# pestpp-opt

May 12, 2019

# 1 Run PESTPP-OPT

In this notebook we will setup and solve a mgmt optimization problem around how much groundwater can be pumped while maintaining sw-gw exchange

```
In [1]: import os
        import shutil
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import matplotlib as mpl
        plt.rcParams['font.size']=12
        import flopy
        import pyemu
flopy is installed in /Users/jeremyw/Dev/gw1876/activities_2day_mfm/notebooks/flopy
In [2]: t_d = "template"
        m_d = "master_opt"
In [3]: pst = pyemu.Pst(os.path.join(t_d, "freyberg.pst"))
        pst.write_par_summary_table(filename="none").sort_index()
Out [3]:
                                                        initial value
                            type transform count
        cn_hk6
                          cn_hk6
                                        log
                                                                    0
        cn hk7
                          cn_hk7
                                        log
                                                                    0
        cn_hk8
                          cn_hk8
                                        log
                                                 1
                                                                    0
        cn_prsity6
                      cn_prsity6
                                                                    0
                                        log
                                                 1
                                                                    0
        cn_prsity7
                      cn_prsity7
                                        log
                                                 1
        cn_prsity8
                      cn_prsity8
                                        log
                                                 1
                                                                    0
                                                                    0
        cn_rech4
                        cn_rech4
                                        log
                                                 1
                        cn_rech5
                                                 1
                                                             -0.39794
        cn_rech5
                                        log
        cn_ss6
                          cn_ss6
                                        log
                                                 1
                                                                    0
                                                                    0
        cn ss7
                          cn_ss7
                                        log
                                                 1
        cn_ss8
                          cn_ss8
                                        log
                                                 1
                                                                    0
                                                 1
                                                                    0
        cn_strt6
                     cn_strt6
                                        log
```

| cn_strt7    | cn_strt7    | log | 1   | 0             |
|-------------|-------------|-----|-----|---------------|
| cn_strt8    | cn_strt8    | log | 1   | 0             |
| cn_sy6      | cn_sy6      | log | 1   | 0             |
| cn_sy7      | cn_sy7      | log | 1   | 0             |
| cn_sy8      | cn_sy8      | log | 1   | 0             |
| cn_vka6     | cn_vka6     | log | 1   | 0             |
| cn_vka7     | cn_vka7     | log | 1   | 0             |
| cn_vka8     | cn_vka8     | log | 1   | 0             |
| drncond_k00 | drncond_k00 | log | 10  | 0             |
| flow        | flow        | log | 1   | 0             |
| gr_hk3      | gr_hk3      | log | 705 | 0             |
| gr_hk4      | gr_hk4      | log | 705 | 0             |
| gr_hk5      | gr_hk5      | log | 705 | 0             |
| gr_prsity3  | gr_prsity3  | log | 705 | 0             |
| gr_prsity4  | gr_prsity4  | log | 705 | 0             |
| gr_prsity5  | gr_prsity5  | log | 705 | 0             |
| gr_rech2    | gr_rech2    | log | 705 | 0             |
| gr_rech3    | gr_rech3    | log | 705 | 0             |
|             |             |     |     |               |
| gr_strt5    | gr_strt5    | log | 705 | 0             |
| gr_sy3      | gr_sy3      | log | 705 | 0             |
| gr_sy4      | gr_sy4      | log | 705 | 0             |
| gr_sy5      | gr_sy5      | log | 705 | 0             |
| gr_vka3     | gr_vka3     | log | 705 | 0             |
| gr_vka4     | gr_vka4     | log | 705 | 0             |
| gr_vka5     | gr_vka5     | log | 705 | 0             |
| pp_hk0      | pp_hk0      | log | 32  | 0             |
| pp_hk1      | pp_hk1      | log | 32  | 0             |
| pp_hk2      | pp_hk2      | log | 32  | 0             |
| pp_prsity0  | pp_prsity0  | log | 32  | 0             |
| pp_prsity1  | pp_prsity1  | log | 32  | 0             |
| pp_prsity2  | pp_prsity2  | log | 32  | 0             |
| pp_rech0    | pp_rech0    | log | 32  | 0             |
| pp_rech1    | pp_rech1    | log | 32  | 0             |
| pp_ss0      | pp_ss0      | log | 32  | 0             |
| pp_ss1      | pp_ss1      | log | 32  | 0             |
| pp_ss2      | pp_ss2      | log | 32  | 0             |
| pp_strt0    | pp_strt0    | log | 32  | 0             |
| pp_strt1    | pp_strt1    | log | 32  | 0             |
| pp_strt2    | pp_strt2    | log | 32  | 0             |
| pp_sy0      | pp_sy0      | log | 32  | 0             |
| pp_sy1      | pp_sy1      | log | 32  | 0             |
| pp_sy2      | pp_sy2      | log | 32  | 0             |
| pp_vka0     | pp_vka0     | log | 32  | 0             |
| pp_vka1     | pp_vka1     | log | 32  | 0             |
| pp_vka2     | pp_vka2     | log | 32  | 0             |
| strk        | strk        | log | 40  | 0             |
| welflux     | welflux     | log | 2   | 0 to 0.176091 |
|             | "OTTTUN     | 6   | _   | 2 22 3.170001 |

