# pestpp-glm

May 31, 2019

# 1 PESTPP-GLM

In this notebook, we will run PESTPP-GLM in standard parameter estimation mode and regularization mode. In both cases, we will use the baked-in bayes-linear posterior monte carlo analysis to get posterior forecast PDFs. We will use the prior monte carlo outputs as the prior forecast PDF.

```
In [1]: import os
    import shutil
    import numpy as np
    import pandas as pd
    import matplotlib.pyplot as plt
    import matplotlib as mpl
    plt.rcParams['font.size']=12
    import flopy
    import pyemu
```

flopy is installed in /Users/jeremyw/Dev/gw1876/activities\_2day\_mfm/notebooks/flopy

#### 1.1 SUPER IMPORTANT: SET HOW MANY PARALLEL WORKERS TO USE

```
In [2]: num_workers = 20
In [3]: t_d = "template"
        m_d = "master_glm"
In [4]: pst = pyemu.Pst(os.path.join(t_d, "freyberg.pst"))
        pst.write_par_summary_table(filename="none")
Out[4]:
                             type transform count
                                                          initial value \
        cn_strt8
                         cn_strt8
                                         log
                                                   1
                                                                       0
                                                                       0
        gr_vka3
                          gr_vka3
                                         log
                                                705
                           cn_hk7
                                                                       0
        cn_hk7
                                         log
                                                   1
                                                                       0
        pp_ss1
                           pp_ss1
                                         log
                                                 32
                                                705
                                                                       0
        gr_hk3
                           gr_hk3
                                         log
        cn_strt6
                         cn_strt6
                                         log
                                                   1
                                                                       0
        cn_vka7
                                                   1
                                                                       0
                          cn_vka7
                                         log
                                                                       0
        cn_vka6
                          cn_vka6
                                                   1
                                         log
```

pp_ss0	pp_ss0	log	32	0
gr_rech2	gr_rech2	log	705	0
gr_prsity5	gr_prsity5	log	705	0
gr_vka4	gr_vka4	log	705	0
gr_prsity3	gr_prsity3	log	705	0
gr_sy4	gr_sy4	log	705	0
gr_strt3	gr_strt3	log	705	0
pp_rech1	pp_rech1	log	32	0
drncond_k00	drncond_k00	log	10	0
cn_rech5	cn_rech5	log	1	-0.39794
gr_ss3	gr_ss3	log	705	0
gr_strt5	gr_strt5	log	705	0
pp_vka0	pp_vka0	log	32	0
gr_hk4	gr_hk4	log	705	0
gr_prsity4	gr_prsity4	log	705	0
pp_vka2	pp_vka2	log	32	0
			705	0
gr_ss5	gr_ss5	log	705	0
gr_sy5	gr_sy5	log		
pp_ss2	pp_ss2	log	32	0
pp_sy0	pp_sy0	log	32	0
gr_vka5	gr_vka5	log	705	0
cn_ss6	cn_ss6	log	1	0
• • •	• • •	• • •	• • •	• • •
cn_rech4	cn_rech4	log	1	0
cn_vka8	cn_vka8	log	1	0
cn_sy8	cn_sy8	log	1	0
pp_strt1	pp_strt1	log	32	0
${\tt cn\_prsity7}$	${\tt cn\_prsity7}$	log	1	0
pp_prsity1	pp_prsity1	log	32	0
flow	flow	log	1	0
cn_sy7	cn_sy7	log	1	0
gr_strt4	gr_strt4	log	705	0
cn_sy6	cn_sy6	log	1	0
gr_hk5	gr_hk5	log	705	0
gr_ss4	gr_ss4	log	705	0
gr_rech3	gr_rech3	log	705	0
pp_prsity0	pp_prsity0	log	32	0
welflux_k02	welflux_k02	log	6	0
pp_strt2	pp_strt2	log	32	0
cn_hk8	cn_hk8	log	1	0
strk	strk	log	40	0
cn_ss8	cn_ss8	log	1	0
pp_hk1	pp_hk1	log	32	0
pp_nki pp_rech0	pp_nki pp_rech0	log	32	0
pp_recho pp_prsity2	pp_recho pp_prsity2	log	32	0
cn_strt7	cn_strt7		1	0
		log	32	0
pp_vka1	pp_vka1	log		0
cn_prsity6	cn_prsity6	log	1	U

