Let $X = X_1 \times \ldots X_N$ with each component having the same distribution, π, such that the joint probability density function on X is given by

$$\pi(x_1, \ldots, x_N) = \prod_{n=1}^{N} \pi(x_n).$$

Additionally consider the weighted summation function

$$s_w : (x_1, \ldots, x_N) \mapsto \sum_{n=1}^{N} w_n \, x_n.$$

If the component distribution is represented by a Gaussian probability density function with locations μ_n and scales σ_n then the pushforward distribution of the joint distribution along s_w is

$$\pi(s) = \mathcal{N}\left(s; \sum_{n=1}^{N} w_n \, \mu_n, \sqrt{\sum_{n=1}^{N} w_n^2 \, \sigma_n^2} \right).$$

In the special case of identically distributed components the pushforward distribution becomes

$$\pi(s) = \mathcal{N}\left(s; \sum_{n=1}^{N} w_n \cdot \mu, \sqrt{\sum_{n=1}^{N} w_n^2} \cdot \sigma \right).$$

If the component distribution is represented by a Cauchy probability density function with locations μ_n and scales σ_n then the pushforward distribution of the joint distribution along s_w is

$$\pi(s) = \mathcal{C}\left(s; \sum_{n=1}^{N} w_n \, \mu_n, \sum_{n=1}^{N} w_n \, \sigma_n \right).$$

In the special case of identically distributed components the pushforward distribution becomes

$$\pi(s) = \mathcal{N}\left(s; \sum_{n=1}^{N} w_n \cdot \mu, \sum_{n=1}^{N} w_n \cdot \sigma \right).$$

Notice the difference – in the Cauchy case the scales add linearly with the weights while in the Gaussian case the scales add quadratically with the weights. This has an important difference when considering averages with $w_n = 1/N$. Here $\sum_{n=1}^{N} w_n = 1$ and the distribution of the average for the Cauchy is the same as the distribution for any individual, where as the distribution for the Gaussian average concentrates around the common location.