|             | upper bound | lower bound | standard deviation |
|-------------|-------------|-------------|--------------------|
| cn_hk6      | 1           | -1          | 0.5                |
| cn_hk7      | 1           | -1          | 0.5                |
| cn_hk8      | 1           | -1          | 0.5                |
| cn_prsity6  | 0.176091    | -0.30103    | 0.11928            |
| cn_prsity7  | 0.176091    | -0.30103    | 0.11928            |
| cn_prsity8  | 0.176091    | -0.30103    | 0.11928            |
| cn_rech4    | 0.0791812   | -0.09691    | 0.0440228          |
| cn_rech5    | -0.09691    | -1          | 0.225772           |
| cn_ss6      | 1           | -1          | 0.5                |
| cn_ss7      | 1           | -1          | 0.5                |
| cn_ss8      | 1           | -1          | 0.5                |
| cn_strt6    | 0.0211893   | -0.0222764  | 0.0108664          |
| cn_strt7    | 0.0211893   | -0.0222764  | 0.0108664          |
| cn_strt8    | 0.0211893   | -0.0222764  | 0.0108664          |
| cn_sy6      | 0.243038    | -0.60206    | 0.211275           |
| cn_sy7      | 0.243038    | -0.60206    | 0.211275           |
| cn_sy8      | 0.243038    | -0.60206    | 0.211275           |
| cn_vka6     | 1           | -1          | 0.5                |
| cn_vka7     | 1           | -1          | 0.5                |
| cn_vka8     | 1           | -1          | 0.5                |
| drncond_k00 | 1           | -1          | 0.5                |
| flow        | 0.09691     | -0.124939   | 0.0554622          |
| gr_hk3      | 1           | -1          | 0.5                |
| gr_hk4      | 1           | -1          | 0.5                |
| gr_hk5      | 1           | -1          | 0.5                |
| gr_prsity3  | 0.176091    | -0.30103    | 0.11928            |
| gr_prsity4  | 0.176091    | -0.30103    | 0.11928            |
| gr_prsity5  | 0.176091    | -0.30103    | 0.11928            |
| gr_rech2    | 0.0413927   | -0.0457575  | 0.0217875          |
| gr_rech3    | 0.0413927   | -0.0457575  | 0.0217875          |
|             |             |             |                    |
| gr_strt5    | 0.0211893   | -0.0222764  | 0.0108664          |
| gr_sy3      | 0.243038    | -0.60206    | 0.211275           |
| gr_sy4      | 0.243038    | -0.60206    | 0.211275           |
| gr_sy5      | 0.243038    | -0.60206    | 0.211275           |
| gr_vka3     | 1           | -1          | 0.5                |
| gr_vka4     | 1           | -1          | 0.5                |
| gr_vka5     | 1           | -1          | 0.5                |
| pp_hk0      | 1           | -1          | 0.5                |
| pp_hk1      | 1           | -1          | 0.5                |
| pp_hk2      | 1           | -1          | 0.5                |
| pp_prsity0  | 0.176091    | -0.30103    | 0.11928            |
| pp_prsity1  | 0.176091    | -0.30103    | 0.11928            |
| pp_prsity2  | 0.176091    | -0.30103    | 0.11928            |
| pp_rech0    | 0.0413927   | -0.0457575  | 0.0217875          |
| 11          | 0.0110021   | 0.020.010   | 0.022.010          |

```
pp_rech1
                        0.0413927
                                            -0.0457575
                                                                     0.0217875
pp_ss0
                                 1
                                                    -1
                                                                           0.5
                                 1
                                                    -1
                                                                           0.5
pp_ss1
                                                    -1
                                                                           0.5
pp_ss2
                                 1
pp_strt0
                        0.0211893
                                            -0.0222764
                                                                     0.0108664
                        0.0211893
                                            -0.0222764
                                                                     0.0108664
pp_strt1
pp_strt2
                        0.0211893
                                            -0.0222764
                                                                     0.0108664
pp_sy0
                         0.243038
                                              -0.60206
                                                                      0.211275
                         0.243038
                                              -0.60206
                                                                      0.211275
pp_sy1
pp_sy2
                         0.243038
                                              -0.60206
                                                                      0.211275
pp_vka0
                                 1
                                                     -1
                                                                           0.5
                                 1
                                                     -1
pp_vka1
                                                                           0.5
                                 1
                                                     -1
                                                                           0.5
pp_vka2
                                 2
                                                     -2
strk
                                                        0.0752575 to 0.11928
welflux
              0.176091 to 0.30103
                                    -0.30103 to
welflux k02
                                                     -1
                                                                           0.5
                                 1
[65 rows x 7 columns]
```

define our decision varible group and also set some ++args. Conceptually, we are going to optimize current pumping rates to make sure we meet ecological flows under both historic (current) conditions and scenario (future) conditions. Remember the scenario is an extreme 1-year drought so if we pump too much now, the system will be too low to provide critical flows if next year is an extreme drough - transient memory!

```
In [4]: pst.pestpp_options = {}
    #dvg = ["welflux_k02", "welflux"]
    dvg = ["welflux_k02"]
    pst.pestpp_options["opt_dec_var_groups"] = dvg
    pst.pestpp_options["opt_direction"] = "max"
```