welflux	welflux	log	2	0 +0	0.176091	
cn_hk6	cn_hk6	log	1	0 00	0.170031	
	pp_hk0	_	32		0	
pp_hk0		log				
pp_hk2	pp_hk2	log	32		0	
			-	, ,		
0	upper			er bound	standard	deviation
cn_strt8	0.02	11893	-0.	0222764		0.0108664
gr_vka3		1		-1		0.5
cn_hk7		1		-1		0.5
pp_ss1		1		-1		0.5
gr_hk3		1		-1		0.5
cn_strt6	0.02	11893	-0.	0222764		0.0108664
cn_vka7		1		-1		0.5
cn_vka6		1		-1		0.5
pp_ss0		1		-1		0.5
gr_rech2	0.04	13927	-0.	0457575		0.0217875
gr_prsity5	0.1	76091	_	-0.30103		0.11928
gr_vka4		1		-1		0.5
gr_prsity3	0.1	76091	_	-0.30103		0.11928
gr_sy4		43038		-0.60206		0.211275
gr_strt3		11893		0222764		0.0108664
pp_rech1		13927		0457575		0.0217875
drncond_k00	0.04	1	0.	-1		0.0217075
cn_rech5	-0.4	09691		-1		0.225772
	-0.0					
gr_ss3	0.00	1	0	-1		0.5
gr_strt5	0.02	11893	-0.	0222764		0.0108664
pp_vka0		1		-1		0.5
gr_hk4		1		-1		0.5
gr_prsity4	0.1	76091	-	-0.30103		0.11928
pp_vka2		1		-1		0.5
gr_ss5		1		-1		0.5
gr_sy5	0.2	43038	-	-0.60206		0.211275
pp_ss2		1		-1		0.5
pp_sy0	0.2	43038	-	-0.60206		0.211275
gr_vka5		1		-1		0.5
cn_ss6		1		-1		0.5
cn_rech4	0.079	91812	-	-0.09691		0.0440228
cn_vka8		1		-1		0.5
cn_sy8	0.24	43038	_	-0.60206		0.211275
pp_strt1	0.02	11893	-0.	0222764		0.0108664
cn_prsity7		76091		-0.30103		0.11928
pp_prsity1		76091		-0.30103		0.11928
flow		09691		0.124939		0.0554622
cn_sy7		43038		-0.60206		0.211275
gr_strt4		11893		0222764		0.0108664
cn_sy6		43038		-0.60206		0.211275
on_by 0	0.2	10000		0.00200		0.211210

1

log

cn\_ss7

cn\_ss7

0

gr_hk5	1	-1	0.5
gr_ss4	1	-1	0.5
gr_rech3	0.0413927	-0.0457575	0.0217875
pp_prsity0	0.176091	-0.30103	0.11928
welflux_k02	1	-1	0.5
pp_strt2	0.0211893	-0.0222764	0.0108664
cn_hk8	1	-1	0.5
strk	2	-2	1
cn_ss8	1	-1	0.5
pp_hk1	1	-1	0.5
pp_rech0	0.0413927	-0.0457575	0.0217875
pp_prsity2	0.176091	-0.30103	0.11928
cn_strt7	0.0211893	-0.0222764	0.0108664
pp_vka1	1	-1	0.5
cn_prsity6	0.176091	-0.30103	0.11928
cn_ss7	1	-1	0.5
welflux	0.176091 to 0.30103	-0.30103 to 0	0.0752575 to 0.11928
cn_hk6	1	-1	0.5
pp_hk0	1	-1	0.5
pp_hk2	1	-1	0.5

[65 rows x 7 columns]

