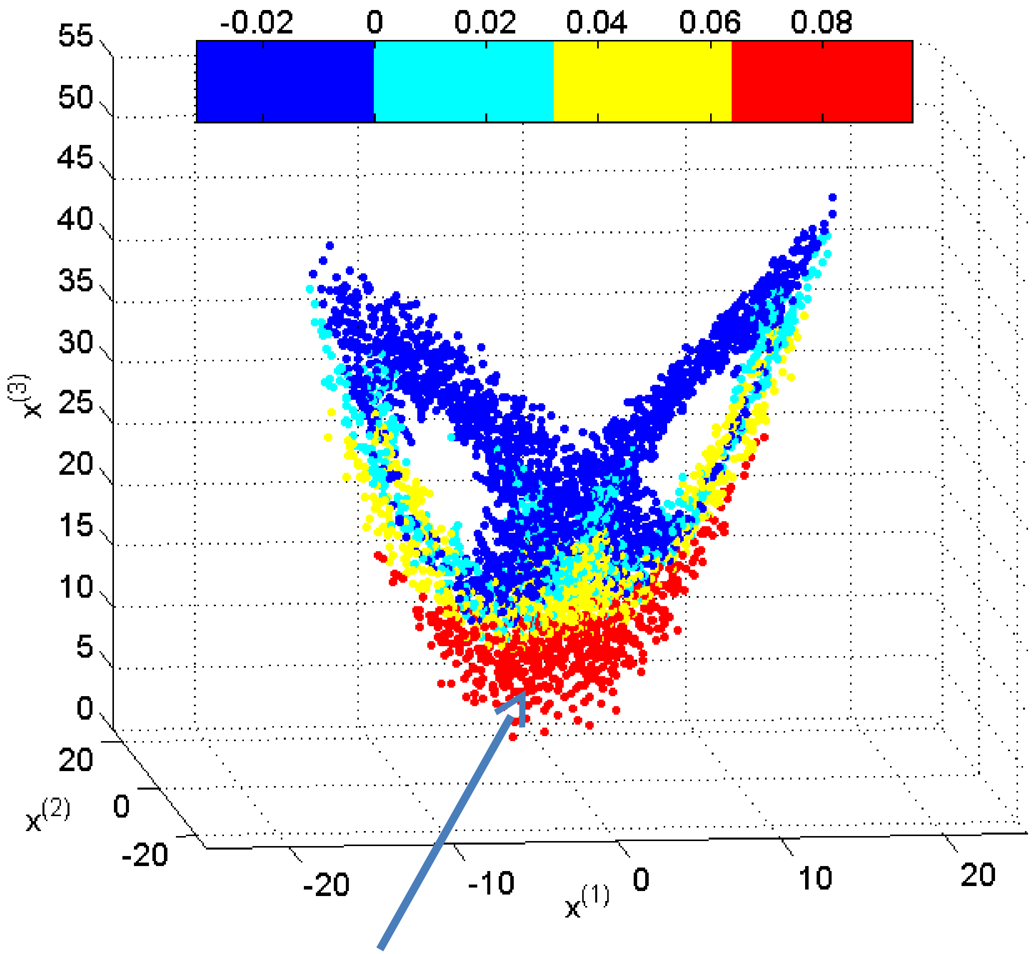


'cold' regime

Short forecast/assimilation window
~ 8 steps, the evolution is quasilinear

Long forecast/assimilation window
~ 20 steps or more, nonlinearity is appreciated

1. Lorenz 1963 3-variable model



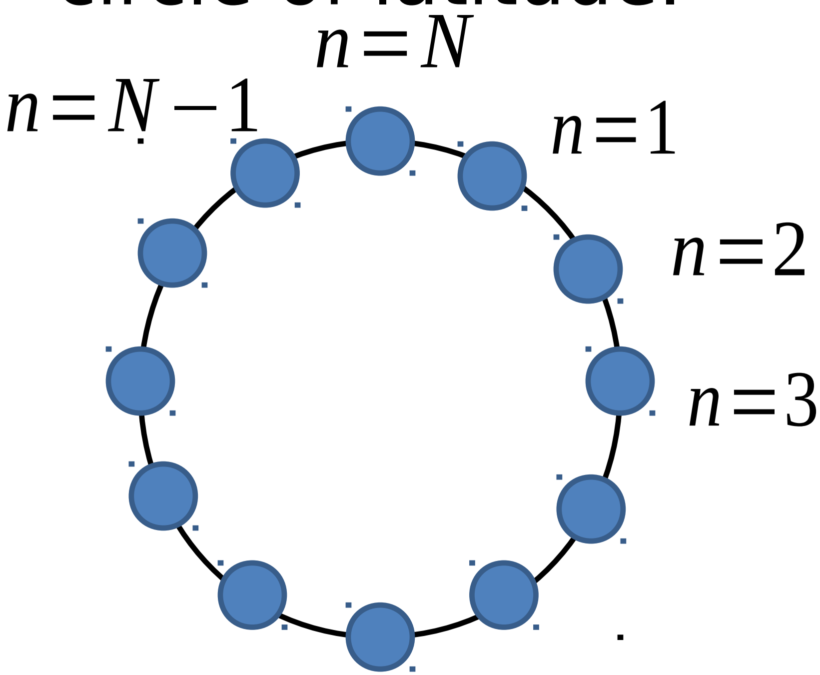
This region is challenging!