For the first run, we wont use chance constraints, so just fix all non-decision-variable parameter. We also need to set some realistic bounds on the welflux multiplier decision variables. Finally, we need to specify a larger derivative increment for the decision variable group

```
In [5]: par = pst.parameter_data
    par.loc[:,"partrans"] = "fixed"

#turn off pumping in the scenario
par.loc["welflux_001","parlbnd"] = 0.0
par.loc["welflux_001","parval1"] = 0.0
dvg_pars = par.loc[par.pargp.apply(lambda x: x in dvg),"parnme"]
par.loc[dvg_pars,"partrans"] = "none"
par.loc[dvg_pars,"parlbnd"] = 0.0
par.loc[dvg_pars,"parubnd"] = 3.0
par.loc[dvg_pars,"parval1"] = 1.0

pst.rectify_pgroups()
pst.parameter_groups.loc[dvg,"inctyp"] = "absolute"
```

```
pst.parameter_groups.loc[dvg,"inctyp"] = "absolute"
        pst.parameter_groups.loc[dvg,"derinc"] = 0.25
        pst.parameter_groups.loc[dvg,:]
Out[5]:
                                    inctyp derinc derinclb forcen
                                                                      derincmul \
                       pargpnme
        pargpnme
        welflux_k02 welflux_k02 absolute
                                              0.25
                                                         0.0
                                                                            2.0
                                                              switch
                       dermthd splitthresh splittreldiff splitaction
        pargpnme
        welflux_k02 parabolic
                                    0.00001
                                                      0.5
                                                              smaller
                                                                         NaN
```

#### 1.0.1 define constraints

model-based and prior information constraints are identified in pestpp-opt by an obs group that starts with "less\_than" or "greater\_than" and a weight greater than zero. So first, we turn off all of the weights and get names for the sw-gw exchange forecasts (funny how optimization turns forecasts into constraints...)

```
In [6]: obs = pst.observation_data
        obs.loc[:, "weight"] = 0.0
        swgw_const = obs.loc[obs.obsnme.apply(lambda x: "fa" in x and( "hw" in x or "tw" in x)
        obs.loc[swgw_const,:]
Out [6]:
                                obsnme
                                            obsval weight obgnme
        obsnme
        fa_hw_19791230 fa_hw_19791230 -620.62265
                                                       0.0 flaqx
                                                                     NaN
        fa_hw_19801229 fa_hw_19801229
                                         109.75435
                                                       0.0 flaqx
                                                                     NaN
        fa_tw_19791230 fa_tw_19791230
                                        1239.63960
                                                       0.0 flaqx
                                                                     NaN
        fa_tw_19801229 fa_tw_19801229
                                        1936.06560
                                                       0.0 flagx
                                                                     NaN
```

We need to change the obs group (obgnme) so that pestpp-opt will recognize these two model outputs as constraints. The obsval becomes the RHS of the constraint. We also need to set a lower bound constraint on the total abstraction rate (good thing we included all those list file budget components as observations!)

```
In [7]: obs.loc[swgw_const,"obgnme"] = "less_than"
    obs.loc[swgw_const,"weight"] = 1.0

# we must have at least 300 m3/day of flux from gw to sw
# for historic and scenario periods
# and for both headwaters and tailwaters
    obs.loc[swgw_const,"obsval"] = -300

# tot_abs_rate = ["flx_wells_19791230"]#, "flx_wells_19801229"]
# obs.loc[tot_abs_rate,"obgnme"] = "less_than"
# obs.loc[tot_abs_rate,"weight"] = 1.0
# obs.loc[tot_abs_rate,"obsval"] = -900.0
# pst.less_than_obs_constraints
```

Now we need to define a minimum total pumping rate, otherwise this opt problem might yield a solution that doesn't give enough water for the intended usage. We will do this through a prior information constraint since this just a sum of decision varible values - the required minimum value will the sum of current pumping rates:

```
In [8]: pyemu.pst_utils.pst_config["prior_fieldnames"]
Out[8]: ['pilbl', 'equation', 'weight', 'obgnme']
```

Since all pumping well are using the same rate, we can just use a 1.0 multiplier in front of each wel.flux decision variable. If that is not the case, then you need to set the multipliers to be more meaningful

```
In [9]: pi = pst.null_prior
        pi.loc["pi_1","obgnme"] = "greater_than"
        pi.loc["pi_1","pilbl"] = "pi_1"
        pi.loc["pi_1","equation"] = " + ".join(["1.0 * {0}".format(d) for d in dvg_pars]) +\
                                     " = {0}".format(par.loc[dvg_pars,"parval1"].sum())
        pi.loc["pi_1","weight"] = 1.0
        pi.equation["pi_1"]
Out[9]: '1.0 * wf0200090016 + 1.0 * wf0200110013 + 1.0 * wf0200200014 + 1.0 * wf0200260010 + 1
In [10]: pst.prior_information
Out[10]:
                     obgnme pilbl \
         pi_1 greater_than pi_1
         pi_1 1.0 * wf0200090016 + 1.0 * wf0200110013 + 1.0 * wf0200200014 + 1.0 * wf02002600
               weight
                1.0
         pi_1
In [11]: pst.control_data.noptmax = 1
         pst.write(os.path.join(t_d,"freyberg_opt.pst"))
         pyemu.os_utils.start_slaves(t_d,"pestpp-opt","freyberg_opt.pst",num_slaves=10,master_opt.pst")
noptmax:1, npar_adj:6, nnz_obs:4
  Let's load and inspect the response matrix
```

```
In [12]: jco = pyemu.Jco.from_binary(os.path.join(m_d, "freyberg_opt.1.jcb")).to_dataframe().lo
        jco
Out[12]:
                        wf0200090016 wf0200110013 wf0200200014 wf0200260010 \
                           137.57200
                                         126.32400
                                                        46.30000
                                                                      21.90800
        fa_hw_19791230
        fa_hw_19801229
                                          28.65600
                                                        12.03600
                                                                      12.29200
                            22.58400
        fa_tw_19791230
                                          14.53516
                                                        93.28136
                                                                      92.42320
                             6.50728
```

| fa_tw_19801229 | 4.10836      | 7.60104      | 15.29948 | 30.88604 |
|----------------|--------------|--------------|----------|----------|
|                |              |              |          |          |
|                | wf0200290006 | wf0200340012 |          |          |
| fa_hw_19791230 | 18.12000     | 4.8320       |          |          |
| fa_hw_19801229 | 13.12800     | 3.3560       |          |          |
| fa_tw_19791230 | 71.84608     | 82.9612      |          |          |
| fa_tw_19801229 | 34.79872     | 17.5232      |          |          |