# 1.1.1 reduce the number of adjustable parameters

This is the painful part: we cant use 10K+ pars because we cant wait around for that many runs and then the linear algebra of factoring a 10k+ by 10K+ matrix is also difficult. So that means we need to fix a lot a parameters #frownyface

```
Out[8]: array(['pp_hk0', 'pp_hk1', 'pp_hk2', 'pp_prsity0', 'pp_prsity1',
                                                'pp_prsity2', 'pp_rech0', 'pp_rech1', 'pp_ss0', 'pp_ss1', 'pp_ss2',
                                                'pp_strt0', 'pp_strt1', 'pp_strt2', 'pp_sy0', 'pp_sy1', 'pp_sy2',
                                                'pp_vka0', 'pp_vka1', 'pp_vka2'], dtype=object)
         Fix the storage pilot points - we still have layer-scale storage pars adjustable
 \label{eq:continuous_part}  \mbox{In [9]: $\#s\_pars = par.loc[par.pargp.apply(lambda x: "pp" in x and ("ss" in x or "sy" in x)), "partial partial pa
                         #par.loc[s_pars, "partrans"] = "fixed"
                         pst.npar_adj
Out[9]: 719
In [10]: adj_par = par.loc[par.partrans=="log",:]
                            adj_par.pargp.value_counts().sort_values()
Out[10]: cn_strt8
                                                                               1
                            cn_sy6
                                                                               1
                            cn_vka6
                                                                               1
                            cn_vka7
                                                                               1
                            cn hk6
                                                                               1
                            flow
                                                                               1
                            cn_strt6
                                                                               1
                            cn_hk7
                                                                               1
                            cn_rech5
                                                                               1
                            cn_prsity6
                                                                               1
                            cn_ss7
                                                                               1
                                                                               1
                            cn_sy7
                            cn_strt7
                                                                               1
                            cn ss6
                                                                               1
                            cn_ss8
                                                                               1
                            cn_prsity8
                                                                               1
                            cn_hk8
                                                                               1
                            cn_rech4
                                                                               1
                            cn_vka8
                                                                               1
                            cn_sy8
                                                                               1
                            cn_prsity7
                                                                               1
                            welflux
                            welflux_k02
                                                                               6
                            drncond_k00
                                                                            10
                            pp_hk1
                                                                            32
                                                                            32
                            pp_ss0
                                                                            32
                            pp_vka1
                                                                            32
                            pp_prsity1
                                                                            32
                            pp_rech1
                                                                            32
                            pp_vka0
                            pp_ss2
                                                                            32
                            pp_sy0
                                                                            32
```

32

pp\_ss1

```
32
pp_strt2
                32
pp_sy1
pp_hk2
                32
pp_strt1
               32
pp_strt0
                32
                32
pp_rech0
pp_prsity0
                32
pp_hk0
                32
                32
pp_prsity2
pp_vka2
                32
                32
pp_sy2
                40
strk
Name: pargp, dtype: int64
```

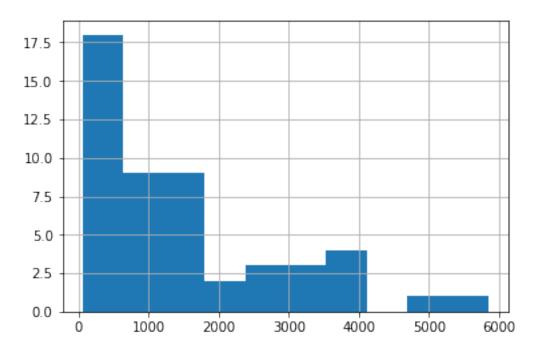
25

57.504700

fix the future recharge pilot points, vka in layers 1 and 3 and the initial condition pilot points (we still have layer-scale pars for each of these types)

Ok, thats better...so lets run PESTPP-GLM. We will use a single "base parameter" jacobian matrix as the basis for 6 super parameter iterations. Then we will draw 100 realizations from the FOSM posterior parameter covariance matrix and run those 100 realizations to get the psoterior forecast PDFs

```
49
       59.965027
48
       79.076502
22
      121.743995
12
      129.003257
21
      235.929537
10
      271.640362
26
      303.531649
42
      328.530748
29
      370.831442
24
      473.160536
47
      474.512084
33
      495.605323
44
      505.914044
5
      507.399642
1
      537.591081
9
      608.619145
19
      616.414116
46
      680.674779
37
      805.598204
dtype: float64
```



Here we see the distribution of phi values across the 100 posterior realizations. Should we accept all of these??? The theoretical phi we should accept is number of nonzero obs (14).