The difficulty of the data assimilation problem depends on the current position within the attractor.

Dark blue denotes regions where perturbations **decay**, **red** denotes regions where perturbations **grow fast**. (figure plotted using bred vectors, reproduced from Evans *et al*, 2004).

2. Lorenz 1996 model

- This cyclic model intends to emulate the behaviour of a meteorological variable in a circle of latitude.



$$\dot{x}^{(q)} = \underbrace{\left(x^{(q+1)} - x^{(q-2)}\right) x^{(q-1)}}_{\text{advection}} \underbrace{- x^{(q)}}_{\text{diffusion}} + \underbrace{F}_{\text{forcing}}$$

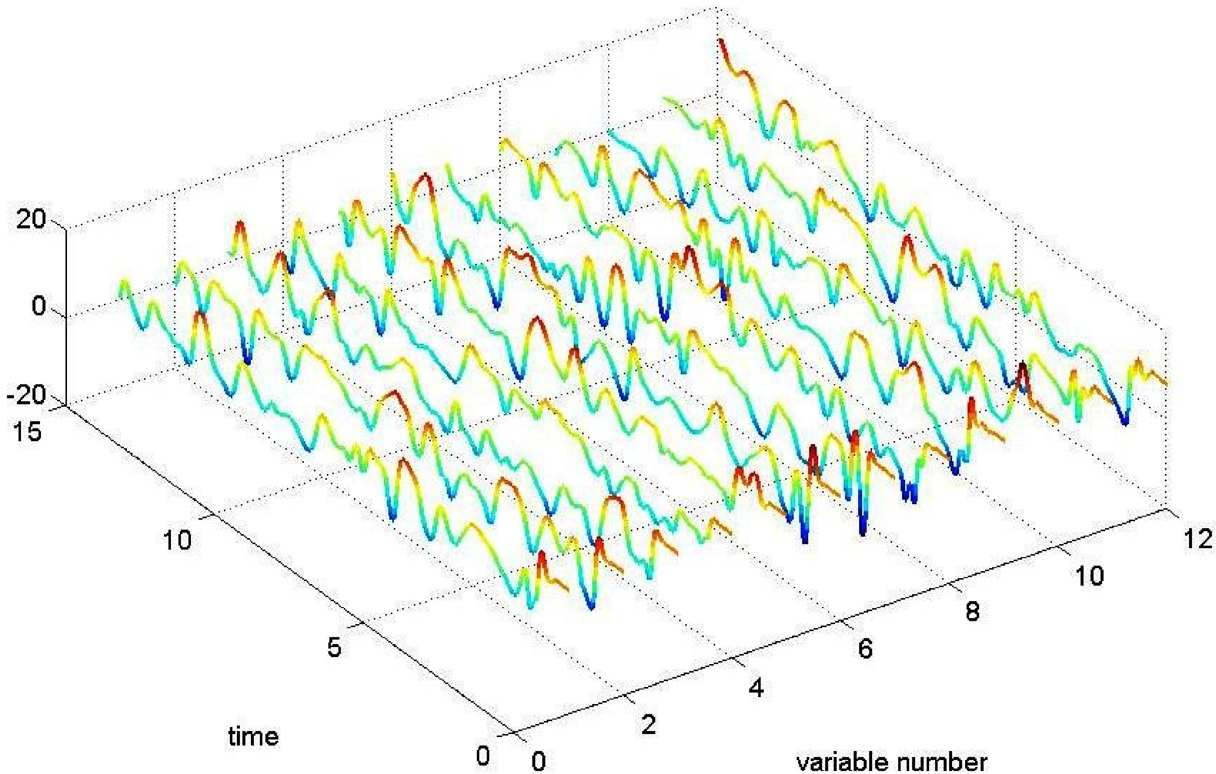
$$q = 1, 2, \ldots, N$$

$$x^{(q)} \equiv x^{(\mathrm{mod}(q,N))}$$

Chaos will appear for $F > 5$ and $N \geq 12$

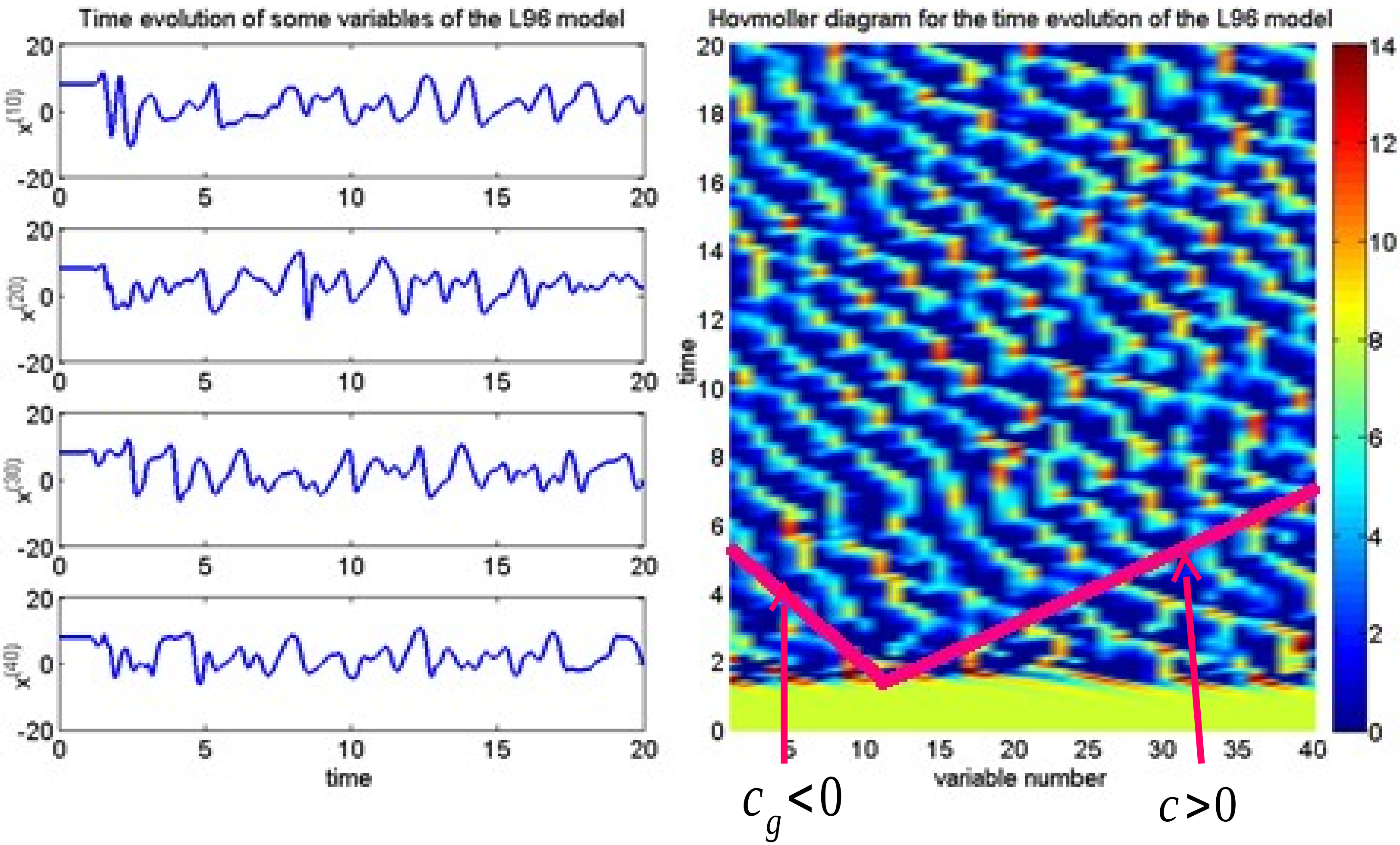
2. Lorenz 1996 model

This model can be started from rest, when $x^{(q)} = F, \forall q$. A tiny perturbation to any variable will grow and spin the system.



Time evolution Lorenz 1996 model with N=12 variables. The model was integrated with RK4 and a time step of 0.025.

2. Lorenz 1996 model



Time evolution of some variables (left) and Hovmoller diagram (right) for the Lorenz 1996 model with *N*=40 variables. Notice the presence of ‘waves’ with positive phase speed and negative group speed.

2. Lorenz 1996 model

- In this model, we will usually use assimilation windows of two time steps, in which the behaviour is quasilinear.

- In the ensemble framework, this model will allow us to test **localization** and **inflation** when using less ensemble members than state variables.

References

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