We see the transient effects in the nonzero value between current pumping rates (columns) and scenario sw-gw exchange (rows from 1980)

Let's also load the optimal decision variable values:

#### 9.693626734842857

fa\_tw\_19801229

```
Out [13]:
                                     parval1 scale offset
                            parnme
        parnme
        wf0200090016 wf0200090016 3.000000
                                                1.0
                                                        0.0
        wf0200110013 wf0200110013 3.000000
                                                1.0
                                                        0.0
        wf0200200014 wf0200200014 3.000000
                                                1.0
                                                        0.0
        wf0200260010 wf0200260010 0.000000
                                                1.0
                                                        0.0
        wf0200290006 wf0200290006 0.000000
                                                1.0
                                                        0.0
        wf0200340012 wf0200340012 0.693627
                                                1.0
                                                        0.0
```

The sum of these values is the optimal objective function value. However, since these are just mulitpliers on the pumping rate, this number isnt too meaningful. Instead, lets look at the residuals file

```
In [14]: pst = pyemu.Pst(os.path.join(m_d, "freyberg_opt.pst"), resfile=os.path.join(m_d, "freyberg_opt.pst")
         pst.res.loc[pst.nnz_obs_names,:]
Out[14]:
                                              group measured modelled residual \
                                   name
         name
         fa_hw_19791230 fa_hw_19791230 less_than
                                                       -300.0 -398.5755
                                                                          98.5755
         fa_hw_19801229 fa_hw_19801229
                                         less than
                                                       -300.0 -656.8370
                                                                         356.8370
                                         less_than
         fa_tw_19791230 fa_tw_19791230
                                                       -300.0 -414.1347
                                                                         114.1347
         fa_tw_19801229 fa_tw_19801229
                                         less than
                                                       -300.0 -299.5820
                                                                          -0.4180
                         weight
         name
         fa_hw_19791230
                            1.0
         fa_hw_19801229
                            1.0
         fa_tw_19791230
                            1.0
```

Sweet as! lots of room in the optimization problem. The bounding constraint is the one closest to its RHS

1.0

### 1.0.2 Opt under uncertainty part 1: FOSM chance constraints

This is where the process of uncertainty quantification/history matching and mgmt optimization meet - worlds collide!

Mechanically, in PESTPP-OPT, to activate the chance constraint process, we need to specific a risk!= 0.5. Risk ranges from 0.001 (risk tolerant) to 0.999 (risk averse). The larger the risk value, the more confidence we have that the (uncertain) model-based constraints are truely satisfied. Here we will start with a risk tolerant stance:

```
In [15]: pst.pestpp_options["opt_risk"] = 0.4
```

For the FOSM-based chance constraints, we also need to have at least one adjustable non-decvar parameter so that we can propogate parameter uncertainty to model-based constraints (this can also be posterior FOSM is non-constraint, non-zero-weight observations are specified). For this simple demo, lets just use the constant multiplier parameters in the prior uncertainty stance:

```
In [16]: cn_pars = par.loc[par.pargp.apply(lambda x: "cn" in x),"parnme"]
         cn_pars
Out[16]: parnme
         hk6_cn
                           hk6_cn
         hk7_cn
                           hk7_cn
         hk8_cn
                           hk8_cn
         prsity6_cn
                       prsity6_cn
         prsity7_cn
                      prsity7_cn
         prsity8_cn
                       prsity8_cn
         rech4_cn
                         rech4 cn
         rech5_cn
                         rech5_cn
         ss6_cn
                           ss6_cn
         ss7_cn
                           ss7_cn
         ss8_cn
                           ss8_cn
         strt6_cn
                         strt6_cn
         strt7_cn
                         strt7_cn
         strt8_cn
                         strt8_cn
         sy6_cn
                           sy6_cn
         sy7_cn
                           sy7_cn
         sy8_cn
                           sy8_cn
         vka6_cn
                          vka6_cn
         vka7_cn
                          vka7_cn
         vka8 cn
                          vka8 cn
         Name: parnme, dtype: object
In [17]: par = pst.parameter_data
         par.loc[cn_pars,"partrans"] = "log"
         pst.control data.noptmax = 1
         pst.write(os.path.join(t_d, "freyberg_opt_uu1.pst"))
noptmax:1, npar_adj:26, nnz_obs:4
```

