Time Series Analysis Workshop

Overview of some methods for detecting step changes

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Motivation

Test processing

Markov switching

Heavy-tailed state space models

Outline

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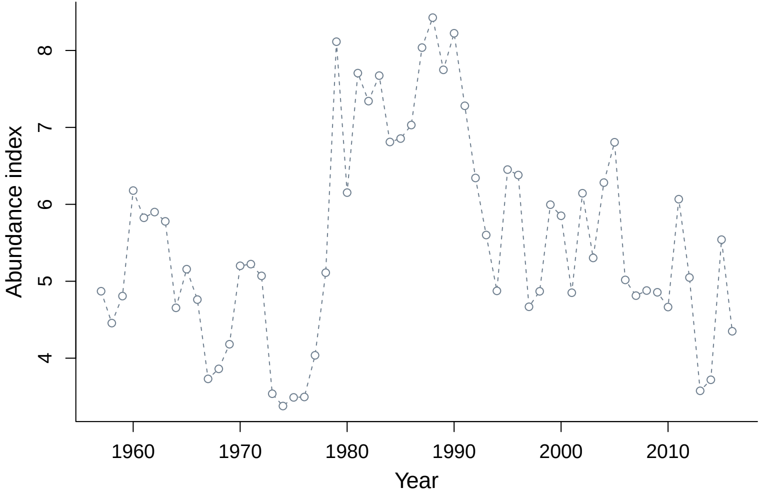
A time series



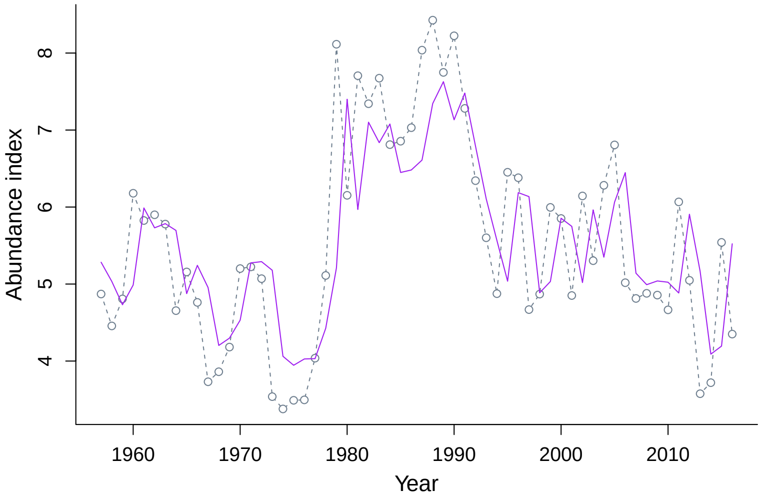
A time series

What if we tried a non-zero mean first-order autoregressive model AR(1)?

$$y_t = \bar{y} + \phi(y_{t-1} - \bar{y}) + \eta_t, \quad \eta_t \sim \mathrm{N}(0, \sigma_\eta^2)$$

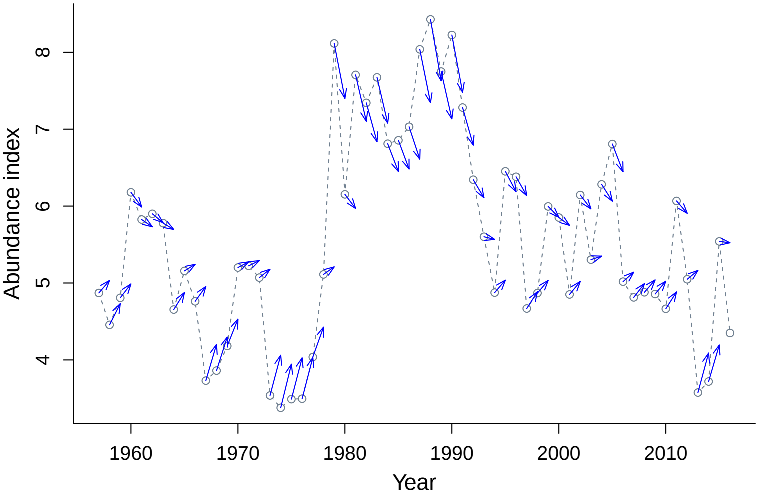
	Estimate	SE
$\bar{y}$	5.47	0.41
ϕ	0.73	0.086
σ_η^2	0.81	

