Optimal Armageddon Bidding in Chess

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Many rapid chess tournaments these days are run as a series of mini-matches with "armageddon" tiebreakers. In armageddon, players bid for the amount of time they will have for the game, which is usually played without an increment.[1] The bidding winner is the player who bids the *lower* time; they then get to choose which color they will have in the game, with black having draw odds.

For example, suppose that in a 10-minute armageddon game, Player A bids 7:30 and Player B bids 8:00. Then Player A wins the bidding portion and gets to choose which color they want in the game. Say they choose Black (which is the overwhelmingly usual choice). Then the players play a single game where Player B is white and starts with 10 minutes for the game, while Player A is black and starts with 7:30. For Player B to be victorious in the match, they would have to win the game, while a draw or black victory is sufficient for Player A to win.

This bidding phase adds an interesting new twist to chess competitions. As a game theorist, I decided to dive in, formulate armageddon bidding as a game in itself, see what the results would be, and from them generate insights and advice for tournament players.

1 Setup and Assumptions

For a game theory analysis, you need three elements: players, strategies and outcomes. First, then, we will assume that there are two players, A and B.

Their strategies are time bids t_A and t_B such that $t_A, t_B \geq 0$, with an overall time limit T, typically 5, 10 or 15 minutes.[2] Whoever bids the lower amount then gets to choose which color $C \in \{B, W\}$ they will use, with ties broken by some mechanism, such as a coin toss.

As for outcomes, the result of any single game is never certain beforehand, but we can assume that there is some probability that a given player is victorious, given the match parameters. The only assumption in the analysis is that a player's chances of winning increase the more time they start with, and

[1] Originally, armageddon games had no increment. But in the current (late 2023) CCT Final they use an increment from move 60 on.

[2] At first I thought that the bids have to be in the range $[0, T]$, but it seems that players can bid more than T if they want. In fact, Magnus Carlsen once bid 15:01 to ensure that he got the white pieces in his game. See the Comments section below.

decrease the more time their opponent starts with, regardless of which color they play with. For instance, if Player A has 9 minutes vs. 15 for player B, the time difference is -6 from Player A's point of view. If it's 12 vs. 15, the difference is -3. If Player A has 15 minutes and Player B has 11, it's +4, and so on. The assumption is that, no matter which color Player A has for the game, their chances of winning increase as this time differential increases.

So a strategy for Player i is a pair (t_i, C_i) giving their time bid and color choice. Then Player A wants to choose a strategy to maximize their probability P_A of winning, while Player B wants to mimimize it.

2 Analysis

In game theory, we first look for the Nash Equilibrium of a game. These are strategies (t_A, C_A) for Player A and (t_B, C_B) for Player B such that neither player can improve their chances of winning by changing strategies. I'll give the results of the analysis as a series of observations.

1. Armageddon bidding is a zero-sum game; what's good for one player is necessarily worse for the other.

2. One consequence of this observation is that there is a particular color C^* that both players will pick if they win the bidding – they will either both choose White or both choose Black. As noted above, in practice Black is almost always chosen due to their having draw odds.

3. The key to the solution is determining what we'll call T^*. This is the time at which a player is indifferent between having T^* minutes for the game with color C^* vs. T for their opponent, or having T themselves and letting their opponent play with time T^* and color C^*.

   To determine T^*, a player should pick a sample bid and ask themselves which color they would rather play with those time odds. For example, in a 10-minute armageddon, say we start by considering a bid of 6 minutes. The question is, do we think we have a greater chance of winning with 6 minutes versus 10 as Black, or 10 minutes versus 6 as White? If we would choose Black, then we should lower our bid; if white, we should raise it. When the player thinks the have the (more or less) exact same chance winning with Black or White, they have arrived at T^*.

4. It might be a bit surprising that T^* is unique and the same for both players, but it follows from the zero-sum nature of the game. Figure 1 shows the derivation of T^* in graphic format.[3] The horizontal line in the middle of the figure is equidistant from the two probability curves, meaning that the probability P_A of Player A winning is the same, whether they win with a bid of T^* and take Black or lose to a bit of T^*, play with White and start with T minutes.

[3]In the graph, $\overline{T}$ is the time differential. To get T^*, just subtract $\overline{T}$ from T. So for a 10-minute game with $\overline{T} = 3$ minutes, T^* would be 7 minutes.

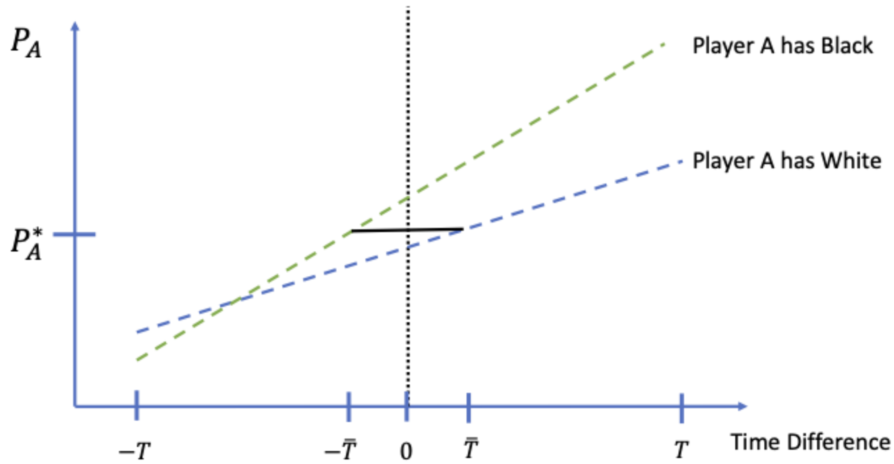

Figure 1: Equilibrium illustrating $T^* = T - \overline{T}$

5. The Nash Equilibrium of the game, then, is for both players to use the strategy (T^*, C^*), and the winner is determined by the tie-breaking mechanism.

3 Comments

1. Once a player figures out what T^* is, they should use this as a *minimum* bid; it makes no sense to bid lower. Assume, for instance, that in a 10-minute game T^* is 7:00, and you're thinking of bidding 6:45 instead.

   There are a few possibilities: a) You would have won with a bid of 7:00 and you still win with 6:45. Then you've just given yourself 15 seconds less for the game. b) You would lose the bidding with both 7:00 and 6:45 because your opponent bid lower than 6:45. Then lowering your bid makes no difference. c) You would have lost with a bid of 7:00 but now win with 6:45 because your opponent bid something like 6:50. Then you are worse off, because, by the definition of T^*, your probability of winning with White and 10 minutes vs. 6:50 is higher than the probability of winning with Black with 6:45 vs. 10 minutes. Overall, if lowering your bid below T^* makes any difference to the outcomes, it makes you worse off in expectation.

2. On the other hand, you might consider shading your bid a little higher than T^*. Same setup as above, but you're considering a bid of 7:15. Then if you would have won with 7:00 and still win with 7:15, you've given yourself 15 more seconds and come out ahead. However, If you would have won before but lose now, because your opponent bids something like

7:10, then you're worse off.

3. There's another aspect of the game worth mentioning, which is that so far we have been implicitly assuming full information; that is, all the parameters of the