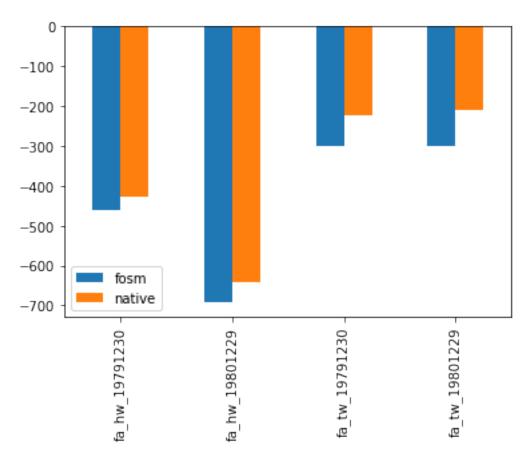
So now we need to not only fill the response matrix (between dec vars and constraints) but we also need to fill the jacobian matrix (between parameters and constraints).

```
In [18]: pyemu.os_utils.start_slaves(t_d, "pestpp-opt", "freyberg_opt_uu1.pst", num_slaves=20, mas:
In [19]: pst = pyemu.Pst(os.path.join(m_d, "freyberg_opt_uu1.pst"), resfile=os.path.join(m_d, "freyberg_opt_uu1.pst")
         pst.res.loc[pst.nnz_obs_names,:]
Out[19]:
                                             group measured modelled residual \
                                   name
         name
         fa_hw_19791230 fa_hw_19791230 less_than
                                                       -300.0 -426.6231 126.6231
         fa_hw_19801229 fa_hw_19801229 less_than
                                                       -300.0 -640.6620 340.6620
         fa_tw_19791230 fa_tw_19791230 less_than
                                                       -300.0 -223.3317 -76.6683
         fa_tw_19801229 fa_tw_19801229 less_than
                                                       -300.0 -208.3292 -91.6708
                         weight
         name
         fa_hw_19791230
                            1.0
         fa_hw_19801229
                            1.0
         fa_tw_19791230
                            1.0
         fa_tw_19801229
                            1.0
In [20]: par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_uu1.1.par"))
         print(par_df.loc[dvg_pars,"parval1"].sum())
         par_df.loc[dvg_pars,:]
12.500896512450069
Out [20]:
                                      parval1 scale offset
                             parnme
         parnme
         wf0200090016 wf0200090016 3.000000
                                                  1.0
                                                          0.0
         wf0200110013 wf0200110013 3.000000
                                                  1.0
                                                          0.0
         wf0200200014 wf0200200014 1.281895
                                                  1.0
                                                          0.0
         wf0200260010 wf0200260010 0.000000
                                                 1.0
                                                          0.0
         wf0200290006 wf0200290006 2.219002
                                                  1.0
                                                          0.0
         wf0200340012 wf0200340012 3.000000
                                                  1.0
                                                          0.0
```

We now see how taking a risk tolerant stance allows for more pumping but that we have only a 40% chance of actually satisfying the sw-gw constraints (see how the model simulated value is actually in violation of the -300 constraint RHS. Lets check the residuals that include the FOSM-based chance constraint shift:

```
-300.0 -693.498392
                                                                  393.498392
fa_hw_19801229
                fa_hw_19801229
                                less_than
                                              -300.0 -299.758707
                                                                    -0.241293
fa_tw_19791230
                fa_tw_19791230
                                less_than
fa_tw_19801229
                fa_tw_19801229
                                less_than
                                              -300.0 -299.676541
                                                                   -0.323459
                weight
name
fa_hw_19791230
                   1.0
fa_hw_19801229
                   1.0
                   1.0
fa_tw_19791230
fa_tw_19801229
                   1.0
```

In [22]: ax = pd.DataFrame({"native":pst.res.modelled, "fosm":res\_df.modelled}).loc[pst.nnz\_obs]



## 1.0.3 Opt under uncertainty part 2: ensemble-based chance constraints

PESTPP-OPT can also skip the FOSM calculations if users specify model-based constraint weights as standard deviations (e.g. uncertainty in the forecasts/constraints). These can be derived from existing ensembles (oh snap!)

```
In [24]: pr_std = obs_df.std().loc[pst.nnz_obs_names]
        pr_std
Out[24]: fa_hw_19791230
                           412.618168
        fa_hw_19801229
                           561.005021
        fa_tw_19791230
                           485.503449
        fa_tw_19801229
                           587.832373
        dtype: float64
In [25]: pst.observation_data.loc[pst.nnz_obs_names,"weight"] = pr_std.loc[pst.nnz_obs_names]
        pst.pestpp_options["opt_std_weights"] = True
        pst.write(os.path.join(t_d, "freyberg_opt_uu2.pst"))
noptmax:1, npar_adj:26, nnz_obs:4
In [26]: pyemu.os_utils.start_slaves(t_d,"pestpp-opt","freyberg_opt_uu2.pst",num_slaves=10,mas
In [27]: par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_uu2.1.par"))
        print(par_df.loc[dvg_pars,"parval1"].sum())
        par_df.loc[dvg_pars,:]
13.190634229136908
Out [27]:
                                    parval1 scale offset
                             parnme
        parnme
        wf0200090016 wf0200090016 3.000000
                                                 1.0
                                                         0.0
        wf0200110013 wf0200110013 3.000000
                                                 1.0
                                                         0.0
         wf0200200014 wf0200200014 0.000000
                                                 1.0
                                                         0.0
         wf0200260010 wf0200260010 1.190634
                                                1.0
                                                         0.0
         wf0200290006 wf0200290006 3.000000
                                                 1.0
                                                         0.0
         wf0200340012 wf0200340012 3.000000
                                                 1.0
                                                         0.0
```

Why is the objective function higher when we use the ensemble-based constraint uncertainty compared to the FOSM constraint uncertainty? remember how many more parameters were used in the ensemble analyses compared to just the hand full of constant by layer parameters???