AR(1) fit



Fit diagnostics

One-step ahead forecast errors



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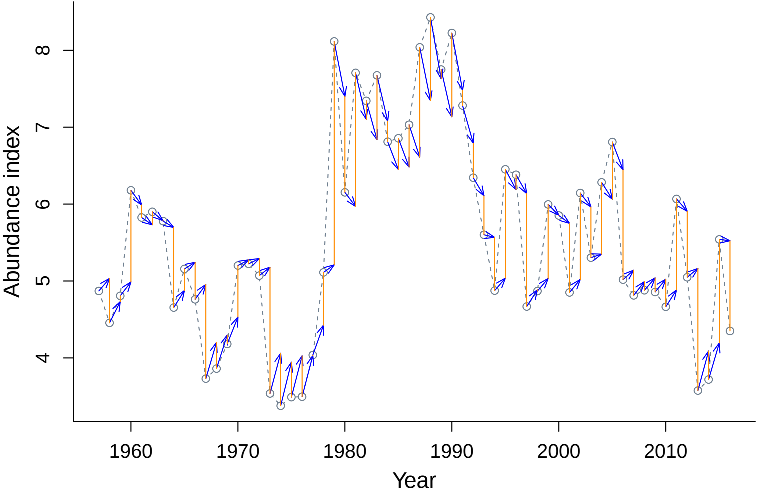

Fit diagnostics

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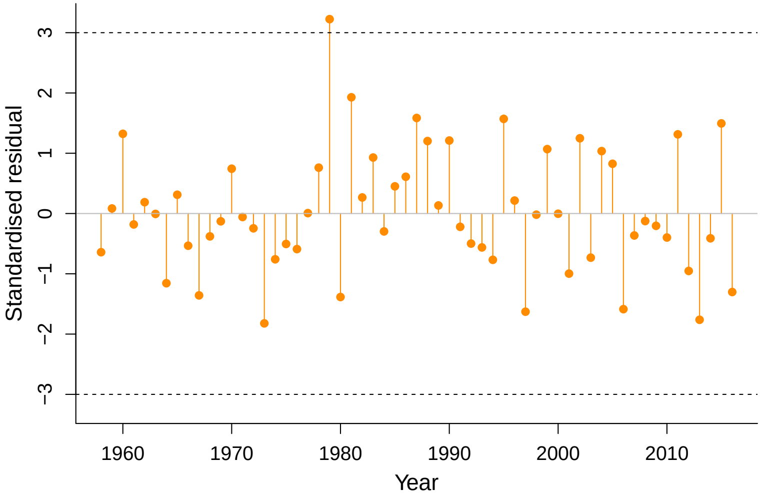
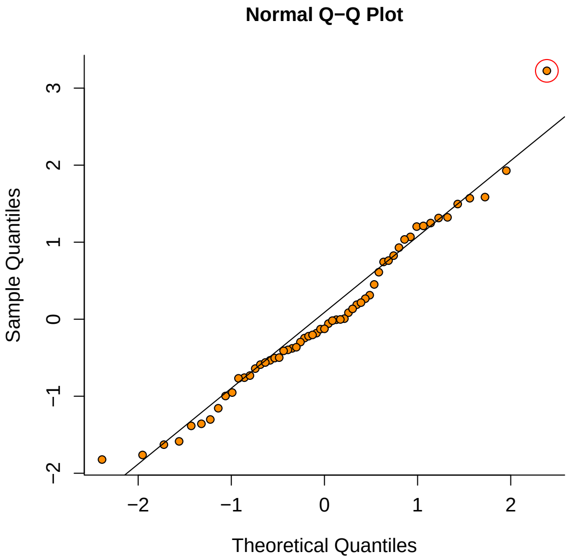


