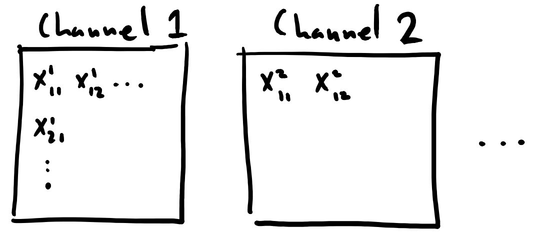
Convolution and filters

Wednesday, May 22, 2019 2:08 PM

Image X



def Filter bank are predefined images (small) s.t. each element $\varphi \in \Phi$ is applied to the image

$$\langle \varphi, X \rangle = \sum_{\substack{j,k \\ \uparrow \text{pixels}}} \varphi_{jk} X_{jk} \quad \text{(target channels)}$$

ex Gabor filter, wavelength, λ, orientation, θ, phase offset, ψ, bandwidth, σ, and aspect ratio, γ

$$\tilde{x} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} x$$

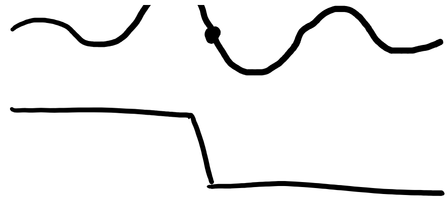
↙ position in image

$$g(x; \lambda, \theta, \ldots) = \exp\left(-\frac{\tilde{x}_1^2 + \gamma^2 \tilde{x}_2^2}{2\sigma^2}\right) \cos\left(\frac{2\pi}{\lambda}\tilde{x}_1 + \psi\right)$$

Designed to detect edges

In 1D φ  $\langle \varphi, X \rangle \longrightarrow \gg 0$

In 1D φ



x $\qquad \xrightarrow{\langle \varphi, x \rangle} \gg 0$

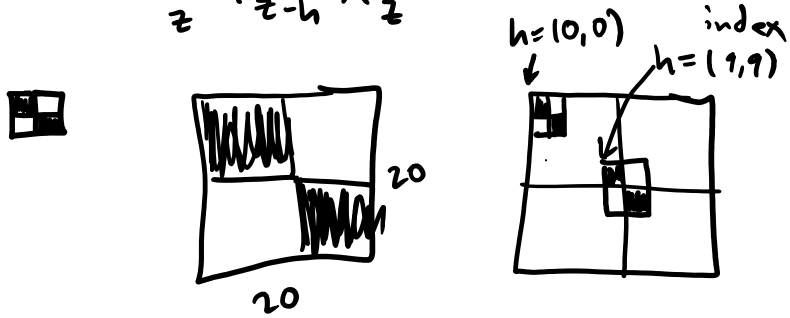
Given filter, g, can center anywhere by shifting $(S_{\Delta x}\, g)(x) = g(x - \Delta x)$

Apply $\langle S_h \varphi, X \rangle = \sum_z (S_h \varphi)_z X_z$

$= \sum_z \varphi_{z-h} X_z$

pixel ($z = (j,k)$) index

$\varphi =$



def images $G * X$ is the convolution of G and X

$$(G * X)_h = \sum_z G_{h-z} X_z$$

(can define $G_{h-z} = \varphi_{z-h}$)

"full" convolution $X_z = 0$ for z outside of bounding box

makes image of same size

"valid" convolution $(G * X)_h$ is only defined when $S_h G$ is contained in domain of X

Gabor filters and SIFT filters (& wavelets) was the state of art w/ images

$X \longrightarrow$ Filter bank Vectorize $\longrightarrow$ Classifier

Can we apply large filter banks?

<u>thm</u> [Convolution Thm]

$$G * X = DFT^{-1}\big(DFT(G) \cdot DFT(X)\big)$$

The fast Fourier tranform (FFT) computes DFT, DFT^{-1} in $O(p \log p)$

p ← # of pixels

Convolutional NNets

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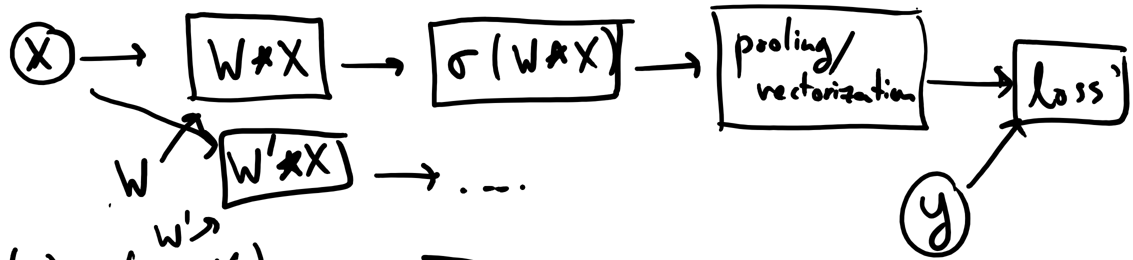
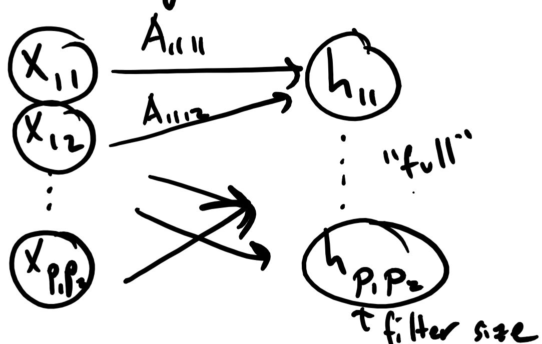


(1) $$(W * X)_{ij} = \sum_{kl} W_{i-k,\, j-l}\, X_{kl}$$

$$= \sum_{kl} A_{ij,k,l}\, X_{k,l}$$

$$A_{ij,k,l} = W_{i-k,\, j-l}$$

Convolution layer can be though of as a "fully" connected layer but



parameters are shared

$$A_{ijkl} = W_{i-k,\, j-l}$$
$$= W_{(i+1)-(k+1),\, j-l}$$
$$= A_{i+1,\, j,\, k+1,\, l}$$

\# of parameters in fully conn. is $(p_1 \cdot p_2)^2$

$p_1 \times p_2$ image $\quad H_1 \times H_2$ filter

\# parameters in convolution $H_1 \cdot H_2$

and interactions are sparse

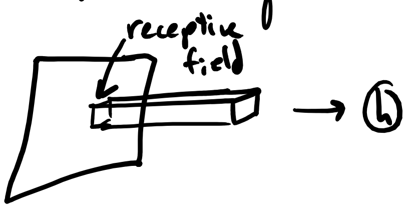
$$A_{ijkl} = 0 \text{ if } |i-k|,\ |j-l| > H_1, H_2$$

Convolution is <u>equivariant</u> to translation

Convolution is equivariant to translation

$$S_t(X \star W) = (S_t X) \star W \quad \text{for "full"}$$

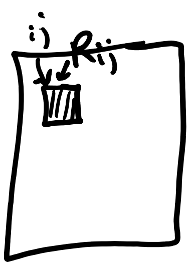
def (Receptive field) of a unit are input pixels that it depends on



def $\sum_{k\ell} X_{i+k,\,j+\ell} W_{k,\ell}$ is called "cross-correlation"

pooling apply a fixed transformation to $T(Y_{R_{ij}})$

T = max or average — rectangle at i,j



▷ strides only every k^{th} i / j are used