### 1.0.4 Super secret mode

turns out, if the opt problem is truely linear, we can reuse results of a previous PESTPP-OPT run to modify lots of the pieces of the optimization problem and resolve the optimization problem without running the model even once! WAT!? This is done by specifying some additional ++ args (and copying some files around)

pst.write(os.path.join(m\_d,"freyberg\_opt\_restart.pst"))

```
noptmax:1, npar_adj:26, nnz_obs:4
In [29]: pyemu.os_utils.run("pestpp-opt freyberg_opt_restart.pst",cwd=m_d)
In [30]: par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_restart.1.par"))
         print(par_df.loc[dvg_pars,"parval1"].sum())
         par df.loc[dvg pars,:]
13.190634229136908
Out [30]:
                                      parval1 scale offset
                             parnme
         parnme
         wf0200090016 wf0200090016 3.000000
                                                  1.0
                                                          0.0
         wf0200110013 wf0200110013 3.000000
                                                 1.0
                                                          0.0
         wf0200200014 wf0200200014 0.000000
                                                  1.0
                                                          0.0
         wf0200260010 wf0200260010 1.190634
                                                 1.0
                                                          0.0
                                                 1.0
                                                          0.0
         wf0200290006 wf0200290006 3.000000
         wf0200340012 wf0200340012 3.000000
                                                 1.0
                                                          0.0
  Oh snap! that means we can do all sort of kewl optimization testing really really fast...we can
test a (slightly) risk averse stance too:
In [31]: pst.pestpp_options["opt_risk"] = 0.51
         pst.write(os.path.join(m_d,"freyberg_opt_restart.pst"))
         pyemu.os_utils.run("pestpp-opt freyberg_opt_restart.pst",cwd=m_d)
         par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_restart.1.par"))
         print(par_df.loc[dvg_pars,"parval1"].sum())
         par_df.loc[dvg_pars,:]
noptmax:1, npar_adj:26, nnz_obs:4
8.831235846269639
Out[31]:
                             parnme parvall scale offset
         parnme
         wf0200090016 wf0200090016 3.000000
                                                  1.0
                                                          0.0
         wf0200110013 wf0200110013 3.000000
                                                 1.0
                                                          0.0
         wf0200200014 wf0200200014 2.831236
                                                 1.0
                                                          0.0
```

Lets use the functionality to evaluate how our OUU problem changes if we use posterior standard deviations - this is a critically important use of the uncertainty analysis from history matching:

1.0

1.0

1.0

0.0

0.0

0.0

wf0200260010 wf0200260010 0.000000

wf0200290006 wf0200290006 0.000000

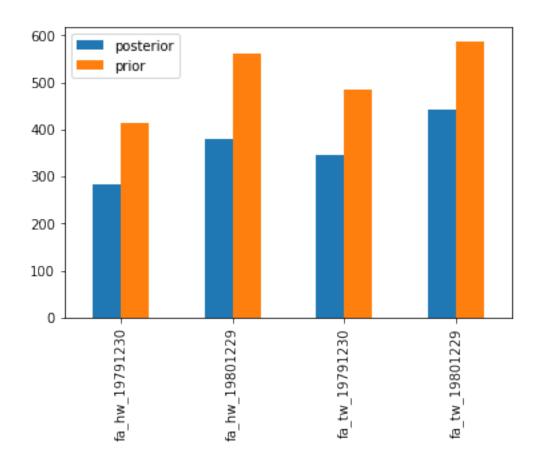
wf0200340012 wf0200340012 0.000000

How much lower is the posterior standard deviations are compared to the prior?

```
In [33]: pd.DataFrame({"prior":pr_std,"posterior":pt_std}).plot(kind="bar")
```

Out[33]: <matplotlib.axes.\_subplots.AxesSubplot at 0x1125b8550>

dtype: float64



This implies that the chance constraints (which express the important model input uncertainty propagated to the forecast/constraints) is significantly lower, meaning uncertainty has less "value" in the optimization objective function

```
In [34]: pst.observation_data.loc[pst.nnz_obs_names,"weight"] = pt_std.loc[pst.nnz_obs_names]
        pst.observation_data.loc[pst.nnz_obs_names,"weight"]
Out[34]: obsnme
        fa_hw_19791230
                           284.228976
        fa_hw_19801229
                           379.936322
        fa_tw_19791230
                           345.713016
        fa_tw_19801229
                           442.676742
        Name: weight, dtype: float64
In [35]: pst.write(os.path.join(m_d,"freyberg_opt_restart.pst"))
        pyemu.os_utils.run("pestpp-opt freyberg_opt_restart.pst",cwd=m_d)
        par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_restart.1.par"))
        print(par_df.loc[dvg_pars,"parval1"].sum())
        par_df.loc[dvg_pars,:]
noptmax:1, npar_adj:26, nnz_obs:4
9.060317215500618
Out [35]:
                            parnme
                                     parval1 scale offset
        parnme
        wf0200090016 wf0200090016 3.000000
                                                 1.0
                                                         0.0
        wf0200110013 wf0200110013 3.000000
                                                 1.0
                                                         0.0
        wf0200200014 wf0200200014 3.000000
                                                 1.0
                                                         0.0
         wf0200260010 wf0200260010 0.000000
                                                 1.0
                                                         0.0
         wf0200290006 wf0200290006 0.000000
                                                 1.0
                                                         0.0
         wf0200340012 wf0200340012 0.060317
                                                 1.0
                                                         0.0
In [36]: pyemu.pst_utils.read_resfile(os.path.join(m_d, "freyberg_opt_restart.1.est+fosm.rei"))
Out [36]:
                                   name
                                             group measured
                                                                modelled
                                                                            residual \
        name
         fa_hw_19791230 fa_hw_19791230 less_than
                                                      -300.0 -394.290117
                                                                           94.290117
                                         less_than
        fa_hw_19801229 fa_hw_19801229
                                                      -300.0 -649.942826 349.942826
        fa_tw_19791230 fa_tw_19791230
                                        less_than
                                                      -300.0 -457.945198 157.945198
        fa_tw_19801229 fa_tw_19801229
                                         less_than
                                                      -300.0 -300.000000
                                                                            0.000000
                             weight
        name
        fa_hw_19791230
                         284.228976
        fa_hw_19801229
                         379.936322
        fa_tw_19791230
                         345.713016
        fa_tw_19801229 442.676742
```

Again we see that scenarion tail water flux is the binding constraint. So! Lets reformulate the problem to be constrained by the total sw-gw flux across all reaches instead of splitting into headwaters and tailwaters. Good thing we have added the list file budget components to the control file!