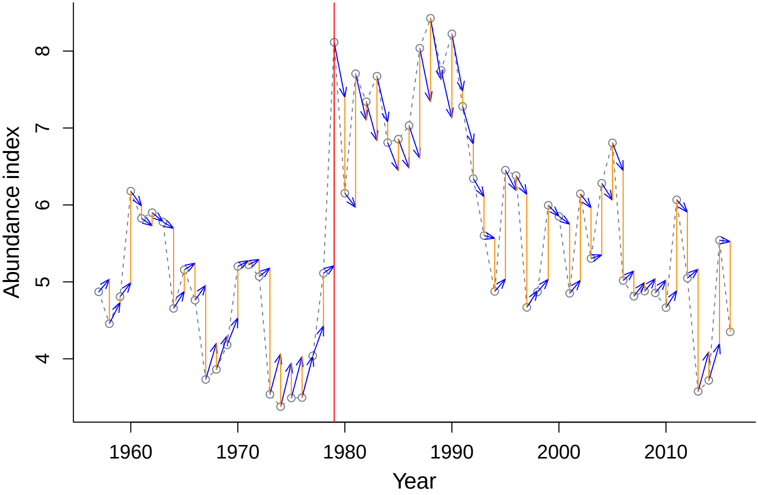
Fit diagnostics

Outlier(s)



Fit diagnostics

Outlier(s)



Motivation

- More happening than a non-zero mean AR(1) process can capture
- Need to account for the large changes in the dynamics
- Goals
  - Detect points of change in <u>components</u> of time series
  - Without a priori knowledge of the location of such points
  - Focus on time series models

Outline

Tests

Batch processing

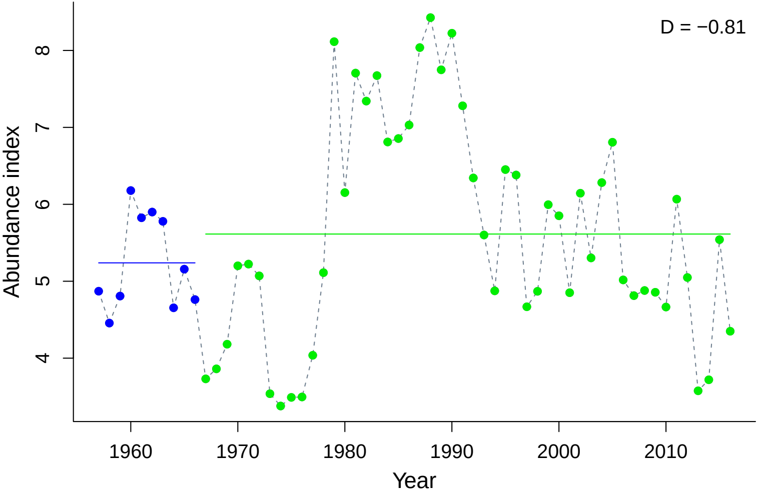
Tests hypotheses of a change at a given point k by comparing the two sets $\{y_{1\ldots k-1}\}$, $\{y_{k\ldots n}\}$. Note full sequence is of a fixed length n.

