Bayes' theorem with Gaussian mixtures

Earth Observation Summer School 2018

1 Exercises

1. Create a Gaussian prior with mean $\mu_p = 2$ and standard deviation $\sigma_p = 1$. Create an observation $y = -2$, that has an *unbiased* Gaussian error with standard deviation $\sigma_o = 1$. When these are combined using Bayes' theorem, how do you expect the posterior to be? Use Bayes' theorem and check the result.

2. Imagine we have the same observation $y = -2$, but its precision has doubled. What is the new standard deviation σ_o now? Run Bayes' theorem and compare the result with the previous one.

3. Now change the observation to $y = -1$ and make it less certain, i.e. $\sigma_o = 2$. How does the posterior look now?

4. Let us make things more interesting. Construct a bi-modal prior pdf, with modes in $\{-2, 2\}$, same coefficients (what should they be), and same standard deviations 1 for both components of the mixture. Let the observation be in $y = 2$, $\sigma_o = 1$. How do you expect the posterior to be? Run the program and check.

5. Keep the characteristics of the observations fixed and now vary the prior. Start by changing the coefficients (α's) for the bi-modal distribution. What do these coefficients do to both prior and the posterior?

6. Try to create the following prior pdf's: (a) a tri-modal prior, (b) a uni-modal prior that is asymmetric. How do the posteriors look like in each case?