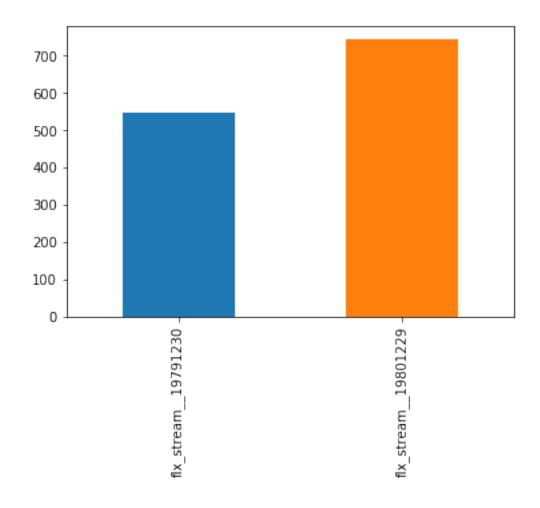
```
In [37]: pst = pyemu.Pst(os.path.join(m_d, "freyberg_opt_restart.pst"))
        obs = pst.observation_data
        obs.loc[pst.nnz_obs_names, "obgnme"] = "sw-gw"
        obs.loc[pst.nnz_obs_names, "weight"] = 0.0

In [38]: tot_swgw = obs.loc[obs.obgnme=="flx_stream_", "obsnme"]

In [39]: obs.loc[tot_swgw, "obgnme"] = "less_than"
        obs.loc[tot_swgw, "weight"] = 1.0
        obs.loc[tot_swgw, "weight"] = obs_df.std().loc[pst.nnz_obs_names]
        obs.loc[tot_swgw, "obsval"] = -600

In [40]: obs_df.std().loc[pst.nnz_obs_names].plot(kind="bar")

Out[40]: <matplotlib.axes._subplots.AxesSubplot at 0x18270ec6a0>
```



Since we want to find the most risk averse stance that is still feasible we will run a sweep of risk values:

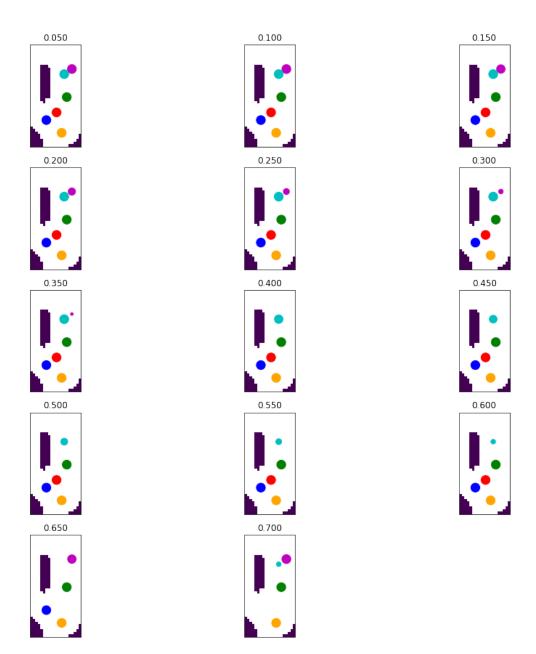
```
In [41]: par_dfs = []
        res_dfs = []
         risk_vals = np.arange(0.05, 1.0, 0.05)
         for risk in risk_vals:
             #try:
                 os.remove(os.path.join(m_d, "freyberg_opt_restart.1.est+fosm.rei"))
             #except:
                 pass
             pst.pestpp_options["opt_risk"] = risk
             pst.pestpp_options["opt_skip_final"] = True
             pst.write(os.path.join(m_d,"freyberg_opt_restart.pst"))
             pyemu.os_utils.run("pestpp-opt freyberg_opt_restart.pst",cwd=m_d)
             par_df = pyemu.pst_utils.read_parfile(os.path.join(m_d, "freyberg_opt_restart.1.page)
             par_df = par_df.loc[dvg_pars,:]
             #when the solution is infeasible, pestpp-opt writes extreme negative values
             # to the par file:
             if par_df.parval1.sum() < 6.0:</pre>
                 print("infeasible at risk",risk)
                 break
             res_df = pyemu.pst_utils.read_resfile(os.path.join(m_d, "freyberg_opt_restart.1.es"
             res_df = res_df.loc[pst.nnz_obs_names,:]
             res_dfs.append(res_df.modelled)
             par_dfs.append(par_df.parval1)
         # process the dec var and constraint dataframes for plotting
         risk_vals = risk_vals[:len(par_dfs)]
         par_df = pd.concat(par_dfs,axis=1).T
         par_df.index = risk_vals
         par_df.index = par_df.index.map(lambda x: "{0:0.3f}".format(x))
         res_df = pd.concat(res_dfs,axis=1).T
         res_df.index = risk_vals
         res_df.index = res_df.index.map(lambda x: "{0:0.3f}".format(x))
noptmax:1, npar_adj:26, nnz_obs:2
```

```
noptmax:1, npar_adj:26, nnz_obs:2
infeasible at risk 0.750000000000001
In [42]: colors = ["m","c","g","r","b","orange"]
         fig, axes = plt.subplots(2,1,figsize=(15,8))
         par_df.plot(kind="bar",ax=axes[0],alpha=0.75,color=colors).legend(bbox_to_anchor=(1.2
         axes[0].set_ylabel("individual pumping rates")
         axes[0].set_xticklabels([])
         res_df.plot(kind="bar",ax=axes[1],alpha=0.75).legend(bbox_to_anchor=(1.2, 0.5))
         axes[1].plot(axes[1].get_xlim(),[-600,-600],"r--",lw=3)
         axes[1].set_ylabel("sw-gw flux")
         axes[1].set_xlabel("risk")
Out[42]: Text(0.5, 0, 'risk')
      E 1.5
                                                                              wf0200090016
wf0200110013
       1.0
                                                                              wf0200200014
                                                                              wf0200260010
      -250
     μmχ
      -1000
                                                                         flx_stream_19791230
flx_stream_19801229
      -1250
      -1500
               0.100
                                                                     0.700
                                          0.400
In [43]: m = flopy.modflow.Modflow.load("freyberg.nam",model_ws=t_d)
         wf_par = pst.parameter_data.loc[dvg_pars,:].copy()
         wf_par.loc[:,"k"] = wf_par.parnme.apply(lambda x: int(x[2:4]))
         wf_par.loc[:,"i"] = wf_par.parnme.apply(lambda x: int(x[4:8]))
         wf_par.loc[:,"j"] = wf_par.parnme.apply(lambda x: int(x[8:]))
         wf_par.loc[:,"x"] = wf_par.apply(lambda x: m.sr.xcentergrid[x.i,x.j],axis=1)
         wf_par.loc[:,"y"] = wf_par.apply(lambda x: m.sr.ycentergrid[x.i,x.j],axis=1)
```