Tests can be

- Parameteric (e.g., Student's t-test)
- Non-parameteric (e.g., Mann-Whitney)

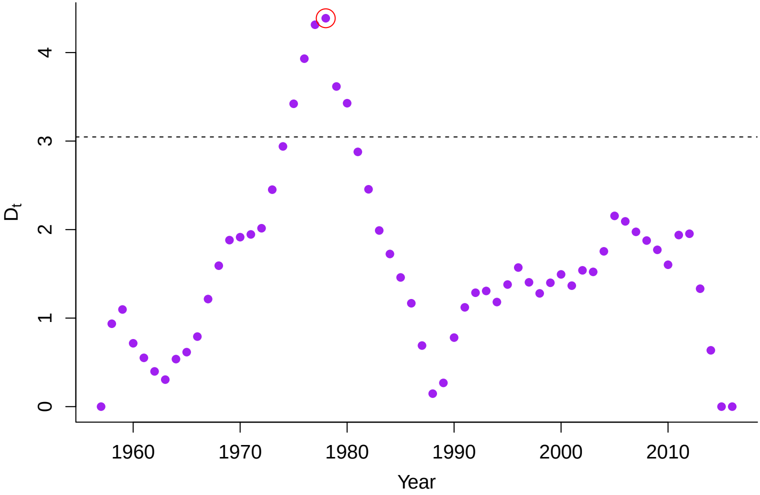
Student t-tests

Say for $k = 10$, testing the hypothesis of no difference in the means



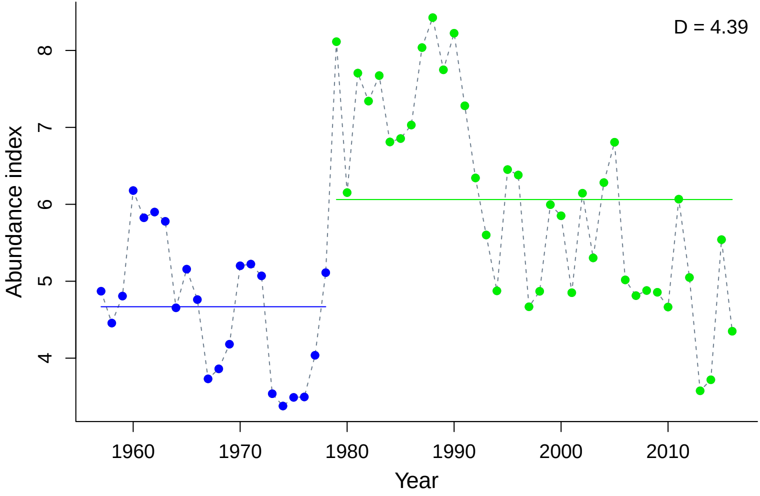
Student t-tests

Test statistics and threshold (maintains $\alpha = 0.05$ for given n)



Student t-tests

Maximum single changepoint significant difference at $k = 22$



Student t-tests

Batch processing

Performs poorly in detecting shifts at the start and end points of the series

Sequential processing

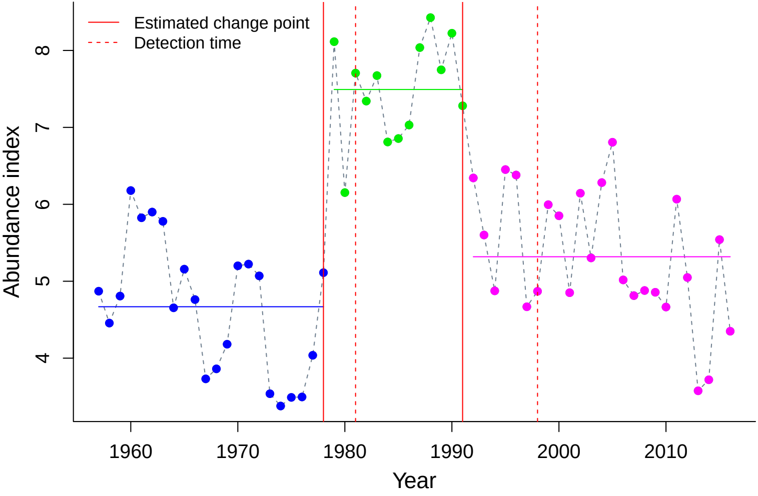
Sequential processing

Tests hypotheses of a change at a given point k by comparing the two sets $\{y_{1\ldots k-1}\}$, $\{y_{k\ldots t}\}$. Note t changes.

