→ EARTH OBSERVATION SUMMER SCHOOL

Earth System Monitoring & Modelling

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Data assimilation practicals

Recapitulation of Ensemble Kalman Filters

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Kalman filter

The **Kalman filter** is a **best linear unbiased** (BLUE) estimator. It relies in l**inear algebra** to update the best guess (**mean**) and **covariance** of the state variable in the **analysis step**.

$$\bar{\mathbf{x}}^a = (\mathbf{I} - \mathbf{KH})\,\bar{\mathbf{x}}^b + \mathbf{Ky}$$ ← Analysis equation for the **mean**

$$\mathbf{A} = (\mathbf{I} - \mathbf{KH})\,\mathbf{B}$$ ← Analysis equation for the **covariance**

The **Kalman gain** is:

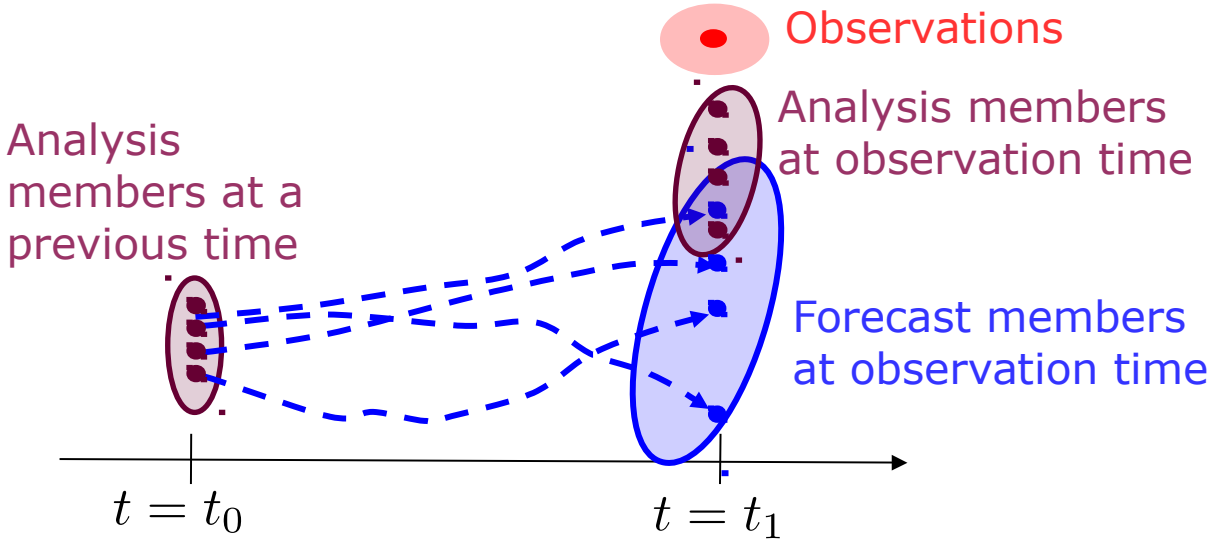
$$\mathbf{K} = \mathbf{BH}^{\mathbf{T}} \left(\mathbf{HBH}^{\mathbf{T}} + \mathbf{R}\right)^{-1}$$

In the **univariate case** and **observing directly** this is simply a **ratio of variances**:

$$k = \frac{b^2}{b^2 + r^2}$$

Ensemble Kalman Filter

The **Ensemble Kalman filter** is a **Monte-Carlo implementation** of the **Kalman filter**. It evolves a family (**ensemble**) of solutions and applies the **KF analysis equations using sample estimators**.



The **Kalman Filter** and the **EnKF evolve mean and covariance in time** -in different ways- in the **forecast step**, and **update** them in the **analysis step**.