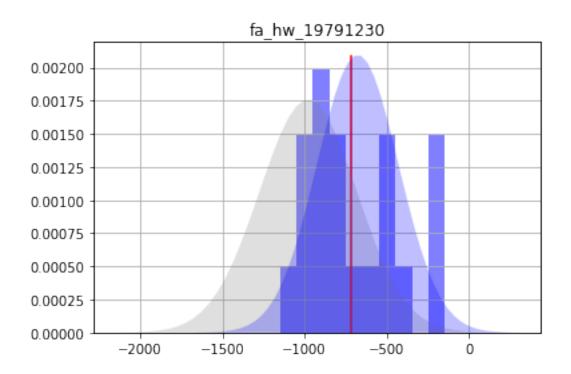
To get a "posterior" ensemble, we need to throw out the realizations with large phi - lets just take the 20 best:

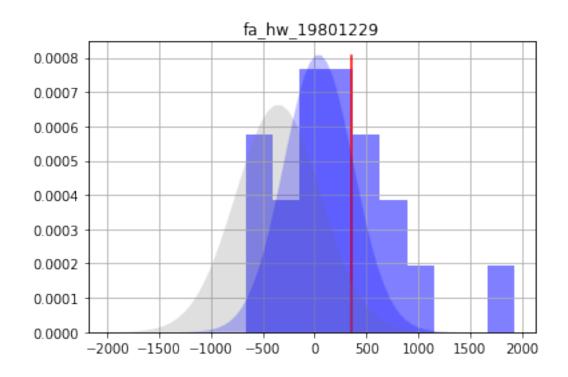
In [16]: oe\_pt = oe.loc[oe.phi\_vector.sort\_values().index[:20],:] #just take the 20 lowest phi

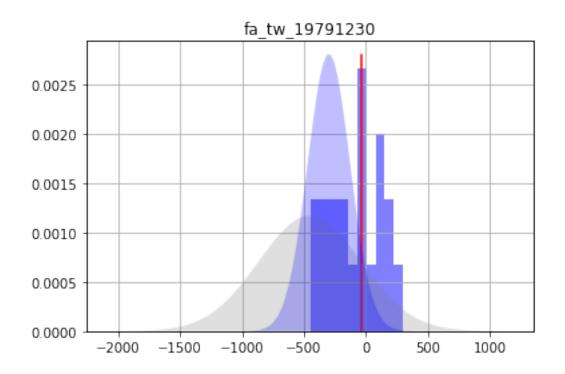
We can also load and plot the FOSM forecast results along side of the ensemble results:

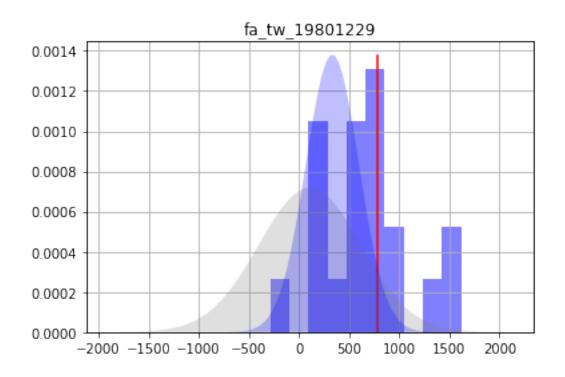
```
In [17]: f_df = pd.read_csv(os.path.join(m_d, "freyberg_pp.pred.usum.csv"),index_col=0)
         f_df.index = f_df.index.map(str.lower)
         f_df
Out [17]:
                             prior_mean prior_stdev prior_lower_bound \
         name
                              -977.2390
                                           295.32800
         fa_hw_19791230
                                                             -1567.8900
         fa_hw_19801229
                              -351.2160
                                           409.77000
                                                             -1170.7600
         fa_tw_19791230
                              -453.0330
                                           409.35100
                                                             -1271.7400
         fa_tw_19801229
                               108.9600
                                           506.73200
                                                              -904.5040
         hds_00_013_002_000
                                39.6102
                                             3.96314
                                                                 31.6840
         hds_00_013_002_001
                                38.3838
                                             4.05782
                                                                 30.2681
         part_status
                                 2.0000
                                             0.00000
                                                                 2.0000
         part_time
                               907.7020
                                           570.98600
                                                              -234.2690
                             prior_upper_bound post_mean post_stdev \
         name
         fa_hw_19791230
                                     -386.5840 -675.0430 247.837000
         fa_hw_19801229
                                      468.3240
                                                  37.6125 335.549000
         fa_tw_19791230
                                      365.6690 -299.4690 170.577000
         fa_tw_19801229
                                     1122.4200
                                                333.7460 263.672000
         hds_00_013_002_000
                                       47.5365
                                                 37.6359
                                                           0.273990
         hds_00_013_002_001
                                                  36.4927
                                       46.4994
                                                             0.687833
         part_status
                                        2.0000
                                                   2.0000
                                                             0.000000
                                     2049.6700
         part_time
                                                 703.9470 436.945000
                             post_lower_bound post_upper_bound
         name
         fa_hw_19791230
                                   -1170.7200
                                                      -179.3700
         fa_hw_19801229
                                                       708.7100
                                    -633.4850
         fa_tw_19791230
                                    -640.6230
                                                        41.6858
         fa_tw_19801229
                                    -193.5970
                                                       861.0890
         hds_00_013_002_000
                                      37.0879
                                                        38.1838
         hds_00_013_002_001
                                      35.1171
                                                        37.8684
         part_status
                                       2.0000
                                                         2.0000
         part_time
                                    -169.9420
                                                      1577.8400
In [18]: obs = pst.observation_data
         fnames = pst.pestpp_options["forecasts"].split(",")
         for forecast in fnames:
             ax = plt.subplot(111)
             oe_pt.loc[:,forecast].hist(ax=ax,color="b",alpha=0.5,normed=True)
             ax.plot([obs.loc[forecast,"obsval"],obs.loc[forecast,"obsval"]],ax.get_ylim(),"r"
             axt = plt.twinx()
             x,y = pyemu.plot_utils.gaussian_distribution(f_df.loc[forecast,"prior_mean"],f_df
             axt.fill_between(x,0,y,facecolor="0.5",alpha=0.25)
```

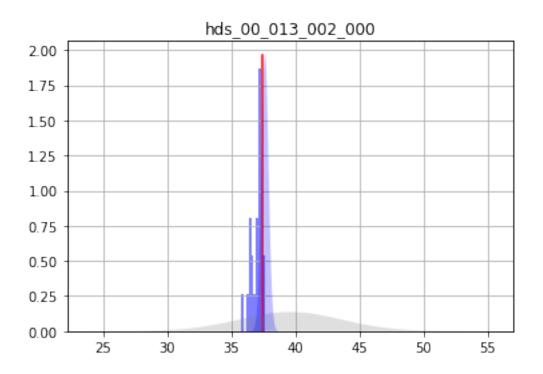
```
x,y = pyemu.plot_utils.gaussian_distribution(f_df.loc[forecast,"post_mean"],f_df.faxt.fill_between(x,0,y,facecolor="b",alpha=0.25)
axt.set_ylim(0,axt.get_ylim()[1])
axt.set_yticks([])
ax.set_title(forecast)
plt.show()
```