- Shows estimated changepoint and time of detection
- Easily incorporates multiple changepoints

Sequential processing

Multiple changepoints



Sequential processing

Sequential processing

- Parameteric tests perform poorly in the presence of autocorrelation
- Can be sensitive to assumption on average run length

Test processing

- Can be implemented in R package `cpm` (install for tomorrow)
- Parametric tests don't work well with autocorrelation (ubiquitous) unless pre-whitened
- Leaves room for development in time series direction

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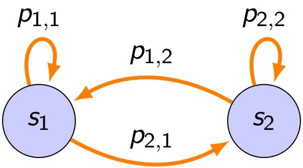
Heavy-tailed state space models

Markov switching

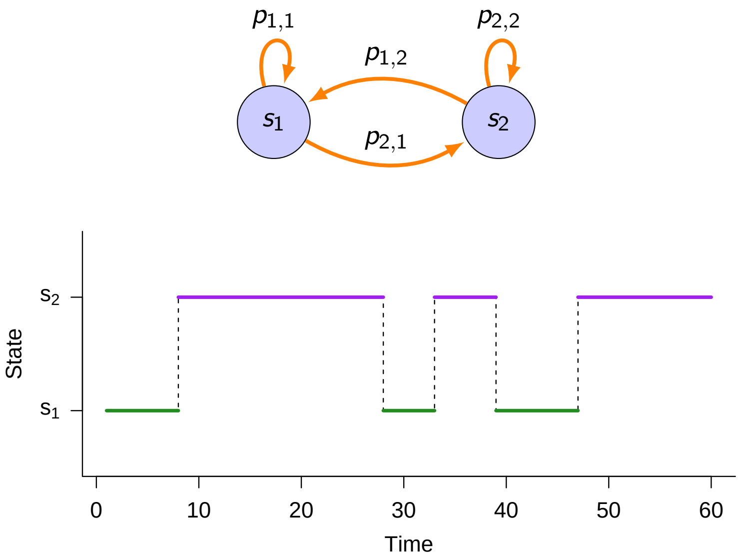
Markov switching

System has states; transition between these states governed by a Markov switching process.



$$P(S_t = i | S_{t-1} = j, S_{t-2} = j, \ldots S_{t-1} = i) = P(S_t = i | S_{t-1} = j)$$