Analysis step in the KF and EnKF

In the **Kalman filter**, **mean** and **covariance** are updated in the **analysis step**.

$$\bar{\mathbf{x}}^b \rightarrow \bar{\mathbf{x}}^a \qquad \mathbf{P}^b \rightarrow \mathbf{P}^a$$

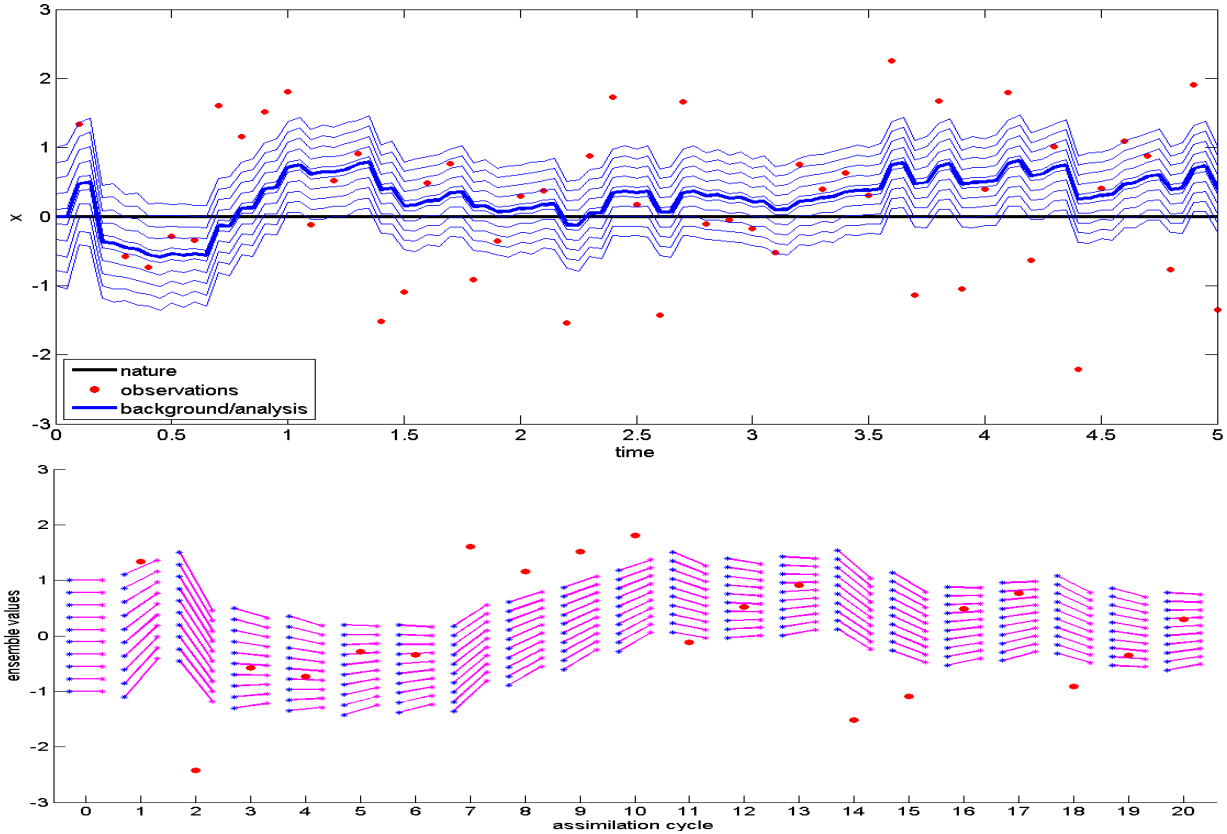
In the **ensemble Kalman filter**, we need to **update** *Ne* **ensemble members from background to analysis**, while **respecting the KF analysis equations**.

$$\{\mathbf{x}^{(1),b}, \mathbf{x}^{(2),b}, \ldots, \mathbf{x}^{(N_e),b}\} \rightarrow \{\mathbf{x}^{(1),a}, \mathbf{x}^{(2),a}, \ldots, \mathbf{x}^{(N_e),a}\}$$

There is **not a unique way** to do this. We have **different flavours**.

Example using ETKF

$$x_t = (1+a)x_{t-1}$$



Forecast and **analysis** steps (cycled)

Only **analysis** steps