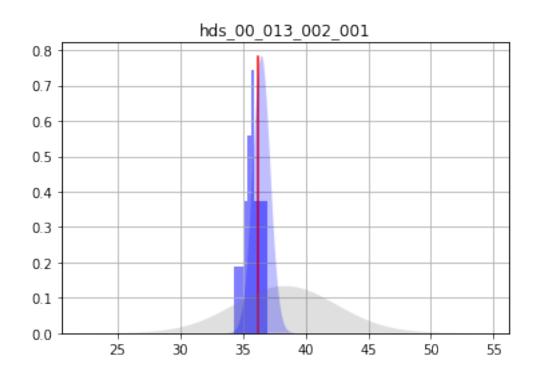


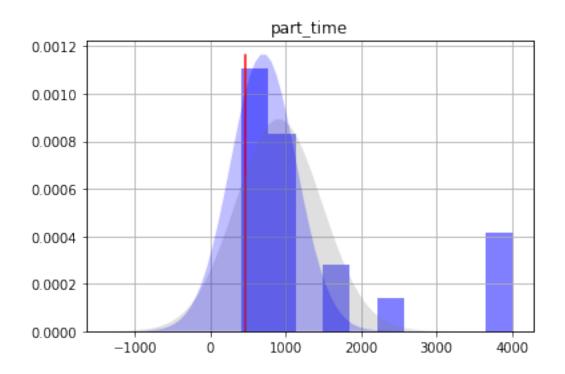


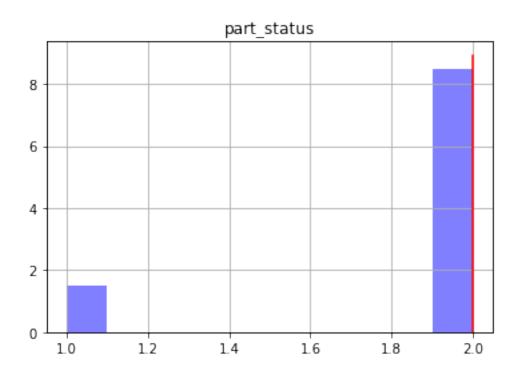












## 1.1.2 Setup of Tikhonov regularization

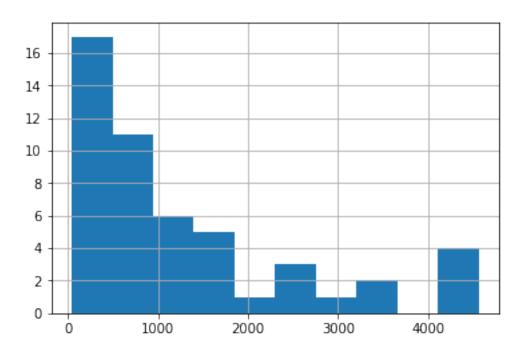
Now lets setup and use some formal regularization to bring the final phi up to around 14. We will use first-order regularization based on the covariance matrix we build earlier:

```
In [19]: cov = pyemu.Cov.from_binary(os.path.join(t_d,"prior_cov.jcb"))
new binary format detected...
In [20]: pyemu.helpers.first_order_pearson_tikhonov(pst,cov)
getting CC matrix
processing
In [21]: pst.prior_information.head()
Out [21]:
                                                                equation
                                                                            obgnme \
        pilbl
                1.0 * \log(dc0000390005) - 1.0 * \log(dc0000390006) = 0.0
                                                                          regul_cc
         pcc_1
         pcc_2 = 1.0 * log(dc0000390005) - 1.0 * log(dc0000390007) = 0.0
                                                                          regul_cc
         pcc_3 1.0 * log(dc0000390005) - 1.0 * log(dc0000390008) = 0.0
                                                                          regul_cc
         pcc_4 = 1.0 * log(dc0000390005) - 1.0 * log(dc0000390009) = 0.0
                                                                          regul_cc
         pcc_5 = 1.0 * log(dc0000390005) - 1.0 * log(dc0000390010) = 0.0
                                                                          regul_cc
```

```
pilbl
                         weight
         pilbl
         pcc_1 pcc_1 0.904837
         pcc_2 pcc_2 0.818731
         pcc_3 pcc_3 0.740818
         pcc_4 pcc_4 0.670320
         pcc_5 pcc_5 0.606531
In [22]: shutil.copy2(os.path.join(m_d, "freyberg_pp.jcb"),os.path.join(t_d, "restart_pp.jcb"))
Out[22]: 'template/restart_pp.jcb'
In [23]: pst.pestpp_options["base_jacobian"] = "restart_pp.jcb"
         pst.reg_data.phimlim = pst.nnz_obs
         pst.reg_data.phimaccept = pst.reg_data.phimlim * 1.1
         pst.write(os.path.join(t_d,"freyberg_pp.pst"))
noptmax:3, npar_adj:527, nnz_obs:14
In [24]: pyemu.os_utils.start_slaves(t_d, "pestpp-glm", "freyberg_pp.pst", num_slaves=num_workers
                                    master dir=m d)
In [25]: df = df=pd.read_csv(os.path.join(m_d,"freyberg_pp.post.obsen.csv"),index_col=0)
         oe = pyemu.ObservationEnsemble.from_dataframe(pst=pst,df=df)
In [26]: ax = oe.phi_vector.hist()#bins=np.linspace(0,100,20))
         oe.phi_vector.sort_values().iloc[:20]
Out[26]: real_name
         35
                43.758830
         10
                66.299790
         46
               71.376560
         12
               118.219687
         24
               119.728221
         8
               140.461733
         22
              176.808587
         21
               200.494414
         47
               225.582200
         30
               255.737081
         26
               273.099728
         44
               290.829433
         48
               291.287982
         5
               321.916424
               420.424853
         23
         38
               430.891678
         49
               432.520905
         29
               522.375353
```