Markov switching

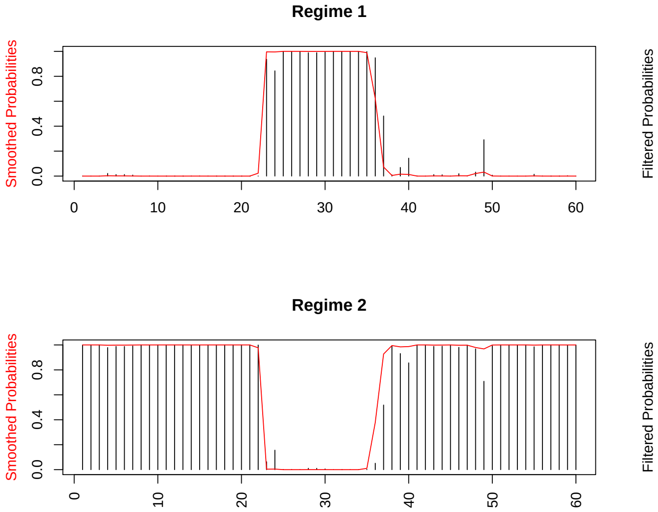


Markov switching

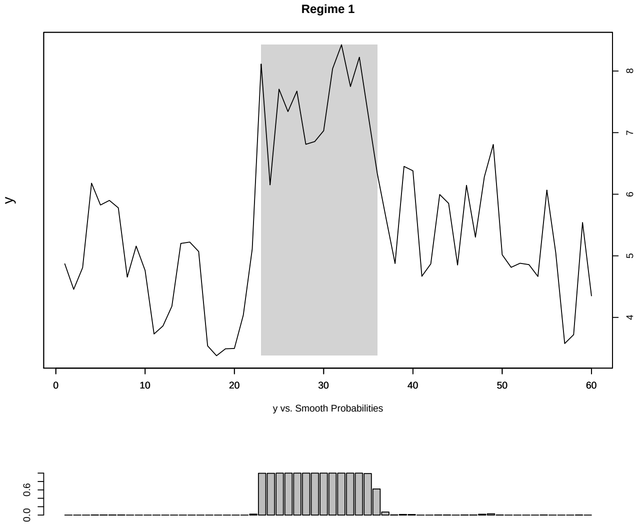
State is unobserved and must therefore be estimated, e.g., via EM algorithm

- Expected probability of state membership for each observation using a given set of parameters (via Bayes theorem)
- Maximise parameters conditional on expected state membership
- Can be implemented in the R package: `MSwM`

Markov switching fit



Markov switching fit



Markov switching fit

- Can be generalised to many within-state models (e.g., ARMA class)
- Can incorporate different covariate effects in each state
- Could be extended to semi-Markov models with expected durations
- Have been extended to vector autoregressive processes

Markov switching

Markov switching

- Well developed theoretical foundation
- Good candidate method for further work
- Performance should be simulation tested (goes for all methods)

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Heavy-tailed state space model

Exploring models such as

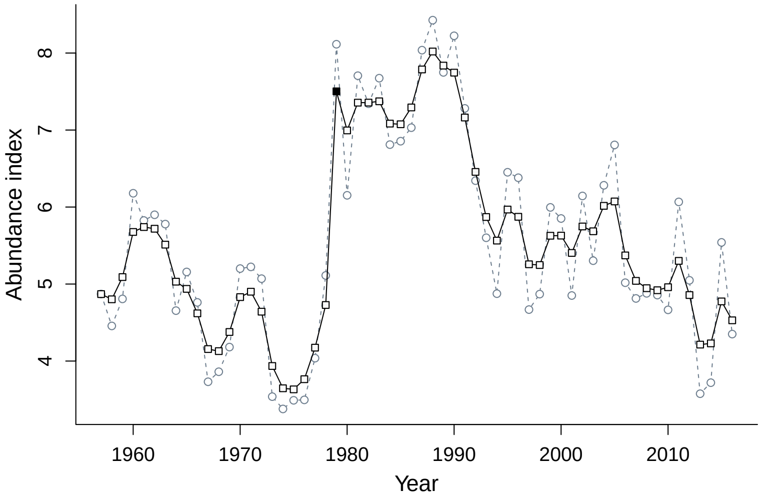
Process equation:	$x_t = \phi x_{t-1} + \eta_t$
Measurement equation:	$y_t = x_t + \varepsilon_t$

where (currently)

$$\eta_t \sim pN(0, \sigma^2_{eta}) + (1-p)N(0, (1+\delta)\sigma^2_{eta})$$
$$\epsilon_t \sim N(0, \sigma^2_{eta})$$

Model developed and estimated in `TMB`

Heavy-tailed state space model



Heavy-tailed state space model

Heavy-tailed state space model

- Moderately well developed theoretical framework
- Estimation code is somewhat involved (C++)
- Good candidate method for further work and comparison
- Needs to be simulation tested against known dynamics
- Think about p_t

Additional methods (not covered)

- Machine learning techniques
  - Significance of change a challenge?
- Additive models
  - Low-order penalties could allow for sudden jumps not overly smoothed as happens with higher order smoothers
- Vector autoregressive processes (multivariate setting)

Acknowledgements

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- Project steering committee

References

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