ib = m.bas6.ibound[0].array

```
ib = np.ma.masked_where(ib!=0,ib)
fig,axes = plt.subplots(5,int(np.ceil(par_df.shape[0]/5)),figsize=(15,15))
axes = axes.flatten()
for risk,ax in zip(par_df.index,axes):
    ax.set_aspect("equal")
    #ax = plt.subplot(111,aspect="equal")
    ax.imshow(ib,extent=m.sr.get_extent())
    ax.scatter(wf_par.x,wf_par.y,s=par_df.loc[risk,wf_par.parnme].values*50,c=colors)
    ax.set_xticks([])
    ax.set_yticks([])
    ax.set_title(risk)

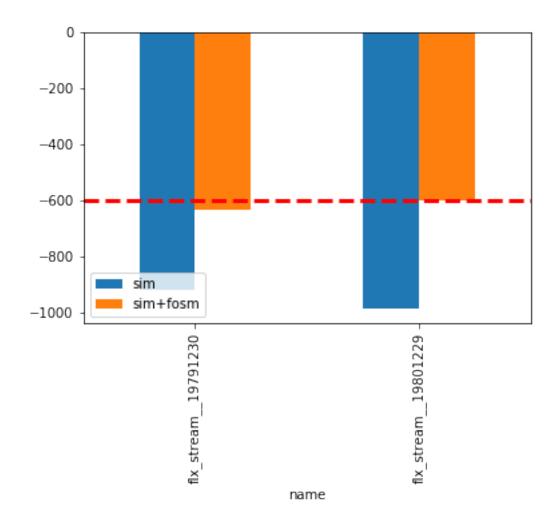
for i in range(par_df.shape[0],axes.shape[0]):
    ax = axes[i]
    ax.axis("off")
```



how about those figures!!!

How slick was that! no more model runs needed and yet we transformed the OUU problem (by swapping constraints) and solved for a much more risk averse stance! Just to make sure, lets run the model with the most risk-averse decision variables:

```
res_df = pyemu.pst_utils.read_resfile(os.path.join(m_d, "freyberg_opt_restart.1.sim+form)
         res_df = res_df.loc[pst.nnz_obs_names,:]
         res_df
noptmax:1, npar_adj:26, nnz_obs:2
Out [44]:
                                                         group measured
                                               name
                                                                            modelled \
         name
         flx_stream__19791230 flx_stream__19791230 less_than
                                                                  -600.0 -634.322727
         flx_stream__19801229 flx_stream__19801229 less_than
                                                                  -600.0 -599.651787
                                residual
                                              weight
         name
         flx_stream__19791230 34.322727
                                          544.898308
         flx_stream__19801229 -0.348213 741.892020
In [45]: # load the actual model simulated outputs
        res_df_sim = pyemu.pst_utils.read_resfile(os.path.join(m_d, "freyberg_opt_restart.1.sin")
         res_df_sim = res_df_sim.loc[pst.nnz_obs_names,:]
         ax = pd.DataFrame({"sim":res_df_sim.modelled, "sim+fosm":res_df.modelled}).plot(kind=")
         ax.plot(ax.get_xlim(),[-600,-600],"r--",lw=3)
Out[45]: [<matplotlib.lines.Line2D at 0x1828bd3828>]
```



Here we can see the cost of uncertainty - we have to simulate a greater flux from gw to sw to make sure (e.g. be risk averse) that the flux from gw to sw is actually at least 600 m3/day

# 2 FINALLY!!!

We now see the reason for high-dimensional uncertainty quantification and history matching: to define and then reduce (through data assimulation) the uncertainty in the model-based constraints (e.g. sw-gw forecasts) so that we can find a more risk-averse management solution - we can use to model to identify an optimal pumping scheme to provide the volume of water needed for supply/ag but also provide assurances (at the given confidence) that ecological flows will be maintained under both current conditions and in the event of an extreme 1-year drought. BOOM!