41 527.057867 33 566.113001 dtype: float64

fa\_hw\_19791230



Same as before, to get a "posterior" ensemble, we need to throw out the realizations with large phi - lets just take the 20 best:

```
In [27]: oe_pt = oe.loc[oe.phi_vector.sort_values().index[:20],:]
In [28]: f_df = pd.read_csv(os.path.join(m_d,"freyberg_pp.pred.usum.csv"),index_col=0)
         f_df.index = f_df.index.map(str.lower)
         f_df
Out [28]:
                             prior_mean prior_stdev prior_lower_bound \
         name
         fa_hw_19791230
                              -977.2390
                                            295.32800
                                                              -1567.8900
         fa_hw_19801229
                                            409.77000
                                                              -1170.7600
                              -351.2160
         fa_tw_19791230
                                            409.35100
                                                              -1271.7400
                              -453.0330
         fa_tw_19801229
                               108.9600
                                            506.73200
                                                               -904.5040
         hds_00_013_002_000
                                39.6102
                                              3.96314
                                                                 31.6840
         hds_00_013_002_001
                                              4.05782
                                                                 30.2681
                                38.3838
         part_status
                                 2.0000
                                              0.00000
                                                                  2.0000
                               907.7020
                                            570.98600
                                                               -234.2690
         part_time
                             prior_upper_bound post_mean post_stdev \
         name
```

-386.5840 -640.3370 250.802000

```
fa_tw_19791230
                                      365.6690 -290.4730 170.616000
         fa_tw_19801229
                                     1122.4200 184.3570 263.999000
         hds_00_013_002_000
                                       47.5365
                                                  37.7893
                                                             0.274188
         hds_00_013_002_001
                                       46.4994
                                                  36.9350
                                                             0.687976
         part_status
                                        2.0000
                                                   2.0000
                                                             0.000000
         part_time
                                     2049.6700
                                                 966.2250 437.215000
                             post_lower_bound post_upper_bound
         name
         fa_hw_19791230
                                   -1141.9400
                                                      -138.7340
         fa_hw_19801229
                                    -774.7640
                                                       585.1190
         fa_tw_19791230
                                    -631.7040
                                                        50.7581
         fa_tw_19801229
                                    -343.6410
                                                       712.3540
         hds_00_013_002_000
                                      37.2409
                                                        38.3377
         hds_00_013_002_001
                                      35.5591
                                                        38.3110
         part_status
                                       2.0000
                                                         2.0000
                                      91.7946
                                                      1840.6600
         part_time
In [29]: obs = pst.observation_data
         fnames = pst.pestpp_options["forecasts"].split(",")
         for forecast in fnames:
             ax = plt.subplot(111)
             oe_pt.loc[:,forecast].hist(ax=ax,color="b",alpha=0.5,normed=True)
             ax.plot([obs.loc[forecast,"obsval"],obs.loc[forecast,"obsval"]],ax.get_ylim(),"r"
             axt = plt.twinx()
             x,y = pyemu.plot_utils.gaussian_distribution(f_df.loc[forecast,"prior_mean"],f_df
             axt.fill_between(x,0,y,facecolor="0.5",alpha=0.25)
             x,y = pyemu.plot_utils.gaussian_distribution(f_df.loc[forecast,"post_mean"],f_df.
             axt.fill_between(x,0,y,facecolor="b",alpha=0.25)
             axt.set_ylim(0,axt.get_ylim()[1])
             axt.set_yticks([])
             ax.set_title(forecast)
             plt.show()
```

468.3240

fa\_hw\_19801229

-94.8227 339.971000

